Simultaneous extraction of m_t and α_s from differential $t\bar{t}$ distributions

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Simultaneous extraction of m_t and α_s from differential $t\bar{t}$ distributions

- 8 TeV data from ATLAS and CMS collected in Run 1
- Differential distributions of tops reconstructed from lepton+jets analyses, common binning
- Transverse momentum p_t^T, invariant mass $M_{t\bar{t}}$, single and pair rapidities y_t,$y_{t\bar{t}}$
- Absolute and normalised distributions—separate data sets from ATLAS, only normalised from CMS (absolute inferred)

Fit methodology

Least squares extraction for normalised and absolute distributions

$$\zeta_i = \zeta_i^{\mathrm{data}} - \zeta_i^{\mathrm{theory}}$$

$$\chi^2_{\mathrm{norm}} = \frac{1}{N_{\mathrm{data}}}\left[\sum_{i,j=1}^{N_{\mathrm{data}}-1} \zeta_i C_{ij}^{-1} \zeta_j \quad + \frac{(\sigma_{\mathrm{NNLO}} - \sigma_{\mathrm{data}})^2}{\delta\sigma^2_{\mathrm{data}}}\right]$$

$$\chi^2_{\mathrm{abs}} = \frac{1}{N_{\mathrm{data}}}\sum_{i,j=1}^{N_{\mathrm{data}}} \zeta_i C_{ij}^{-1} \zeta_j$$

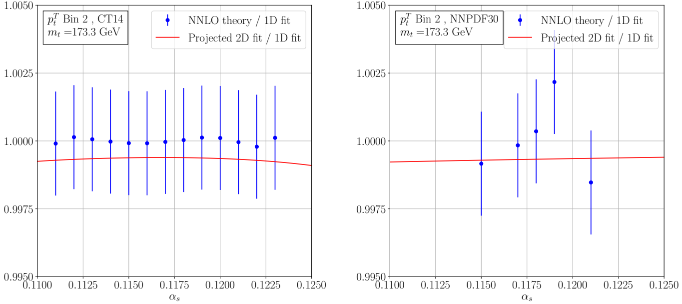
- Measured values of $\sigma_{t\bar{t}}$ taken from separate 8 TeV ATLAS/CMS measurements [1]
- Theory values of $\sigma_{t\bar{t}}$ calculated using `top++2.0` at NNLO with NNLL resummation of soft gluons

[1] 1406.5375,1603.02303

Theory input

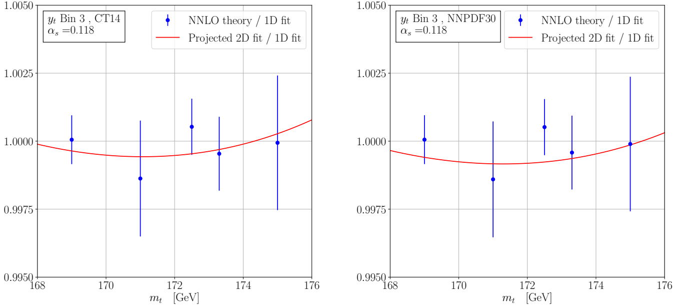
- For each distribution, need differential cross section in each bin as a function of α_s, m_t
- Precompute each bin weight on a grid of α_s, m_t and interpolate parameter dependence
- α_s dependence determined by PDF set, so procedure needs to be done for each choice
- Possible through use of FastNLO tables for values of $m_t = \{169.0, 171.0, 172.5, 173.3, 175.0\}$ GeV
- 3 sets chosen, CT14, NNPDF3.0, NNPDF3.1
- Different parametrisations chosen for each distribution, PDF choice

Assessing fit quality



- Factorised form taken for 2D parametrisation, $f(\alpha_s, m_t) = g(\alpha_s)h(m_t)$
- Points removed and fit redone to ensure no overfitting

