GRAVITATIONAL PERTURBATIONS OF EXT KERR

REVISITED

Island Hopping 2020

Based on V. Godet, J. Simon, W. Song, B. Yu (to appear)

Also (5D BHs) J. Pedraza, C. Toldo, E. Verhijden (WiP)

Refs on nAdS$_2$+Rotation
1707.03380
1807.06988
1906.09083
2011.01944
2011.10001

Intro: JT gravity has been central

Quantum / holographic of (n-ext) BHS

Where does JT sit in the context of grav pert of kerr?

☆ Imprints of near horizon effects on UV $\Rightarrow$ gluing

☆ New features due to rotation $\Rightarrow$ holographic BH chemistry

Two aspects about JT & n-ext BHs

1) $AdS_2 \to nAdS_2$ $\qquad S_{BH} \to S_{BH,0} + \delta S_{BH,0}$

2) Φ : JT field $\qquad \int \Phi (R+2)$

- $\nabla_a \nabla_b \Phi - g_{ab} \Box_2 \Phi + g_{ab} \Phi = 0$
- $\Delta = 2$
- size of sphere (e.g. RN)

Extreme Kerr & NHEK

$$ds^2 = -\frac{\rho^2(r,\theta)\,\Delta(r)}{\Sigma(r,\theta)}dt^2 + \rho^2(r,\theta)\left(\frac{dr^2}{\Delta(r)} + d\theta^2\right)$$

$$+\frac{\sin^2\theta}{\rho^2(r,\theta)}\Sigma(r,\theta)\left(d\phi - \frac{2aMr}{\Sigma(r,\theta)}dt\right)^2 \qquad \Delta(r) = r^2 - 2Mr + a^2$$

Extreme: $M = a \equiv M_0$

NHEK: $M = a$ & decop. limit $(\lambda \to 0)$

$$r \to M_0 + \lambda r\,, \quad t \to 2M_0^2\, t/\lambda\,, \quad \phi \to \phi + M_0 t/\lambda$$

$$ds^2 = M_0^2(1+\cos^2\theta)\ \Big(\underbrace{-r^2dt^2 + \frac{dr^2}{r^2}}_{AdS_2} + d\theta^2\Big)$$

$$+\,4M_0^2\frac{\sin^2\theta}{(1+\cos^2\theta)}\ \underbrace{(d\phi + r\,dt)^2}_{S^1\text{ fiber}}$$

Gravitational Perturbations: Teukolsky (NP), Wald, ...

* Focus NHEK
* Focus on pert indep ϕ
* Weyl scalars $\psi_i \quad i = 0, \dots, 4$

$$\psi_0 = C_{\mu\nu\alpha\beta}\, \ell^\mu m^\nu \ell^\alpha m^\beta$$

$$\psi_4 = C_{\mu\nu\alpha\beta}\, n^\mu \bar{m}^\nu n^\alpha \bar{m}^\beta$$

→ @ linear order inv. tetrad rot, gauge inv.

→ Teukolsky master eqn

Possible type of perturbations

1) Grav. waves $\rightsquigarrow$ $\Psi_4 \neq 0$ $\Psi_0 \neq 0$
2) Changes M, J
3) Diffeo

$\Big]\to$ on Kerr $\Psi_4 = 0 = \Psi_0$

4) NUT charges

$$ds^2 = M_0^2 (1 + \cos^2\theta + \chi) \Big(\underbrace{-r^2 dt^2 + \frac{dr^2}{r^2}}_{AdS_2} + d\theta^2 \Big)$$

$$+ 4M_0^2 \frac{\sin^2\theta}{(1+\cos^2\theta + \chi)} \Big(\underbrace{d\phi + r\,dt}_{S^1 \text{ fiber}} + A_a dx^a \Big)^2$$

$\chi(\underbrace{x^a}_{AdS_2}, \theta)$: linear modes

$A_a(x^a, \theta) \rightsquigarrow A_a(\chi)$

Relation to Ψ_0 & Ψ_4

$$\Psi_0 = -\frac{1}{2\sin^2\theta} \, l^a l^b \nabla_a \nabla_b \chi$$

$$l^2 = 0 = n^2$$
$$l \cdot n = -1$$

$$\Psi_4 = -\frac{1}{2M_0^4 (1+i\cos\theta)^4} \, n^a n^b \nabla_a \nabla_b \chi$$

Linear E.O.M :

$$\chi(x^a, \theta) = \sin^2\theta \sum_k \underbrace{S_k(\theta)}_{\text{spheroidal harmonic}} \chi_k(x^a)$$

$$\boxed{\Box_2 \chi_k = k\, \chi_k}$$

$$\Delta_\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4} + k} \qquad \text{BF} \quad 4k > -1$$

