To average or not to average

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Based on:
- Alex Belin, JdB, arXiv:2006.05499
- Alex Belin, JdB, Pranyal Nayak, Julian Sonner, to appear

Island Hopping 2020
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Gravity is a low-energy effective field theory.

This would normally imply that gravity has no access to or possesses information about energies $E \gtrsim \Lambda_{\rm UV}$

The existence of a consistent UV completion does however impose constraints on low-energy effective field theory (cf swampland)

A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi '06
Camanho, Edelstein, Maldacena, Zhiboedov '14
+many more

Gravity is also very different from standard low-energy effective field theory.

It knows for example about (cf Daniel Harlow's talk)

- Black hole entropy – the high temperature partition function
- The partition function on various Euclidean manifolds in AdS/CFT
- The page curve

Penington '19
Almheiri, Engelhardt, Marolf, Maxfield '19
Penington, Shenker, Stanford, Yang '19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '19

From a field theory point of view, these seem to all be related to UV data.

Exactly how much information is available in gravity as a low-energy effective field theory may depend on what we mean by the latter:

- Do we view it as a “complete” theory as in JT gravity or in 3d gravity?
- Should we take all semiclassical saddles seriously? In particular, how serious should we take wormhole solutions?
- Are correlators with exponentially long timescales part of the low-energy effective field theory?
- Does it include non-perturbative objects such as D-branes?

Depending on which ingredients we include, we may find different dual descriptions, which may be averaged or not.

The role of saddles in computations in Euclidean low-energy effective field theory:

- Computations which produce exponentially small quantities $\sim e^{-N}$ are naively not to be trusted in EFT
- In many cases the leading saddle persists in the full UV complete theory and yields reliable answers (cf Hawking-Page phase transition in AdS/CFT)
- If there are several saddles with an exponential hierarchy, the subleading saddles do not yield a priori meaningful answers in EFT.
- Subleading saddles may or may not persist as saddles in the full UV theory – could depend on properties of UV completion and whether or not averaging is involved.

To capture all these features (i.e. a dual description of a low-energy effective field theory with gravity), we possibly need a new framework, one that combines precision low-energy data with approximate statistical high-energy data.

This dual could involve an averaging over theories, involve chaos, random matrix theory, causal symmetry breaking… (Julian Sonner's talk, discussion session on Monday..)

In this talk: show that modeling various features of gravitational theories using a combination of ETH and statistics of OPE coefficients goes a long way.

Details will depend on which ingredients to include in the gravitational theories.

The Eigenstate Thermalization Hypothesis (ETH) is a well-known hypothesis which connects precision low-energy data with statistical high-energy data:

$$\langle E_i|O_a|E_j\rangle = \delta_{ij} f_a(\bar{E}) + e^{-S(\bar{E})/2} g_a(\bar{E},\Delta E) R^a_{ij}$$

Deutsch '91
Srednicki '94

$f_a(\bar{E})$: one point functions of simple operators
$g_a(\bar{E},\Delta E)$: two point functions of simple operators
R^a_{ij} : Gaussian random variables

$$\langle R^a_{ij}\rangle = 0, \qquad \langle R^a_{ij} R^b_{kl}\rangle = \delta^{ab}\delta_{il}\delta_{jk}$$

We will restrict to CFTs. Recall that CFTs are completely described in terms of a spectrum of scaling dimensions Δ_i (energies) plus Operator Product Expansion coefficients C_{ijk}

We distinguish between “light operators” with label a,b,… and “heavy operators” with label i,j,k,…

The HHLL four-point correlator is often studied and compared to the two-point function of light fields in a black hole background.

Notice that we can interpret ETH as the statement that C_{aij} is a Gaussian random variable with amplitude

$$\langle E_i|O_a|E_j\rangle = \delta_{ij} f_a(\bar{E}) + \overbrace{e^{-S(\bar{E})/2} g_a(\bar{E}, \Delta E)} R^a_{ij}$$

We propose that this generalizes to other OPE coefficients as well, leading to the OPE Randomness Hypothesis:

$C_{ijk}, C_{ija}, C_{iab}$ are random variables in the heavy Indices with an approximate Gaussian distribution

Intuition is based on the fact that it is very difficult to distinguish high-energy states.

