Wormholes, random matrices, and (non-)factorization in d>2

Kristan Jensen (University of Victoria)

Island Hopping

November 19, 2020

Based on work with Jordan Cotler:

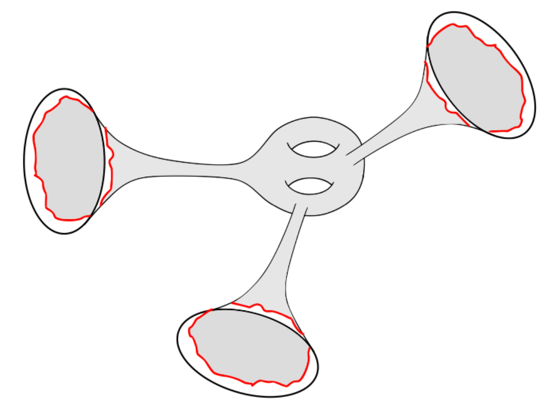
2006.08648, 2007.15653, 2010.02241, 2011.XXXXX

Wormholes and averaging in gravity

Duality between JT and RMT

[Saad, Shenker, Stanford] '19

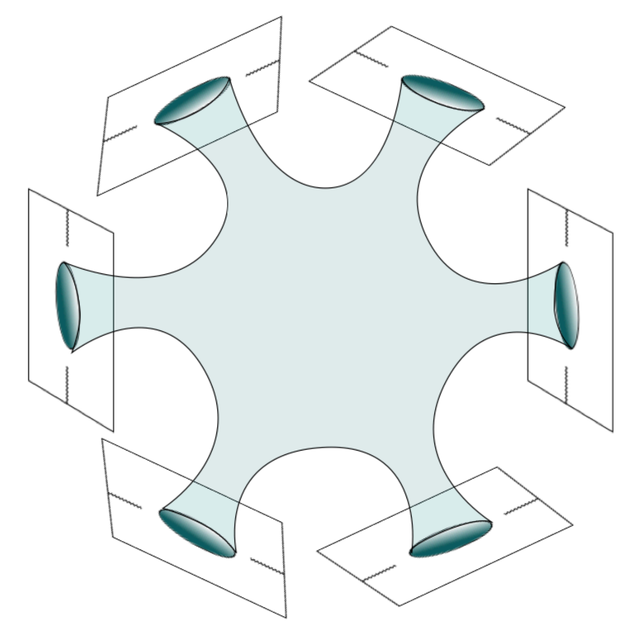
$$Z_{g=2,n=3}(\beta_1, \beta_2, \beta_3) \quad =$$



Replica wormholes & Page curve

[Almheiri, Engelhardt, Marolf, Maxfield] [Penington] '19

West/East coast papers '19



Hilbert space of baby universes, null states

[Marolf, Maxfield] '20

Transition amplitudes in de Sitter JT, bra-ket wormholes

[Maldacena, Turiaci, Yang], [Cotler, KJ, Maloney] '19,

[Anous, Kruthoff, Mahajan], [Chen, Gorbenko, Maldacena] '20

Some questions

1. Whence pure 2+1-dimensional quantum gravity? Is it a consistent theory of gravity, with an ensemble dual?

2. What can we say about wormholes and factorization in standard AdS/CFT? In particular, what about d>2?

3. Non-saddles are important in JT gravity. Can we find the “most important” non-saddles in ordinary Einstein gravity? Embed them into supergravity?

The plan:

1. **Pure 3d gravity and random matrix statistics.**
2. Wormholes in d>2 as constrained instantons.
3. Embedding d>2 wormholes into AdS/CFT.

Pure 3d gravity

Classical model is exactly soluble on disk $\times\, \mathbb{R}$ (global AdS_3).

No bulk gravitons.

Boundary excitations (boundary gravitons) with power-counting renormalizable interactions $\sim \dfrac{G}{L} \sim \dfrac{1}{c}$.

Analogue of Schwarzian description, a boundary model closely related to the quantization of coadjoint orbits of Virasoro.
[Alekseev, Shatashvili] '89, [Cotler, KJ] '18

The status of the quantum theory is unclear.

Torus partition function

Global AdS_3

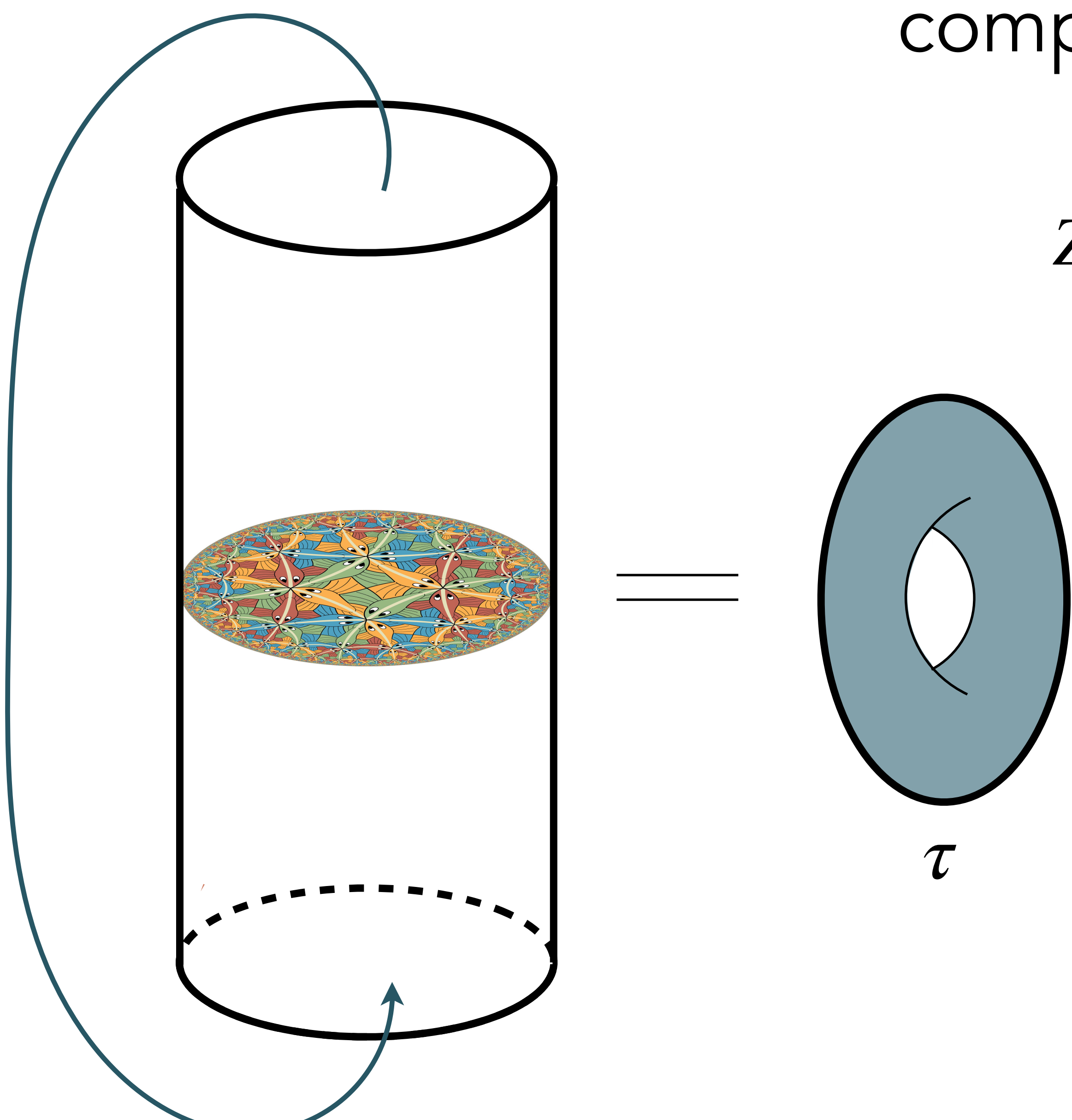
Classical phase space of metrics continuously connected to it is:
[Banados] '99, [Maloney, Witten] '07

$$\text{Diff}(\mathbb{S}^1) / PSL(2;\mathbb{R}) \times \text{Diff}(\mathbb{S}^1) / PSL(2;\mathbb{R})$$

Quantizing this phase space produces a Hilbert space = vacuum Verma module of Virasoro^2. [Witten] '89, [Maloney, Witten] '07

Torus partition function

$$D \times \mathbb{S}^1 = \mathbb{H}^3/\mathbb{Z}$$



Analytically continuing to imaginary time, compactifying, path integral gives trace:

$$Z(\tau, \bar{\tau}) = \mathrm{tr}_{\mathscr{H}}\left(q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{c}{24}}\right) = |\chi_{0,c}(\tau)|^2$$

Sum over choice of contractible cycle on boundary torus:

$$Z_{D\times\mathbb{S}^1}(\tau, \bar{\tau}) = \sum_{\gamma \in PSL(2;\mathbb{Z})/\Gamma_\infty} Z(\gamma\tau, \gamma\bar{\tau})$$

Maloney-Witten partition function.