1) Grav waves

$\Longrightarrow$ S_k are assoc Legendre func.

$$k = \ell(\ell+1) \qquad \ell = 2, 3, \dots$$

$$\Delta = \ell + 1 \geq 3$$

2) $k = 2$, $\Delta = 2$: JT sector (1st half)

$$\boxed{\Box_2 \chi_2 = 2 \chi_2} \quad (1)$$

$$S_2(\theta) = \frac{s_1}{\sin^2\theta} + s_2 (2 + \cos\theta) \tan^2\left(\frac{\theta}{2}\right)$$ $\rightsquigarrow$ poles $\theta = 0, \pi$

$$\Psi_0 = -\frac{1}{2 \sin^2\theta} \underbrace{\ell^a \ell^b \nabla_a \nabla_b \chi_2}_{=0 \;(2)} S_2(\theta) \sin^2\theta$$

$$\Psi_4 = -\frac{1}{2M_0^4 (1 + i\cos\theta)^4} \overbrace{n^a n^b \nabla_a \nabla_b \chi_2} S_2(\theta) \sin^2\theta$$

(1) + (2) $\Longrightarrow$ JT eom ☺

Notes :

1) (1) + (2) does not imply χ is a diff

$$ds^2 = M_0^2 \left(1+\cos^2\theta + \chi\right)\Big(\underbrace{-r^2 dt^2 + \frac{dr^2}{r^2}}_{AdS_2} + d\theta^2\Big)$$

$$+ 4M_0^2 \frac{\sin^2\theta}{\left(1+\cos^2\theta + \chi\right)} \big(\underbrace{d\phi + r\,dt}_{S^1 \text{ fiber}} + A_a dx^a\big)^2$$

2) χ is not controlling size S^2

3) Geom conical singularity

$$\underset{\theta = 0}{ds^2 =} \;\cdots + M_0^2 (2+\chi)\, d\theta^2 + M_0^2\, \theta^2 d\phi^2 (2-\chi) + \cdots$$

singular.

Second Half of JT :

$$ds^2 = M_0^2 \left(1+\cos^2\theta\right)\Big(\underbrace{-\left(1+\psi(\Phi)\right) r^2 dt^2 + \frac{dr^2}{r^2}}_{AdS_2} + d\theta^2\Big)$$

$$+ 4M_0^2 \frac{\sin^2\theta\,(1+\Phi)}{\left(1+\cos^2\theta\right)} \big(\underbrace{d\phi + r\,dt}_{S^1 \text{ fiber}} + A_a^{(\Phi)} dx^a\big)^2$$

$\Phi(x^a)$: size of S^2 $\Rightarrow$ induce via diffeo

$$\xi = -\frac{1}{2}\phi \left(-\partial_r \Phi\, \partial_t + \partial_t \Phi\, \partial_r + r^2 \partial_r\left(\frac{\Phi}{r}\right) \partial_\phi\right) - \frac{1}{2r^2} \partial_t \Phi\, \partial_r$$

Φ obeys JT bc $g_{\mu\nu}$ to not depend on ϕ

Return to our issue of conical sing (inc. both χ + Φ)

$$ds^2 = \dots + M_0^2(2+\chi)\, d\theta^2 + M_0^2\, \theta^2 d\phi^2 (2-\chi+2\Phi) + \dots$$

$\theta = 0$

$$\boxed{\chi = \Phi}$$

JT sector in Kerr is composed by two effects

↪ non-single valued diff Φ ⇝ sphere

↪ mode $(\psi_{0,4})$ with $k=2$, χ ⇝ frozen

Observations

1) Φ is dominating $S_{BH} = \frac{A_H}{4}$ + Free energy

2) Iyer-Wald charges, both Φ & χ contribute

$$E \to E_0 + C T^2$$

Conclusion :

1) Gluing to UV of JT sector

RN: Birkhoff's Thm $\rightsquigarrow$ Diff + SM

Kerr: if $\Phi = \chi$ $\rightsquigarrow$ Diff + SM

2) 5D BHs : Φ, χ $\Rightarrow$ control on non-linear effects couplings between Φ + matter.

3) Applications + Directions $\rightsquigarrow$ super radiance

Interplay of Φ with SR mode

$\rightsquigarrow$ logs corrections

1205.0971
1505.01156
1801.01926

2003.02860
2011 06038

$\rightsquigarrow$ I-W capture, effects AD
(wip AA, SD, BM + 1911.11434)
2010.08761