Islands beyond AdS

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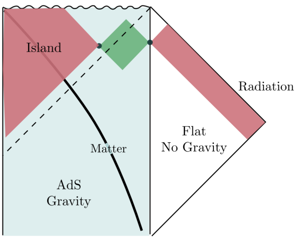
University of Pennsylvania

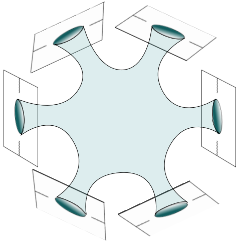
Island Hopping 2020: from Wormholes to Averages
November 17, 2020

[Hartman, Shaghoulian, Strominger 2004.13857]

[Hartman, Jiang, Shaghoulian 2008.01022]

Introduction





- Page curve for Hawking radiation from holography [Penington; Almheiri, Engelhardt, Marolf, Maxfield] and Euclidean gravity path integral [Penington, Shenker, Stanford, Yang; Almheiri, Hartman, Maldacena, ES, Tajdini]

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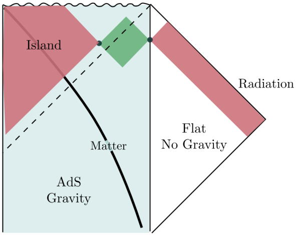

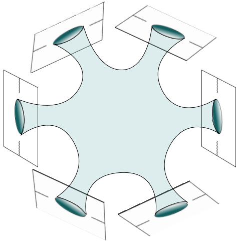

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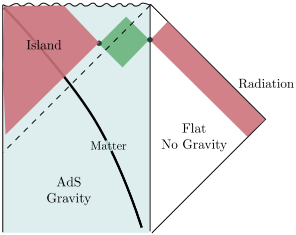

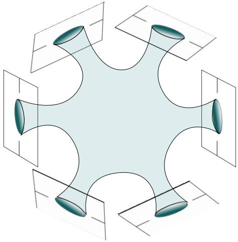

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- These approaches may be the same via Euclidean gravity = holography [Harlow, ES]
- Euclidean gravity approach is generalizable to other spacetimes; let's consider flat/cosmological spacetimes ($\longrightarrow$ holography?)

Flat space

CGHS/RST model:

$$I_{\rm cl} = \int d^2x\sqrt{-g}\Big[e^{-2\phi}(R+4(\nabla\phi)^2+4)\underbrace{-\frac{c}{24}\phi R}_{\rm RST}\Big] + I_{\rm CFT}[g]$$

Conformal anomaly leads to

$$I_{\rm anom} = -\frac{c}{96\pi}\int d^2x\sqrt{-g}\,R\Box^{-1}R$$

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Quantum effective action exact at large c. Equivalent to flat-space JT

$$I = \int d^2x\sqrt{-g}\,\left[\Phi R + 1\right] + I_{\rm CFT}[g]$$

Conformal gauge

$$ds^2 = -e^{2\rho}dx^+dx^-$$

$U(1)$ current $\partial_\mu(\rho-\phi)$ equivalent to Φ shift symmetry.

Solutions: linear dilaton vacuum

Field redefinitions and further gauge-fixing:

$$\partial_+\partial_-\Omega = -1\,, \qquad \partial_\pm^2\Omega = -T^{\text{flat}}_{x^\pm x^\pm}$$

Restrict to $\Omega \geq 1/4$.

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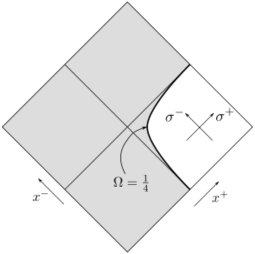
Linear dilaton vacuum:

$$\Omega = -x^+x^- - \frac{1}{4}\log(-4x^+x^-)\,, \qquad ds^2 = \frac{dx^+dx^-}{x^+x^-} = -d\sigma^+d\sigma^-$$

$$T_{x^\pm x^\pm} = T^{\text{flat}}_{x^\pm x^\pm} - ((\partial_+\rho)^2 - \partial_+^2\rho) \quad \Longrightarrow \quad T_{x^\pm x^\pm} = T_{\sigma^\pm\sigma^\pm} = 0$$