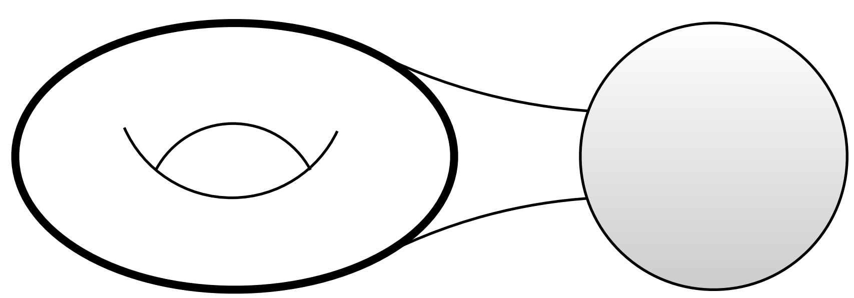
Torus partition function

Modular sum diverges. Regularized result is *not* a partition function for a compact, unitary 2d CFT.

- continuous d.o.s.
- negative norm states (see also [Benjamin, Ooguri, Shao, Wang], '19, [Benjamin, Collier, Maloney] '20).

More importantly, this is an incomplete computation.
We should (?) sum over all ways of filling in the boundary torus.

$$Z(\tau, \bar{\tau}) =$$



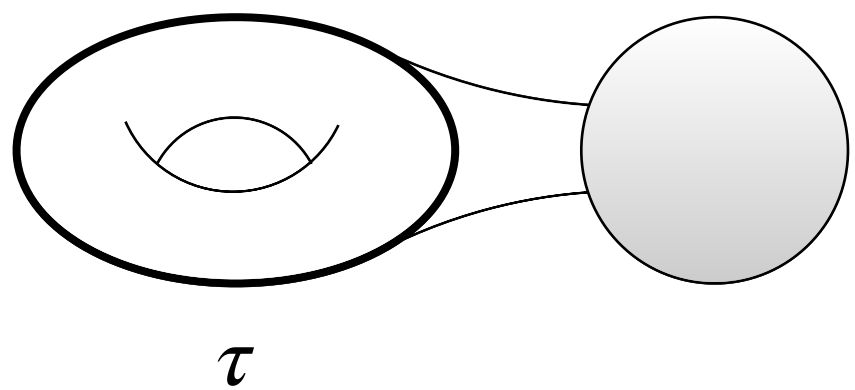
τ

Significant evidence that contributions from Seifert manifolds lead to $\rho \geq 0$ near BTZ threshold.
[Maxfield, Turiaci] '20

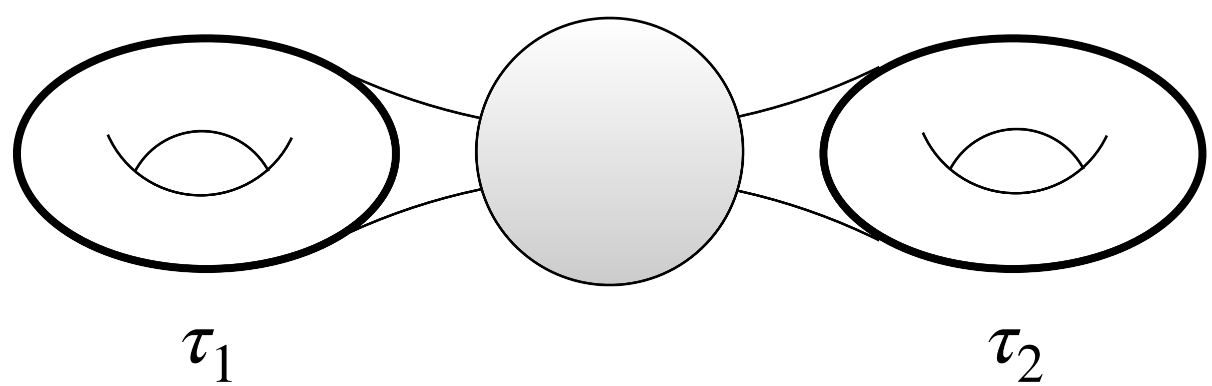
An ensemble dual?

Natural to conjecture that perhaps pure 3d gravity is a consistent theory of gravity after all, with an ensemble dual.*

$$\langle Z(\tau, \bar{\tau}) \rangle \stackrel{?}{=}$$



$$\langle Z(\tau_1, \bar{\tau}_1) Z(\tau_2, \bar{\tau}_2) \rangle \stackrel{?}{=}$$



etc.

*Sweeping under the rug that we don't know of chaotic c>1 CFTs out of which to make an ensemble.

An ensemble dual?

This conjecture has been floating in the air.

- See de Boer's talk, [Cotler, KJ] '20, also [Maxfield, Turiaci] '20
- Cf. Narain ensembles

  [Afkhami-Jeddi, Cohn, Hartman, Tadjini], [Maloney, Witten] '20

Consistent with some known facts:

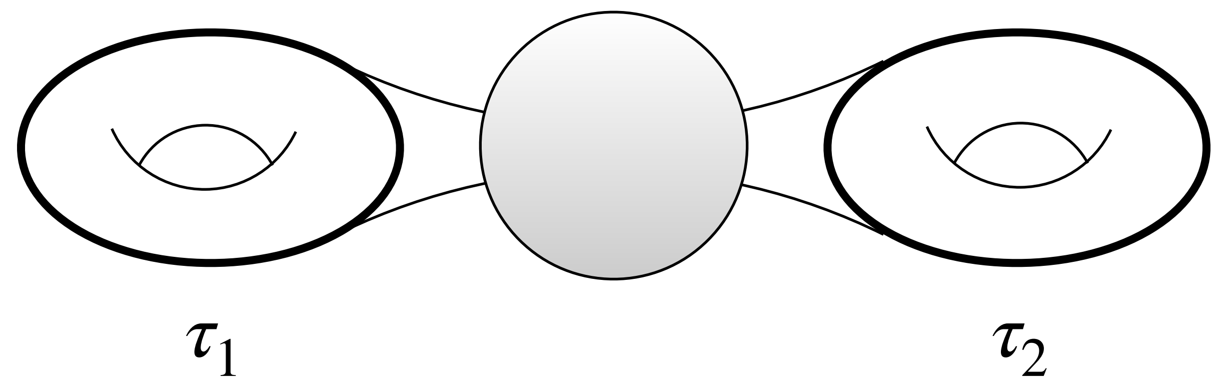
- Continuous d.o.s.
- Some $\mathbb{H}^3/\Gamma$ wormhole saddles with higher genus boundary.

