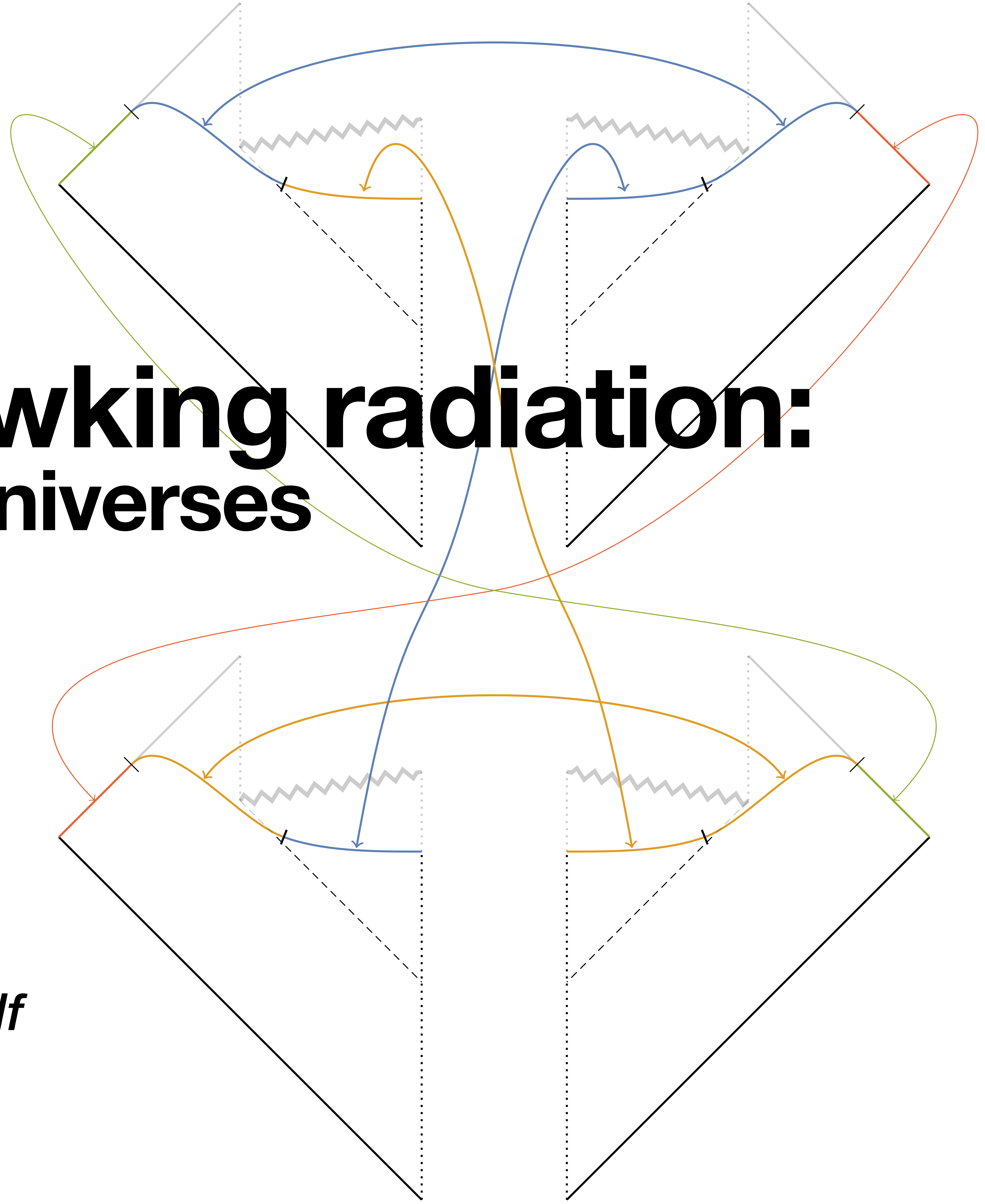
Observations of Hawking radiation:
the Page curve and baby universes



Island Hopping 2020

[2010.06602],[2002.08950] with Don Marolf

Black hole information

An old question: is there a standard quantum statistical description underlying black hole thermodynamics?

Should we interpret $S_{\mathrm{BH}} = \frac{A}{4G_N}$ as counting internal states?

A hypothesis: "Bekenstein-Hawking (BH) unitarity"

In order to describe measurements of distant observers, black holes can be modelled as a unitary quantum system with $\dim \mathscr{H} = e^{S_{\mathrm{BH}}}$

Today's talk: test this idea by computing asymptotic **observables**, using only low-energy, **semiclassical gravity**

The von Neumann entropy of Hawking radiation

Perturbative QG: radiation is thermal, so S_{Hawking} steadily increases

BH unitarity: S_{Hawking} is bounded by S_{BH}

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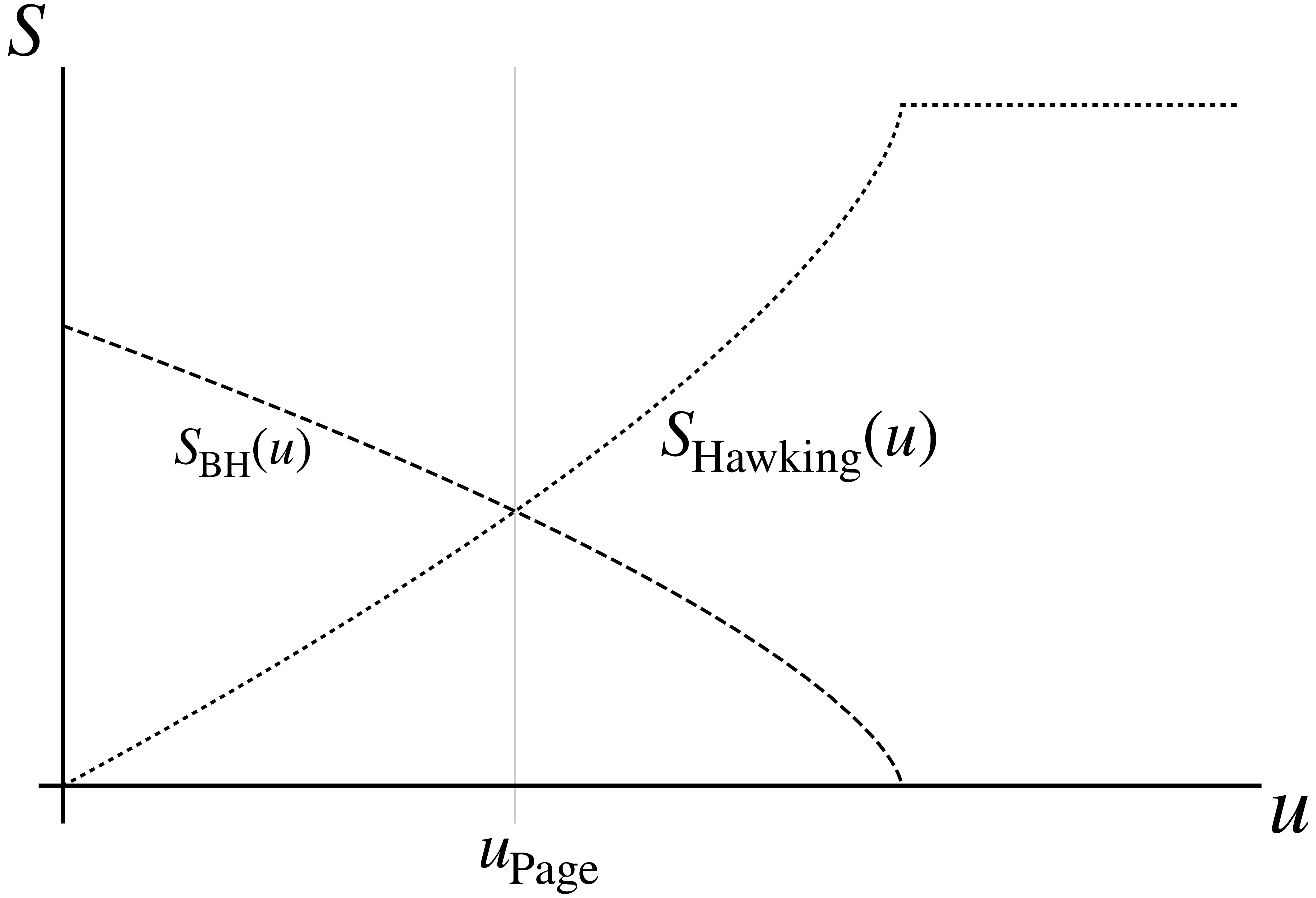

In tension while the black hole is still large!

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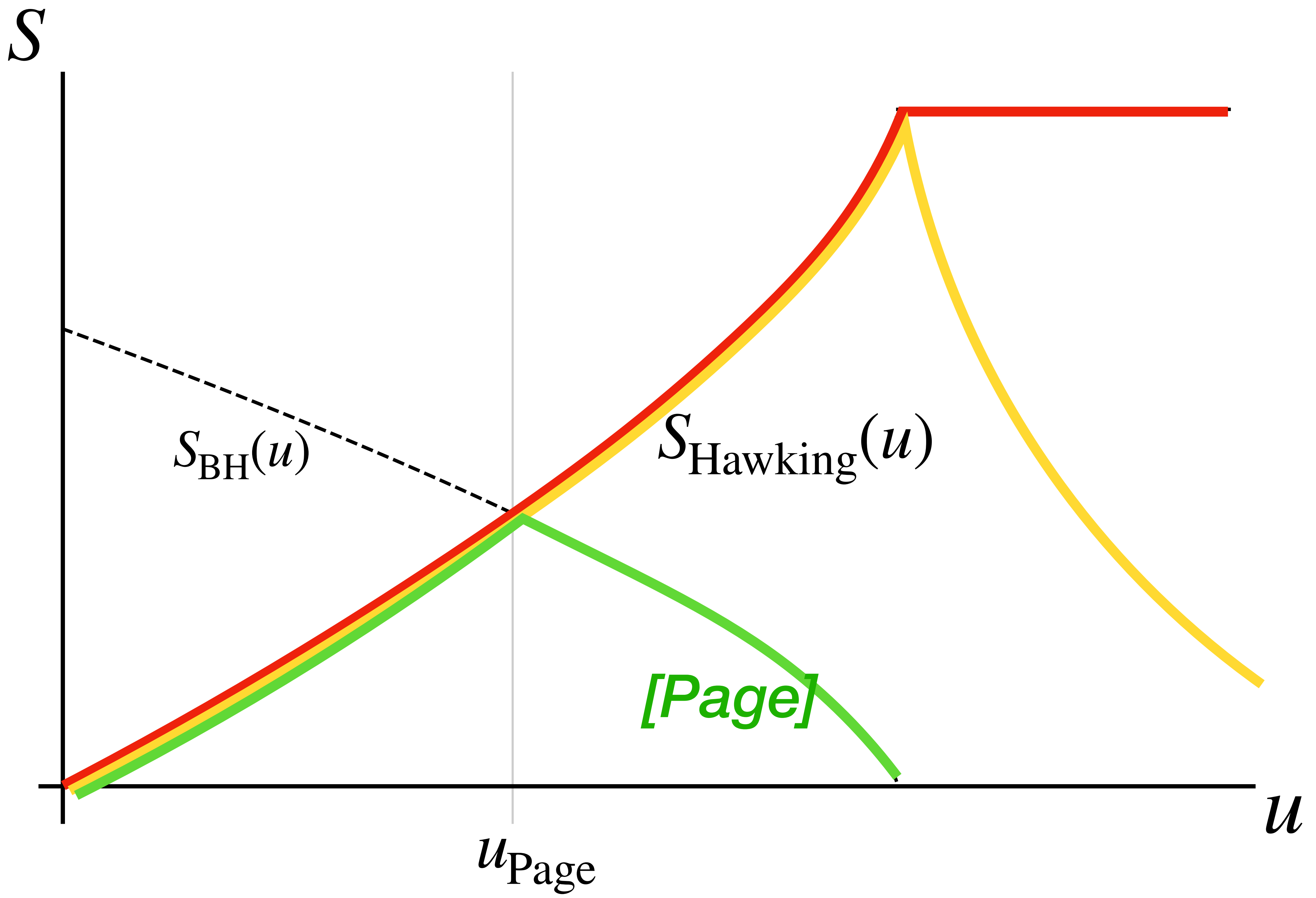


Possibilities:

- Information loss
- Remnants
- BH unitarity

In tension while the black hole is still large!

Conclusions

Semiclassical gravity gives a consistent, coherent picture such that

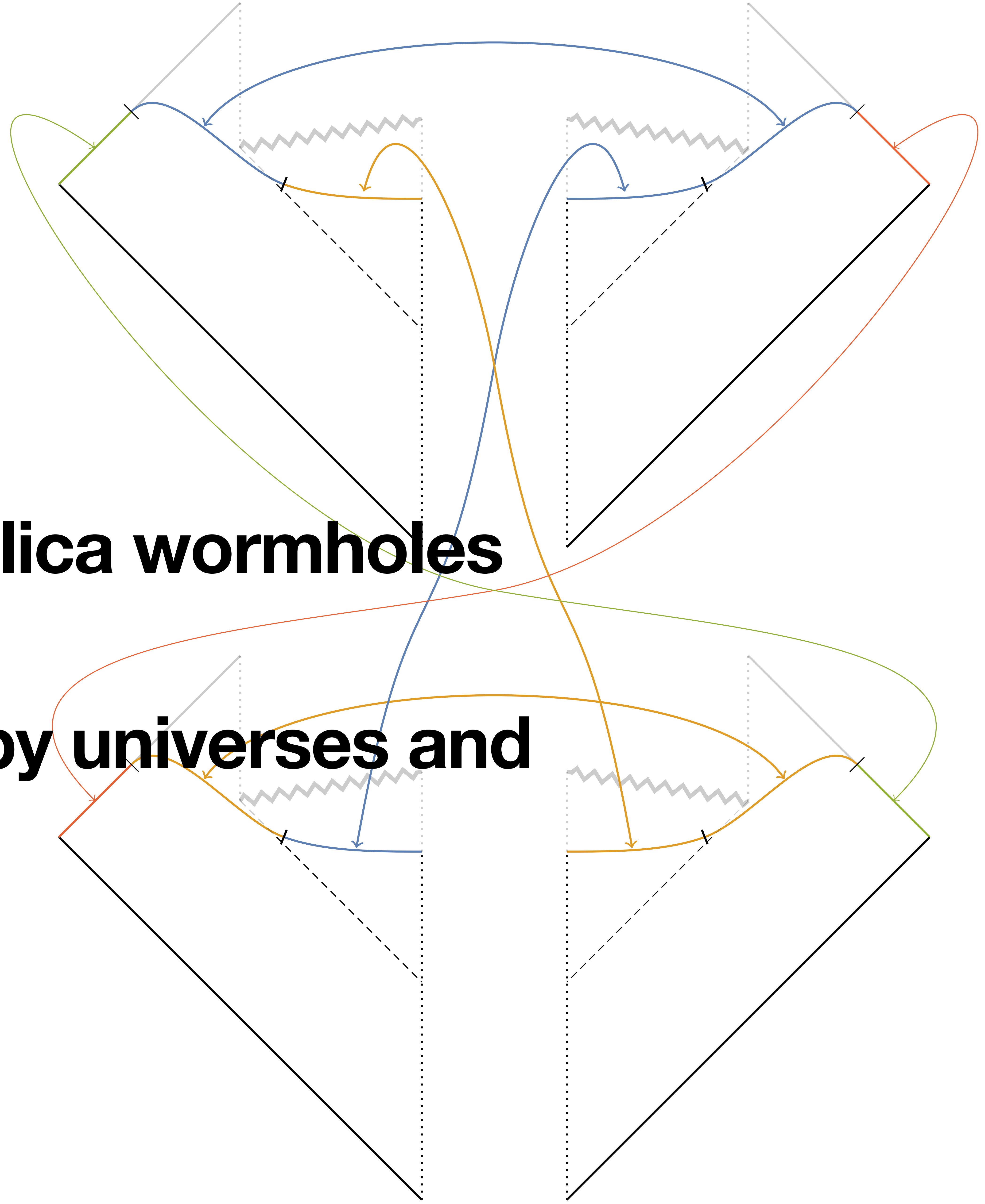
- **Perturbative quantum gravity is correct:** no significant modifications to Hawking's calculation of the state of radiation are known, &
- **BH unitarity is correct:** the outcome of experiments predicted by semiclassical gravity are consistent with $S_{\mathrm{BH}} = \dfrac{A}{4G_N}$ counting states.

How is this possible?

Superselection sectors for the algebra of asymptotic observables.

Large entropy of Hawking radiation: entanglement with "baby universes", or superposition of superselection sectors: **unobservable**!

1. Observables for entropy
2. Semiclassical gravity & replica wormholes
3. Spacetime wormholes, baby universes and superselection sectors



An operational approach

Concentrate on predictions for experiments performed by asymptotic observers

von Neumann entropy is not directly observable!