On OPE coefficients:

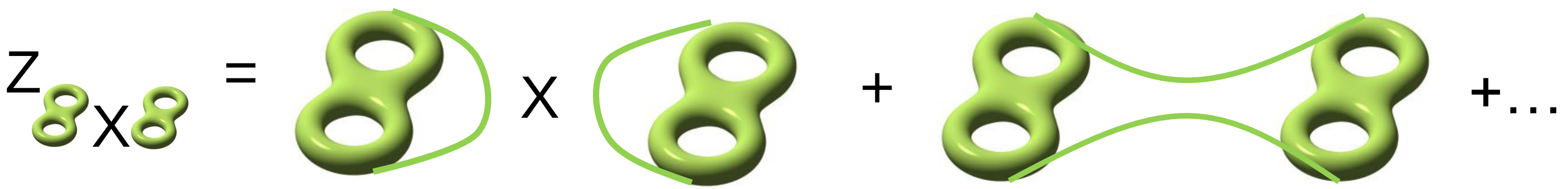
- Distributional properties were first analyzed in Pappadopulo, Rychkov, Espin, Rattazzi ‘12
- In d>2 the diagonal term C_{iia} can be computed from the finite temperature one-point function Gobeil,Maloney,Ng,Wu ’16
- Off-diagonal terms $|C_{ija}|^2 \sim e^{-S}$ were recently obtained from hydrodynamics in d>2 by Delacrétaz ‘20
- In d=2 Collier, Maloney, Maxfield, Tsiares ’19 argued for an extension of ETH based on asymptotics for all cases that involve at least one heavy operator:

$$\overline{C^2_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}}} \approx 16^{-\Delta} e^{-2\pi\sqrt{\frac{c-1}{12}\Delta}} \Delta^{2(\Delta_1+\Delta_2)-\frac{c+1}{4}}, \qquad \Delta \gg c, J, \Delta_i, J_i$$

$$\overline{C^2_{\mathcal{O}_0\mathcal{O}_1\mathcal{O}_2}} \approx e^{-4\pi\sqrt{\frac{c-1}{12}\Delta_1}} \Delta_1^{\Delta_0}, \qquad \Delta_1, \Delta_2 \gg c, J_i, \Delta_0, J_0, |\Delta_1 - \Delta_2|$$

$$\overline{C^2_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}} \approx \left(\frac{27}{16}\right)^{3\Delta_1} e^{-6\pi\sqrt{\frac{c-1}{12}\Delta_1}} \Delta_1^{\frac{5c-11}{36}}, \qquad \Delta_1, \Delta_2, \Delta_3 \gg c, J_i, |\Delta_i - \Delta_j|$$

First case: the genus two wormhole



$$ds^2 = \ell_{\rm ads}^2 (d\tau^2 + \cosh^2 \tau d\Sigma_g^2)$$

Maldacena, Maoz '04

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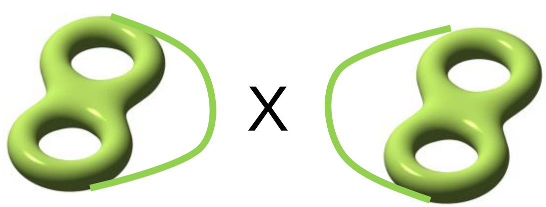
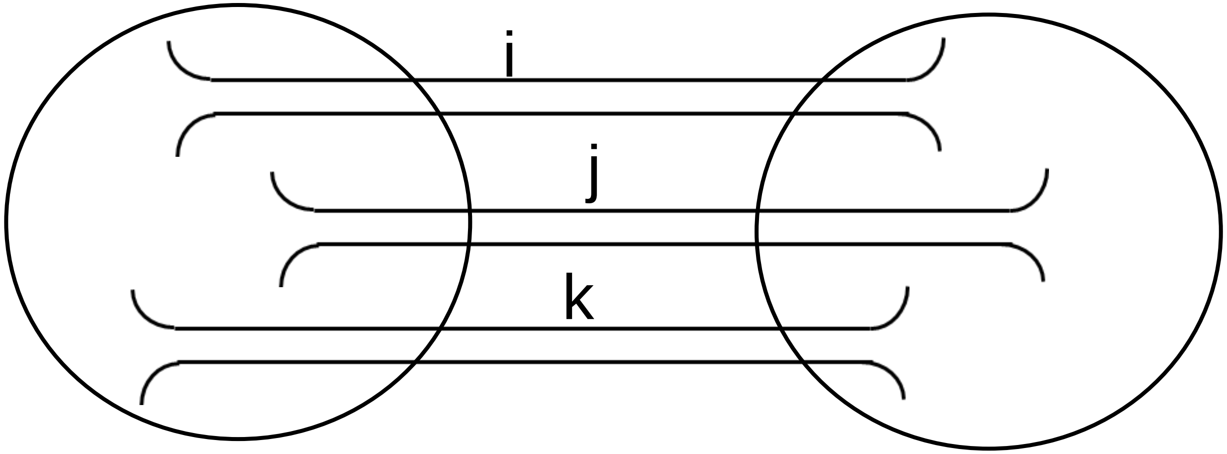


$$\left(1 + b e^{-\frac{c}{2}\frac{\pi^2}{\beta}}\right)$$



CFT genus two partition function

$$Z_{g=2} = \sum_{i,j,k} C_{ijk} C^*_{ijk} q_1^{\Delta_1} q_2^{\Delta_2} q_3^{\Delta_3} \qquad q_i = e^{2\pi i \tau_i}$$