  [Maldacena, Maoz] '04 (One-loop Z given in [Giombi, Maloney, Yin] '08)
- A controlled JT limit [Ghosh, Maxfield, Turiaci] '19

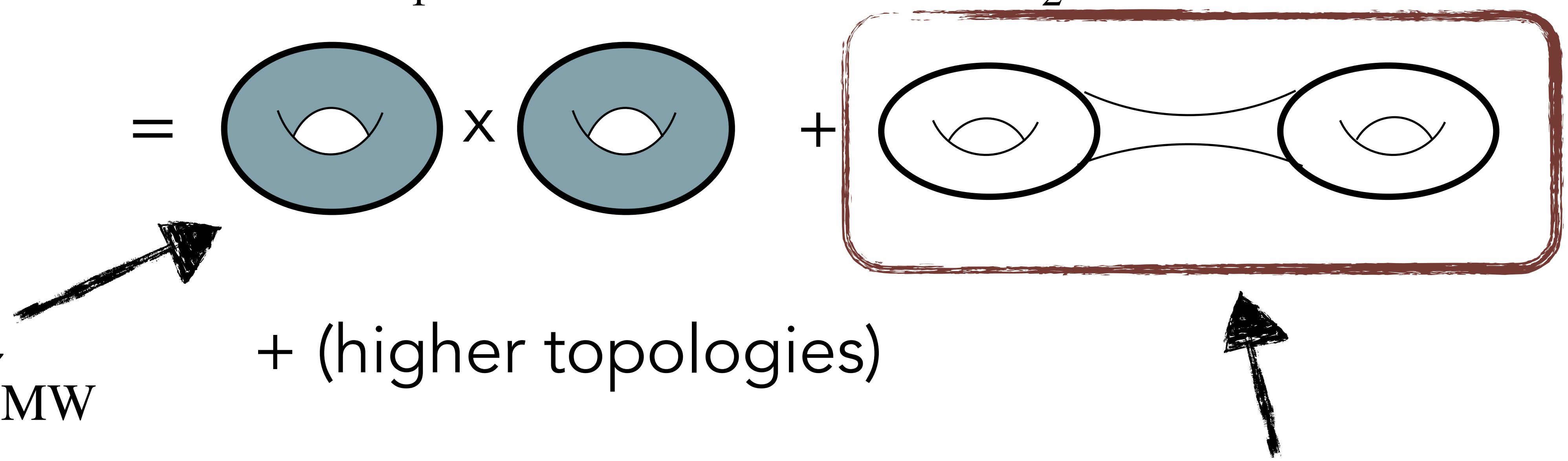
<u>Goal</u>: give a precise computation of a wormhole amplitude,
study fluctuation statistics of putative dual ensemble.

Simplest Euclidean wormhole

$\langle Z(\tau_1, \bar{\tau}_1) Z(\tau_2, \bar{\tau}_2) \rangle =$



$=$



+ (higher topologies)

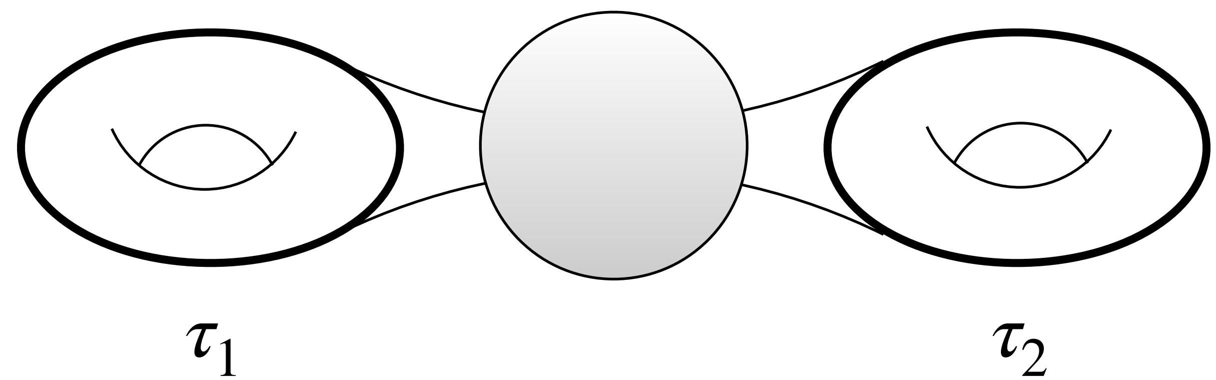
Two copies of Z_{MW}

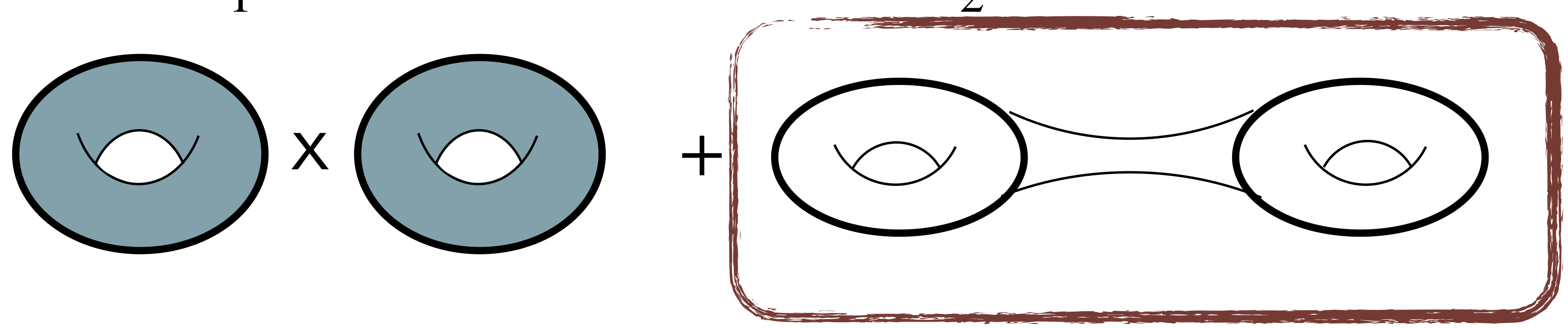
$A \times \mathbb{S}^1 = \mathbb{T}^2 \times I$

[Cotler, KJ] '20

"AdS$_3$ double trumpet"

Simplest Euclidean wormhole

$\langle Z(\tau_1, \bar{\tau}_1) Z(\tau_2, \bar{\tau}_2) \rangle =$ 

$=$ 

+ (higher topologies)

$A \times \mathbb{S}^1 = \mathbb{T}^2 \times I$

[Cotler, KJ] '20

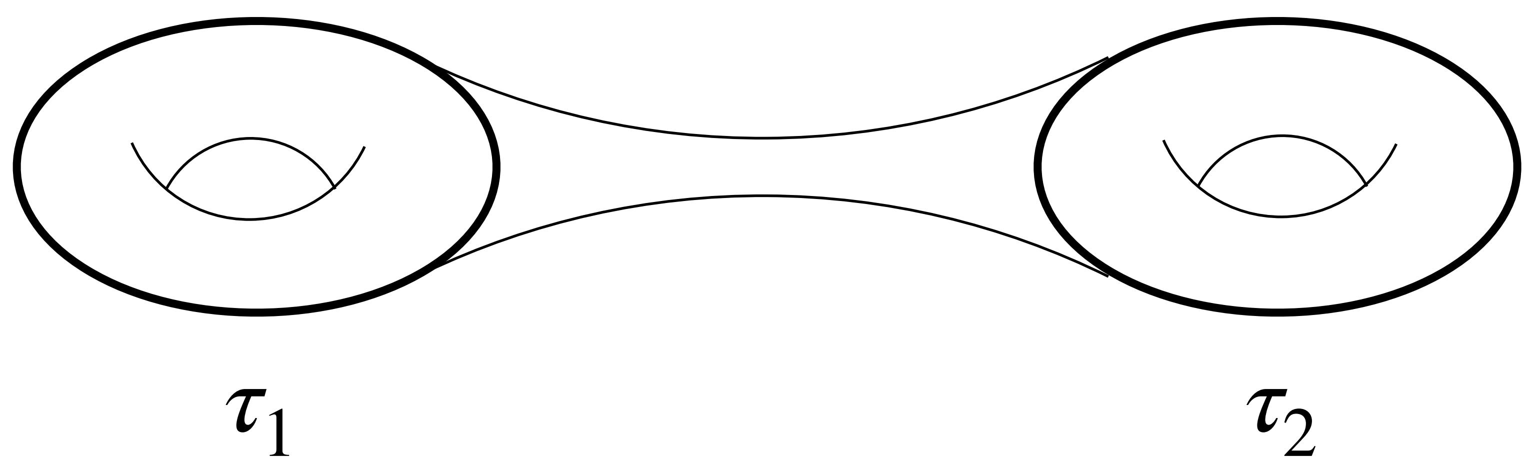
No coupling constant that explicitly suppresses flucs of topology. However, 3d gravity has a JT limit as $\tau \to i\infty$ with $S_0 \propto \sqrt{c\,|\,s\,|}$. [Ghosh, Maxfield, Turiaci] '19

Expect $\langle Z_1 Z_2 \rangle_{\rm conn}$ dominated by $\mathbb{T}^2 \times I$ at low temperature, high spin.

