Based on earlier work by **de Saint-Exupéry**

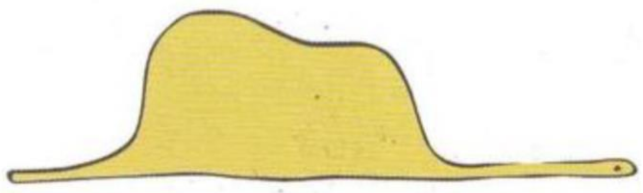
Life without pythons would be so simple

Adam Brown, Hrant Gharibyan, **GP**, Leonard Susskind,

Netta Engelhardt, **GP**, Arvin Shahbazi-Moghaddam

arXiv:1912.00228

ongoing work



Complexity and the AdS/CFT Dictionary

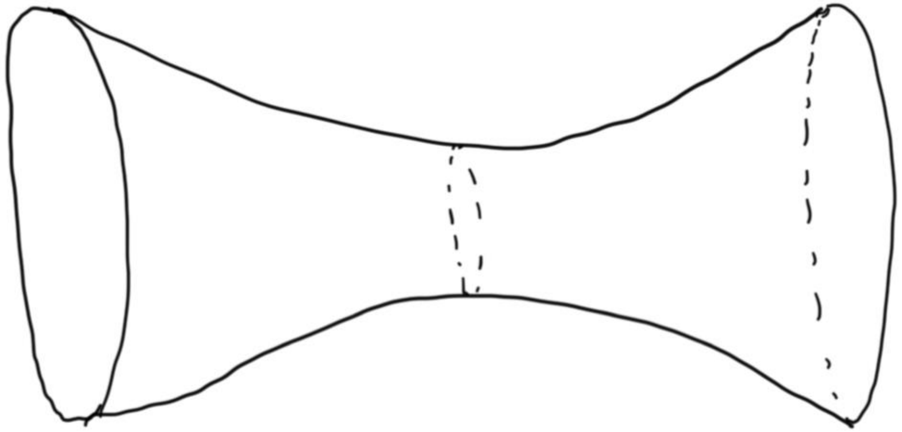
- It's been known for a **long time** that seeming **simple bulk** observables can be dual to **exponentially complicated** boundary observables
- These bulk observables are normally in the interior of black holes
- However, not all interior operators have a complicated dual, e.g. one-sided black hole after the scrambling time: **left-moving** (infalling) interior modes have **simple** reconstructions, **right-moving** interior modes are **complicated**
- Some operators can be simple to reconstruct **globally**, but hard to reconstruct on a **subregion**, e.g. Hayden-Preskill **diary** is hard to reconstruct from the Hawking radiation, but easy to reconstruct with control of the black hole as well (just reverse time)

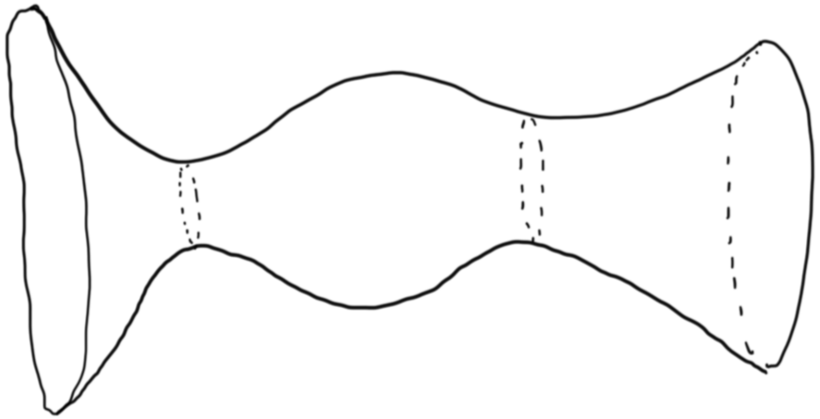
Python vs no python?

- **Aim**: understand why some observables are simple to reconstruct and some are hard
- **Conjecture**: observables are simple to reconstruct **if, and only if,** they are not contained in a **python's lunch**
- Some observables may be in a python's lunch w.r.t. a subregion of the boundary, but **not** w.r.t. the entire boundary. This explains their different reconstruction complexities
- Brown, Gharibyan, GP, Susskind (2019): arguments that observables in a lunch are **hard** to reconstruct
- Engelhardt, GP, Shahbazi-Moghaddam (ongoing): arguments that anything not in is **simple** to reconstruct

Still being worked out

What is a python’s lunch?