Assessing fit quality



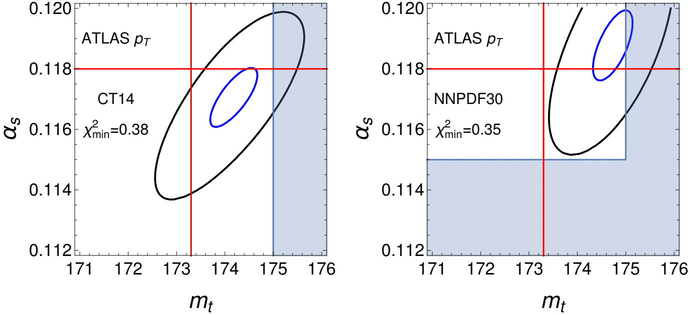
- Factorised form taken for 2D parametrisation, $f(\alpha_s, m_t) = g(\alpha_s)h(m_t)$
- Points removed and fit redone to ensure no overfitting

Results: p_t^T

ATLAS, normalised results

White region: interpolated, blue region: extrapolated

Blue line: $\Delta\chi^2 = 0.1$, black line: $\Delta\chi^2 = 1.0$



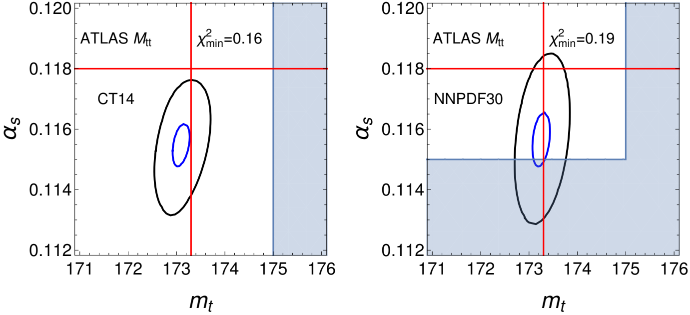
	CT14			NNPDF30		
	α_s	m_t	$\chi^2_{\min}$	α_s	m_t	$\chi^2_{\min}$
p_T^t	0.1171	174.2	0.38	0.1188	174.7	0.35

Results: $M_{t\bar{t}}$

ATLAS, normalised results

White region: interpolated, blue region: extrapolated

Blue line: $\Delta\chi^2 = 0.1$, black line: $\Delta\chi^2 = 1.0$



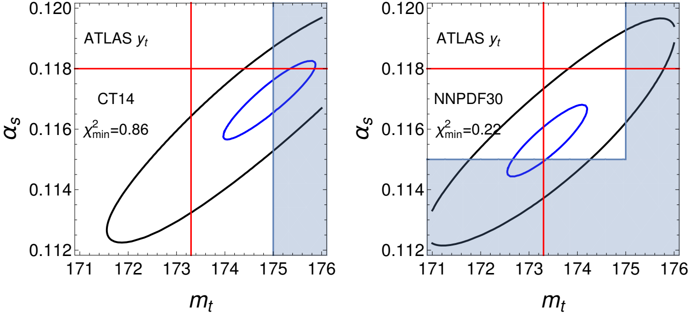
	CT14			NNPDF30		
	α_s	m_t	$\chi^2_{\min}$	α_s	m_t	$\chi^2_{\min}$
$M_{t\bar{t}}$	0.1155	173.1	0.16	0.1157	173.2	0.19

Results: y_t

ATLAS, normalised results

White region: interpolated, blue region: extrapolated

Blue line: $\Delta\chi^2 = 0.1$, black line: $\Delta\chi^2 = 1.0$



	CT14			NNPDF30		
	α_s	m_t	$\chi^2_{\min}$	α_s	m_t	$\chi^2_{\min}$
y_t	0.1171	175.0	0.86	0.1156	173.4	0.22

Combining distributions and experiments

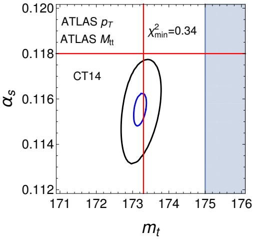
- Combine distributions from ATLAS making use of available correlations between distributions
- Combine distributions from ATLAS and CMS assuming no correlations (luminosity?)