We will study the regime

$$\tau_1 = \tau_2 = \tau_3 = \frac{i\beta}{2\pi}, \qquad \beta \to 0$$

We need an expression for the heavy-heavy-heavy OPE coefficients as random Guassian variables:

$$C_{ijk}\bar{C}_{lmn} = f(\Delta)\mathrm{Sym}_{(ijk),(lmn)}[\delta_{i,l}\delta_{j,m}\delta_{k,n}] + \sqrt{g(\Delta)}S_{ijklmn}$$

From genus two partition function

$$f(\Delta) \simeq \left(\frac{27}{16}\right)^{3\Delta} e^{-3\pi\sqrt{\frac{c}{3}\Delta}}$$

Collier, Maloney, Maxfield, Tsiares '19

From higher genus partition functions; detailed structure of S unclear but

$$\overline{S_{ijklmn}} = 0$$

Compare in ETH:

$$R^a_{ij}R^{b*}_{kl} = \delta^{ab}(\delta_{ik}\delta_{jl}) + hC_a{}^{bc}\left(R^c_{ki}\delta_{jl} + \ldots\right)$$

When inserted in the CFT genus two partition function

$$Z_{g=2} = \sum_{i,j,k} C_{ijk} C^*_{ijk} q_1^{\Delta_1} q_2^{\Delta_2} q_3^{\Delta_3} \qquad q_i = e^{2\pi i \tau_i}$$

Indeed reproduces the genus two partition function

$$Z_{g=2} = \sum_{\Delta} \left(\frac{27}{16}\right)^{-3\Delta} \left(e^{2\pi\sqrt{\frac{c}{3}\Delta}}\right)^3 f(\Delta) e^{-3\beta\Delta} \simeq e^{\frac{c}{4}\frac{\pi^2}{\beta}}$$

descendants

density of states

saddle point

The square of the genus two partition function

$$Z_{g=2\times g=2} = \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta} \right)$$

The square of the genus two partition function

$$Z_{g=2\times g=2} = \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta}\right)\left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta}\right)$$

C: Gaussian variables
Standard Wick contraction

Reproduces $(Z_{g=2})^2$

The square of the genus two partition function

$$Z_{g=2\times g=2} = \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta}\right) \left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta}\right)$$

C: Gaussian variables

Non-Standard Wick contraction

Result:

$$Z^{\text{nonstandard}}_{g=2\times g=2} = \sum_{\Delta} \left(\frac{27}{16}\right)^{-6\Delta} \left(e^{2\pi\sqrt{\frac{c}{3}\Delta}}\right)^3 f(\Delta)^2 e^{-6\beta\Delta}$$

$$\simeq \sum_{\Delta} e^{-6\beta\Delta} = \mathcal{O}(1)$$

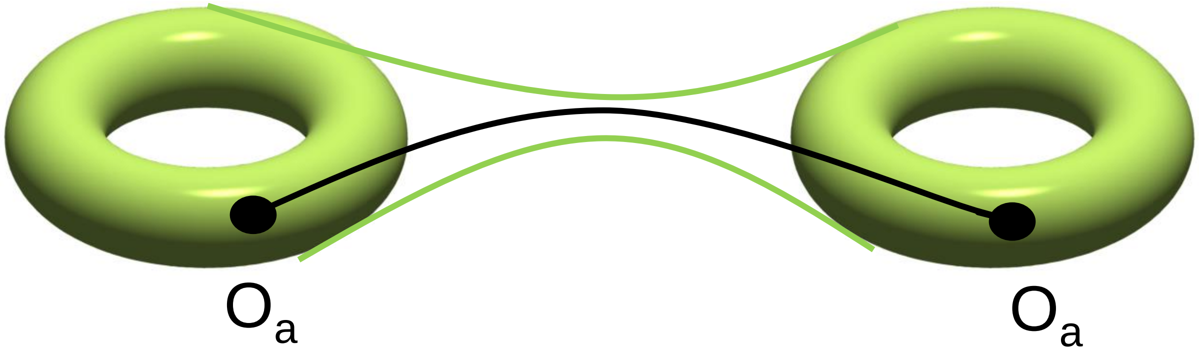
We therefore obtain:

$$Z^{\text{gaussian}}_{g=2\times g=2} \simeq e^{\frac{c}{2}\frac{\pi^2}{\beta}}\left(1 + b' e^{-\frac{c}{2}\frac{\pi^2}{\beta}}\right)$$

The extra O(1) piece makes the answer non-factorizable if moduli dependence is included. Thus the Gaussian approximation gives rise to an apparently non-factorized answer.