Require measurements on **multiple copies** of the state

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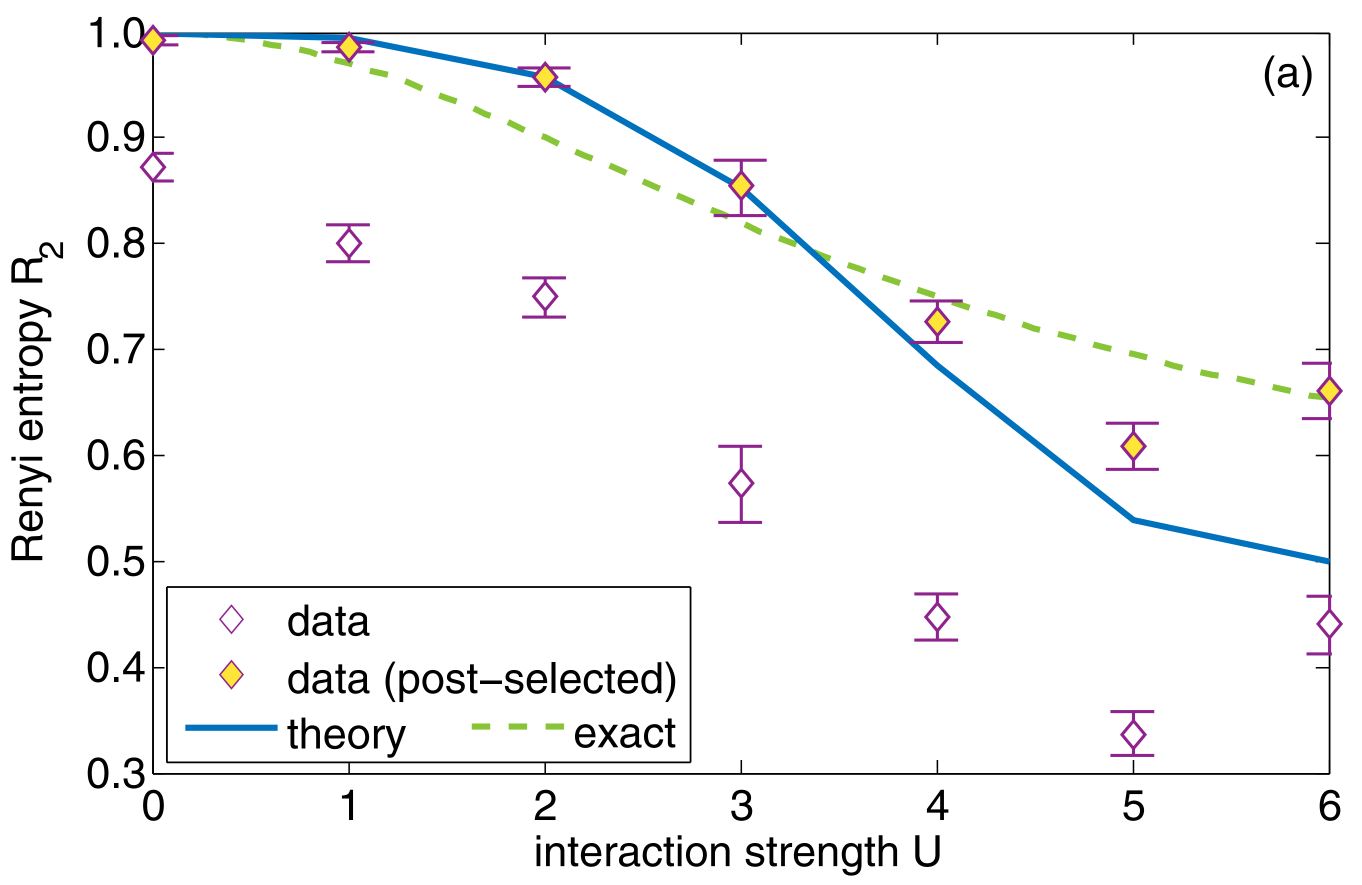

[Linke et al]

Swap test: measure swap operator $\mathcal{S}$

$$\mathcal{S}\,|\psi_1\rangle \otimes |\psi_2\rangle = |\psi_2\rangle \otimes |\psi_1\rangle$$

[Hayden,Preskill]

Expectation value on two copies of a state ρ:

$$\mathrm{Tr}(\mathcal{S}\rho \otimes \rho) = \mathrm{Tr}(\rho^2) = e^{-S_2(\rho)}$$

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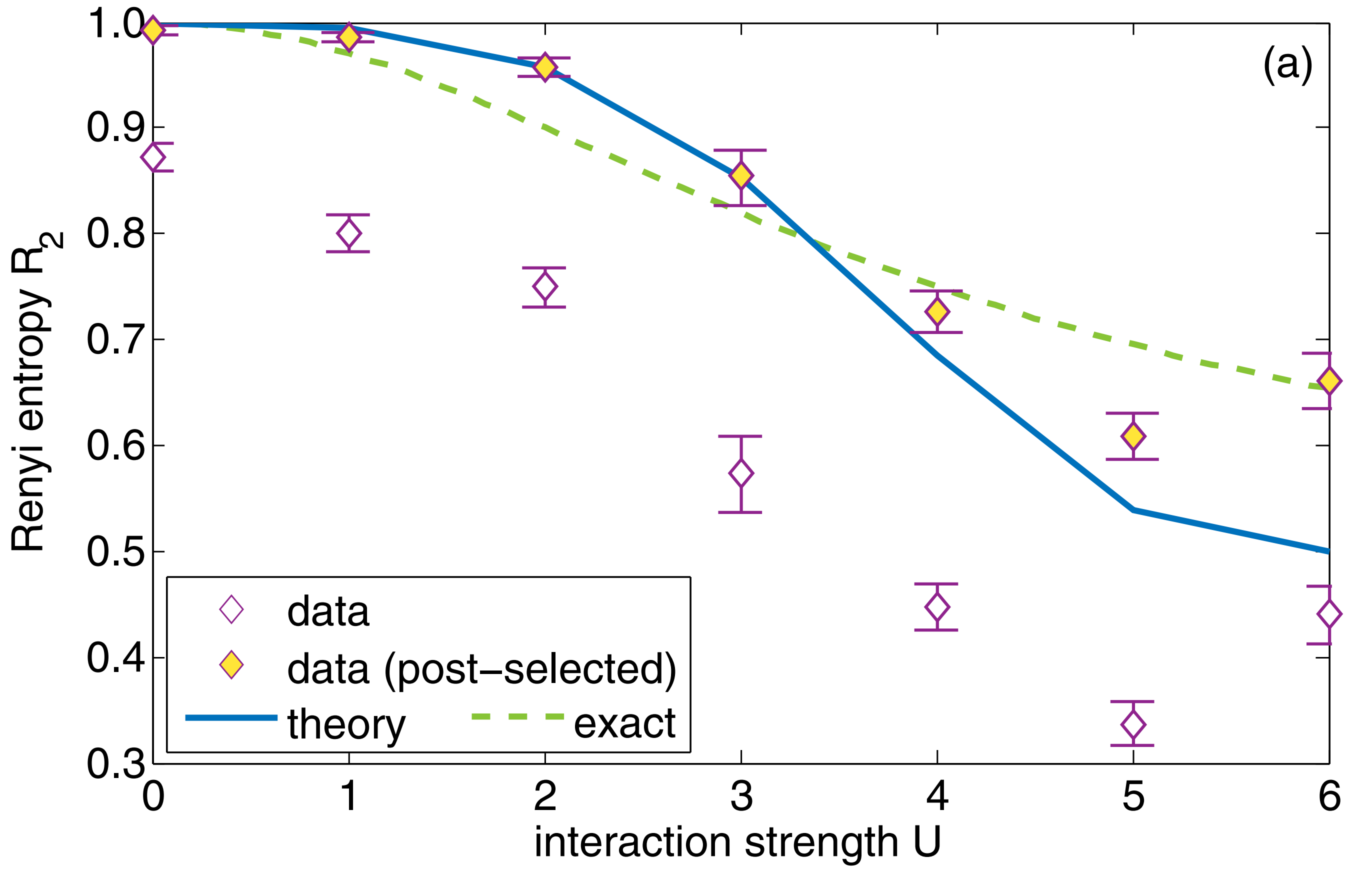

[Linke et al]

More generally:

$$S_n(\rho) = -\frac{1}{n-1}\log \mathrm{Tr}(\rho^n) = -\frac{1}{n-1}\log \mathrm{Tr}(U_\sigma\, \rho^{\otimes n})$$

nth Rényi entropy

Operator enacting cyclic permutation $\sigma = (12\cdots n)$

n copies of state ρ

von Neumann entropy: formal limit

$$S(\rho) = \lim_{n\to 1} S_n(\rho)$$

An operational approach

We'll study "**swap (Rényi) entropies**":

$$S_n^{\text{swap}}(u) = -\frac{1}{n-1} \log \text{Tr}(U_\sigma \, \rho^{(n)}(u))$$

State of Hawking radiation before time u from n identically prepared black holes

Cyclic permutation operator

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Interpretation: entropy deduced by asymptotic observer from measurements

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This will fail: $S^{\text{swap}}(u) \neq S(u)$!

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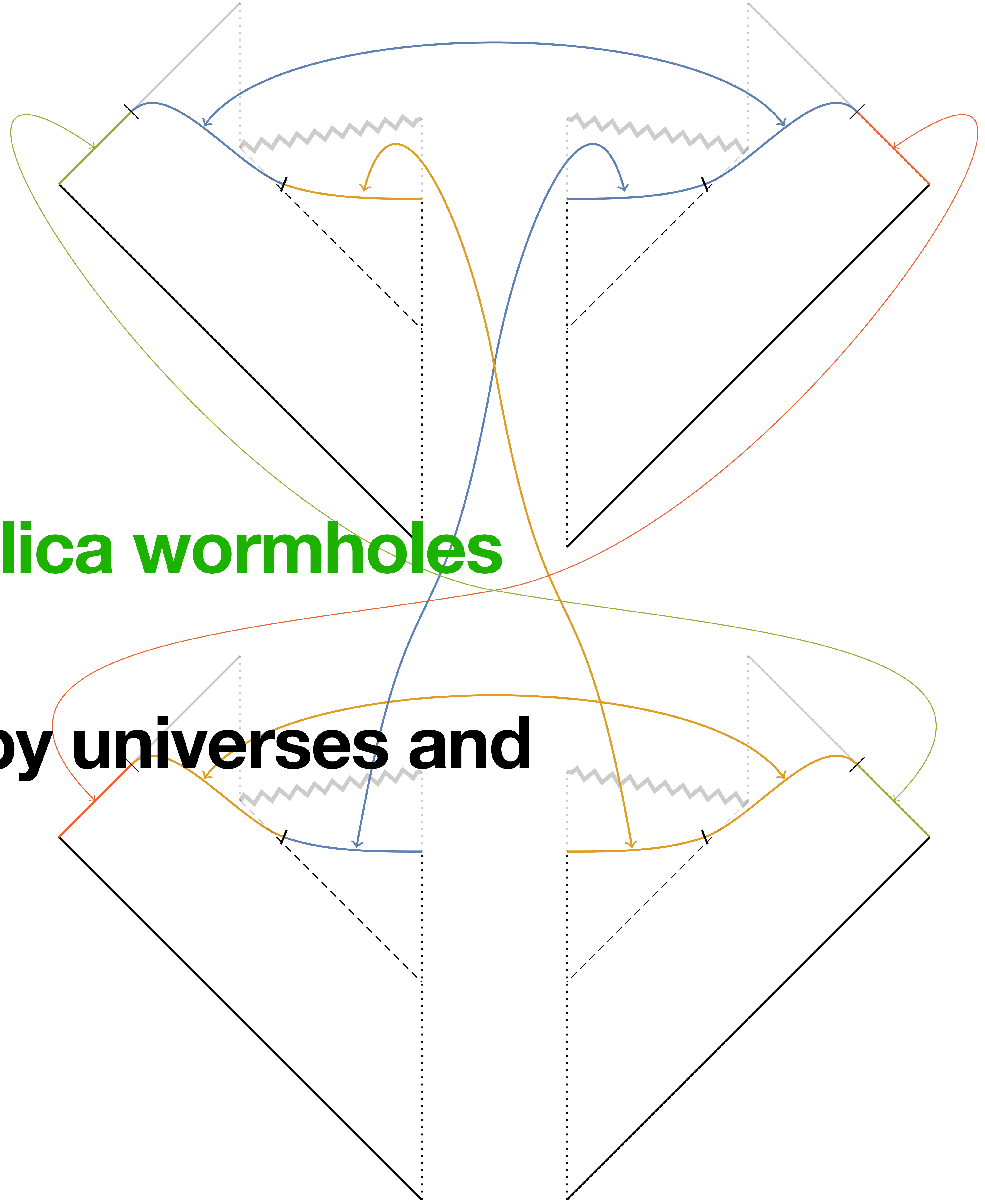
2. Semiclassical gravity & replica wormholes
3. Spacetime wormholes, baby universes and superselection sectors



Framework: semiclassical gravity

Use only low-energy theory of GR + matter

Path integral formulation (Lorentzian):

$$\int \mathscr{D}g\ e^{iS_{\mathrm{EH}}[g]} \int \mathscr{D}\phi\ e^{iS_{\mathrm{matter}}[g,\phi]} = \int \mathscr{D}g\ e^{i(S_{\mathrm{EH}}[g]+S_{\mathrm{eff}}[g])}$$

Semiclassical: saddle-points of gravitational action + matter effective action $S_{\mathrm{EH}}[g] + S_{\mathrm{eff}}[g]$

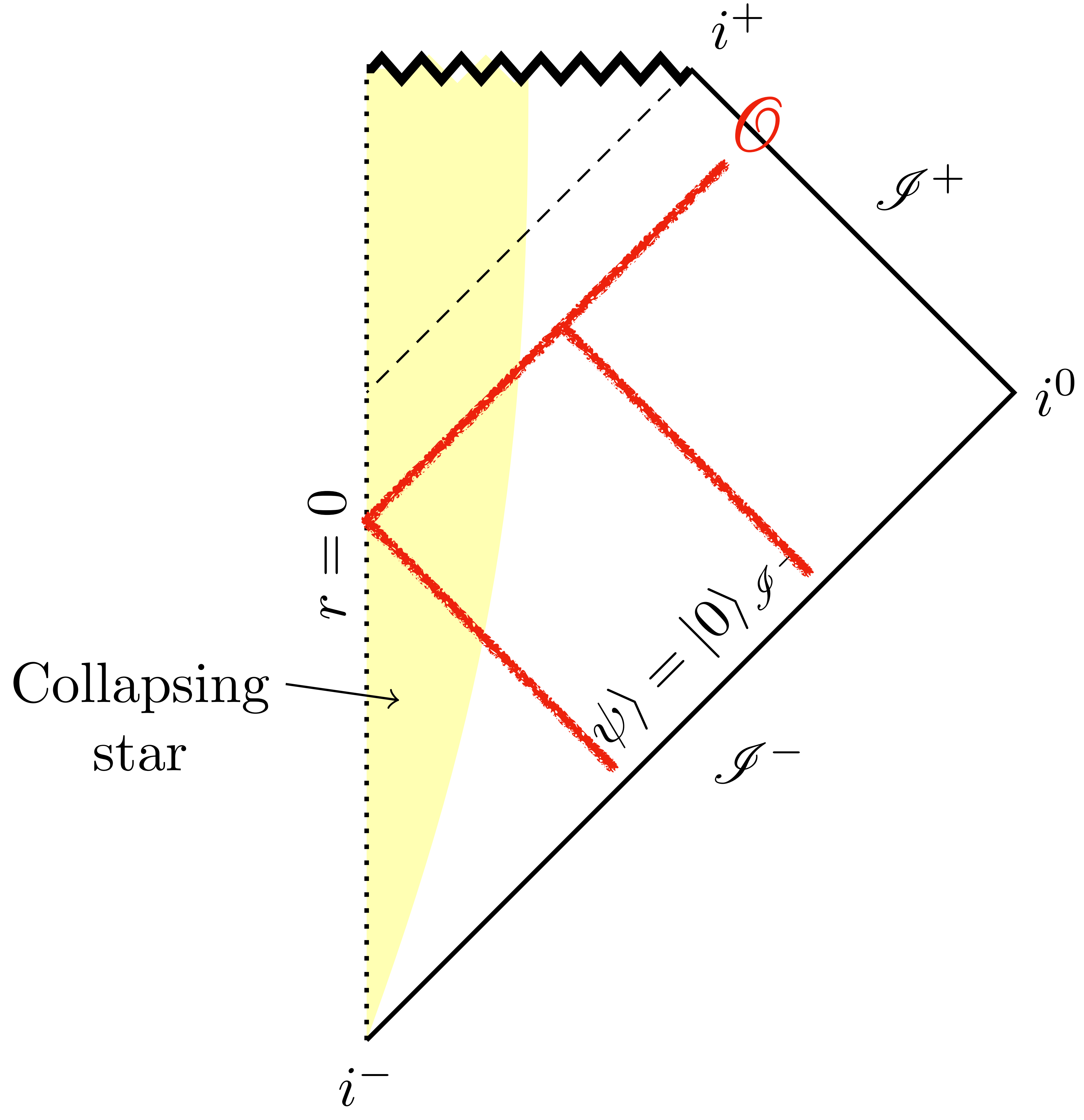
Boundary conditions: fix asymptotic geometry

Saddle-points with any topology allowed

Geometries with regions of strong curvature excluded

Hawking's calculation

QFT on fixed background (canonical)



Compute EV of operator $\mathcal{O}$ on $\mathscr{I}^+$

Heisenberg evolve back to $\mathscr{I}^-$

e.g. free matter $a_m(\mathscr{I}^+) = \sum_n \left(\alpha_{mn}\, a_n(\mathscr{I}^-) + \beta_{mn}\, a_n^\dagger(\mathscr{I}^-)\right)$

Evaluate in initial state (ingoing vacuum)

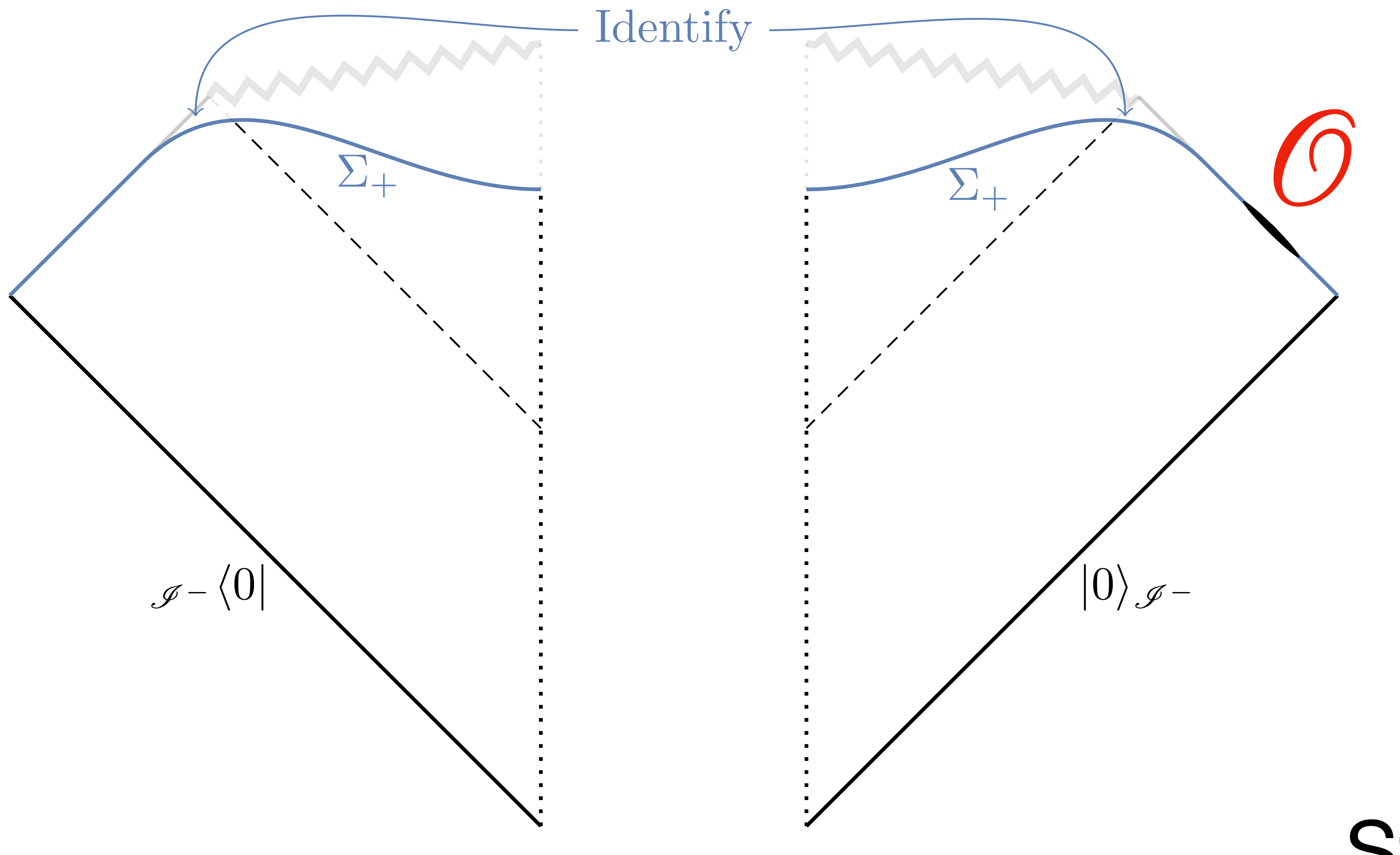
e.g. occupation numbers $\left\langle N_m(\mathscr{I}^+)\right\rangle = \sum_n |\beta_{mn}|^2\,.$

No evolution through strong curvature regions required

Hawking's calculation

QFT on fixed background (path integral)

Integrate over matter fields on "doubled" spacetime:



Compute EV of operator $\mathcal{O}$ on $\mathscr{I}^+$

Heisenberg evolve back to $\mathscr{I}^-$

In-in (Schwinger-Keldysh) formalism

Boundary conditions at $\mathscr{I}^-$: initial state

No need to specify final state

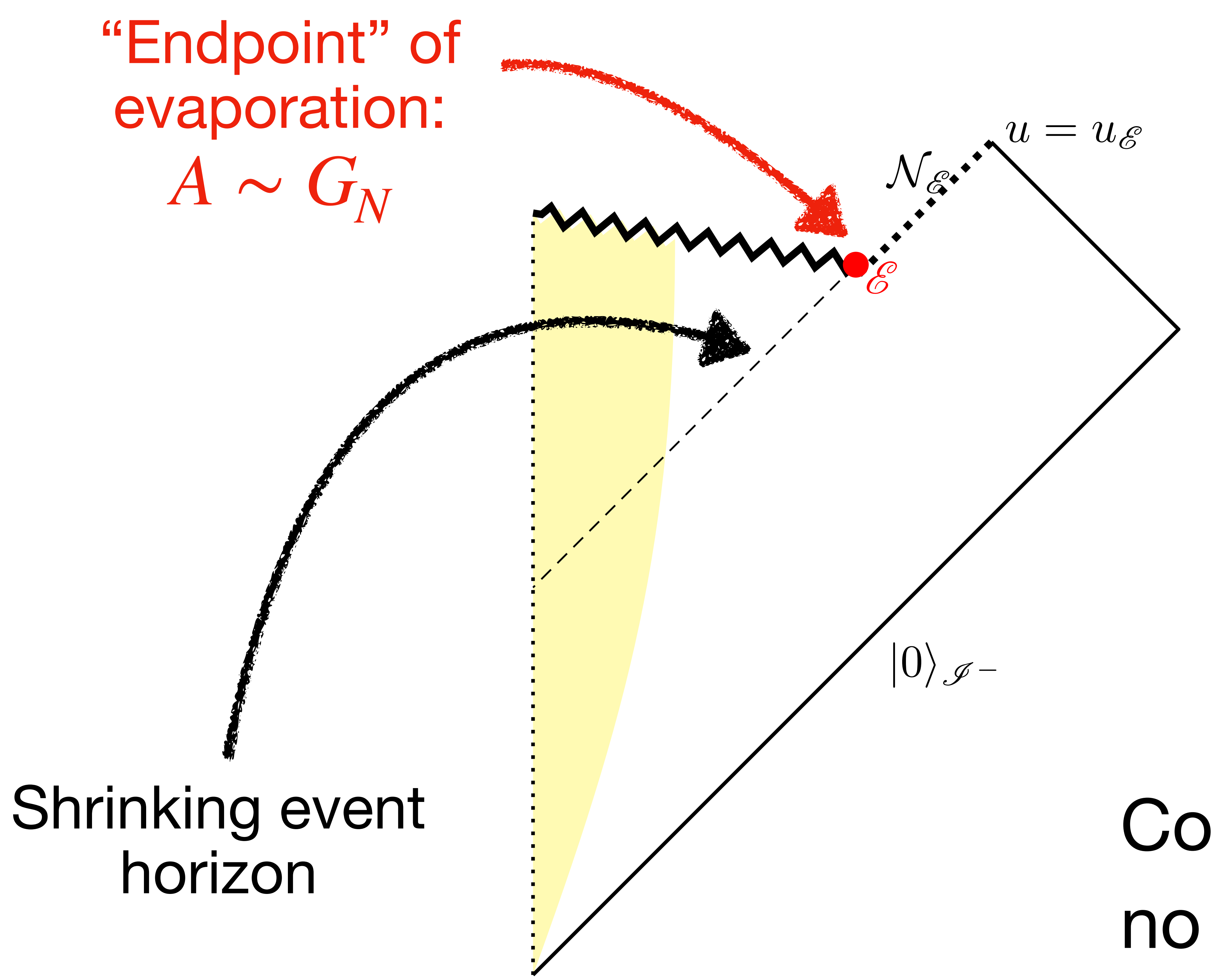
Identify "bra" and "ket" spacetimes on future Cauchy surface Σ_+

Strong curvature regions not part of geometry

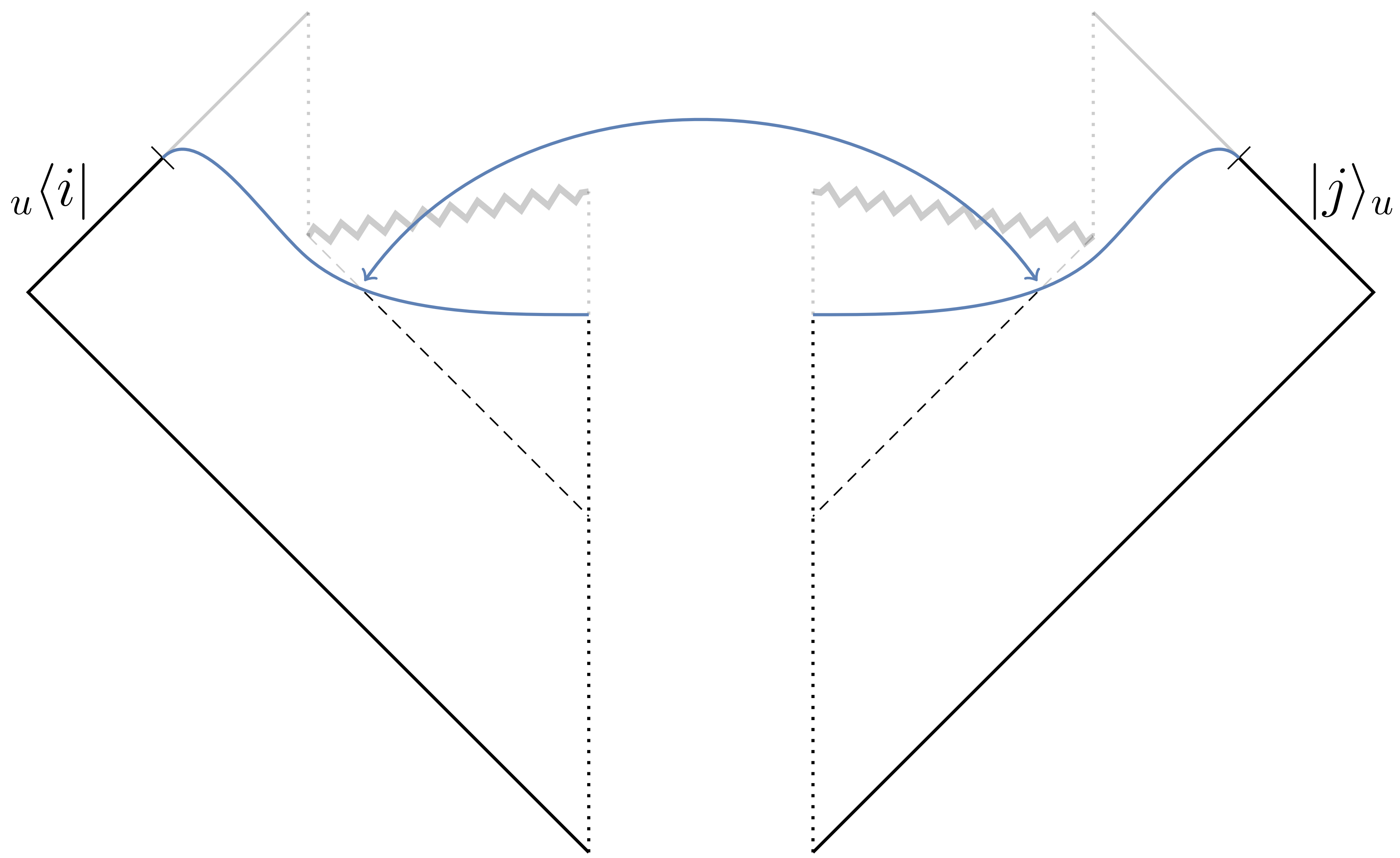
Perturbative quantum gravity

A dynamical metric

Black hole evaporates:



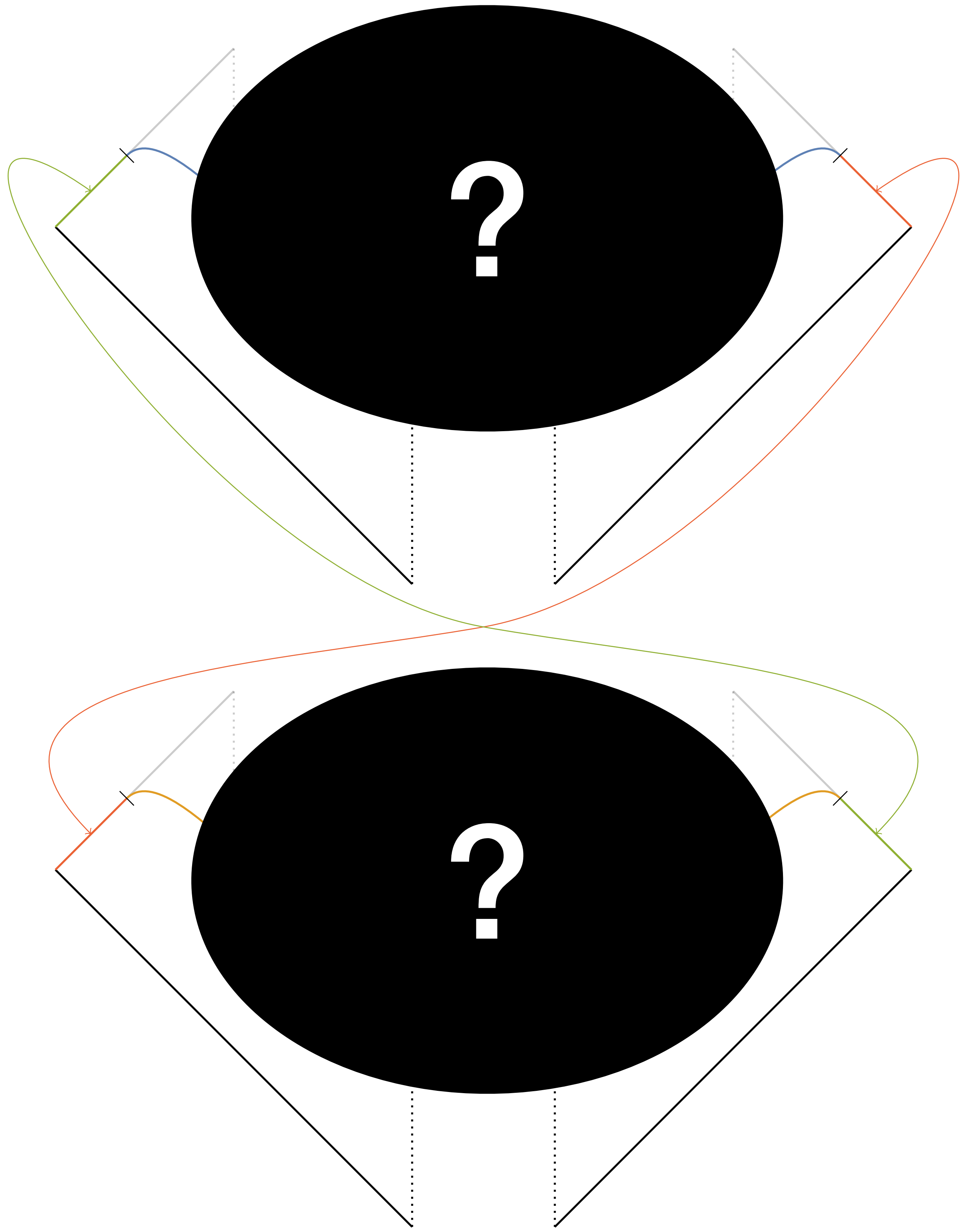
Perturbatively corrected saddle-point for the density matrix of Hawking radiation ${}_u\langle i \,|\, \rho(u) \,|\, j\rangle_u$:



Concentrate on measurements before evaporation ($u < u_{\mathscr{E}}$): no assumptions about endpoint $\mathscr{E}$ or singularity necessary

Computing the swap entropy

Swap operator $\mathcal{S}$ acting on two sets of Hawking radiation, $\rho^{(2)}(u)$:



Boundary conditions for computing the expectation value $\mathrm{Tr}\left(\mathcal{S}\rho^{(2)}(u)\right)$

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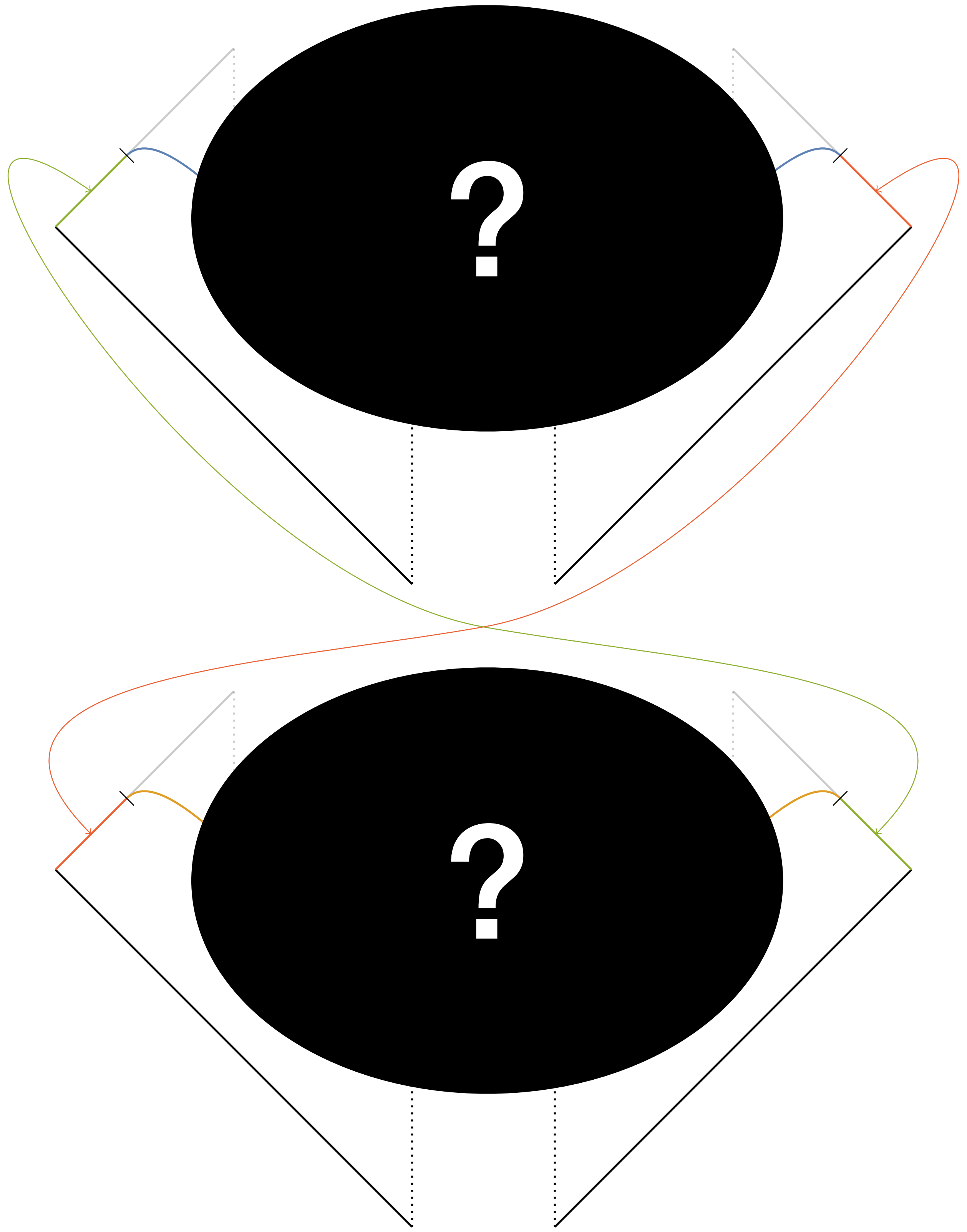

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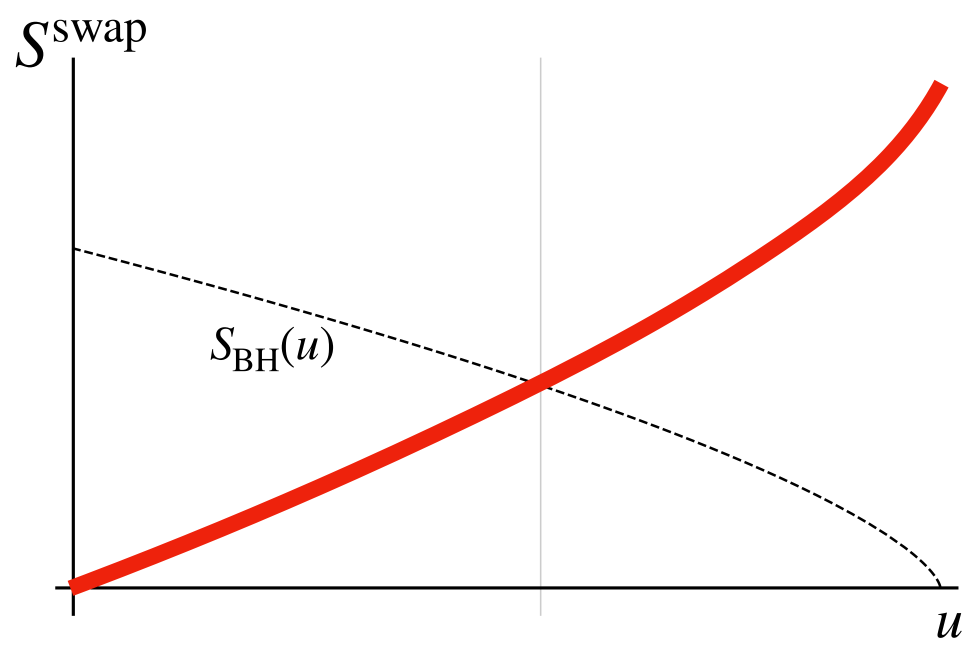
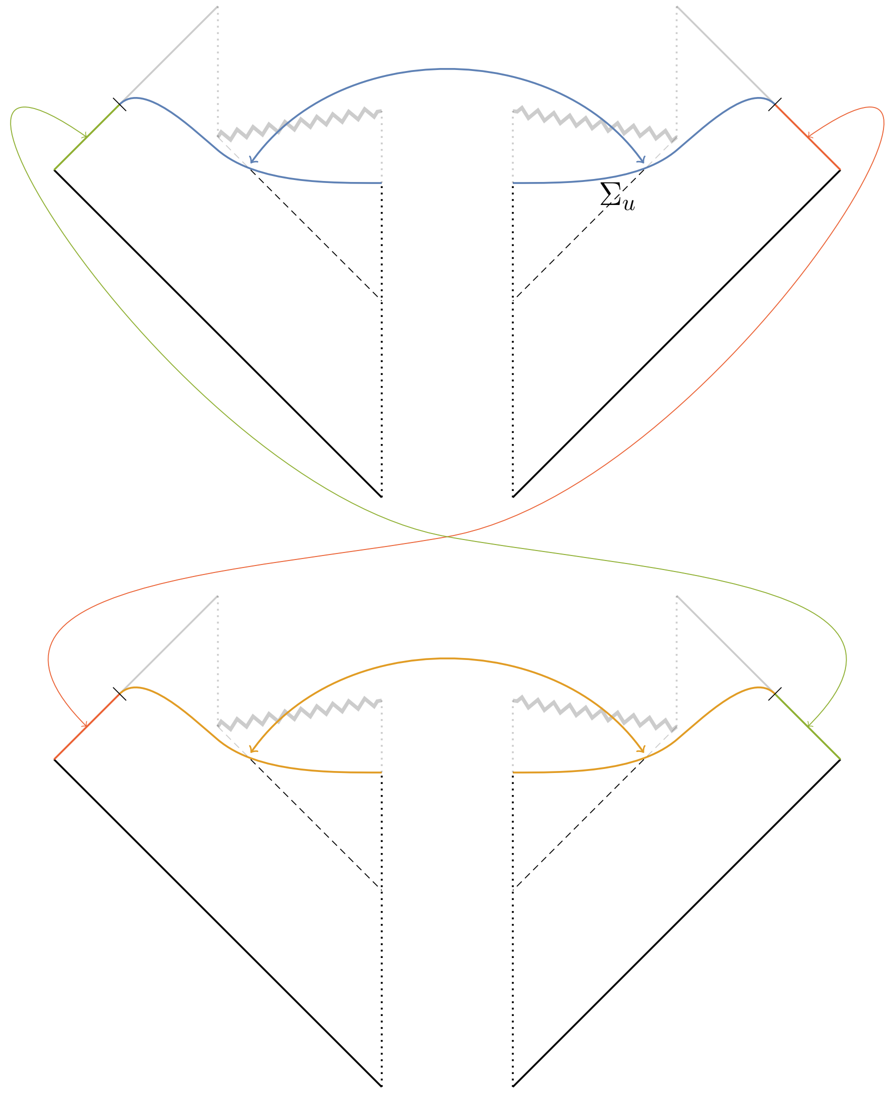
A saddle point:

Two copies of “Hawking” saddle





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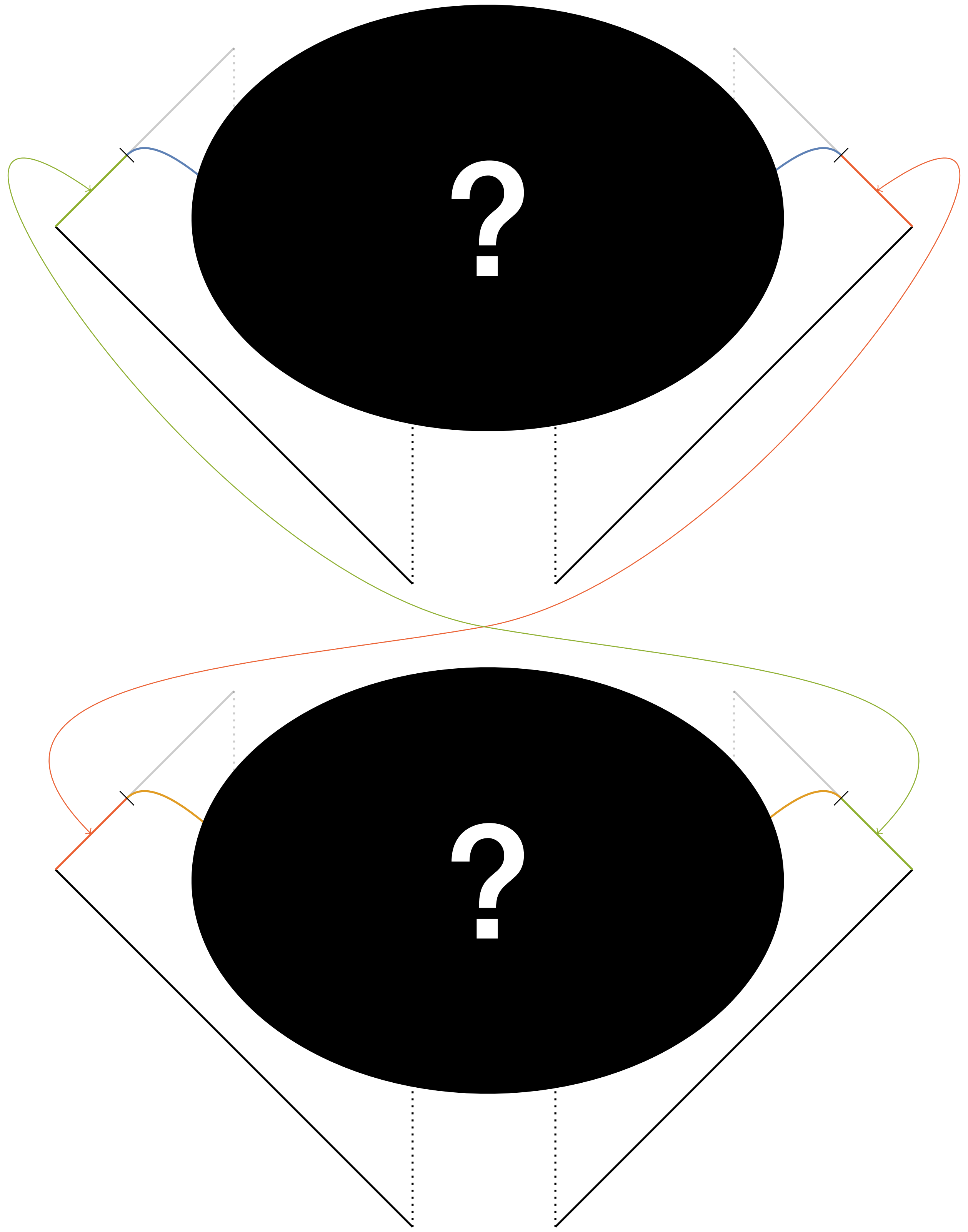

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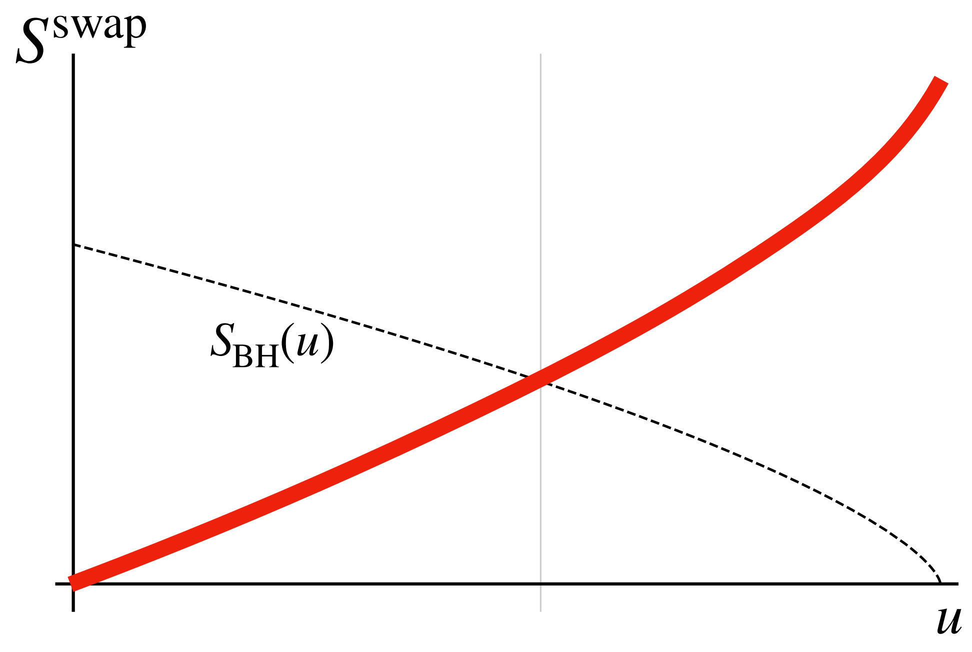
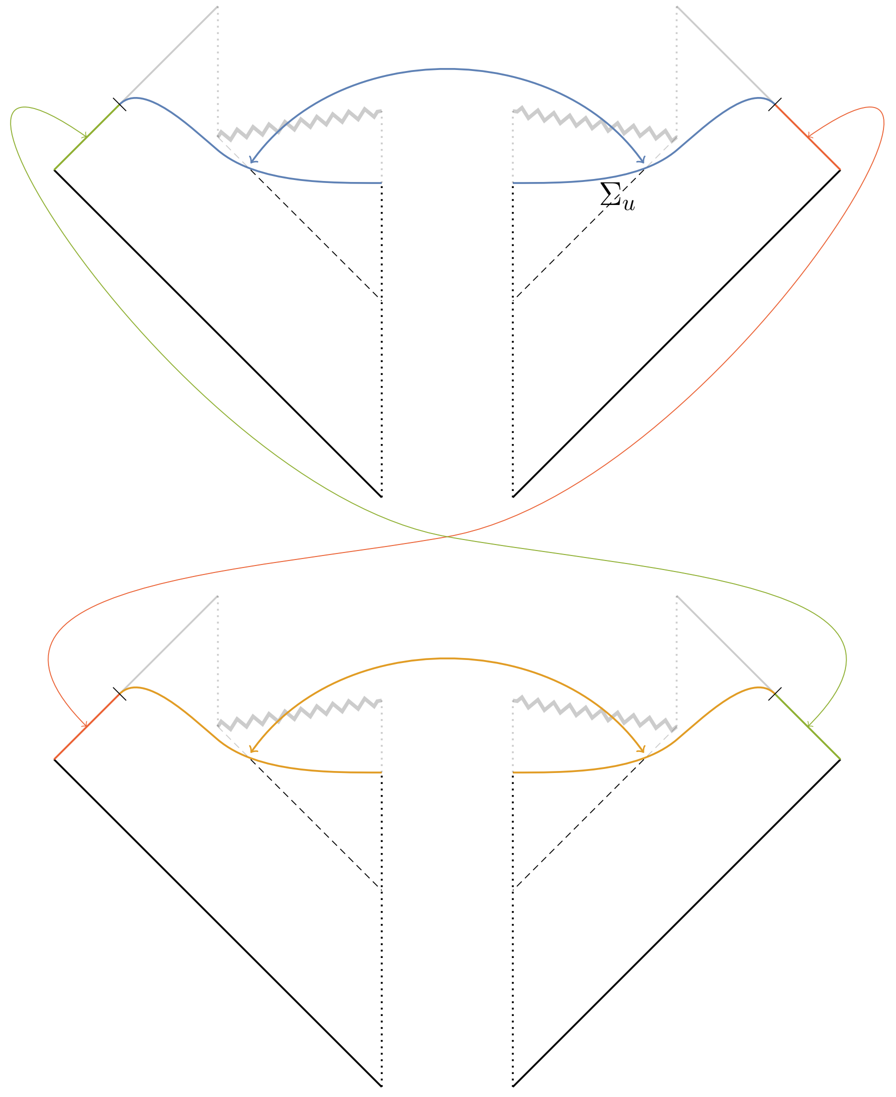
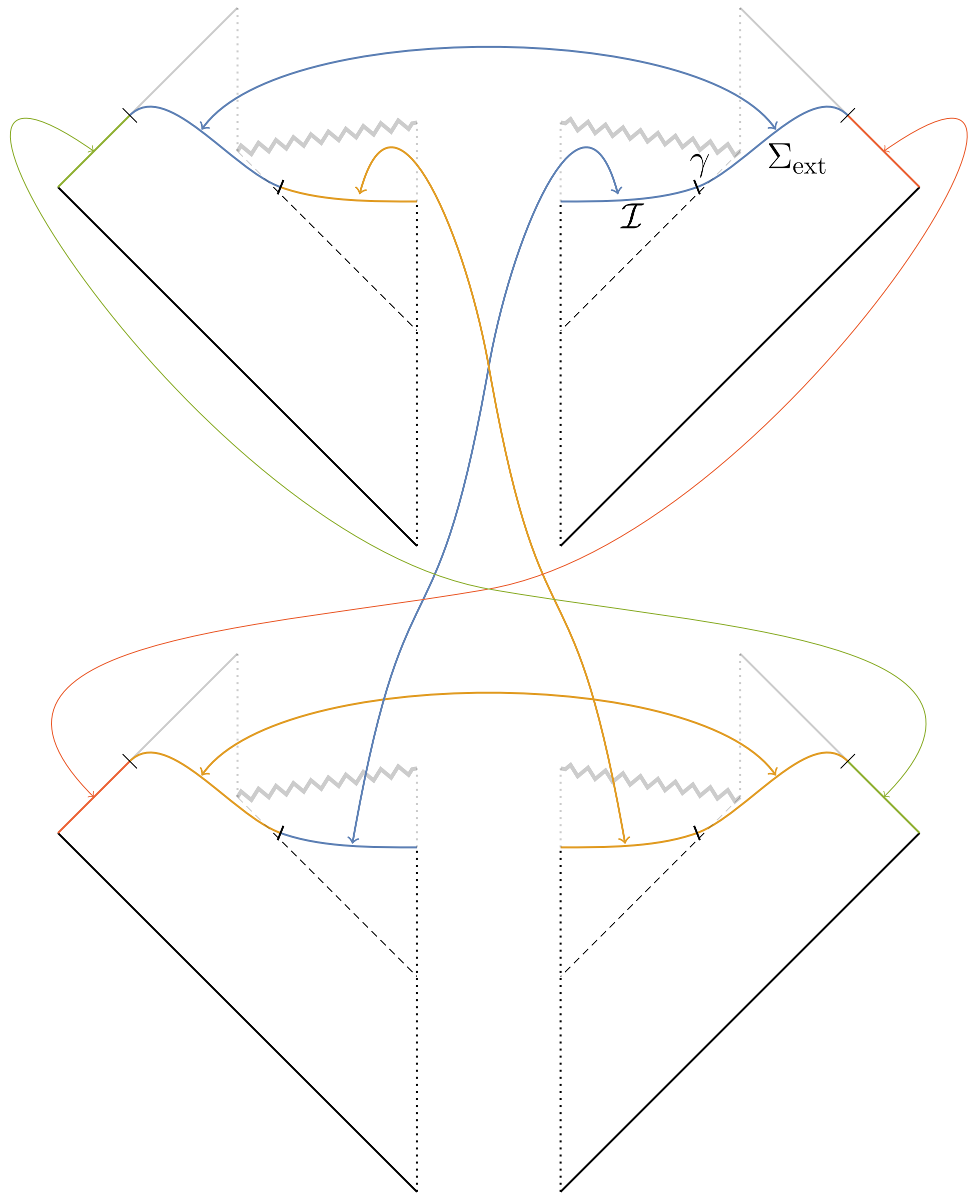
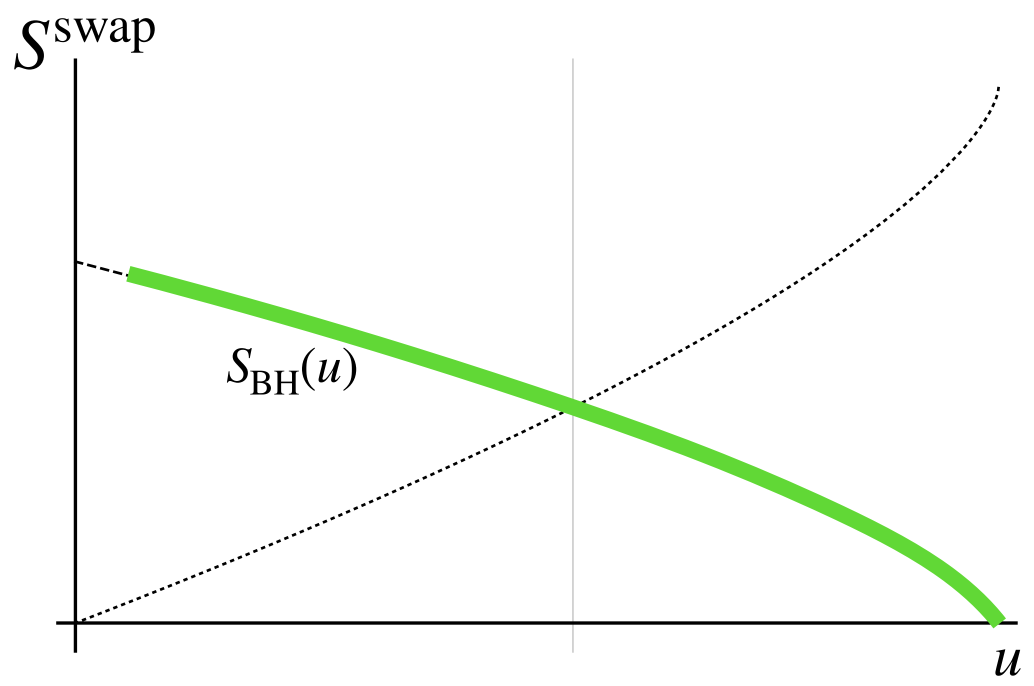




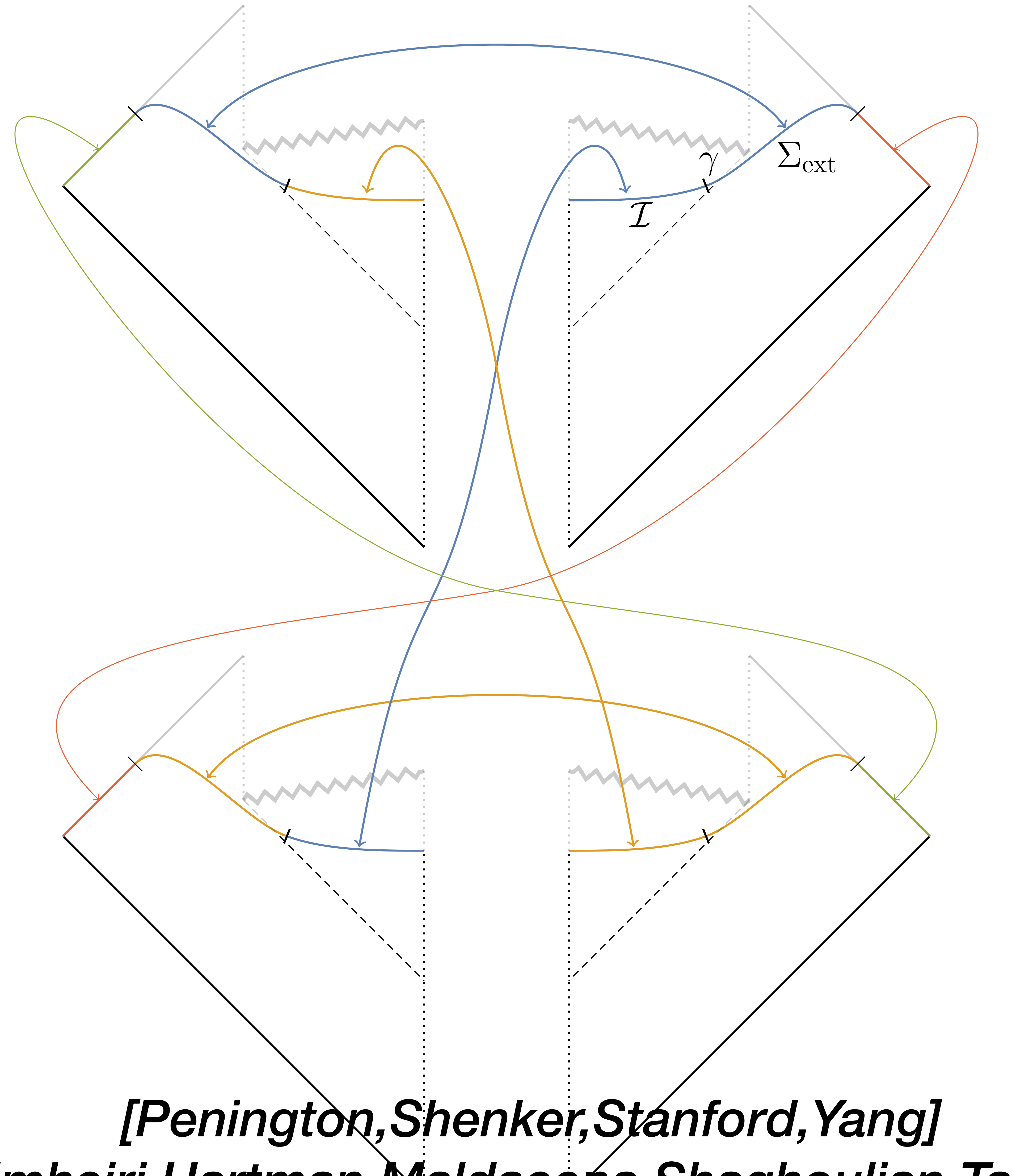


Another saddle!