We would like to compute:

$$Z_{\mathbb{T}^2\times I}(\tau_1, \bar{\tau}_1, \tau_2, \bar{\tau}_2) = \int \frac{[dg]_{\mathbb{T}^2\times I}}{\text{diff}} e^{\frac{1}{16\pi G}\int d^3x \sqrt{g}(R+2)}$$

$=$



Immediate problem: no smooth saddle points!

Hamiltonian path integral

In 2006.08648, we took a somewhat indirect approach.

Work in Lorentzian signature, first-order formalism.

(For global AdS_3, this is an easy way to obtain the phase space, which can then be quantized via canonical or path integral methods.)

$$g_{MN} \to (e^A_M, \omega^B{}_{CN}) \qquad\qquad g_{MN} = \eta_{AB} e^A_M e^B_N$$

Gravitational action becomes first order in derivatives:

$$S = -\frac{1}{16\pi G} \int \varepsilon_{ABC} \left(e^A \wedge (d\omega^{BC} + \omega^B{}_D \wedge \omega^{DC}) + \frac{1}{3} e^A \wedge e^B \wedge e^C \right)$$

Hamiltonian path integral

Action is linear in time derivatives!

We are dealing with a Hamiltonian path integral.

Integrate over trajectories in a phase space, rather than configuration space.

$$S \sim \int dt \left(p_i \dot{q}^i - H(p,q) + \lambda_a \mathscr{C}^a(p,q)\right)$$

The time-components e_0^A and $\omega^A{}_{B0}$ enforce curvature, torsion constraints, analogues to Hamiltonian & momentum in ADM.

Our approach

Our computation contains some subtleties and assumptions.

We start in Lorentzian signature, on annulus x time. We “constrain first.” Solving the constraints leads to a phase space, labeled by boundary configurations of bulk fields, a la CS/WZW.

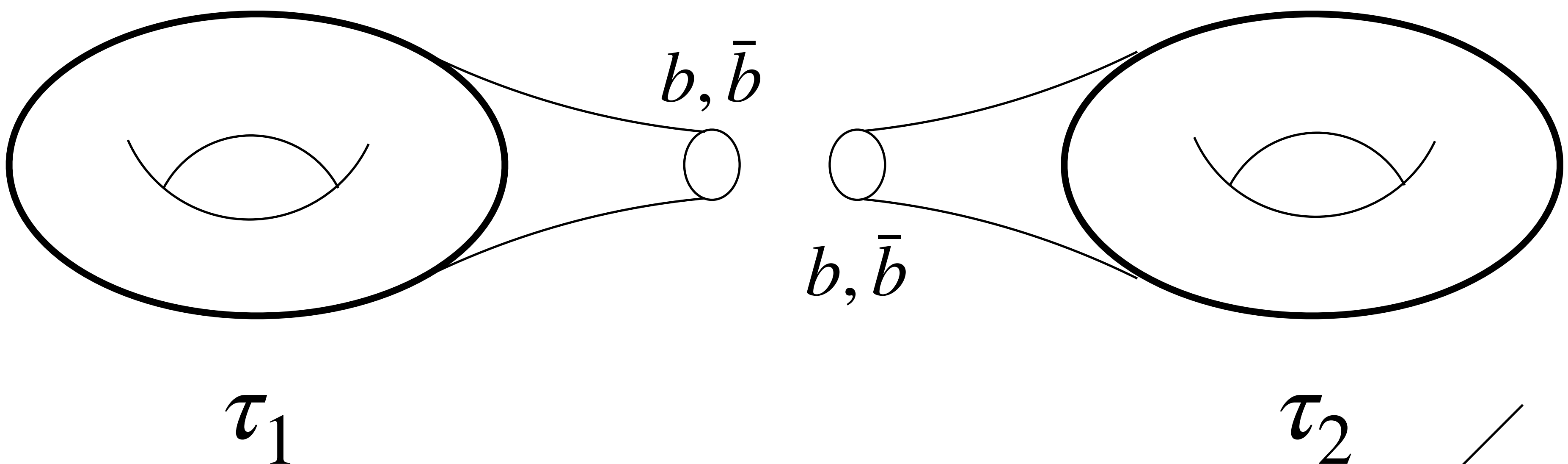
We then perform a path integral quantization of this phase space, and define the Euclidean theory by analytic continuation (rather than starting with Euclidean gravity).

Strictly speaking this is only guaranteed to work semiclassically, as gauge-fixing may modify the constraints, but we have strong reason to believe our answer goes beyond semiclassics, like WZW partition function.

AdS$_3$ double trumpet

At an intermediate point in our computation, our amplitude closely mirrors the "double trumpet" amplitude in JT gravity.

[Saad, Shenker, Stanford] '19



"trumpet"

$$\widetilde{Z} = V_{\varnothing} \int_0^\infty db d\bar{b}\, b\bar{b}\, Z_T(\tau_1, \bar{\tau}_1 \,|\, b, \bar{b})\, Z_T(\tau_2, \bar{\tau}_2 \,|\, b, \bar{b})$$

$$Z_T = \chi_{h,c}(\tau)\chi_{\bar{h},c}(\bar{\tau})$$

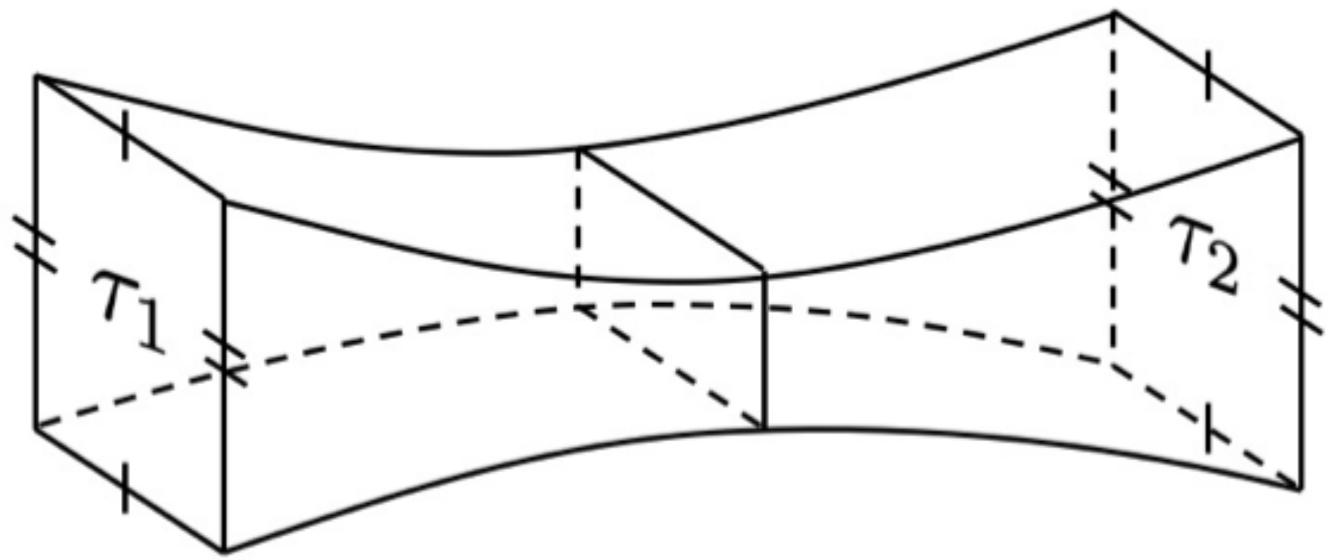
$$h - \frac{c}{24} = \frac{Cb^2 - 1}{24}$$

twist zero modes

pseudomoduli

Amplitude and preamplitude

This Euclidean path integral produces a "preamplitude," the sum over spaces of the sort pictured:



We find

$$\widetilde{Z}(\tau_1, \bar{\tau}_1, \tau_2, \bar{\tau}_2) = \frac{1}{2\pi^2} Z_0(\tau_1, \bar{\tau}_1) Z_0(\tau_2, \bar{\tau}_2) \frac{\text{Im}(\tau_1)\text{Im}(\tau_2)}{|\tau_1 + \tau_2|^2}$$

"Non-compact boson partition function": $Z_0(\tau) = \dfrac{1}{\sqrt{\text{Im}(\tau)}\, |\eta(\tau)|^2}$

Amplitude and preamplitude

This result satisfies an important consistency check.