Evaporating black hole

Shock wave impinges on linear dilaton vacuum

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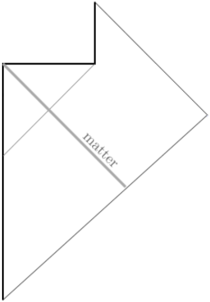

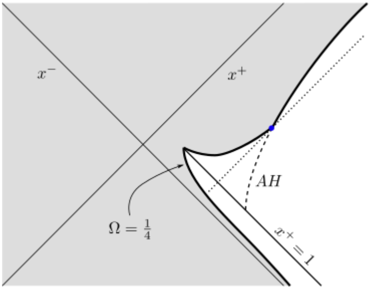

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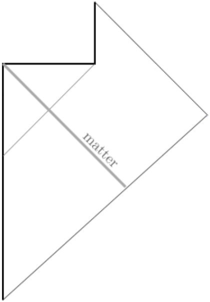

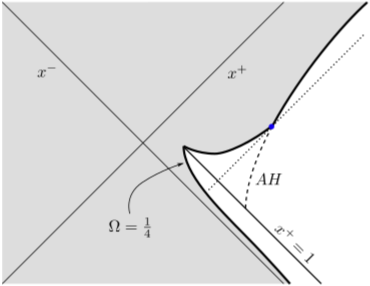

$ds^2 \approx -d\sigma^+ d\tilde{\sigma}^-$ near $\mathcal{I}^+$ with $x^+ = e^{\sigma^+}$, $x^- + M = -e^{-\tilde{\sigma}^-}$, so observer sees

$$T_{\tilde{\sigma}^-\tilde{\sigma}^-} \approx \frac{1}{4}\left(1 - \frac{1}{1 + Me^{\tilde{\sigma}^-}}\right)\,, \qquad \tilde{\sigma}^- \in (-\infty, 4M)$$

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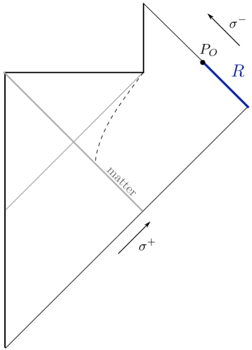

$$S_{\rm QG}(R) = S_{\rm QFT}(R) = \frac{c}{12}\log(1 + M e^{\tilde{\sigma}^-}) + \text{UV/IR divergences}$$

[Fiola, Preskill, Strominger, Trivedi]

Remnants!

Island rule entropy [Hartman, ES, Strominger]

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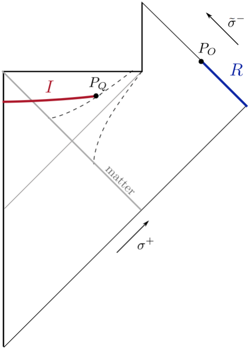
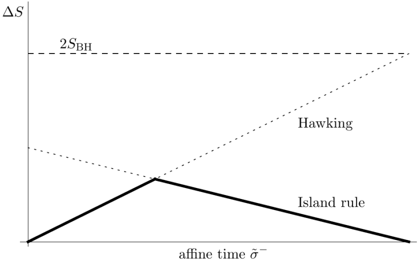
Let's revisit the evaporating black hole! cf [Gautason, Schneiderbauer, Sybesma, Thorlacius; Anegawa, Iizuka]

Evaporating black hole

$\tilde{\sigma}^- \sim O(M)$ with $M \gg 1$:

$$S_{\text{QG}}(R) = \min \text{ext}_I S_{\text{gen}}(I \cup R) = \min \frac{c}{24}\{2\tilde{\sigma}^-,\ 4M - \tilde{\sigma}^-\} + \frac{c}{6} \log \frac{\sigma^+}{\epsilon_{\text{uv}}}$$