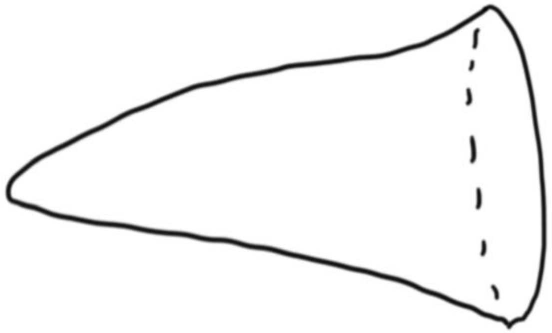


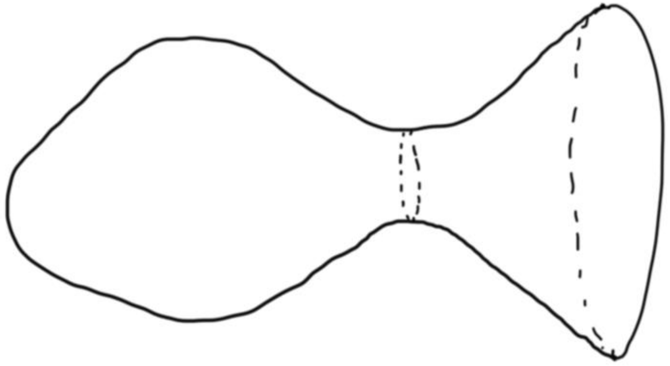


Two-sided

No lunch

Lunch

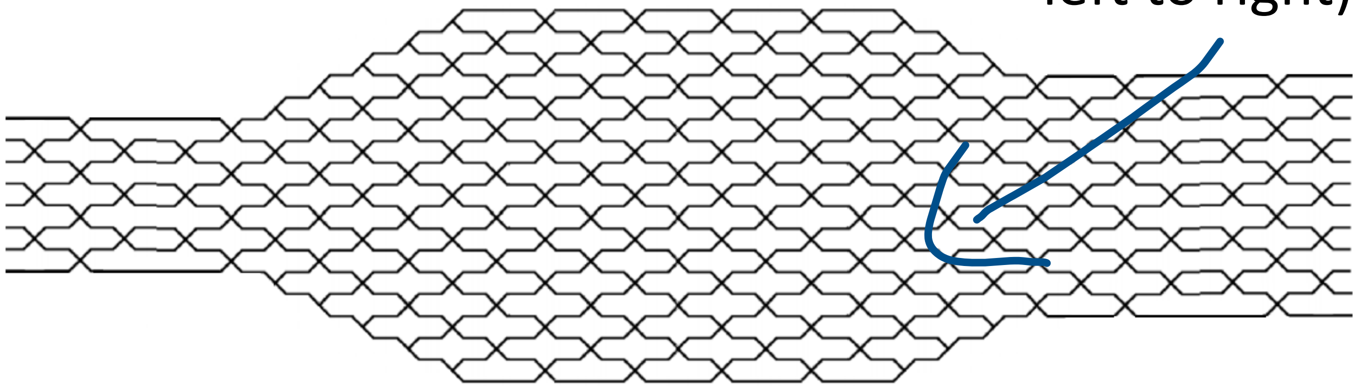


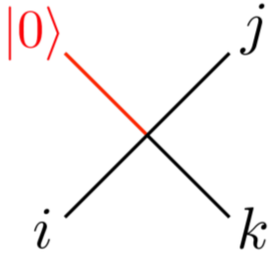


One-sided

The python's lunch (tensor network edition)

Not an isometry (from left to right)



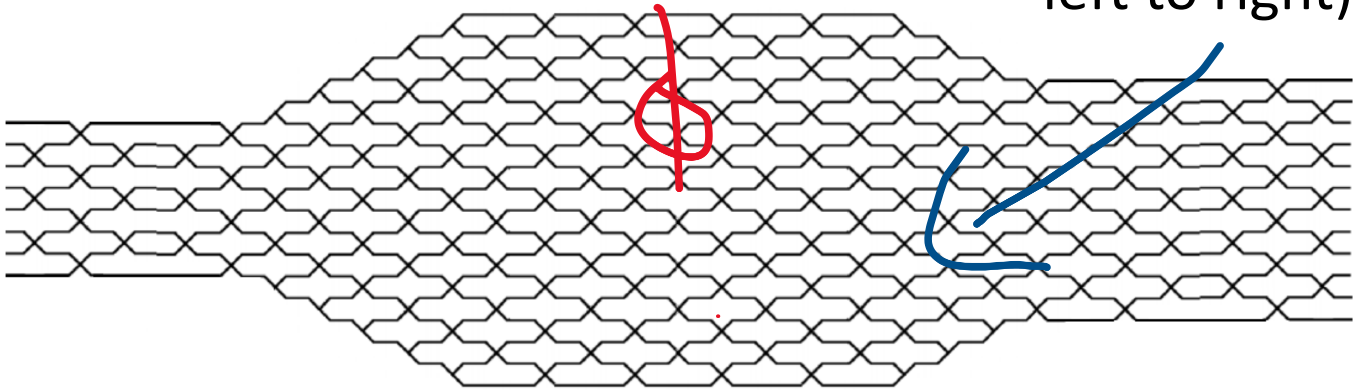


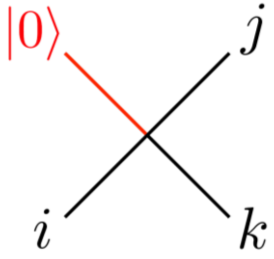
All tensors either *unitaries* or *isometries*

- **Generically**, the entire network will be an **isometry** (up to a very small error) from **left to right**
- However the individual steps are **not** all isometries

Extracting food from the lunch

Not an isometry (from left to right)

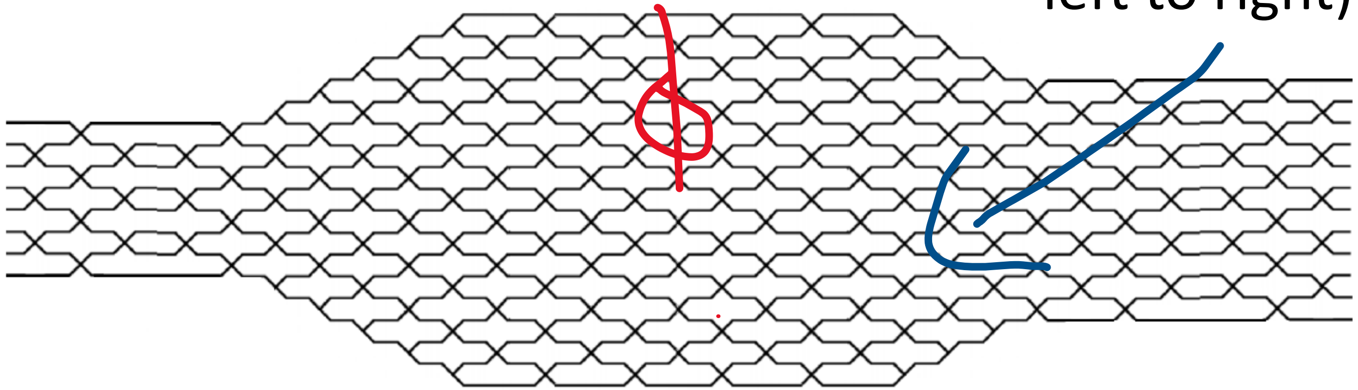




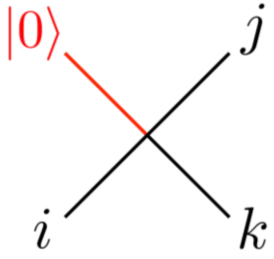
All tensors either *unitaries* or *isometries*

- Suppose that we want to change/measure some bulk operator sitting in the **middle** of the lunch by acting on the **right-hand end** with unitary operators
- Can't just undo the TN as far as the middle of the lunch because it would be **non-unitary**
- Need to undo the entire network and replace with a new version

Complexity of the lunch



Not an isometry (from left to right)