Invariant under simultaneous modular transformations:

$$\widetilde{Z}(\gamma\tau_1, \gamma\bar{\tau}_2, \gamma^{-1}\tau_2, \gamma^{-1}\bar{\tau}_2) = \widetilde{Z}(\tau_1, \bar{\tau}_1, \tau_2, \bar{\tau}_2)$$

The full answer includes a sum over Dehn twists of one boundary torus relative to the other.

$$Z_{\mathbb{T}^2\times I}(\tau_1, \bar{\tau}_1, \tau_2, \bar{\tau}_2) = \sum_{\gamma\in PSL(2;\mathbb{Z})} \widetilde{Z}(\tau_1, \bar{\tau}_1, \gamma\tau_2, \gamma\bar{\tau}_2)$$

Final answer is invariant under independent modular transformations.

Fluctuation statistics

Recall our provisional assumption:

$$\langle Z(\tau_1, \bar{\tau}_1) Z(\tau_2, \bar{\tau}_2) \rangle_{\text{conn}} = Z_{\mathbb{T}^2 \times I} + \text{(other connected geometries)}$$

Useful to consider the primary-counting partition function Z^P.

$$\langle Z^P(\tau_1, \bar{\tau}_1) Z^P(\tau_2, \bar{\tau}_2) \rangle_{\text{conn}} = \frac{1}{2\pi^2 \sqrt{\text{Im}(\tau_1)\text{Im}(\tau_2)}} \sum_{\gamma \in PSL(2;\mathbb{Z})} \frac{\text{Im}(\tau_1)\text{Im}(\gamma\tau_2)}{|\tau_1 + \gamma\tau_2|^2} + (\dots)$$

Next, Fourier transform in $\text{Re}(\tau_1), \text{Re}(\tau_2)$ to work at fixed spin.

Fluctuation statistics

At low temperature, fixed spin, we find

$$\langle Z^P_{s_1}(\beta_1) Z^P_{s_2}(\beta_2)\rangle_{\text{conn}} = \frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2} e^{-2\pi\left(|s_1| - \frac{1}{12}\right)(\beta_1+\beta_2)} \left(\delta_{s_1,s_2} + O(\beta^{-1})\right)$$

Terms in modular sum with at least one S-transformation.

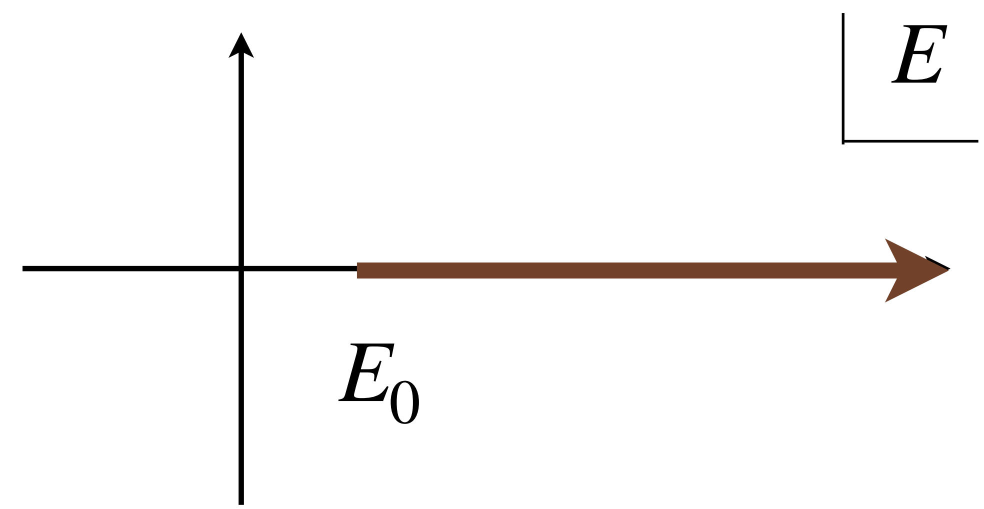
Subleading terms involve sums of Kloosterman sums.

Absolute convergence for nonzero spin. $Z_{0,0} = \ln\Lambda + \bar{Z}_{0,0}(\beta_1,\beta_2)$.

Eigenvalue repulsion in random matrix theory

This function is familiar. It is (after a field redefinition) a two-point function in an ensemble of large Hermitian matrices in the double scaling limit:

$$\langle O(H) \rangle_{\rm MM} = \int dH\, e^{-N \mathrm{tr} V(H)} O(H)\,,$$



$$\langle \mathrm{tr}\left(e^{-\beta_1 H}\right) \mathrm{tr}\left(e^{-\beta_2 H}\right) \rangle_{\rm MM,conn} = \frac{1}{2\pi} \frac{\sqrt{\beta_1 \beta_2}}{\beta_1 + \beta_2} e^{-E_0(\beta_1+\beta_2)} + O(e^{-2S_0})$$

This result is universal, independent of the details of the potential, and steps from eigenvalue repulsion.

Eigenvalue repulsion in random matrix theory

In a random matrix ensemble with a global symmetry, H may be block diagonalized into different superselection sectors, and eigenvalues in different sectors do not repel.

$$\langle \mathrm{tr}\left(e^{-\beta_1 H_{q_1}}\right)\mathrm{tr}\left(e^{-\beta_2 H_{q_2}}\right)\rangle_{\mathrm{MM,conn}} = \frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2}e^{-E_0(\beta_1+\beta_2)}\delta_{q_1q_2} + O(e^{-2S_0})$$

Precisely matches what we find from AdS_3, with spin the conserved quantum number.

Further, the offset $E_0 = 2\pi\left(|s| - \frac{1}{12}\right)$ is the threshold energy of a BTZ black hole with spin s.

Some lessons

We made sense of the wormhole amplitude. If pure 3d gravity is a consistent theory of gravity, this is significant evidence that it is dual to an ensemble.

Fluctuation statistics of BTZ microstates near threshold are precisely described by RMT with Virasoro symmetry!*
The full answer appears to generalize RMT.

In particular, BTZ microstates corresponding to Virasoro primaries at fixed spin exhibit *eigenvalue repulsion*.

*In fact, the entire amplitude may be obtained by taking an RMT ansatz for $\langle Z^P_{s_1}(\beta_1) Z^P_{s_2}(\beta_2)\rangle_{\text{conn}}$, inverse Fourier transform, restore descendants, and then take a modular sum.

Another approach

I don't have the time to discuss it, but I did want to mention that, given my comments about "constrain first," in 2007.15653 we found another way to arrive at $Z_{\mathbb{T}^2 \times I}$.

The idea was to use some physical inputs from 3d gravity to then "bootstrap" the wormhole amplitude.

Most important is that there is a preamplitude $\widetilde{Z}$ invariant under simultaneous modular transformations.

We recovered the result above for $\widetilde{Z}$, using the JT limit of 3d gravity to fix its normalization.

Going forward

So, what about more general manifolds? Is pure 3d gravity a consistent theory?

Answering that question is, to put it mildly, a daunting task.

For torus boundaries, the JT limit should help. Indeed, the "AdS_3 double trumpet" may be reconstructed purely from the JT double trumpet, but it is not yet clear if this should persist beyond $\mathbb{T}^2 \times I$.