So if the genus two wormhole is part of the low-energy effective field theory, this can be captured by assuming OPE coefficients are Gaussian random variables.

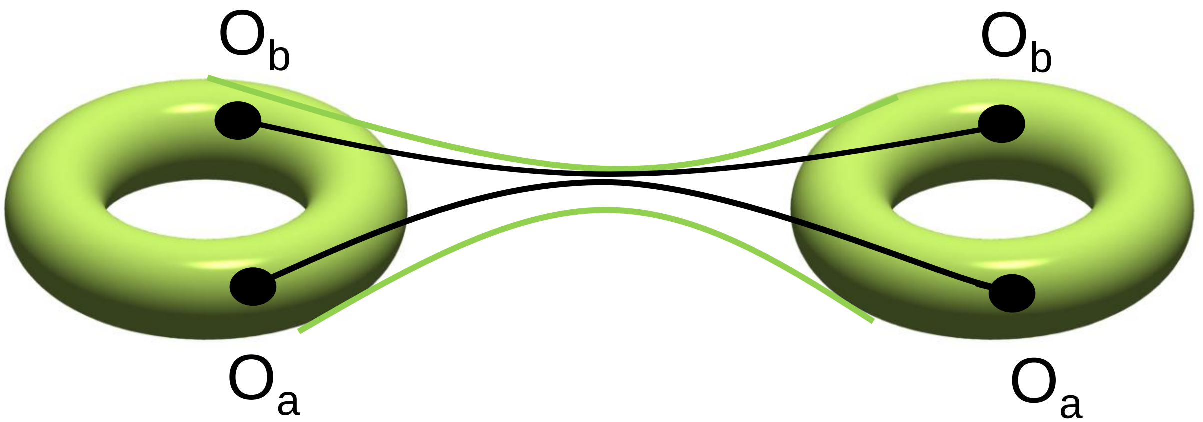
Second example: torus one-point functions



No on-shell gravitational solutions of this type:

$$\langle C_{aii} C_{ajj} \rangle_c = 0$$

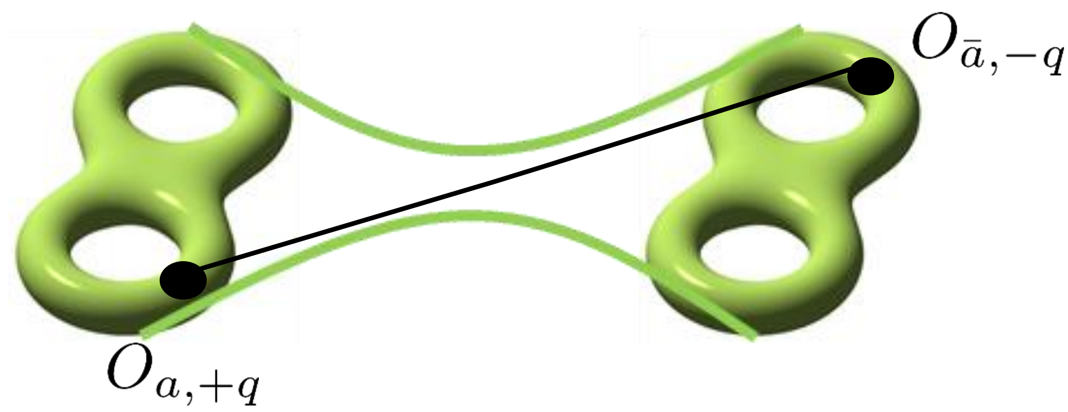
Third example: torus two-point functions



Existence of on-shell gravitational solutions of this type:

$$\langle C_{aij} C_{bji} C_{akl} C_{blk} \rangle_c \neq 0$$

Fourth example: charged correlators in theories with global symmetries Belin, JdB, Nayak, Sonner



Existence of such solutions requires

$$\langle C_{aij}C_{ilk}C_{jkl}C_{\bar{a}i'j'}C_{i'k'l'}C_{j'l'k'}\rangle_c \neq 0$$

But this can only happen if charge conservation does not apply to the “heavy” indices..