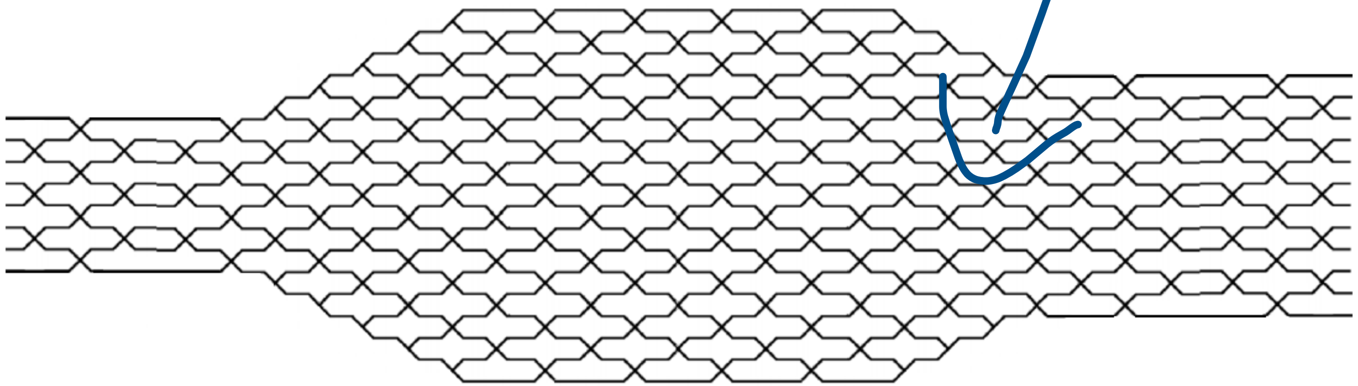


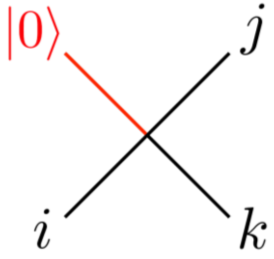
All tensors either *unitaries* or *isometries*

- As a **tensor network**, the map from left to right is pretty **simple**
- However, as an **isometry**, its complexity is determined by the number of simple unitaries (plus ancillas) needed to construct it
- Not the same because of the **non-isometric** parts of the TN!

The python's lunch

Not an isometry (from left to right)





All tensors either *unitaries* or *isometries*

More explicitly,

Simple

Input

$$|\psi\rangle \propto \langle 0|^{\otimes m_R}\, U_{TN} |I\rangle\, |0\rangle^{\otimes m_L}$$

Output

Postselection

Ancilla

NOT UNITARY

$$|\psi\rangle\, |0\rangle^{m_R} = U_{PL}\, |I\rangle |0\rangle^{m_L}$$

Simple or not?

How hard is it to bypass postselection?

- Naïve approach (if input state can be prepared many times and measurements are allowed): keep trying until you get lucky and measure the correct state
- Estimated time is $O(2^{m_R})$ (**exponentially hard**). Also still not really unitary
- Better method: **Grover search**
- First apply U_{TN}. Then apply a *phase of* (-1) if all m_R ancilla qubits in zero state. Apply $U_{TN}^{\dagger}$. Apply *phase of* (-1) if all m_L ancilla qubits in zero state. Repeat $2^{m_R/2}$ times.
- **Still exponentially hard**

Could there be a more efficient way?

- **Maybe**. Complexity theory is hard
- Grover search is the **optimal search strategy**
- However, in this case, we know in advanced what we're searching for so that could mean more efficient approaches exist
- Very strong reasons to think that it cannot normally be done in polynomial time
- This would imply BQP = PostBQP = PP ← Incredibly powerful
- If you suggest that this is true to **Scott Aaronson** he will laugh at you

Two complexity conjectures

A reasonable conjecture: the *unitary complexity* of the **python's lunch** tensor network is generically

$$O(C_{TN} \times 2^{m_R/2})$$

Number of postselected qubits

Complexity of U_{TN}

A **more speculative** conjecture: the same thing is true for python's lunches in gravity

Evaporating black holes

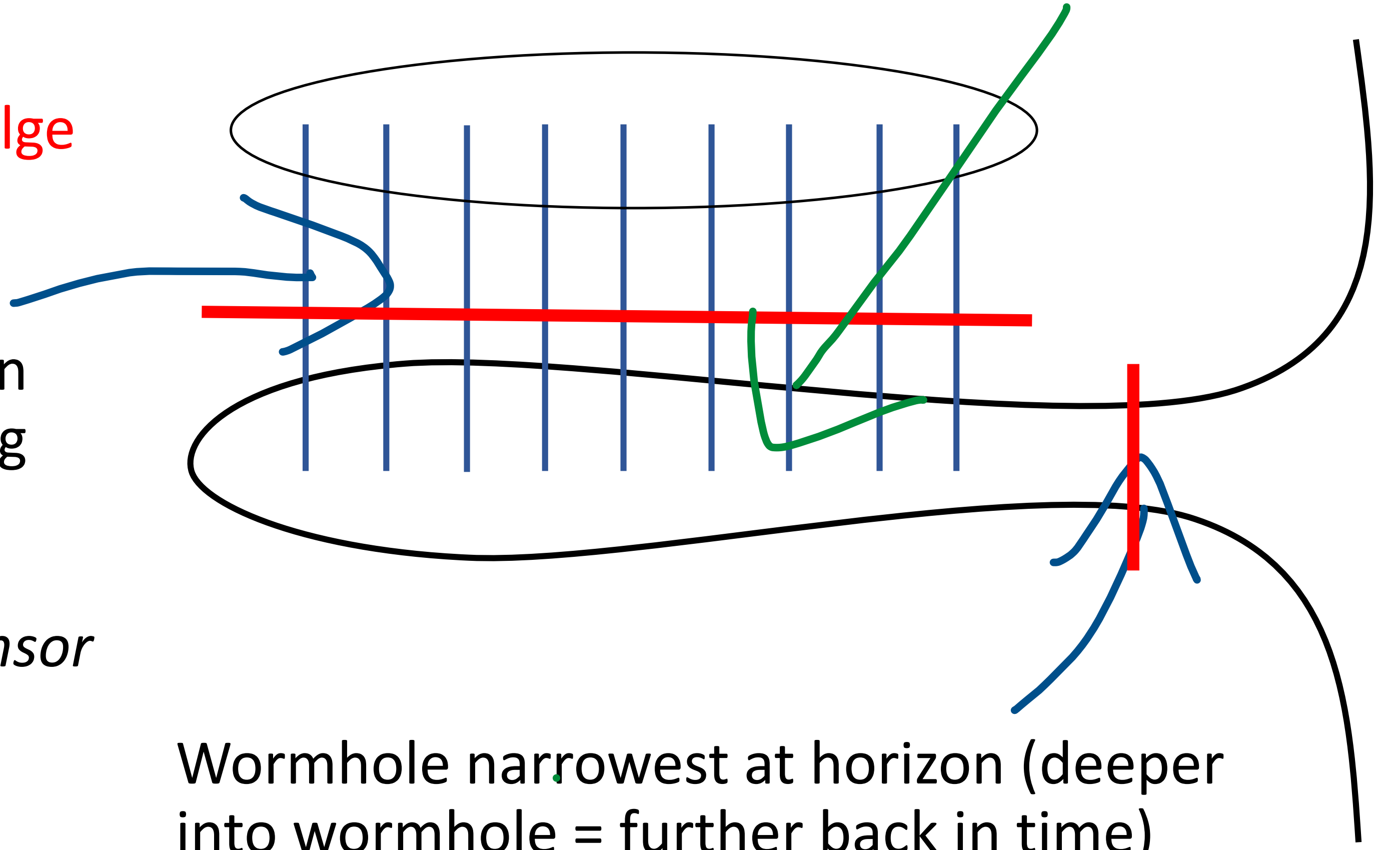
- Harlow-Hayden (2013): performing the **AMPS experiment** (extracting a purification of a late-time Hawking quanta from the early radiation is **exponentially hard** (based on general arguments about scrambling unitaries)
- Modern 'islands' viewpoint: **entanglement wedge reconstruction** of the interior partner of the Hawking mode, using the early radiation, is exponentially hard
- Hayden-Preskill decoding of infalling modes in the island using just the radiation is also expected to be exponentially hard for **very similar** reasons (but global reconstruction is easy).
- Can this be explained by a **python's lunch**?