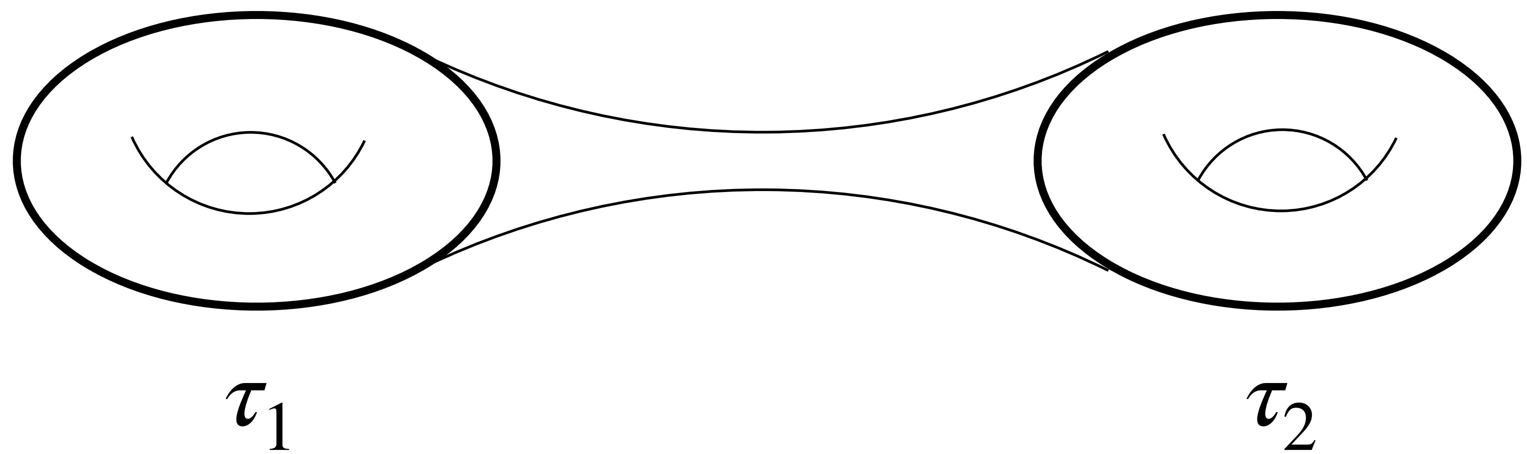
What to make of wormholes with sphere, g>1 boundary? Requires metric approach.

The plan:

1. Pure 3d gravity and random matrix statistics.
2. **Wormholes in d>2 as constrained instantons.**
3. Embedding d>2 wormholes into AdS/CFT.

We would like to compute:

$$Z_{\mathbb{T}^2\times I}(\tau_1, \bar{\tau}_1, \tau_2, \bar{\tau}_2) = \int \frac{[dg]_{\mathbb{T}^2\times I}}{\text{diff}} e^{\frac{1}{16\pi G}\int d^3x \sqrt{g}(R+2)}$$

$=$ 

<u>Hope</u>: develop a framework for computing this amplitude in the second-order formalism, at least to one-loop.

If we can, we can then try to go to d>3, embed into AdS/CFT..

Reminder: non-saddles in JT

Problem: No saddle point solutions.

A thread to follow: In Euclidean JT gravity, $\mathbb{H}^2$ is the only saddle.

Dilaton (after continuing its contour) enforces constraint $R = -2$.

There is a moduli space of $R = -2$ surfaces, on which the JT action varies. Path integral given by an integral over moduli and pseudomoduli. [Saad, Shenker, Stanford] '19

$$Z_{g,n}(\beta_1, \ldots, \beta_n) \propto \int_0^\infty b_1 db_1 \ldots b_n db_n \, V_{g,n}(b_i) \frac{e^{-\frac{b_1^2}{\beta_1}}}{\sqrt{\beta_1}} \ldots \frac{e^{-\frac{b_n^2}{\beta_n}}}{\sqrt{\beta_n}}$$

Reminder: non-saddles in JT

$$Z_{g,n}(\beta_1,..,\beta_n) \propto \int_0^\infty b_1 db_1 .. b_n db_n \, V_{g,n}(b_i) \frac{e^{-\frac{b_1^2}{\beta_1}}}{\sqrt{\beta_1}} \dots \frac{e^{-\frac{b_n^2}{\beta_n}}}{\sqrt{\beta_n}}$$

1. Formal saddle point at $b_i = 0$, where bottlenecks pinch off. Dominant contributions near there.

2. There are configurations at fixed b_i which are saddles wrt all other fluctuations in field space. Z obtained by summing over fluctuations at fixed b_i, then integrate over b_i.

Fixing the b_i amounts to fixing the energy on each boundary. The ensuing spacetime is a "constrained instanton," not a true saddle. [Affleck], '81 [Affleck, Dine, Seiberg] '83

Gravitational constrained instantons

Let's adapt this procedure to d>2 Einstein gravity.

Say, look for wormhole with topology $\mathbb{T}^d \times \mathbb{R}$

$$ds^2 = g_{\rho\rho}(\rho)d\rho^2 + g_{ij}(\rho)dx^i dx^j$$

Reduce on torus to one-dimensional problem in ρ.

Can look for saddle points where we: (c.f. [Stanford] '20)

1. Fix the length between the two boundaries.

2. Fix the energy of the boundary T^{ij}.

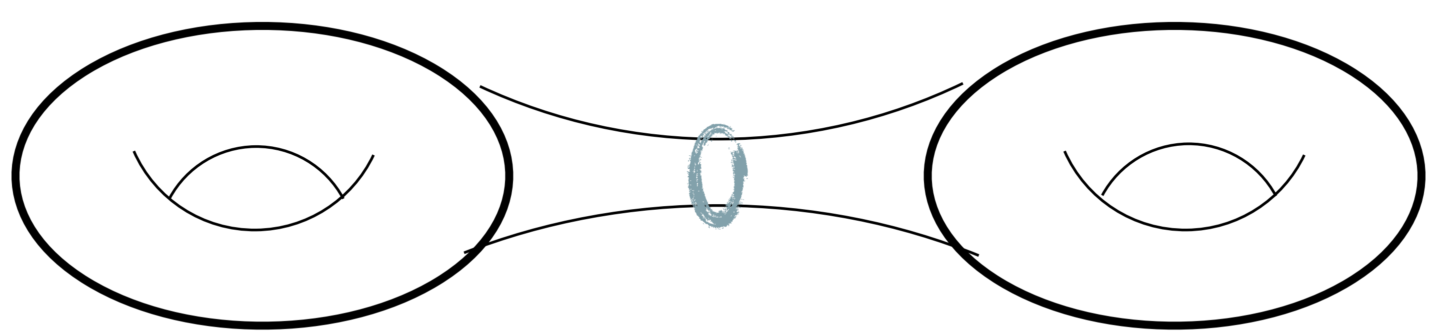
Gravitational constrained instantons

For example:

$$ds^2 = d\rho^2 + b^2 \cosh^{\frac{4}{d}}\left(\frac{d\rho}{2}\right)\delta_{ij}dy^i dy^j \qquad (y^i = x^i + f^i(\rho))$$

Interpolates between tori of equal conformal structure.

Twist zero modes, plus pseudomodulus b. (Bottleneck $\sim b^d$.)



Gravitational constrained instantons

$$ds^2 = d\rho^2 + b^2 \cosh^{\frac{4}{d}}\left(\frac{d\rho}{2}\right)\delta_{ij}dy^i dy^j$$

However we got here, these spaces have interesting properties:

1. EH action evaluates to a pure boundary term, $S_{\text{EH}} \propto \frac{b^d}{G}\text{vol}(\mathbb{T}^d)$.

2. Fluctuations of the metric with fixed BC can be divided into flucs of b, twists (zero modes), everything else $\left(\frac{\delta^2 S_{\text{EH}}}{\delta h^2} \geq 0\right)$.

   So these appear to be important contributions to the connected two-boundary problem.

Gravitational constrained instantons

What about the integral over b?

$$Z = \int db\, \rho(b) e^{-S_{\text{wormhole}}} \mathscr{D}_b^{-1/2} \left(1 + GL_2(b) + G^2 L_3(b) + ..\right)$$

How does one-loop determinant behave as $b \to 0$?

Presumably stringy effects are important in the loop corrections around small wormholes.

What we can in principle reliably obtain is the exponentially suppressed contribution from wormholes with AdS-scale bottlenecks, with $S_{\text{wormhole}} = O(1/G)$.

Other instantons

We can find many other configurations of this sort, e.g.