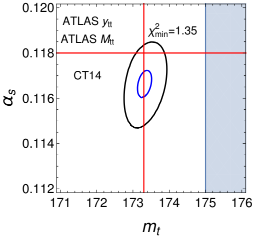
$$\chi^2_{\rm norm} = \frac{1}{(N_{\rm ATLAS}+N_{\rm CMS})}\left(\sum_{i,j=1}^{N_{\rm ATLAS}-1} \zeta_{i,\rm ATLAS}\, C^{-1}_{ij,\rm ATLAS}\zeta_{j,\rm ATLAS} + \sum_{i,j=1}^{N_{\rm CMS}-1} \zeta_{i,\rm CMS}\, C^{-1}_{ij,\rm CMS}\zeta_{j,\rm CMS}\right.$$
$$\left. + \frac{(\sigma_{\rm NNLO}-\sigma_{\rm ATLAS})^2}{\delta\sigma^2_{\rm ATLAS}} + \frac{(\sigma_{\rm NNLO}-\sigma_{\rm CMS})^2}{\delta\sigma^2_{\rm CMS}}\right)$$

$$\chi^2_{\rm abs} = \frac{1}{(N_{\rm ATLAS}+N_{\rm CMS})}\left(\sum_{i,j=1}^{N_{\rm ATLAS}} \zeta_{i,\rm ATLAS}\, C^{-1}_{ij,\rm ATLAS}\zeta_{j,\rm ATLAS} + \sum_{i,j=1}^{N_{\rm CMS}} \zeta_{i,\rm CMS}\, C^{-1}_{ij,\rm CMS}\zeta_{j,\rm CMS}\right)$$

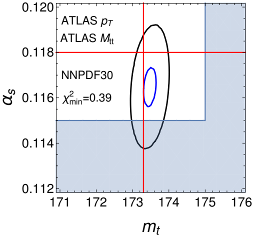
ATLAS p_t^T, $M_{t\bar{t}}$ | ATLAS $M_{t\bar{t}}$, $y_{t\bar{t}}$

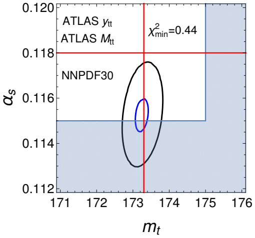
CT14

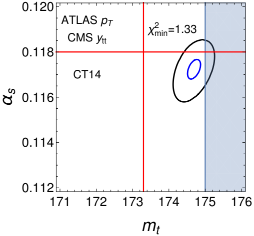
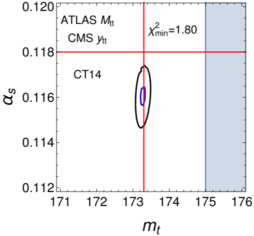
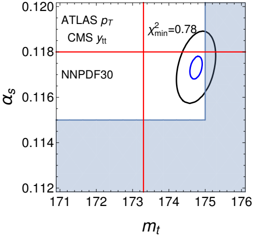
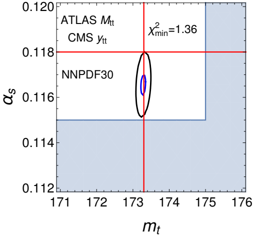




NNPDF3.0





	ATLAS p_t^T, CMS $y_{t\bar{t}}$	ATLAS $M_{t\bar{t}}$, CMS $y_{t\bar{t}}$
CT14		
NNPDF3.0		

Best fit values via averaging

- In light of differences, obtain best quality results via a weighted average.
- Consider only extractions returning $0.115 \leq \alpha_s \leq 0.121$ and $170.0 \leq m_t \leq 176.0$ GeV ($\pm \sim 3\sigma$ around world average).
- Exclude cases where $\chi^2_{\rm min}$ is uniformly bad.