A ‘nice slice’ of an evaporating black hole

For the moment, we consider a single, ‘nice’ Cauchy slice that sticks close to the event horizon (we will define everything **covariantly** later)

Bulge = island

Two constrictions with a bulge in the middle



Bulk entanglement between interior modes and Hawking radiation. Equivalent to classical area *(ER=EPR, Engelhardt-Wall, HaPPY tensor networks)*

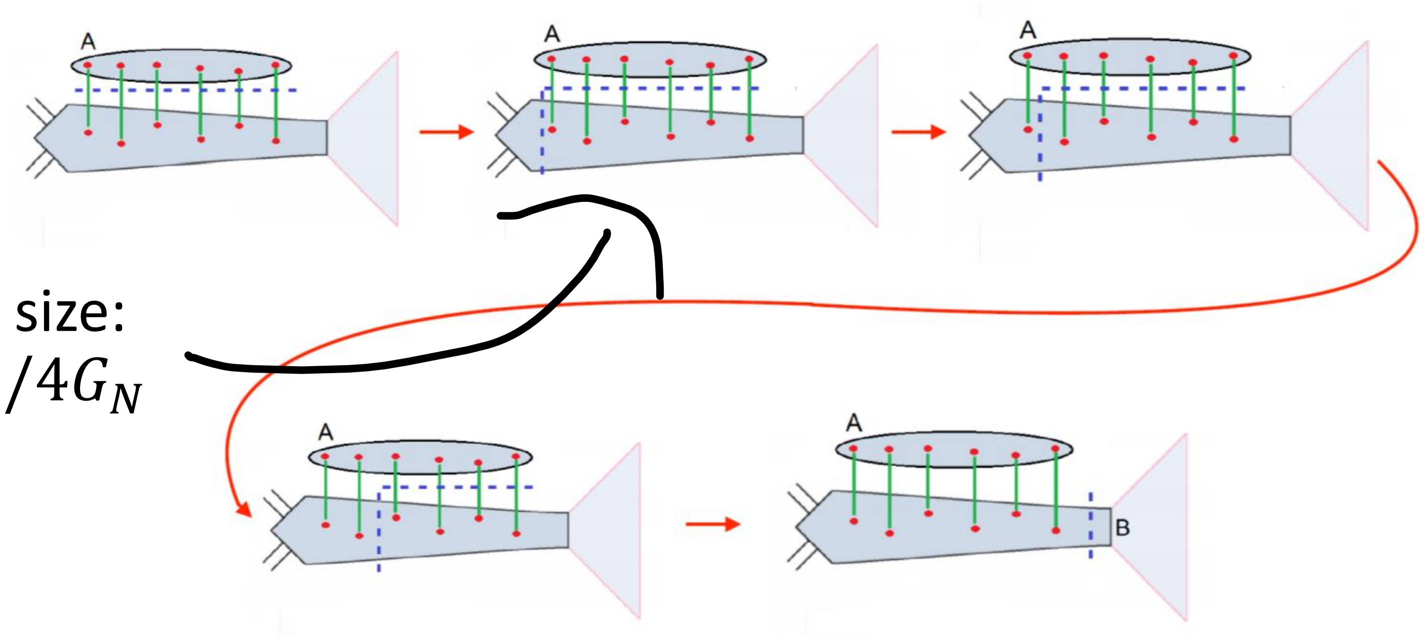
Python’s lunch

Wormhole narrowest at horizon (deeper into wormhole = further back in time)

How big is the python's lunch?

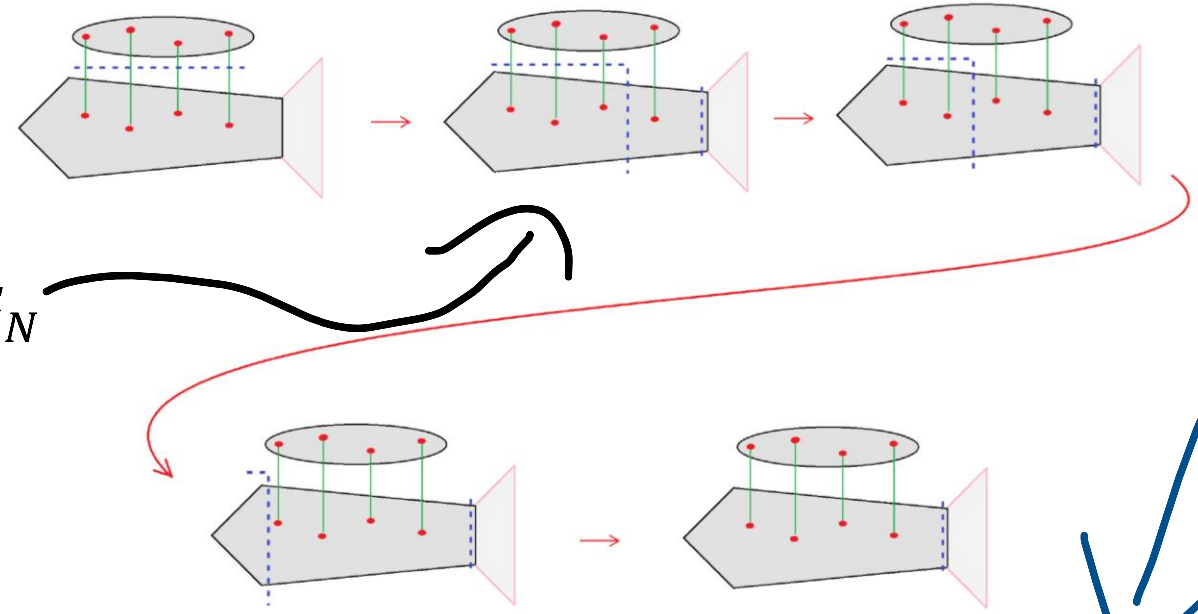
- The **maximum size** of the lunch depends on how you slice it from one constriction to the other
- We want to choose the slicing that **minimizes** this maximum size (this corresponds to the **most efficient** Grover search protocol)
- One option: start at end of the wormhole and move **forwards** along it

Maximum size:
$S_{rad} + A_0/4G_N$



Forwards vs reverse sweep

- Alternative option: start with **double cut** near the horizon, and the move one cut **backwards** along the wormhole



Maximum size:
$S_{rad} + 2A_{hor}/4G_N$

Forwards sweep still dominates when island first forms

More efficient when $A_{hor} < A_0/2$. Note that this transition happens **strictly after** the Page time (defined by $S_{rad} = A_{hor}/4G_N$).