1. Wormholes in 3d which interpolate between tori with τ_1, τ_2, as well as between two spheres.

2. Wormholes with $\mathbb{S}^1_\beta \times \mathbb{T}^{d-1}$ cross-section, which interpolate between boundaries at general β_1, β_2.
   (Ends up being analytic continuation of "double cone" [Saad, Shenker, Stanford] '18)

3. 5d wormholes with $\mathbb{S}^1_\beta \times \mathbb{S}^3$ cross-section.

Other instantons

In cases with $\mathbb{S}^1_\beta \times \mathbb{T}^{d-1}$, $\mathbb{S}^1_\beta \times \mathbb{S}^3$ cross-sections, we have:

$$S_{\text{wormhole}} = (\beta_1 + \beta_2)E\,, \qquad E \sim \frac{b^d}{G}V$$

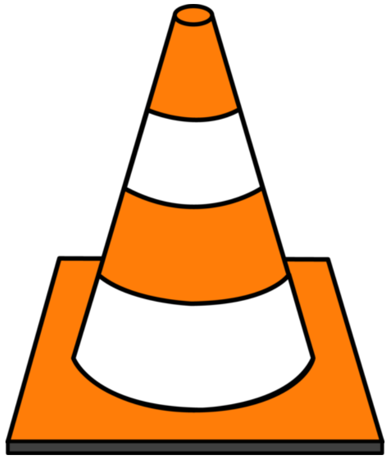
In a natural renormalization scheme, the $\mathbb{S}^1 \times \mathbb{S}^3$ wormhole has $E = b^4 E_0$, with $b \geq 1$ and E_0 the small BH mass threshold.

Recalls how our AdS_3 wormhole "knows" about BTZ threshold.

The plan:

1. Pure 3d gravity and random matrix statistics.
2. Wormholes in d>2 as constrained instantons.
3. **Embedding d>2 wormholes into AdS/CFT.**

Work in progress with Cotler.



Embedding into supergravity

The metrics presented above are rather simple configurations which are asymptotically $\mathbb{H}^{d+1}$ with various boundary geometries (torus, sphere, $\mathbb{S}^1 \times \mathbb{S}^{d-1}$, etc.)

In particular they are not supported by gauge fields or matter.

So it is rather easy to embed them into Freund-Rubin compactifications of 10d or 11d supergravity with an AdS_{d+1} factor.

One simply adds the internal space, constant dilaton (in 10d), and a profile for RR flux* fixed by Gauss' Law.

*AdS_3 compactifications from 10d may be supported by general NS/RR 3-form fluxes

Example: wormholes in $\mathbb{H}_5 \times \mathbb{S}^5$

$$ds^2 = d\rho^2 + b^2 \cosh\left(\frac{2\rho}{L}\right) \delta_{ij} dx^i dx^j + L^2 d\Omega_5^2\,,$$

$$\varphi = g_s$$

$$C_4 = \frac{Lb^4}{8}\left(\frac{3}{L} + \sinh\left(\frac{4\rho}{L}\right) + \frac{1}{8}\sinh\left(\frac{8\rho}{L}\right)\right) dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + \text{(angular)}$$

The wormhole is threaded by RR flux!

Other simple examples with AdS_3, AdS_4, and AdS_7 factors.

Stability?

We should now assess whether these wormholes are stable configurations in supergravity.

Q: Is there a sharp factorization paradox?

Perturbative fluctuations of the 10d, 11d supergravity fields?

For $AdS_5 \times \mathbb{S}^5$, the 5d spectrum includes a tower of minimally coupled scalars with $m^2 \geq -4$, fermions, 5d gauge fields, and of course the metric.

(Non-perturbative) Brane/string nucleation?

Supergravity fluctuations

This story is still developing, and is work in progress.

What we can show:

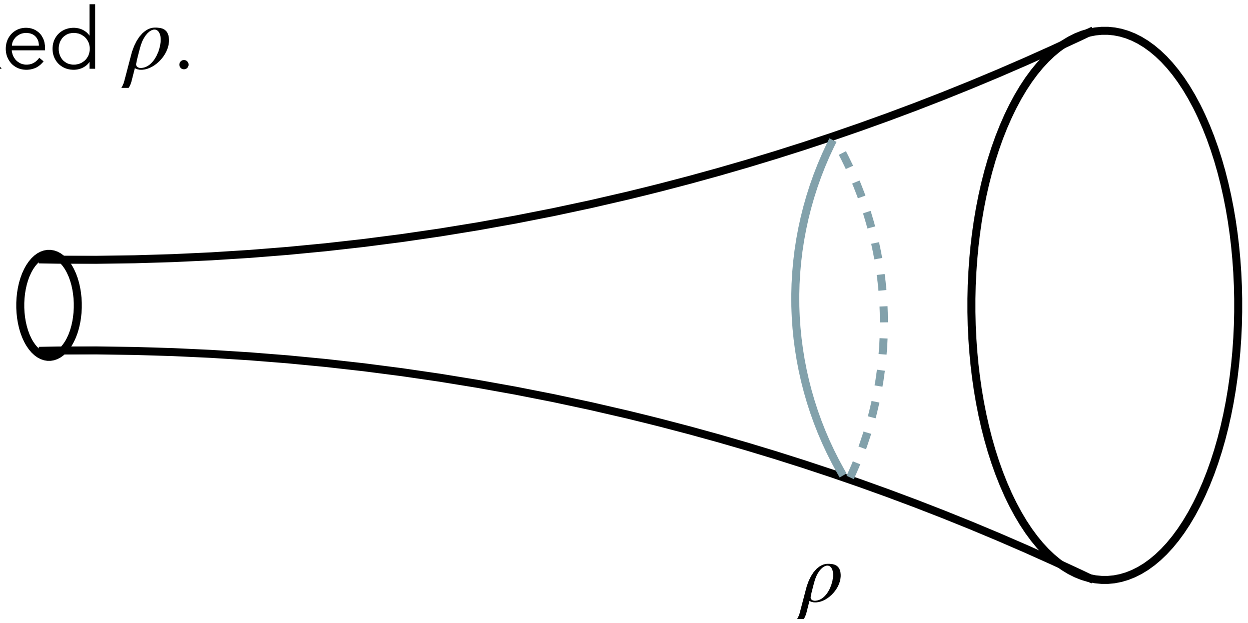
Minimally coupled scalars with $m^2 \geq m^2_{\mathrm{BF}}$ are always stable.
(Essentially due to a result of [Maldacena, Maoz] '04)

YM fields are always stable. (Trivial)

Metric fluctuations are stable for symmetric $\mathbb{T}^d \times I$, and for $\mathbb{S}^1 \times \mathbb{S}^2 \times I$ wormholes in AdS_4 down to relatively small bottleneck, where our numerics our breaking down.

Probe D3-branes and nucleation

Consider putting a probe D3-brane in this geometry, wrapping the torus at fixed ρ.



The brane experiences an attractive gravitational force due to the DBI term, repulsive RR force from WZ term:

$$S_{\rm D3} = T_3 \left(\int d^4x \sqrt{\mathrm{P}[g]} - \int \mathrm{P}[C_4] \right)$$