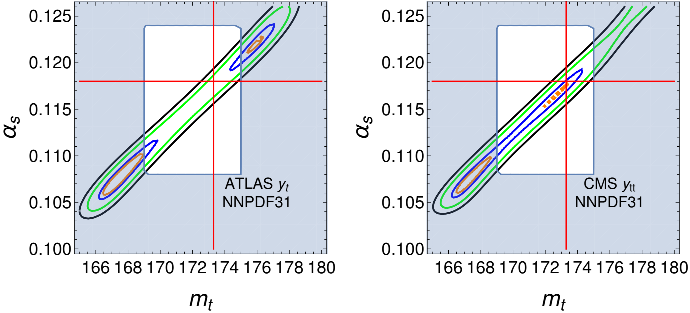
$$\bar{\beta} = \frac{\sum_i u_i \beta_i}{\sum_i u_i}, \qquad u_i = \int_{\chi^2_{{\rm min},i}}^{\infty} f(z; n_{\rm d}) {\rm d}z \tag{1}$$

$$\delta\beta_{\rm stat}^{\rm up/down} = \frac{\sqrt{\sum_i u_i^2 (\delta\beta_i^{\rm up/down})^2}}{\sum_i u_i}, \qquad \delta\beta_{\rm sys} = \frac{\sum_i u_i |\beta_i - \bar{\beta}|}{\sum_i u_i} \tag{2}$$

	CT14		NNPDF3.0	
Extraction	α_s	m_t	α_s	m_t
Single ATLAS	$0.1163^{+0.0015}_{-0.0016}$	$173.8^{+0.8}_{-0.8}$	$0.1164^{+0.0019}_{-0.0019}$	$173.6^{+0.8}_{-0.8}$
ATLAS combination	$0.1159^{+0.0013}_{-0.0014}$	$173.8^{+0.8}_{-0.8}$	$0.1161^{+0.0011}_{-0.0010}$	$173.7^{+0.6}_{-0.6}$

Extractions using NNPDF3.1

- ATLAS y_t and CMS $y_{t\bar{t}}$ used in this work also included in NNPDF3.1 fit
- What effect does this have on extractions from these distributions? Are others affected?



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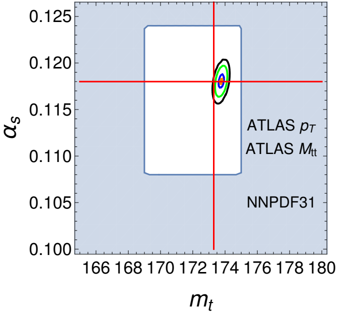

Theory systematic uncertainties

- Given the differences seen between experiments/PDFs/distributions, we neglect all potential sources of systematic uncertainty in the theory predictions.
- Most pragmatic way to include effects due to MHOU is *alla* NNPDF.
  - Assume Gaussian uncertainties, construct a theory covariance matrix.
- More sophisticated approach (see Tackmann, Les Houches 2019):
  - Regard missing higher order terms as nuisance parameters.
  - $\sigma = c_0 + \alpha_s(\mu)[c_1 + \alpha_s(\mu) c_2 + \ldots]$
  - In the simplest case c_2 is a number, more generally a function.
  - Correct correlations obtained, when multiple parameters involved CLT implies total theory uncertainty is Gaussian.

Conclusions

- 8 TeV ATLAS, CMS data compared to NNLO theory to extract α_s, m_t simultaneously.
- Find noticeable differences between
  - ATLAS and CMS
  - Different PDF choices
  - Different distributions

  indicating large sensitivity to all factors—data, PDF and kinematics.
- In order to reconcile differences/arrive at best values,
  - Restrict $0.115 \leq \alpha_s \leq 0.120$ and $170.0 \leq m_t \leq 175.0$ GeV ($\pm \sim 3\sigma$ around world average);
  - Perform a weighted average over various extractions.