To explain these using averaging requires an averaging which breaks the global symmetries.

To reproduce this result, we need a version of ETH which violates charge conservation

$$\langle E_i, q_i | O_{a,Q} | E_j, q_j \rangle = \delta_{ij}\delta_{Q,0} f_a(\bar{E}, \bar{q}) + e^{-S(\bar{E},\bar{q})/2} g_a(\bar{E}, \Delta E, \bar{q}, \Delta q) R^a_{ij}$$

This is different from the more conventional ETH which respects charge conservation

$$\langle E_i, q_i | O_{a,Q} | E_j, q_j \rangle = \delta_{ij}\delta_{Q,0} f_a(\bar{E}, \bar{q}) + e^{-S_{q_i}(\bar{E})} g_a(\bar{E}, \Delta E, q_i, q_j) R^a_{ij} \delta_{q_i, Q+q_j}$$

Which one one should employ depends on e.g. whether in the low-energy theory one can measure the charge of a large black hole exactly.

DISCUSSION

- With OPE coefficients as random variables we can capture many aspects of a gravitational low-energy effective field theory. (cf Pollack, Rozali, Sully, Wakeham '20)

- It is an interesting question what the minimal statistical structure is that we need; is universal wave function statistics sufficient? Belin, JdB, Nayak, Sonner
- Is all we need a version of random matrix theory?

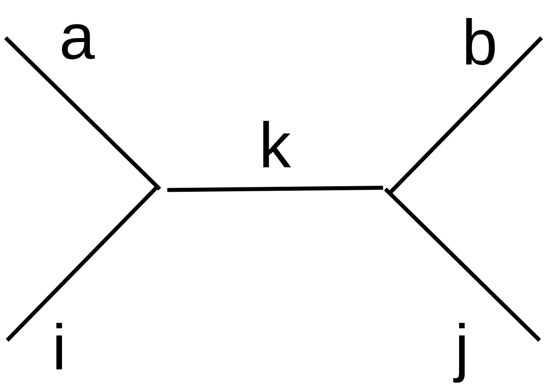
- Is there a notion of averaging which produces the required type of OPE statistics with/without charge?

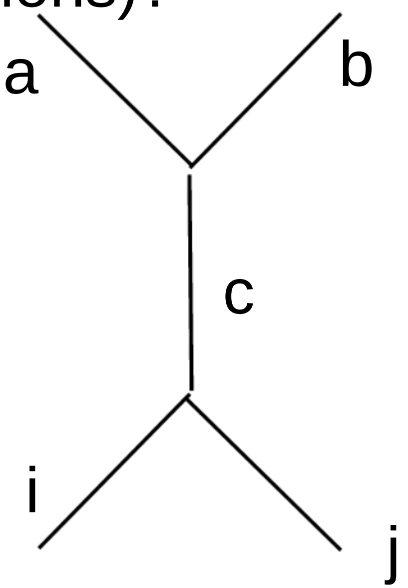
- cf recent work on averaging: Cotler, Jensen; Marolf, Maxfield; Giddings Turiaci; Afkhami-Jeddi, Cohn, Hartman, Tajdini; Maloney, Witten; Stanford – all '20

DISCUSSION

- Are corrections to Gaussianity important? Could be explored by considering moduli dependence, correlation functions on various surfaces, the non-Cardy regime, etc: more details, less universality.

- What are the implications of crossing (is there a statistical solution of the boostrap equations)?





$$R^{a}_{ij} R^{b*}_{kl} = \delta^{ab}(\delta_{ik}\delta_{jl}) + hC_{a}{}^{bc}\left(R^{c}_{ki}\delta_{jl} + \ldots\right)$$

DISCUSSION

- It would be interesting to reobtain the Page curve from this perspective.

- We have focused on saddle points. The could also be off-shell configurations which spoil factorization and contribute, but beyond d=2,3 it seems hard to control such off-shell computations. It is also not clear we should even in principle allow such off-shell configurations in low-energy effective field theory. (cf Cotler, Jensen '20)

DISCUSSION

What is the fate of these wormhole solutions in the UV complete theory?

They could survive and small corrections could restore factorization.

They could also be unstable (eg due to D-branes) and disappear leaving only decoupled solutions.

An interesting analogy is the small D1-D5 black hole, where depending on the duality frame one either sees a small regular black hole, or regular individual microstates, but never both simultaneously (Sen ‘09)