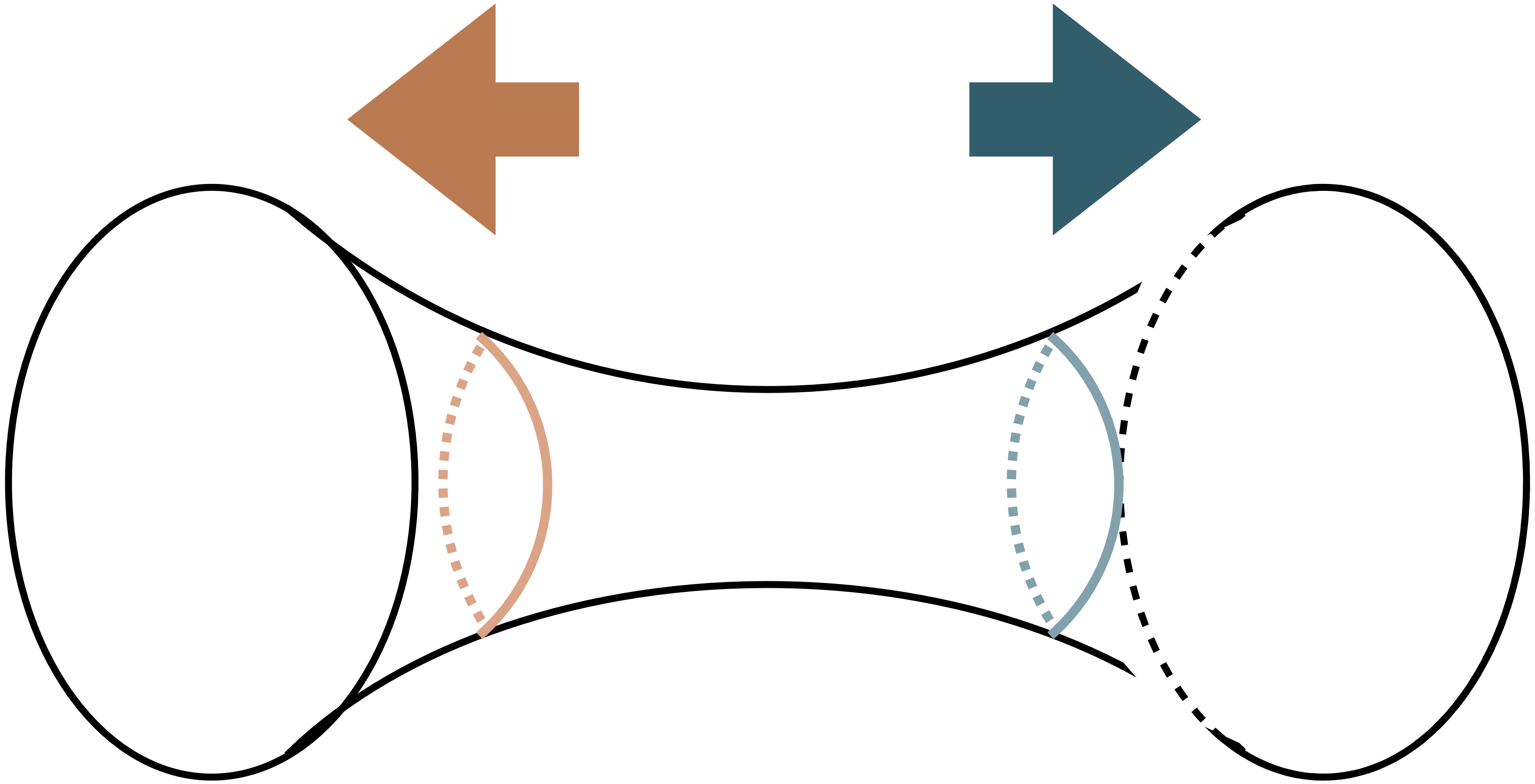
Probe D3-branes and nucleation

In pure $AdS_5 \times \mathbb{S}^5$, these two effects precisely cancel. Probe D3 encodes points on the Coulomb branch with $SU(N) \to SU(N-1) \times U(1)$.

In a black hole background, gravity wins, and probe D3s fall into the BH.

In the wormhole, RR wins, and the probe D3 is repelled infinitely far from the bottleneck.

Probe D3-branes and nucleation



In fact, this leads to a pair production instability of 3-branes in the wormhole. You lose $O(N)$ action by letting a D3-$\overline{\text{D3}}$ pair nucleate, thereby screening the RR flux by two units, which shrinks the size of the bottleneck.

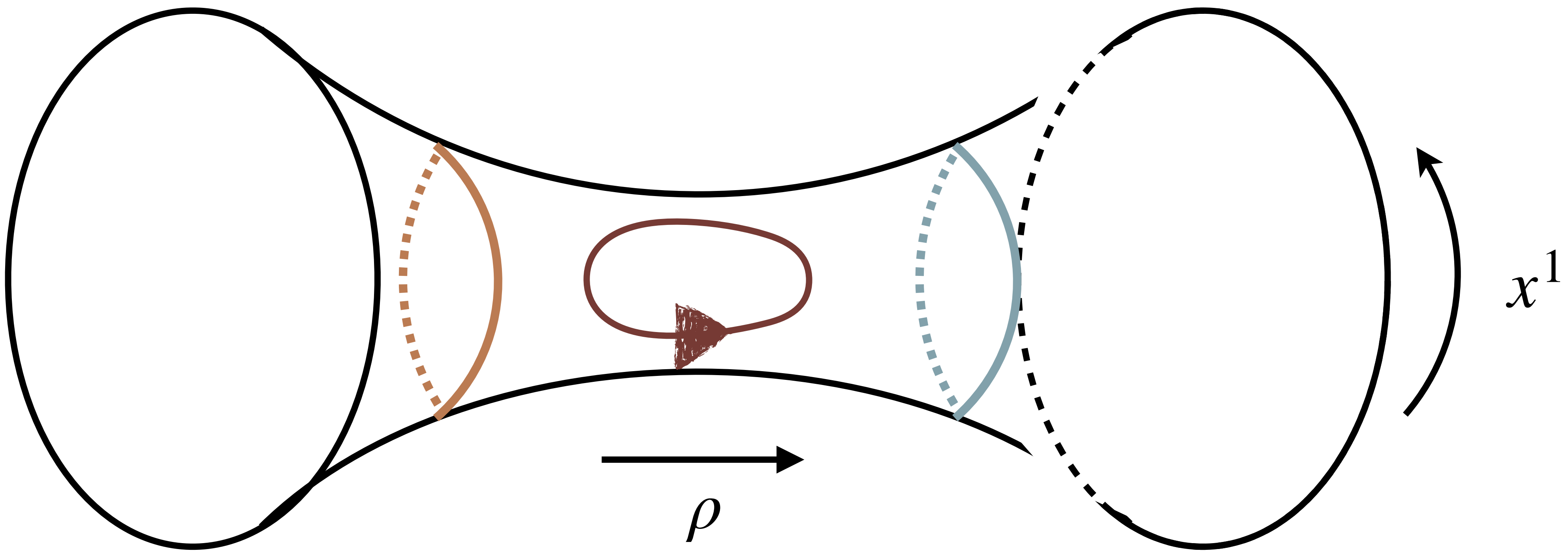
Probe branes and nucleation

D3s are BPS, so $m = q$. It is instructive to study the problem for $m \geq |q|$. (As for nearly BPS branes in $\mathrm{AdS}_3 \times \mathbb{S}^3 \times M_4$.)

For the torus wormholes in any d, we find *two* brane nucleation instabilities for $x = \dfrac{|q|}{m} \gtrsim 0.836$.

1. A pair on opposite sides of the bottleneck, at a position determined by x.
2. See the next slide. A brane instanton, akin to the Schwinger instanton for $e^- - e^+$ production in an electric field.

Probe branes and nucleation



Stringy censorship?

Once embedded into SUGRA, our wormholes are almost always unstable to some brane nucleation.

Tempting to assert a “stringy censorship” proposal for Euclidean wormholes in AdS, but..

Stringy censorship?

We have found two exceptions, both asymp. $\mathbb{H}^5 \times \mathbb{S}^5$:

1. The [Maldacena, Maoz] wormhole with $\mathbb{S}^4$ cross-section.

2. Our wormholes with $\mathbb{S}^1 \times \mathbb{S}^3$ cross-section, and relatively small bottlenecks.

$$ds_{10}^2 = d\rho^2 + \frac{b^2}{4}\frac{\cosh^2(2\rho)}{b^2\cosh(2\rho) - 1}d\tau^2 + \frac{1}{2}(b^2\cosh(2\rho) - 1)d\Omega_3^2 + d\Omega_5^2$$

There are D3-brane nucleation instabilities for $b \geq 1.355$, but no apparent instabilities for smaller b.

What does this mean?

We are inching towards a sharp factorization paradox for $AdS_5 \times \mathbb{S}^5$. It remains to nail down whether SUGRA fluctuations around these wormholes are stable.

A full stability analysis is non-trivial, but tractable.
(The main technical problem is dealing with the negative norm fluctuations of the trace, which couples to other flucs.)

What if these configurations are stable?
Is there a fundamental constraint in string theory which forbids them?

Thank you!