Conclusions

- Averaging procedure performed on different types of extraction and different PDFs results in consistent values.
- Absolute distributions give larger errors in general.
- 'Best' value from combining two ATLAS distributions and averaging CT14 results.
- $\alpha_s = 0.1159^{+0.0013}_{-0.0014}$, $m_t = 173.8^{+0.8}_{-0.8}\mathrm{GeV}$.
- Prospects for future inclusion of theory uncertainties due to MHO.
- Results available at `http://www.precision.hep.phy.cam.ac.uk/results/ttbar-fastnlo/`

	ATLAS					
	CT14			NNPDF30		
	α_s	m_t	$\chi^2_{\min}$	α_s	m_t	$\chi^2_{\min}$
p_T^t	$0.1171^{+0.0020}_{-0.0021}$	$174.2^{+1.0}_{-1.0}$	0.38	$0.1188^{+0.0028}_{-0.0028}$	$174.7^{+1.0}_{-1.0}$	0.35
$m_{t\bar{t}}$	$0.1155^{+0.0020}_{-0.0022}$	$173.1^{+0.6}_{-0.5}$	0.16	$0.1157^{+0.0027}_{-0.0027}$	$173.2^{+0.6}_{-0.5}$	0.19
y_t	$0.1171^{+0.0016}_{-0.0018}$	$175.0^{+1.3}_{-1.3}$	0.86	$0.1156^{+0.0018}_{-0.0018}$	$173.4^{+1.3}_{-1.3}$	0.22
$y_{t\bar{t}}$	$0.1205^{+0.0017}_{-0.0019}$	$176.6^{+1.3}_{-1.2}$	1.76	$0.1150^{+0.0025}_{-0.0024}$	$173.0^{+1.3}_{-1.3}$	0.57
Average	$0.1163^{+0.0015}_{-0.0016}$	$173.8^{+0.8}_{-0.8}$		$0.1164^{+0.0019}_{-0.0019}$	$173.6^{+0.8}_{-0.8}$	

	CMS					
	CT14			NNPDF30		
	α_s	m_t	$\chi^2_{\min}$	α_s	m_t	$\chi^2_{\min}$
p_T^t	$0.1096^{+0.0017}_{-0.0015}$	$169.0^{+0.6}_{-0.6}$	0.68	$0.1109^{+0.0023}_{-0.0022}$	$170.5^{+0.6}_{-0.6}$	0.67
$m_{t\bar{t}}$	$0.1108^{+0.0013}_{-0.0012}$	$168.5^{+0.8}_{-0.8}$	4.43	$0.1055^{+0.0021}_{-0.0020}$	$168.8^{+0.9}_{-0.9}$	2.02
y_t	$0.1100^{+0.0021}_{-0.0018}$	$169.7^{+1.2}_{-1.1}$	2.20	$0.1233^{+0.0022}_{-0.0021}$	$175.3^{+1.1}_{-1.0}$	2.89
$y_{t\bar{t}}$	$0.1191^{+0.0013}_{-0.0015}$	$177.0^{+1.3}_{-1.3}$	1.85	$0.1132^{+0.0016}_{-0.0016}$	$171.8^{+1.2}_{-1.2}$	0.85

$M_{t\bar{t}}$ distribution fits

- Sensitivity to mass dependence in $M_{t\bar{t}}$ concentrated in first bin
  - Calculations with different m_t will show majority of variation here with tail largely unaffected
- Experimental binning begins at 345 GeV which is above threshold for $m_t < 172.5$ GeV. Leads to missed events for $m_t < 172.5$ GeV.
- MC mass 172.5 assumed in published measurements
  - For consistent mass extraction, extrapolations with different vales of m_t needed in MC.
- In our calculations, binning is consistent with experimental binning for all values of m_t (i.e. missed events not included).