"Replica wormhole"



Geometry of a replica wormhole



Split future Cauchy surface in two pieces (Σ_{ext} and I) along a surface γ.

Exterior piece Σ_{ext} identifies "bra" & "ket" as required by boundary conditions

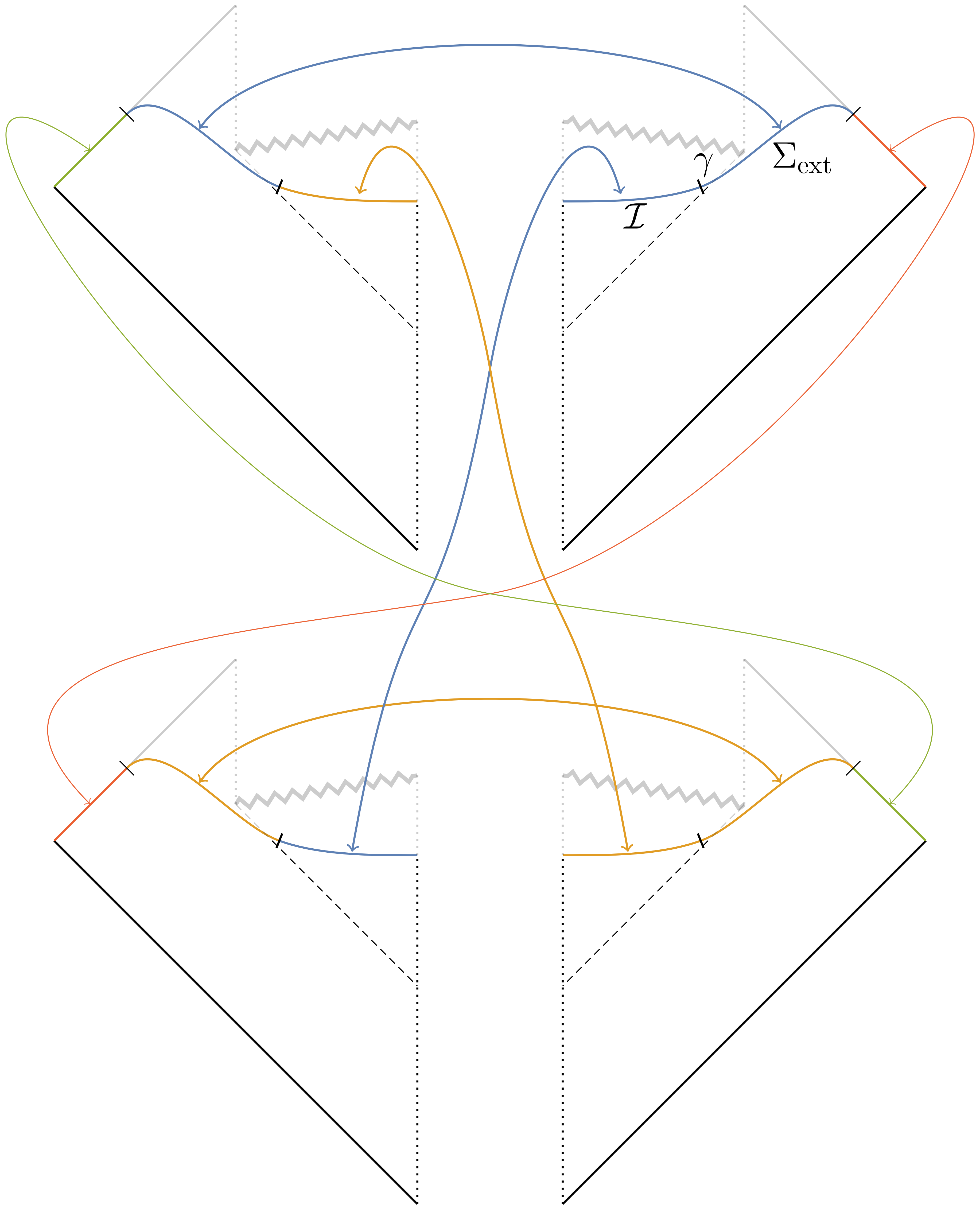
But the "island" I is identified with a "swap":

joins replicas with a "spacetime wormhole"!

Geometry — including location of $\gamma = \partial I$ — dynamically determined to get a saddle

[Penington,Shenker,Stanford,Yang]
[Almheiri,Hartman,Maldacena,Shaghoulian,Tajdini]

Replica wormholes & QES rule for S^{swap}



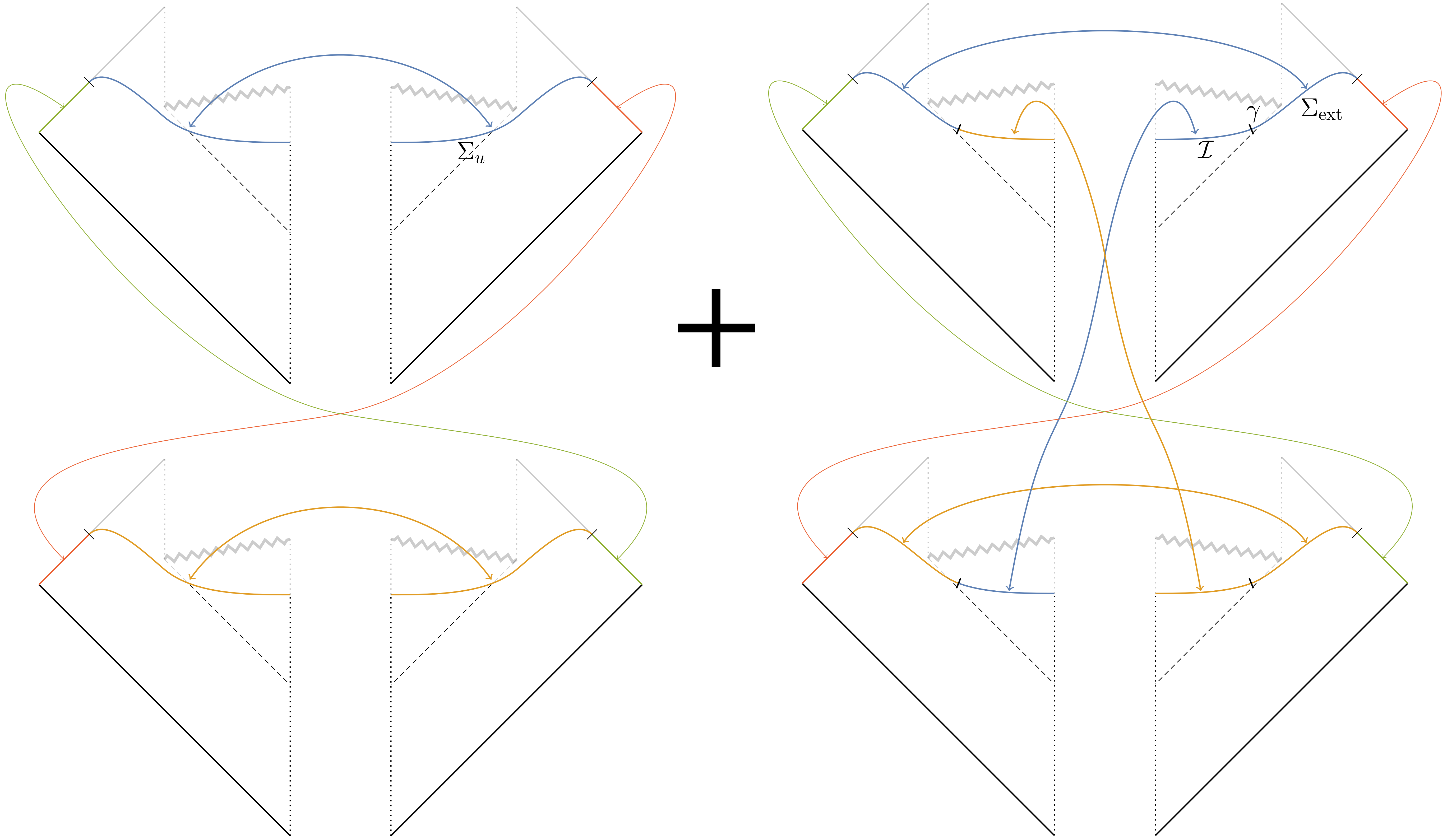
- Hard to construct saddles for integer $n \geq 2$
- Reformulate path integral for $S_n^{\text{swap}}(u)$ to make sense for real $n > 1$ *[Lewkowycz,Maldacena] [Dong,Lewkowycz]*
- Simplifies when $n-1$ is small: saddle-point if γ is a **quantum extremal surface**

$$S^{\text{swap}}(u) \sim \min_I \operatorname{ext} S_{\text{gen}}(I; u)$$

[Ryu,Takayanagi] [Hubeny,Rangamani,Takayanagi] [Faulkner,Lewkowycz,Maldacena][Engelhardt,Wall]

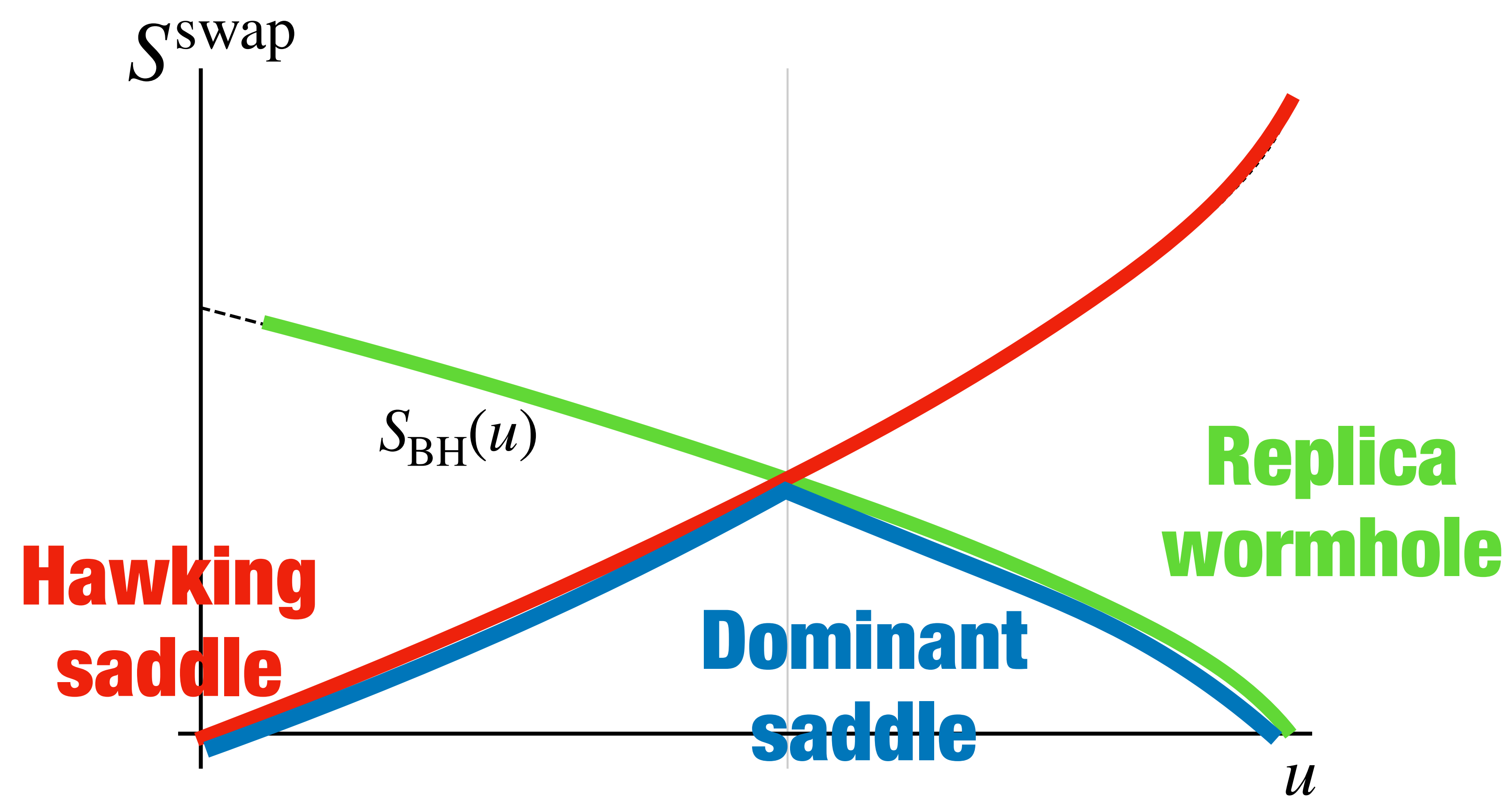
- Finite n: saddle-point geometry is complex. Captures contribution of oscillatory integral over real Lorentzian geometries

Sum of two saddle-points



Dominant at early times   Dominant at late times

Together: Page curve for $S^{\rm swap}(u)$!



Page curve for $S^{\rm swap}(u)$ from first-order phase transition between semiclassical saddle-points

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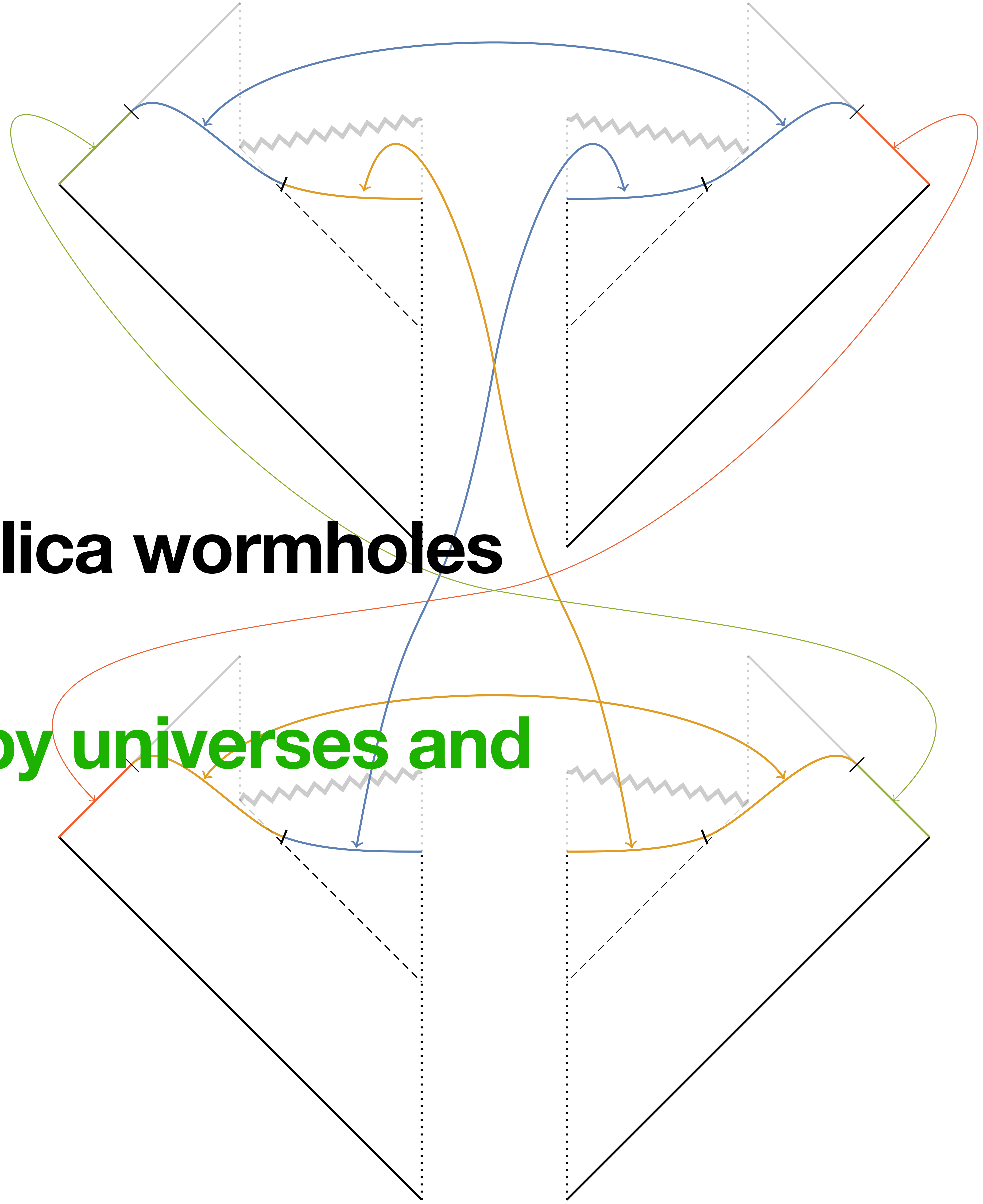
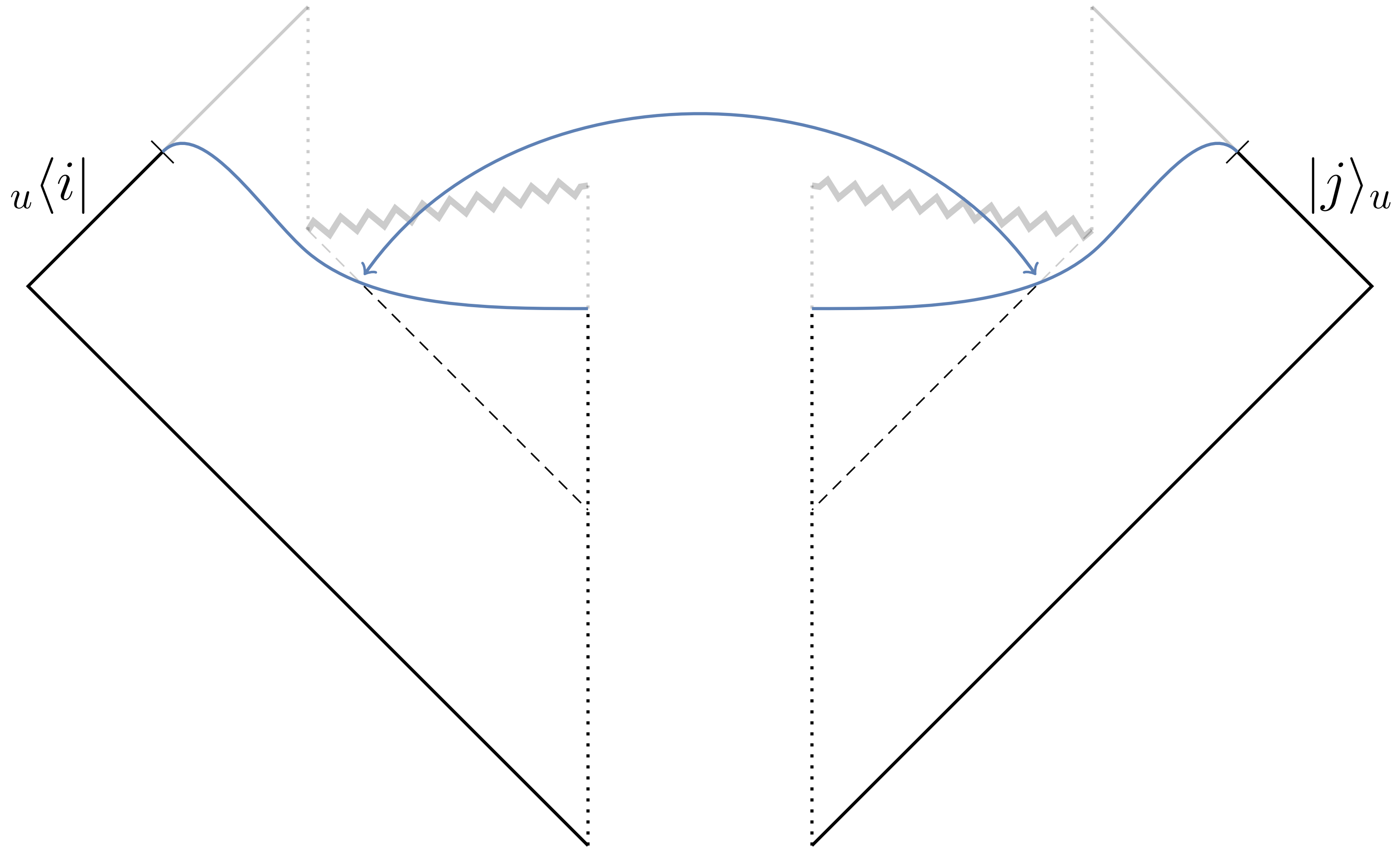
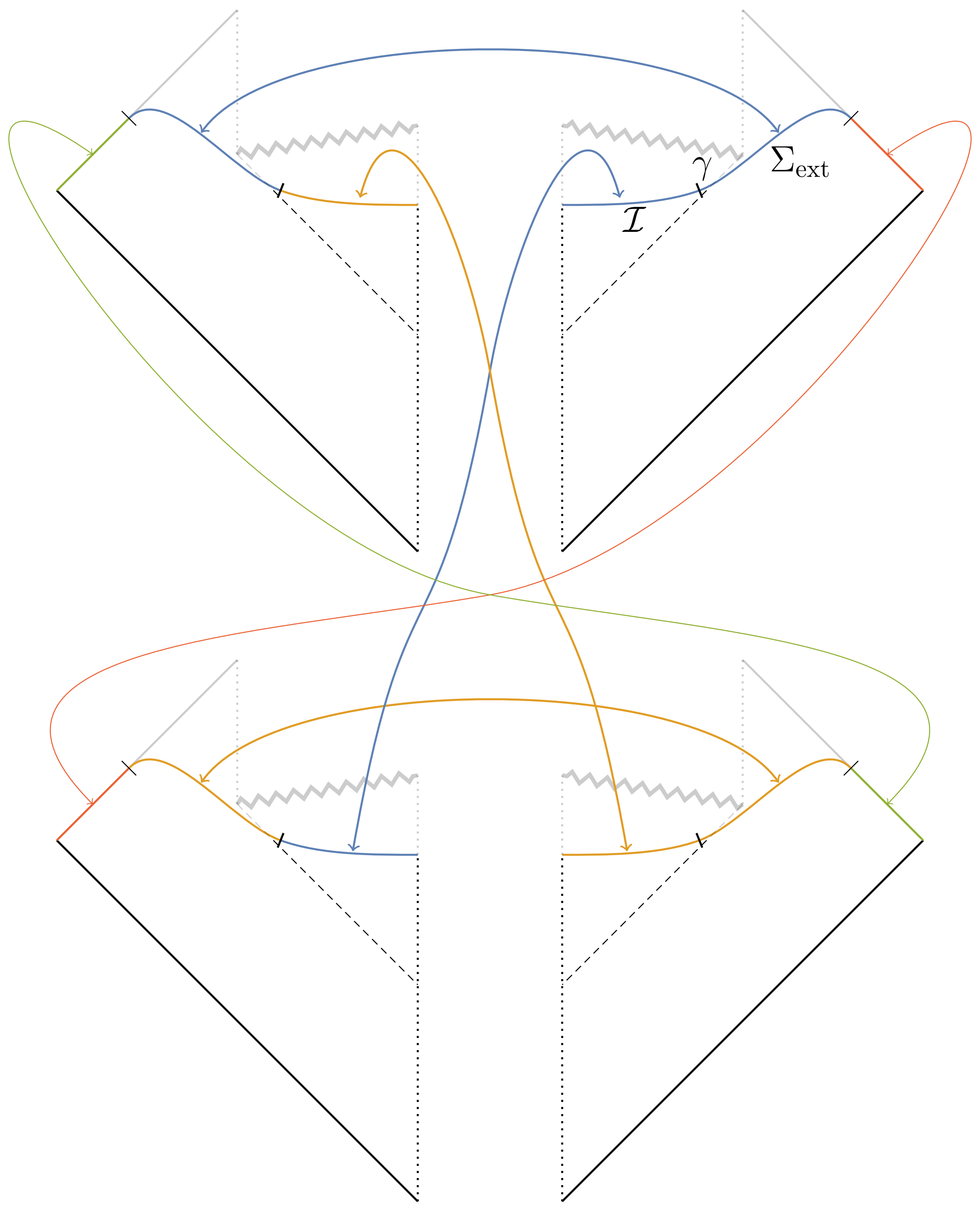