$t_{\text{Page}} = 4M/3 = t_{\text{evap}}/3$: $-\delta S_{\text{BH}} = 2\delta S_{\text{radiation}}$

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Consider general dilaton gravity

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$ds^2 = e^{2\rho} dz d\bar{z}$, place defect (represents ∂I) at $z = 0$ and take $z \sim 0$, $n \sim 1$:

$$\rho \to \rho + (n-1)\delta\rho, \qquad \phi \to \phi + (n-1)\delta\phi, \qquad T_{\mu\nu}^{\text{flat}} \to T_{\mu\nu}^{\text{flat}} + (n-1)\delta T_{\mu\nu}^{\text{flat}}$$

Smoothness on replica manifold $w = z^{1/n}$ implies $\delta\rho \sim -\frac{1}{2}\log(z\bar{z})$.

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Conformal ward identity gives

$$2\pi\delta T^{\text{flat}}_{zz} = \frac{c/12}{z^2} - \frac{\partial S^{\text{flat}}_{CFT}}{z} + \text{reg}\,, \qquad \text{barred}$$

Expanding dilaton in series respecting replica symmetry around $z = 0$ and solving singular terms of constraint equations (g_{zz} and $g_{\bar{z}\bar{z}}$ equations) gives

$$\partial\left(\frac{\text{Area}(\partial I)}{4} + S_{CFT}\right) = \bar{\partial}\left(\frac{\text{Area}(\partial I)}{4} + S_{CFT}\right) = 0\,.$$

Looking for islands [Hartman, Jiang, ES]

How do we guide our search for islands in general dimensions/spacetimes?

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Necessary condition 1 [Almheiri, Engelhardt, Marolf, Maxfield]:

$$\frac{d}{d\lambda_+} S_{\text{gen}}(I \cup R) = 0 \qquad \Longrightarrow$$

$$\frac{d}{d\lambda_+}\left[S_{\text{QFT}}(I) + \frac{\text{Area}(\partial I)}{4}\right] + \frac{d}{d\lambda_+}\left[S_{\text{QFT}}(I \cup R) - S_{\text{QFT}}(I) - S_{\text{QFT}}(R)\right] = 0$$

for outgoing null derivative. Monotonicity of mutual information gives

$$\frac{d}{d\lambda_+} S_{\text{gen}}(I) = \frac{d}{d\lambda_+} I_{\text{QFT}}(I, R) \geq 0$$

For inward null derivative

$$\frac{d}{d\lambda_-} S_{\text{gen}}(I) = \frac{d}{d\lambda_-} I_{\text{QFT}}(I, R) \leq 0$$

So ∂I is in "quantum normal region":

$$\boxed{\pm \frac{d}{d\lambda_\pm} S_{\text{gen}}(I) \geq 0}$$

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Rearrange to get

$$S_{\text{QFT}}(I) - \frac{\text{Area}(\partial I)}{4} > S_{\text{QFT}}(I) + S_{\text{QFT}}(I \cup R) - S_{\text{QFT}}(R)$$

Araki-Lieb $S(I \cup R) \geq |S(I) - S(R)|$ gives

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"Phenomenology": in our past lightcone, for thermal matter entropy, bound violated at $T \sim 1$ TeV.

Looking for islands in the sky

Necessary conditions 1 and 2 **independent of** R:

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∂I must be in intersection of two regions.