How complex is Hayden-Preskill reconstruction?

Intuition from tensor networks: restricted complexity is

Amount of postselection required

$$O\left(C_{TN}\exp\left(\frac{1}{2}\left[S^{gen}_{max} - S^{gen}_{larger\ min}\right]\right)\right)$$

Volume/action = $O(t)$

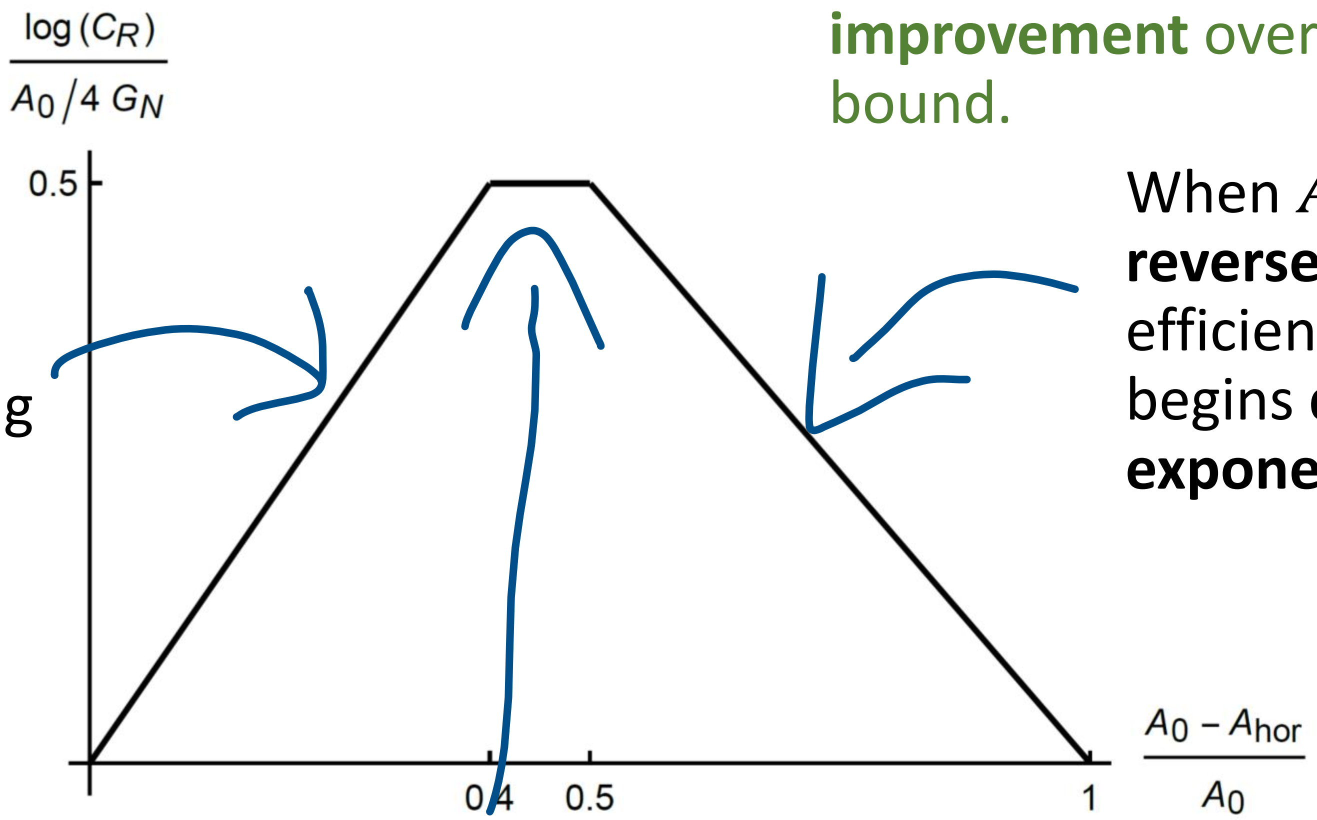
Maximum generalized entropy in the most efficient slicing (**minimax surface**)

Generalised entropy of the larger of the two minima

Complexity of reconstruction on (one of) BH/radiation over time

Agrees with complexity of **Grover search** algorithms in toy models. **Quadratic improvement** over Harlow-Hayden bound.

Complexity (of reconstructing using BH) initially **grows exponentially**



When $A_{hor} < A_0/2$, the **reverse sweep** becomes more efficient and the complexity begins **decreasing exponentially**

After the evaporation is complete, **no lunch, complexity = complexity of global reconstruction =** $\boldsymbol{O(S_{BH})}$.

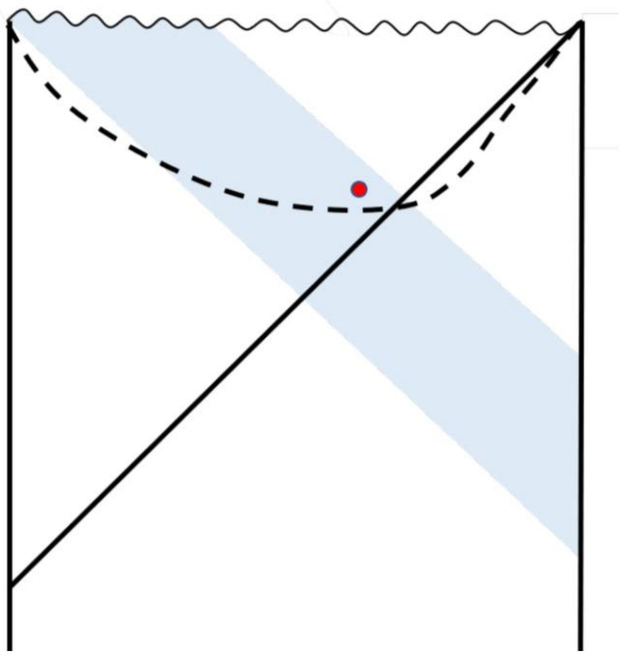
After Page time, larger minimum becomes the **empty surface**. Interior now reconstructable from radiation (not BH). Complexity grows **linearly**