A puzzle



We have not found anything new to modify Hawking's calculation of the state of radiation $_u\langle i\,|\,\rho(u)\,|\,j\rangle_u$



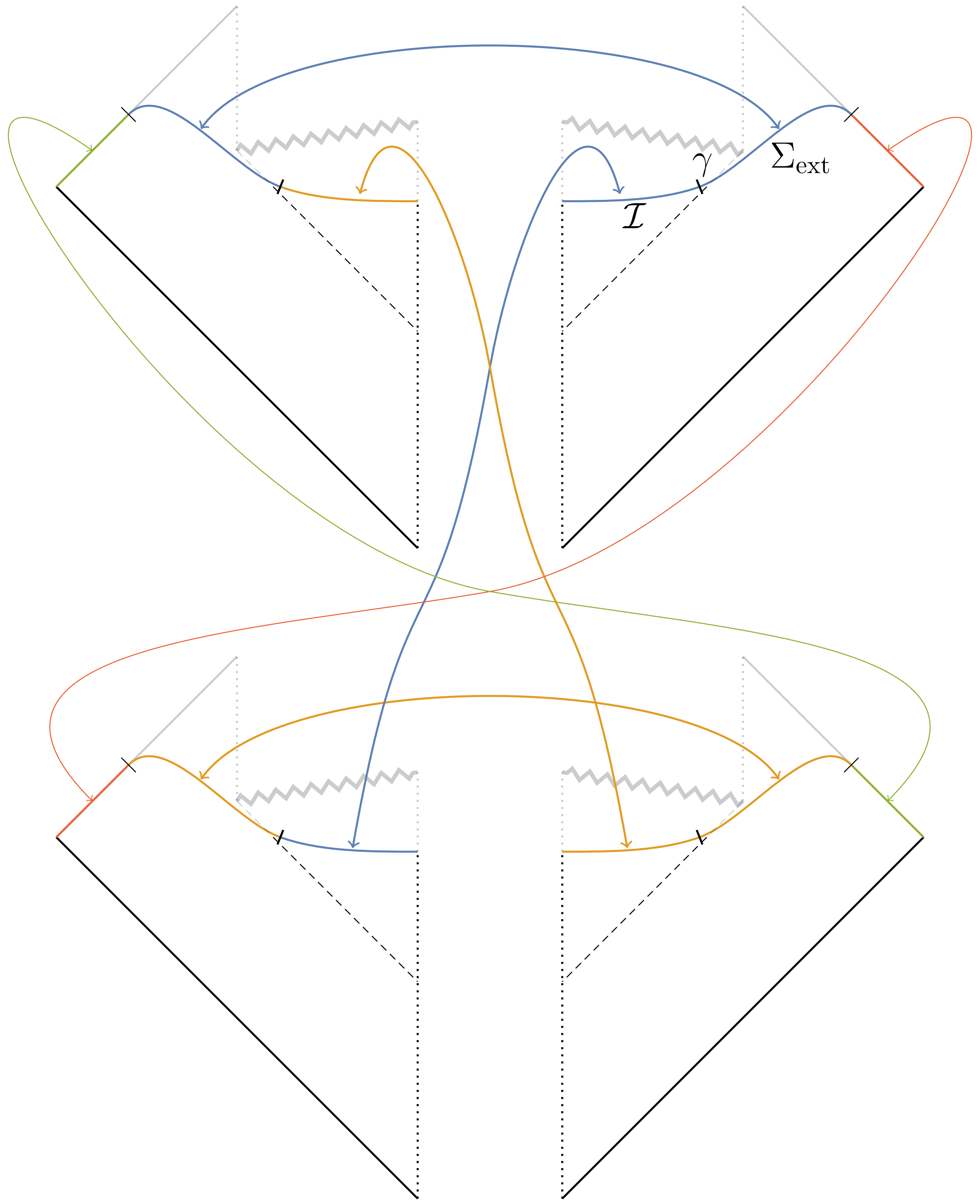
Meanwhile, we have a new saddle-point for the swap $\mathrm{Tr}(\mathcal{S}\rho^{(2)}(u))$!

Resolution: $\rho^{(2)}(u) \neq \rho(u) \otimes \rho(u)$ **!**

For more detail: what is the Hilbert space interpretation of replica wormholes?

Polchinski-Strominger wormholes

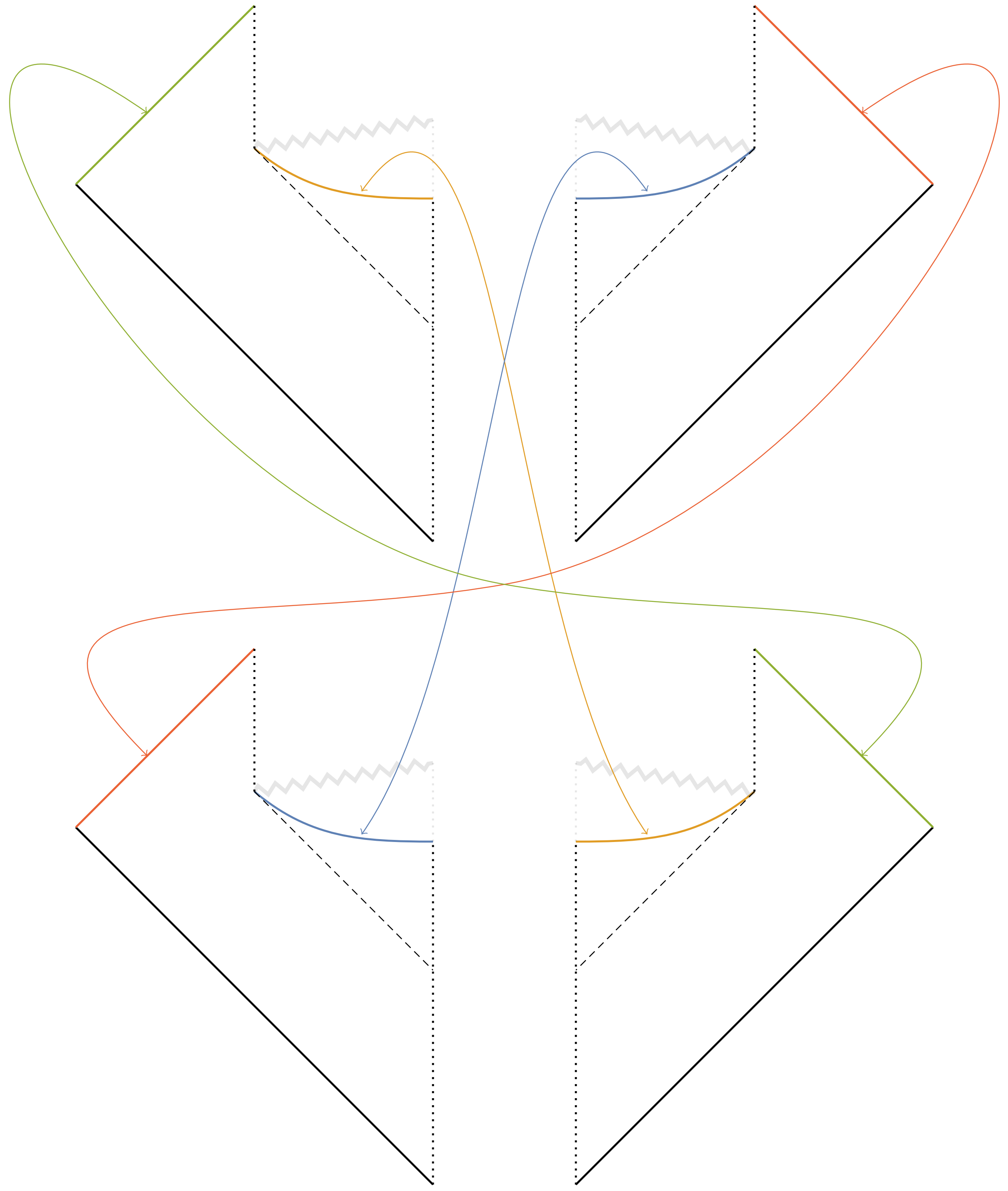
A simplified setting



Extrapolate to late time

$I \longrightarrow \Sigma_{\text{int}}$

Cauchy surface for black hole interior



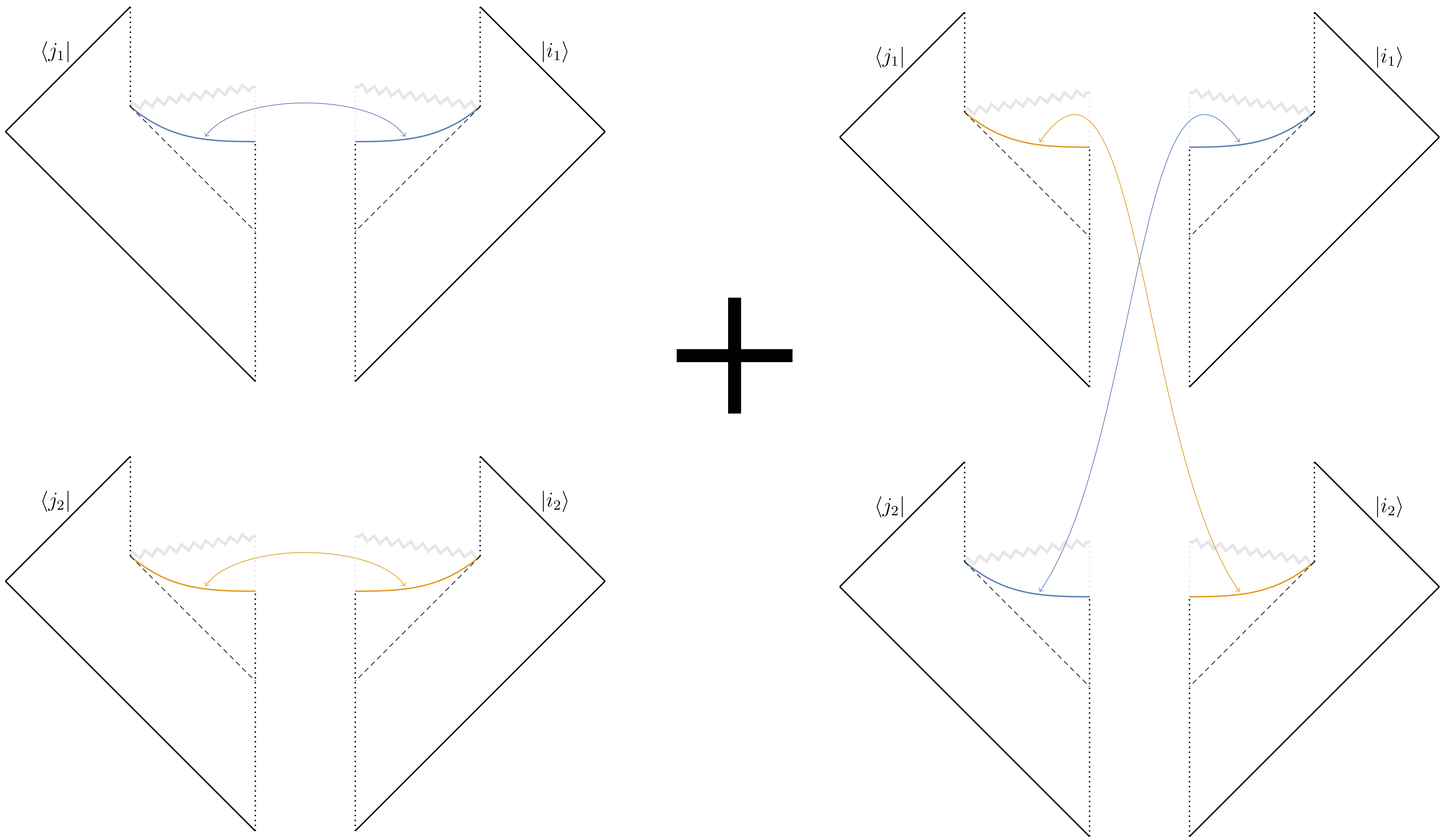
No longer semiclassical. Makes assumptions about evaporation endpoint.

Here: a simplification to explain the main ideas.

Polchinski-Strominger wormholes

Two-copy density matrix on $\mathscr{I}^+$:

$$\langle j_1, j_2 | \rho^{(2)} | i_1, i_2 \rangle = \langle j_1 | \rho_H | i_1 \rangle \langle j_2 | \rho_H | i_2 \rangle + \langle j_2 | \rho_H | i_1 \rangle \langle j_1 | \rho_H | i_2 \rangle$$



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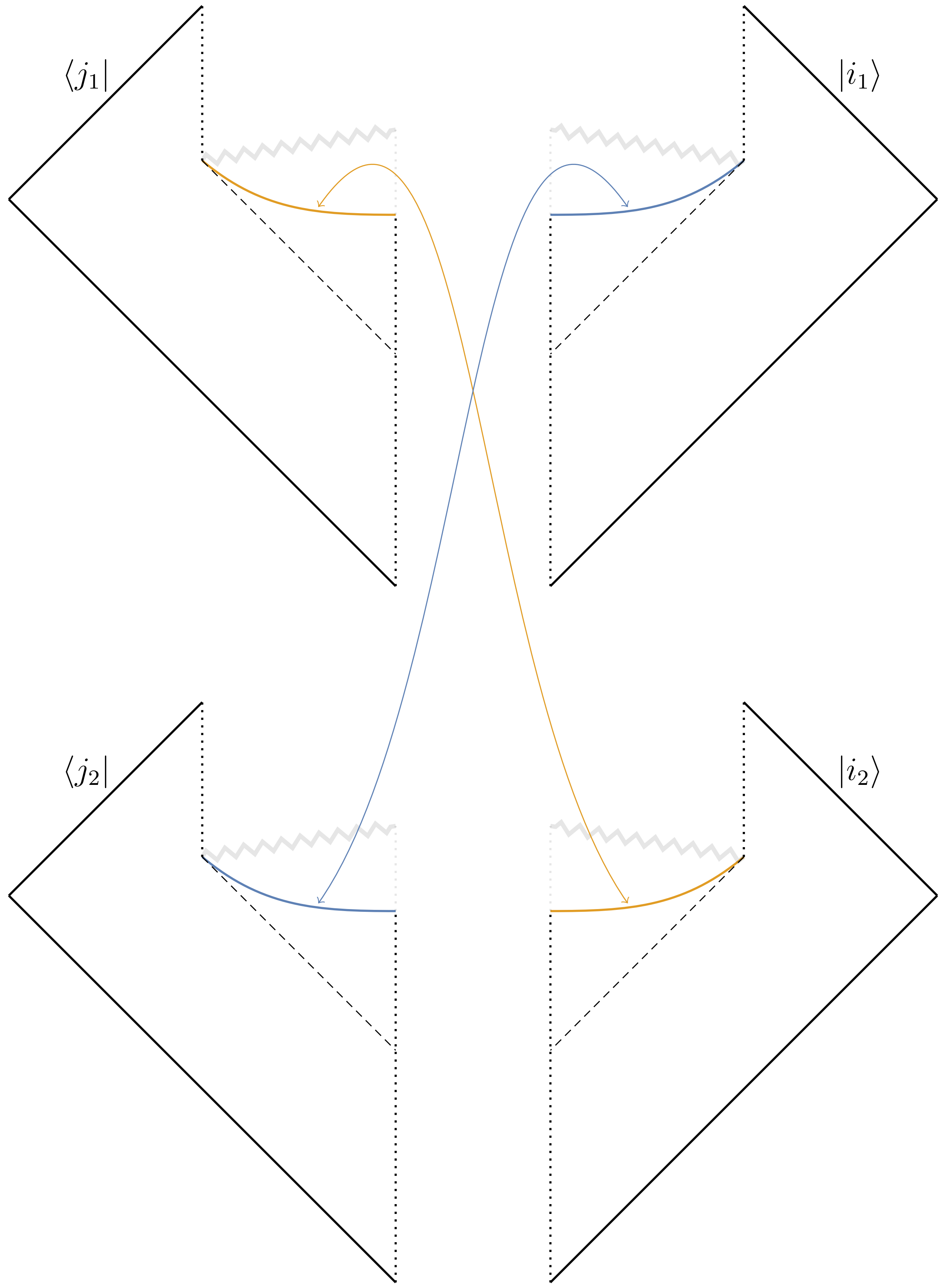