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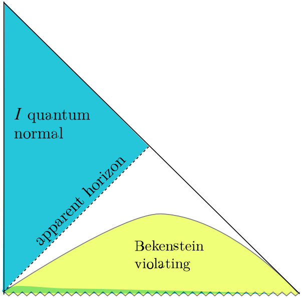

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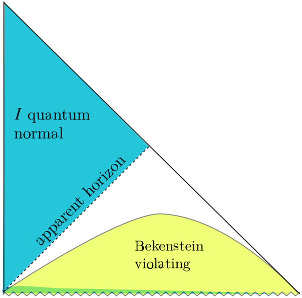

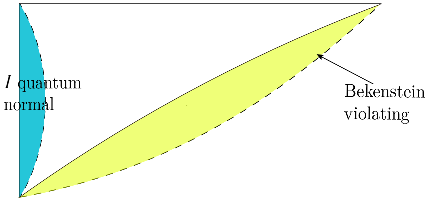

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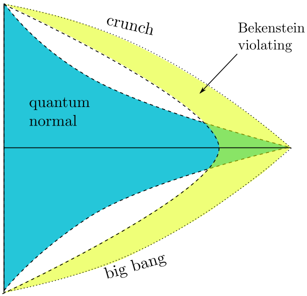

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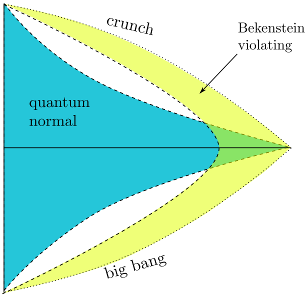

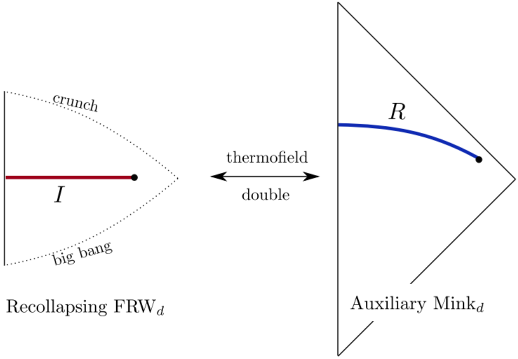

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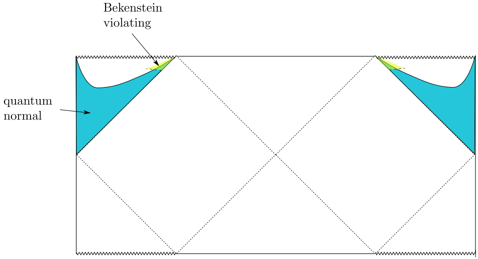

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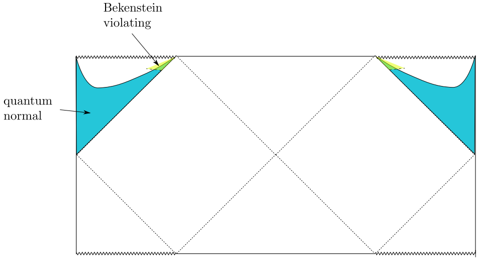

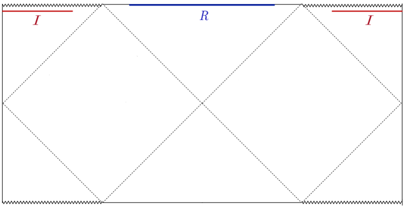

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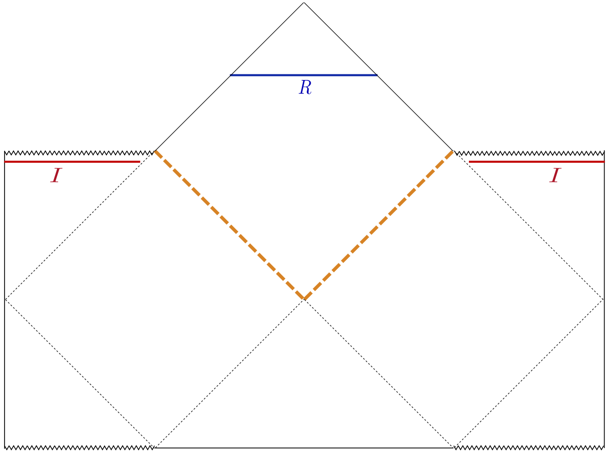

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Hint of dual description, using Euclidean gravity = holography?