Covariant python's lunches

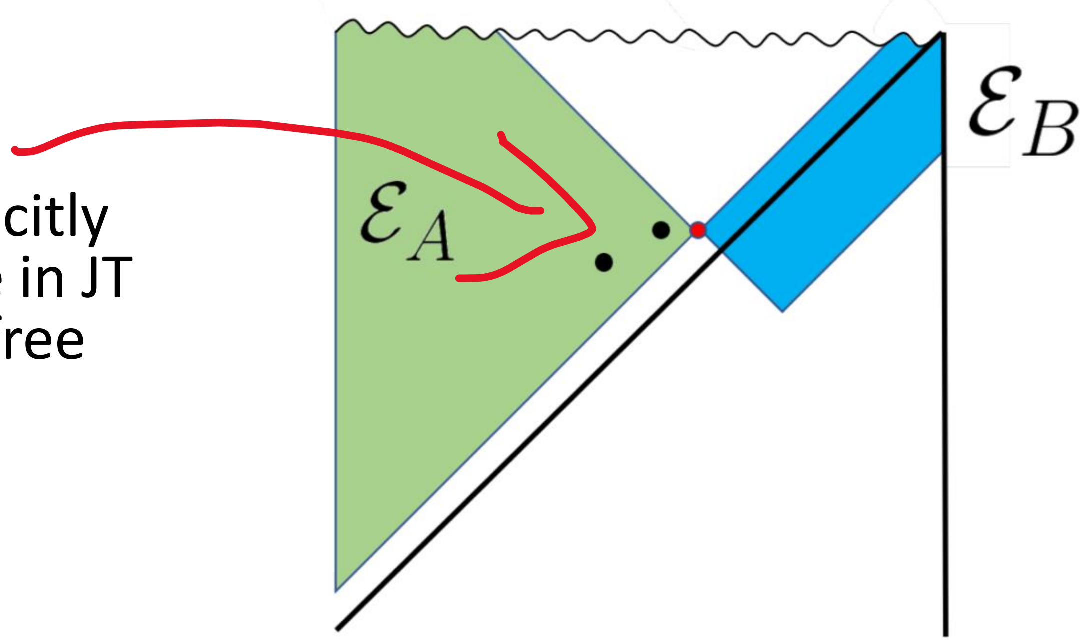
- The covariant surface that corresponds to the **minimal cut** in a tensor network is the **minimal quantum extremal surface** *(Engelhardt, Wall 2014)*.
- This can also be found using a **maximin prescription**: first find the minimal generalised entropy surface within a Cauchy slice, then maximise over all Cauchy slice *(Wall 2012, Akers, Engelhardt, GP, Usatyuk 2019)*.
- Other end of the lunch = a second larger QES
- What is the covariant definition of the maximum size of the lunch?
- For a tensor network, it was a **minimax surface** (minimize the maximum slice over all ways of slicing from one end to the other)
- **Our conjecture:** covariant definition is the **maximinimax surface** (maximise the minimax surface over all Cauchy slices containing the two ends of the lunch)
- Assuming everything is well behaved, this should also be a **quantum extremal surface**.

Covariant python's lunches for evaporating black holes

We can explicitly find quantum extremal surfaces that give the maximum bulge size in the **forwards and reverse sweeps**



Only explicitly calculable in JT gravity + free fermions



Forwards sweep: QES is a **sphere** inside the shell of collapsing matter that formed the black hole

Reverse sweep: QES is the union of **two spheres**, just inside the minimal QES

One-sided python's lunches

- Hayden-Preskill is easy if we are allowed to act on both BH + radiation = there is no python's lunch for the **global system** of BH + radiation
- However it's easy to construct states where this isn't true. For example, allow a black hole to **partially evaporate** and the **measure** all the Hawking radiation. No way to reconstruct interior information without unitarily reversing the **postselection** using **Grover search**
- Also easy to see that the state has a **one-sided python's lunch**, i.e. a **nonempty (nonminimal) QES** associated to the entire boundary Hilbert space
- Note that the QFC implies that you cannot create a new QES (for the entire boundary) by **causal bulk evolution**. Simple boundary operators (which act causally in the bulk) cannot create a one-sided python's lunch

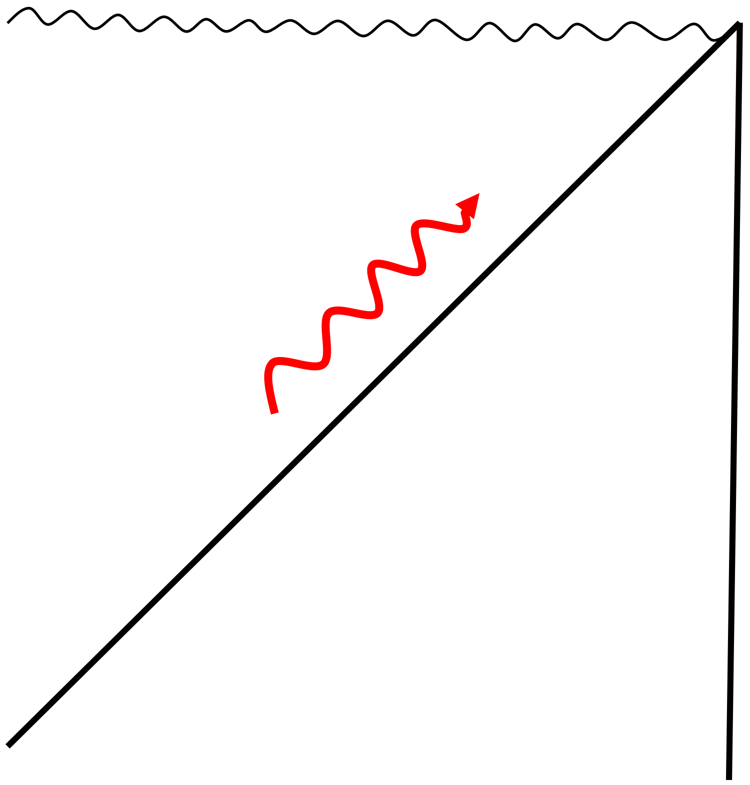
Everything from here onwards is **work in progress** and **in flux**

Life without pythons would be so simple

- **Claim:** pythons' lunches are the **only** source of exponential complexity in the Ads/CFT dictionary
- Will argue:

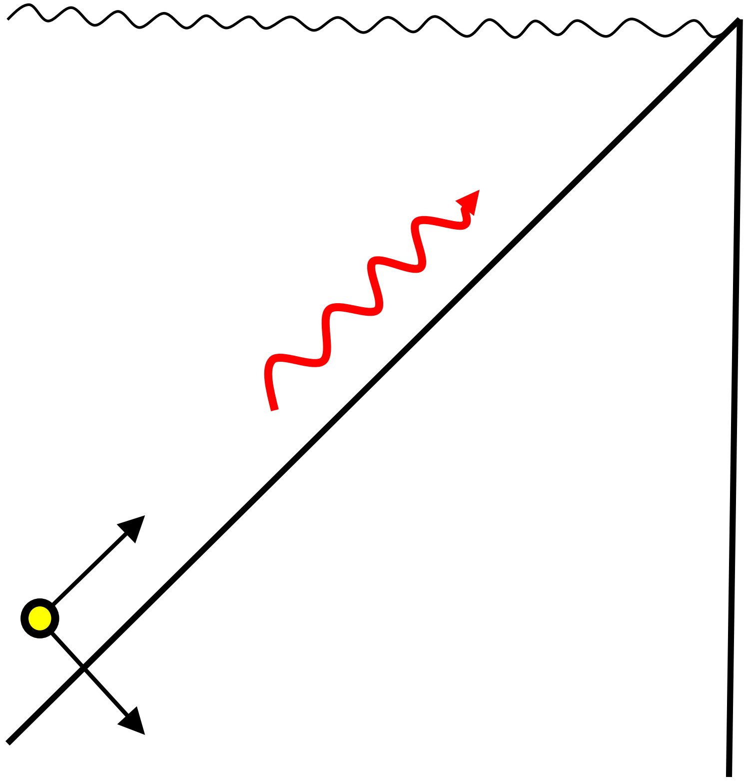
  1. The **counterexample** to this claim that you are probably thinking of is not actually a counterexample
  2. General procedure for reconstructing anything not in a lunch using HKLL-style **causal bulk dynamics** (can never be exponentially complicated)

Finding pythons in unexpected places



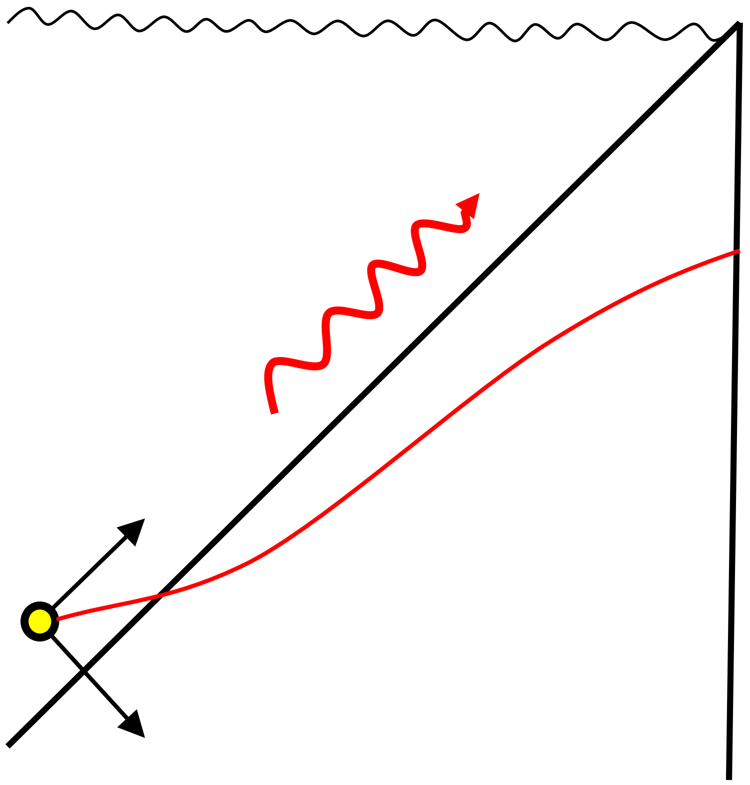
- Classic example of complexity in the dictionary: **interior partner modes** in **nonevaporating** black holes (after the scrambling time)
- No **python's lunch** because constructed by unitary evolution from vacuum state
- **Counterexample?**

A python from a code subspace



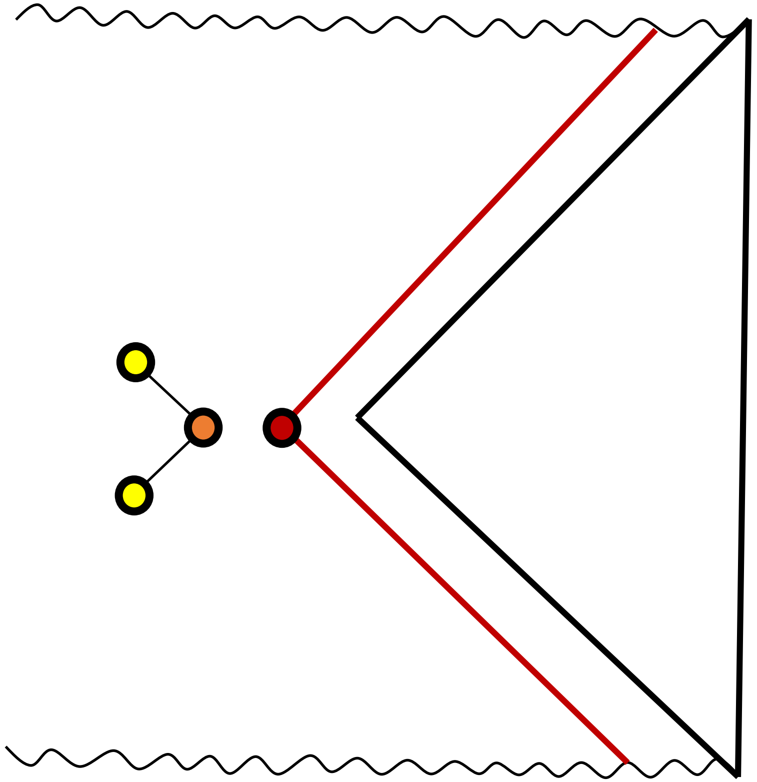
- To talk about bulk reconstruction we can't just consider a **single state**, we need to consider arbitrary (mixed) states in a full **code subspace**
- At the very least, this code subspace needs to allow the interior mode to be in **multiple states**, not just entangled with the Hawking quanta
- Consider a sphere, just inside the horizon, just over one **scrambling time** in the past
- Disentangling the interior mode, decreases the **quantum expansion** in the **past outward direction**
- Causes quantum expansion to be **negative** everywhere in **both** past+future outwards directions