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$$\rho^{(2)} = (1 + \mathscr{S})\, \rho_H \otimes \rho_H$$

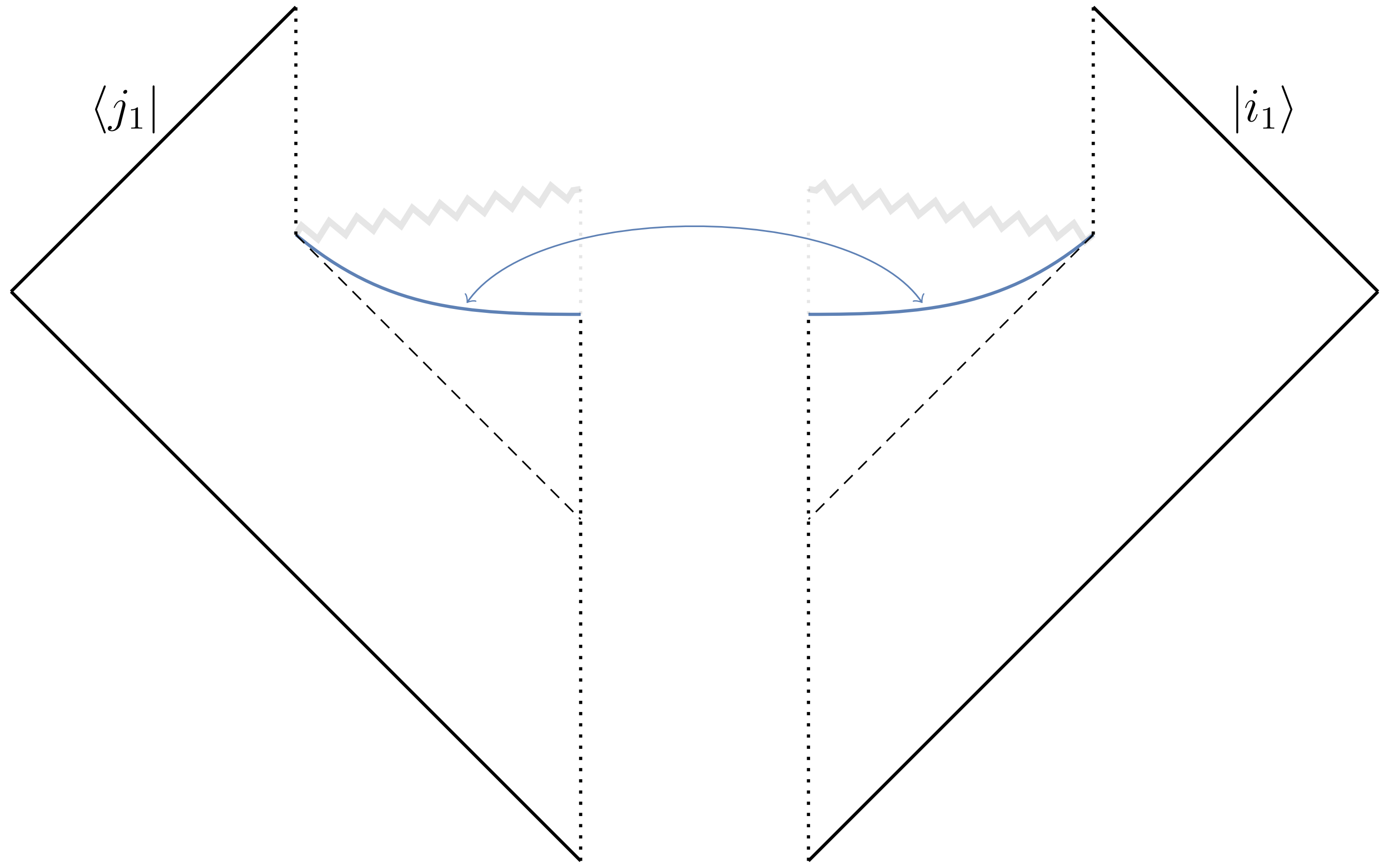
$\rho^{(2)}$ invariant under $\mathscr{S} \implies \left\langle \mathscr{S} \right\rangle = 1 \implies S_2^{\text{swap}} = 0$

Observables match with **pure** state of Hawking radiation

Hilbert space interpretation

Hilbert space interpretation: comes from cutting open path integral

One set of Hawking radiation:

$\langle j_1 | \rho | i_1 \rangle =$ 

"Ket" computes pure state wavefunction on $\mathscr{I}^+$ **and** Σ_{int}:

$$|\psi\rangle = \sum_{i,a} \psi_{ai} |i\rangle_{\mathscr{I}^+} \otimes |a\rangle_{\Sigma_{\text{int}}}$$

Orthonormal basis of states on Σ_{int}

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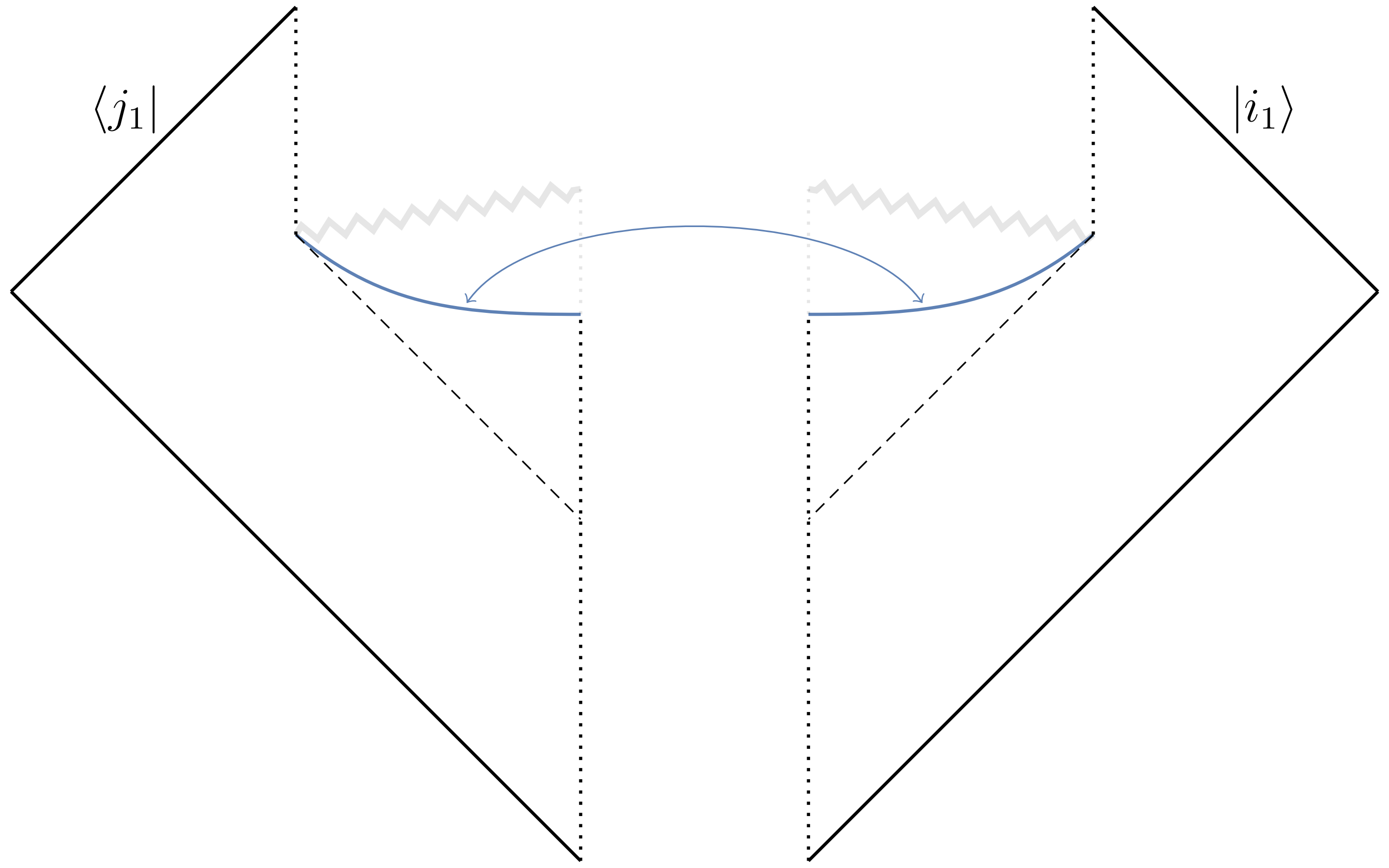

Identification sums over intermediate states on Σ_{int}:

$$\langle j | \rho | i \rangle = \sum_a \bar{\psi}_{aj} \psi_{ai}$$

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$$|\psi\rangle = \sum_{i,a} \psi_{ai} |i\rangle_{\mathscr{I}^+} \otimes |a\rangle_{\Sigma_{\text{int}}}$$

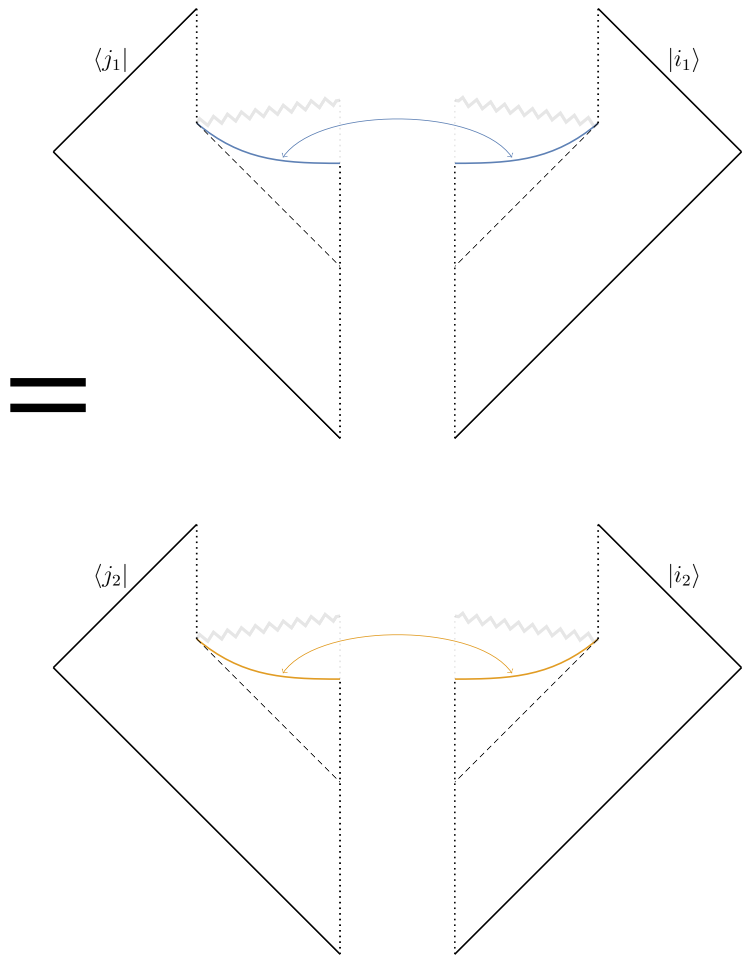
Orthonormal basis of states on Σ_{int}

ρ mixed due to entanglement with "closed universe" Σ_{int}

Wormholes and the BU inner product

Several sets of Hawking radiation:

$\langle j_1, j_2 | \rho^{(2)} | i_1, i_2 \rangle =$



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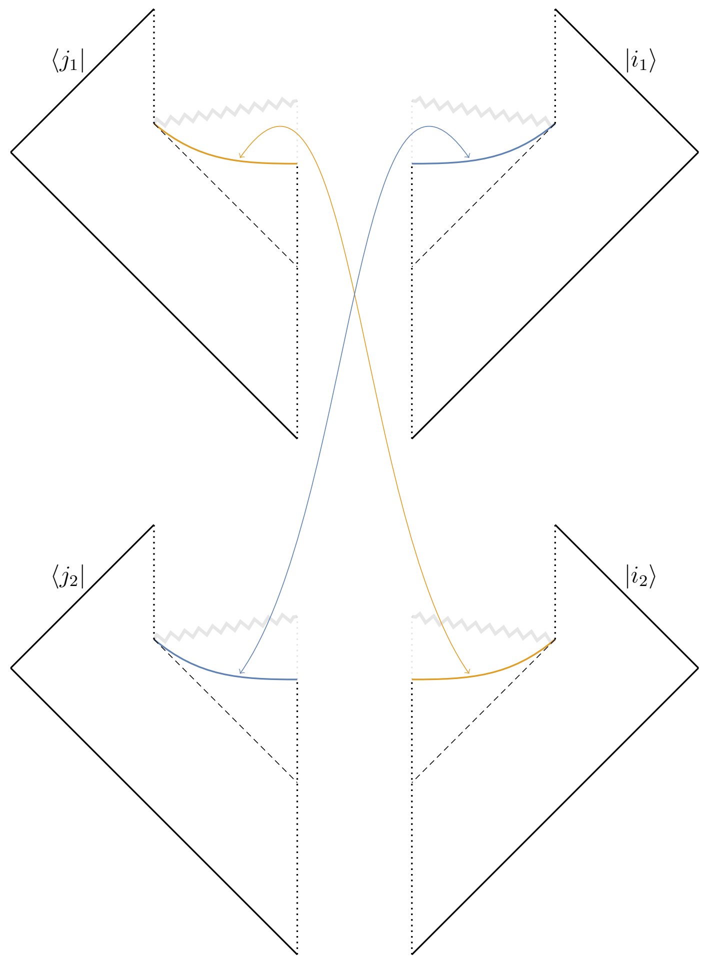

Pure state on 2 copies of $\mathscr{I}^+$ and Σ_{int}:

$$|\psi^{(2)}\rangle = \sum_{\substack{i_1, i_2 \\ a_1, a_2}} \psi_{a_1 i_1} \psi_{a_2 i_2} | i_1, i_2 \rangle_{\mathscr{I}^+} \otimes | a_1, a_2 \rangle_{\text{BU}}$$

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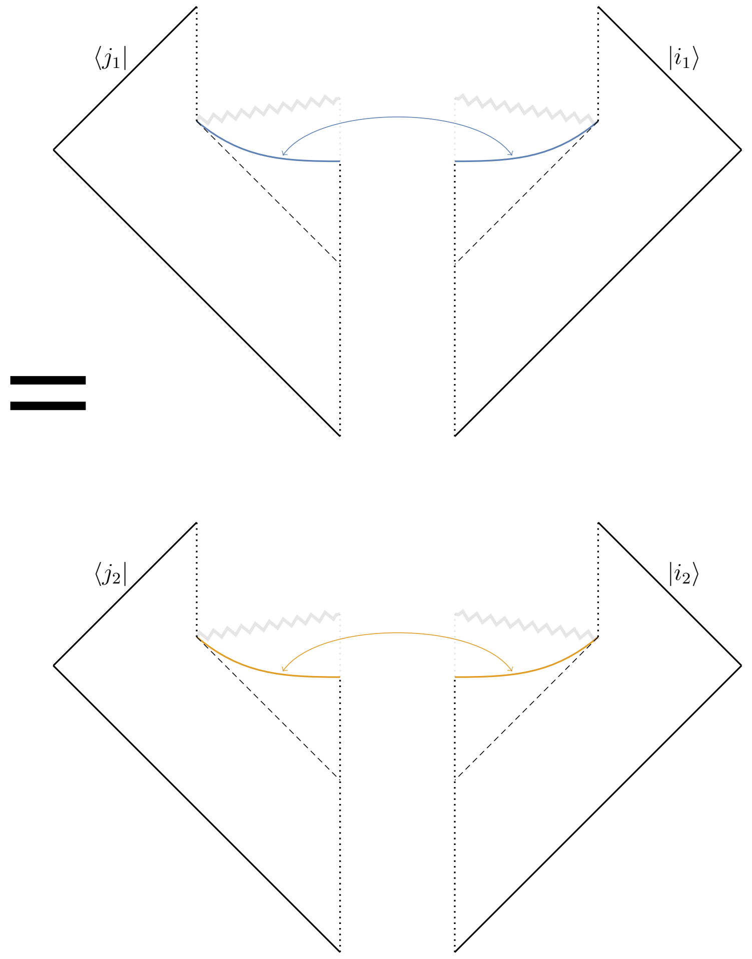

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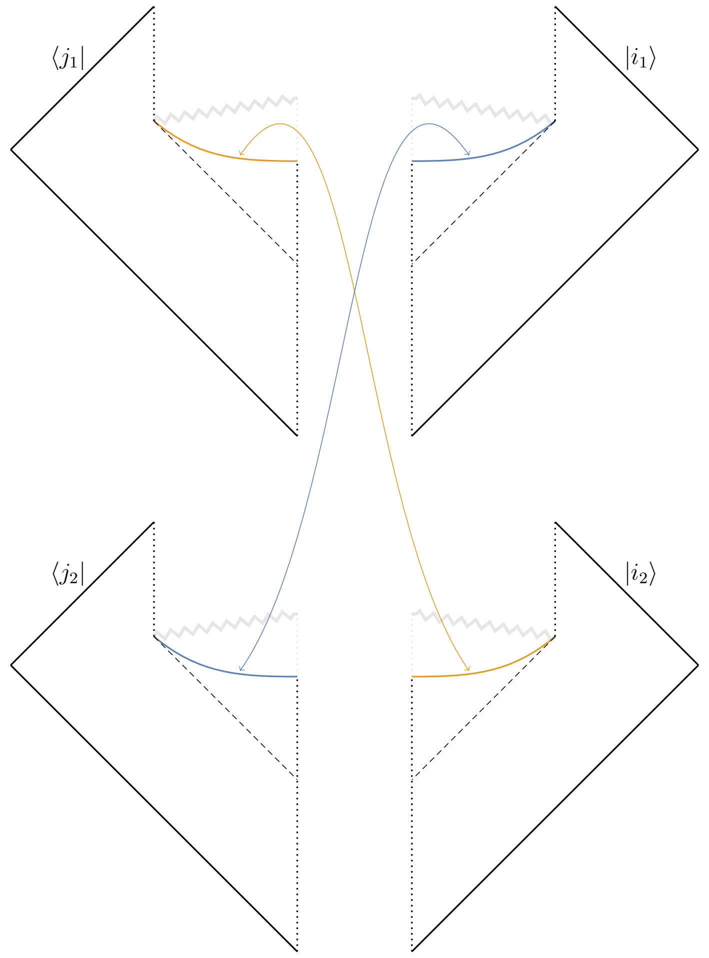

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Inner product on "baby universes" induced by PS wormholes:

$$\langle b_1, b_2 | a_1, a_2 \rangle_{\text{BU}} = \langle b_1 | a_1 \rangle_{\Sigma_{\text{int}}} \langle b_2 | a_2 \rangle_{\Sigma_{\text{int}}} + \langle b_2 | a_1 \rangle_{\Sigma_{\text{int}}} \langle b_1 | a_2 \rangle_{\Sigma_{\text{int}}}$$

Not just factorised inner product on $\mathscr{H}_{\Sigma_{\text{int}}} \otimes \mathscr{H}_{\Sigma_{\text{int}}}$!