Restricted quantum maximin



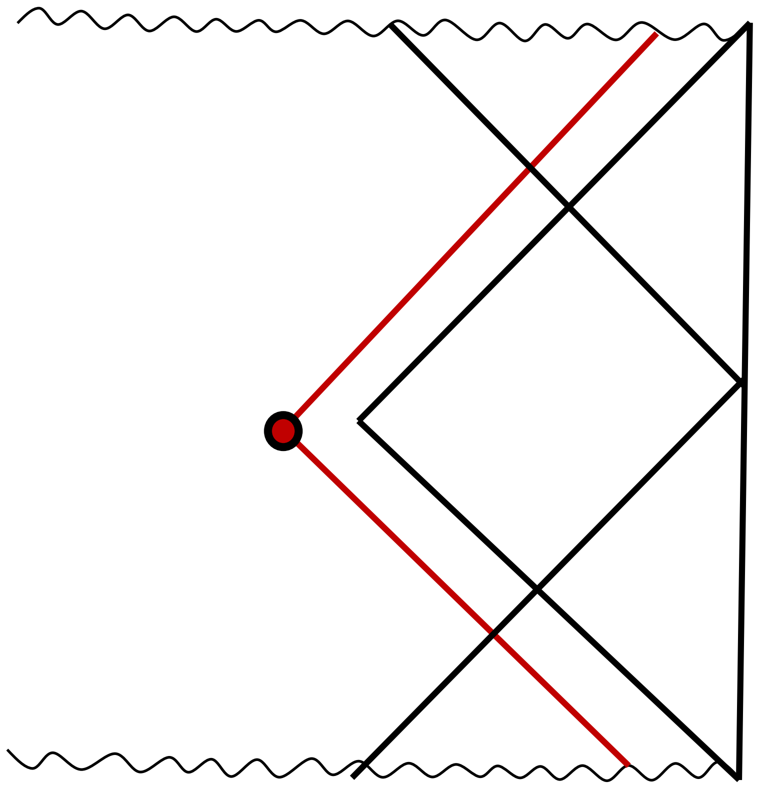
- Want to argue that this implies the existence of a **nonempty QES** with the interior mode in the lunch
- Approach: **maximinimise** S_{gen} over all surfaces on time slices **anchored** on interior sphere + boundary
- Maximin surface cannot intersect interior sphere (or be lightlike separated) because of negative expansions
- Hence maximin surface must be a **QES**
- Outside interior sphere, so interior mode is in the lunch
- **Size of lunch** depends on size of code subspace (complexity agrees with **toy model** results)

A simple wedge



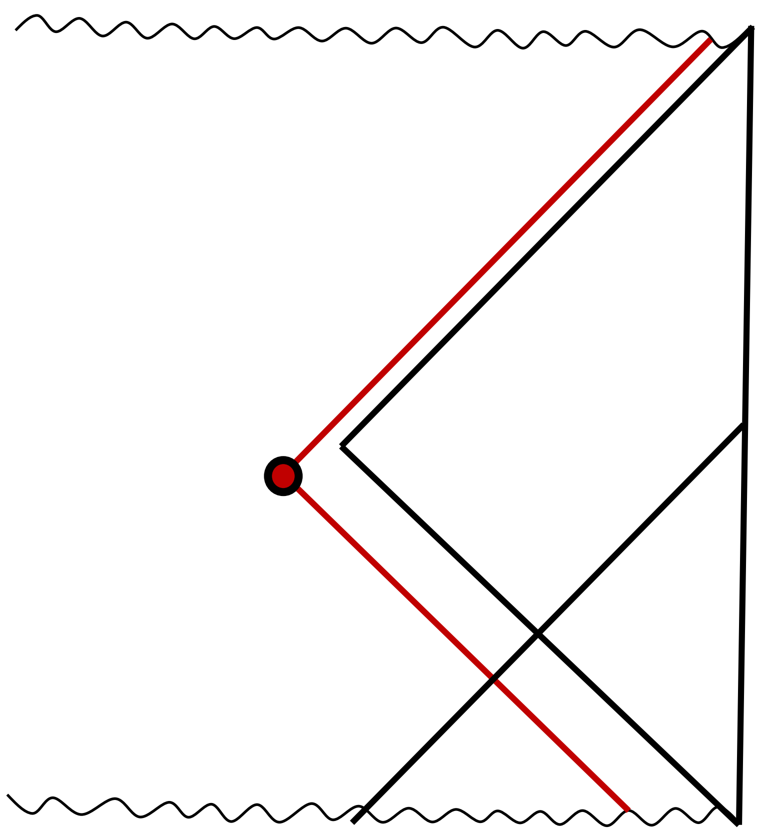
- We want to show that everything outside the **outermost QES** (i.e. outside the lunch) can be easily reconstructed
- Unclear if this is well defined: might be two (partially) timelike-separated/intersecting surfaces.
- **Prove impossible by contradiction**
- Consider surface bounding the **intersection** of the two QES wedges
- This is everywhere either **contained** in one QES or **lightlike-separated** from both
- **Restricted maximin** then gives a new QES **outside** both original QESs.
- We say that the outermost QES bounds the **simple wedge**

The simple wedge vs the causal wedge



- QFC implies that the simple wedge contains the causal wedge
- However in general there will be a **gap** between them
- Everything in the causal wedge is easy to reconstruct (**HKLL**). What about the gap?
- Causal wedge is **teleological** – it depends on the boundary conditions for time evolution
- For example, suppose there was some **matter** that falls into the BH.
- By changing the BCs, we can remove the matter and hence make the causal wedge **larger**

The simple wedge vs the causal wedge



- Cannot generally reach the boundary of the simple wedge in one go, because we can’t get past the **outermost apparent horizon** on the **past causal boundary**
- However by **iteratively** evolving forwards and backwards in time with different boundary conditions, we could hope to gradually **stretch** the causal wedge out all the way to the simple wedge
- Easy to prove this is possible for **classical chiral theories** in 2D
- Hopeful that we can argue it’s true for general states in classical GR (**hard**)
- **Strategy**: show that whenever the causal surface expansion is **positive**, we can consistently **perturb** the boundary conditions to make the old causal horizon slightly **timelike** => causal wedge has expanded
- We have a general procedure that seems to work, but still trying to **rule out** avenues for possible counterexamples

Summary

- We have a **general proposal** for the source of all exponential complexity in the holographic dictionary – **python's lunches**
- Brown, Gharibyan, GP, Susskind (2019): intuition from **tensor networks** suggests that bulk reconstruction within a lunch should be exponentially hard
- Ongoing work: '**secret pythons**' even in examples of exponential complexity where a lunch doesn't seem to exist. Again, gives answers that agrees with toy models
- Easy to see (**QFC**) that nothing **behind** a QES can be reconstructed **causally**
- There exists a well defined '**simple wedge'** bounded by an **outermost QES**. Can everything in this wedge be reconstructed causally (=> simply)?
- Hopefully: **iterative** procedure for **expanding** the causal wedge out towards the simple wedge. Still trying to rule out possible **weird** counterexamples!

Thank you!