Closed universes are indistinguishable bosons

$$\mathscr{H}_{\text{BU}} = \bigoplus_{n=0}^{\infty} \text{Sym}^n \mathscr{H}_{\Sigma_{\text{int}}}$$

Wormholes and the BU inner product

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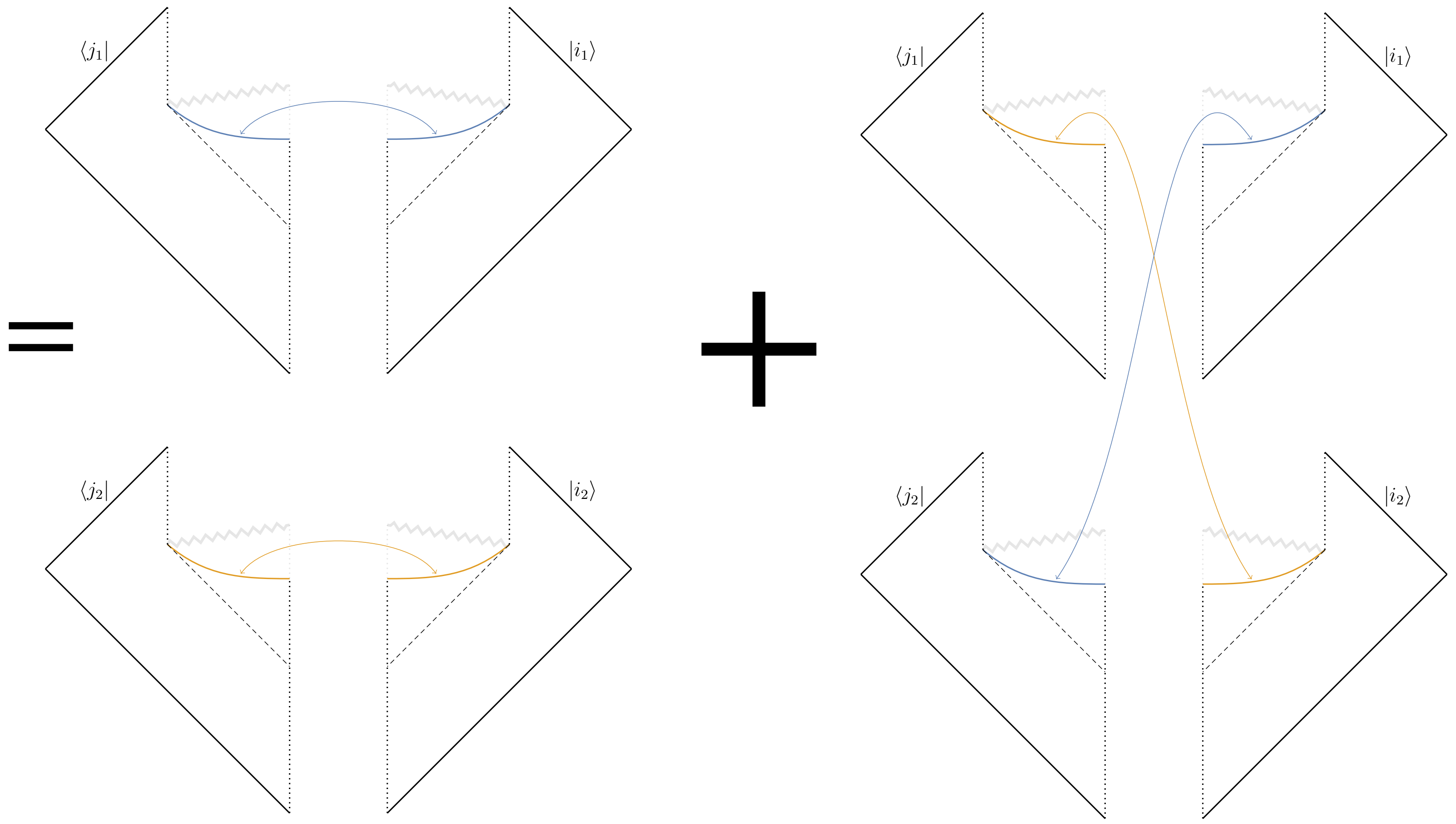
$$\langle j_1, j_2 | \rho^{(2)} | i_1, i_2 \rangle =$$



Pure state on 2 copies of $\mathscr{I}^+$ and Σ_{int}:

$$|\psi^{(2)}\rangle = \sum_{\substack{i_1, i_2 \\ a_1, a_2}} \psi_{a_1 i_1} \psi_{a_2 i_2} | i_1, i_2 \rangle_{\mathscr{I}^+} \otimes | a_1, a_2 \rangle_{\text{BU}}$$

Inner product on “baby universes” induced by PS wormholes:

$$\langle b_1, b_2 | a_1, a_2 \rangle_{\text{BU}} = \langle b_1 | a_1 \rangle_{\Sigma_{\text{int}}} \langle b_2 | a_2 \rangle_{\Sigma_{\text{int}}} + \langle b_2 | a_1 \rangle_{\Sigma_{\text{int}}} \langle b_1 | a_2 \rangle_{\Sigma_{\text{int}}}$$

Not just factorised inner product on $\mathscr{H}_{\Sigma_{\text{int}}} \otimes \mathscr{H}_{\Sigma_{\text{int}}}$!

Closed universes are indistinguishable bosons

$$\mathscr{H}_{\text{BU}} = \bigoplus_{n=0}^{\infty} \text{Sym}^n \mathscr{H}_{\Sigma_{\text{int}}}$$

Wormholes $\longrightarrow$ modified inner product of $\mathscr{H}_{\text{BU}} \longrightarrow \rho^{(2)} \neq \rho \otimes \rho$

Wormholes and the BU inner product

Wormholes $\longrightarrow$ modified inner product of $\mathscr{H}_{\mathrm{BU}} \longrightarrow \rho^{(2)} \neq \rho \otimes \rho$

$$\langle j_1, \ldots, j_n | \rho^{(n)} | i_1, \ldots, i_n \rangle = \sum_{\substack{a_1, \ldots, a_n \\ b_1, \ldots, b_n}} \psi_{a_1 i_1} \bar{\psi}_{b_1 j_1} \cdots \psi_{a_n i_n} \bar{\psi}_{b_n j_n} \langle b_1, \ldots, b_n | a_1, \ldots, a_n \rangle_{\mathrm{BU}}$$

Complicated inner product induced by replica wormholes **(Invariant under permutations)**

Can write any such inner product as $\langle b_1, \ldots, b_n | a_1, \ldots, a_n \rangle_{\mathrm{BU}} = \int d\mu(\alpha) \alpha_{a_1} \cdots \alpha_{a_n} \bar{\alpha}_{b_1} \cdots \bar{\alpha}_{b_n}$

for an appropriate choice of integration measure $d\mu(\alpha)$ for the parameters α_a.

Gives $\langle j_1, \ldots, j_n | \rho^{(n)} | i_1, \ldots, i_n \rangle = \int d\mu(\alpha) \Psi^\alpha_{i_1} \bar{\Psi}^\alpha_{j_1} \cdots \Psi^\alpha_{i_n} \bar{\Psi}^\alpha_{j_n}$ where $\Psi^\alpha_i = \sum_a \psi_{ai} \alpha_a$

$$\Longrightarrow \rho^{(n)} = \int d\mu(\alpha) \big(|\Psi^\alpha\rangle\langle\Psi^\alpha| \big)^{\otimes n}$$

Correlations between copies of Hawking radiation

From replica wormholes, $\rho^{(n)}(u) \neq \left(\rho_{\mathrm{H}}(u)\right)^{\otimes n}$. But correlations have a special form:

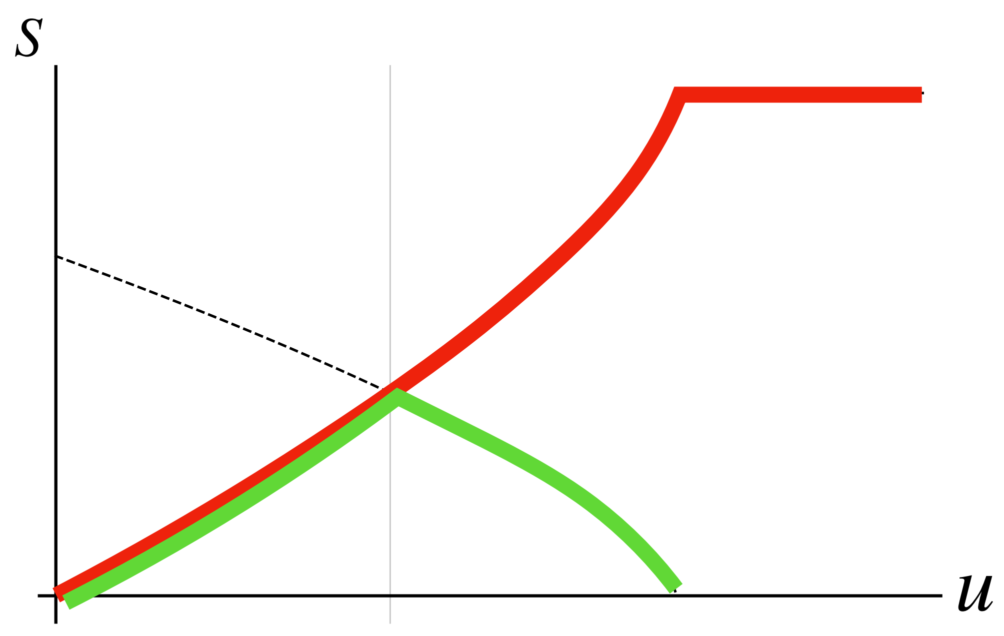
$$\rho^{(n)}(u) = \int d\mu(\alpha) \left(\rho_\alpha(u)\right)^{\otimes n}$$

Classical statistical mixture (ensemble) of possibilities $\rho_\alpha(u)$

One copy of Hawking radiation: $\rho_{\mathrm{H}}(u) = \int d\mu(\alpha)\ \rho_\alpha(u)$

Average over α:
Hawking result

Each fixed α:
Page curve



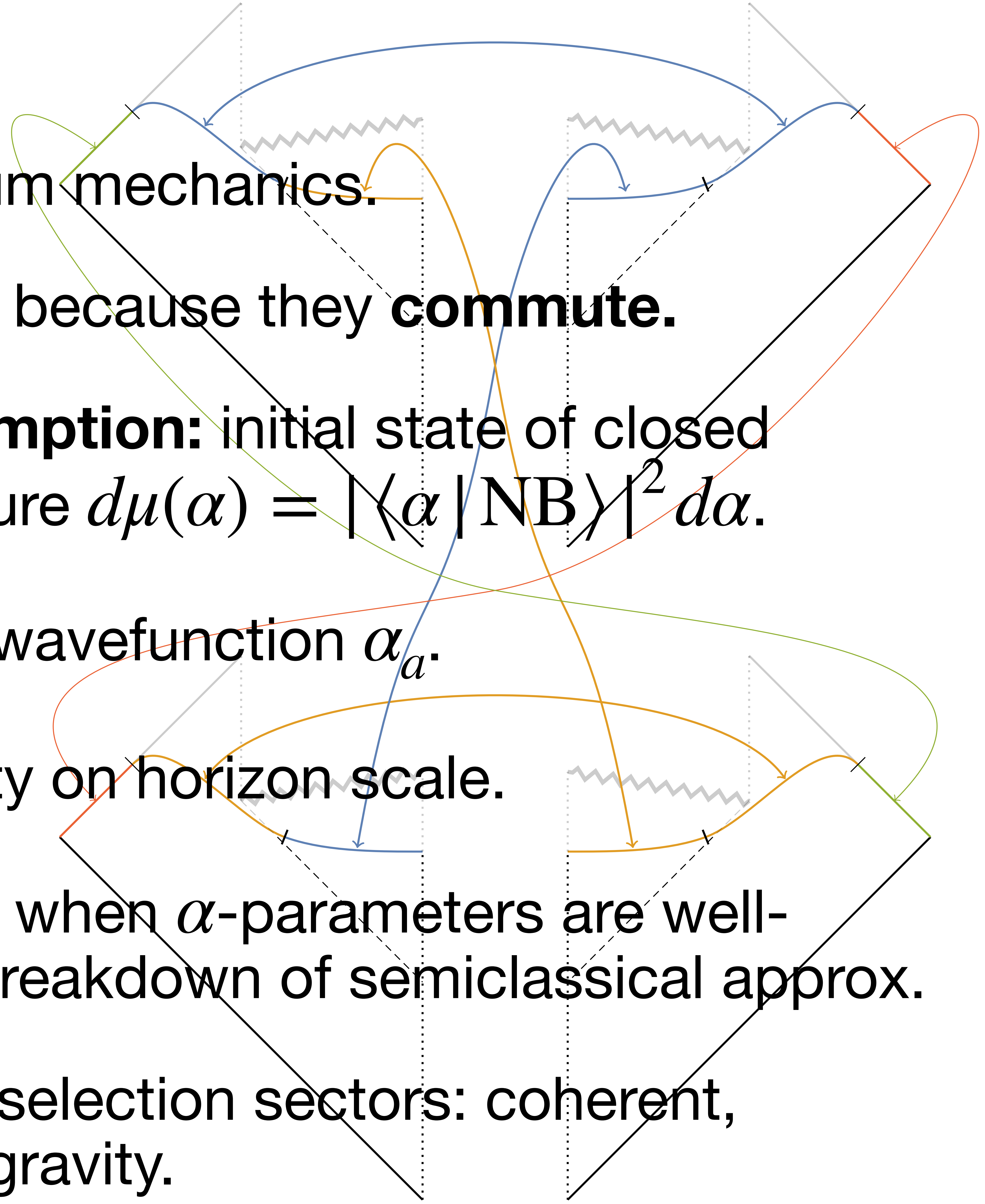
Labels α correspond to a basis of “baby universe” states $|\alpha\rangle \in \mathscr{H}_{\mathrm{BU}}$

Asymptotic observers will “measure” $\alpha \longrightarrow$ **superselection sector**

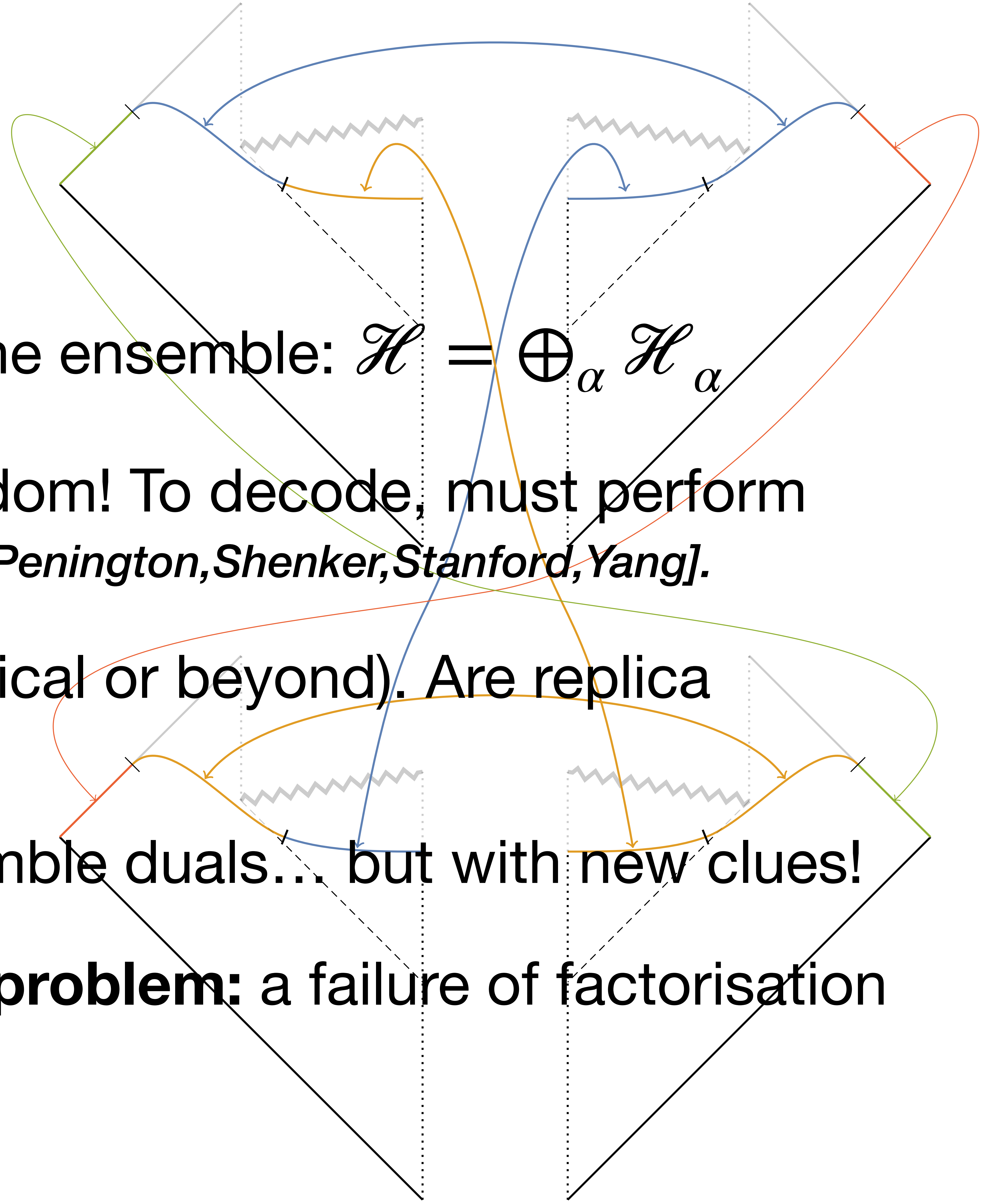
[Coleman][Giddings,Strominger]

Some comments

- This is not a departure from ordinary quantum mechanics.
- Asymptotic observables are **superselected** because they **commute.**
- Hawking's calculation had an **implicit assumption:** initial state of closed universes. No-boundary state $|\mathrm{NB}\rangle$, measure $d\mu(\alpha) = |\langle\alpha|\mathrm{NB}\rangle|^2 d\alpha$.
- α-states resemble **final state**: project onto wavefunction α_a.
- "Integrating out" wormholes $\longrightarrow$ nonlocality on horizon scale.
- Sum over topologies becomes uncontrolled when α-parameters are well-determined: a principled reason to expect breakdown of semiclassical approx.
- Info paradox evaporates **if** we accept superselection sectors: coherent, consistent picture using only semiclassical gravity.



AdS/CFT



- **Ensemble duality:** α-states $\longleftrightarrow$ CFTs in the ensemble: $\mathscr{H} = \bigoplus_\alpha \mathscr{H}_\alpha$
- Quantum code: equivalent to α-state. Random! To decode, must perform many measurements. Compare Petz map ***[Penington,Shenker,Stanford,Yang].***
- To avoid this, need new physics (semiclassical or beyond). Are replica wormholes still "correct"?
- Information problem remains for non-ensemble duals… but with new clues!
- Becomes an instance of the **factorisation problem:** a failure of factorisation between "ket" and "bra" boundaries