MiNNLO$_{PS}$ and prospects for top pair production

Javier Mazzitelli

In collaboration with: Pier Monni, Paolo Nason, Emanuele Re,
Marius Wiesemann and Giulia Zanderighi

Max-Planck-Institut
für Physik

LHC TOP WG meeting, November 24th 2020

Quick overview

- The MiNNLO method for colour singlet production
- q_T-resummation for top-quark pair production
- Extending MiNNLO to the production of top pairs
- Preliminary results

MiNNLO method

- While NNLO QCD corrections for $t\bar{t}$ are available, event generators are at most NLO accurate
- Matching of NNLO QCD calculations to parton showers is crucial for more precise simulations
- Different methods have been proposed so far, all of them applied for the moment only to the production of a colour singlet (F)

[Hamilton, Nason, Oleari, Zanderighi, 2012]
[Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi, 2013]
[Höche, Li, Prestel, 2014]
[Monni, Nason, Re, Wiesemann, Zanderighi, 2019]
[Monni, Re, Wiesemann, 2020]

- We focus on the MiNNLO method, which is built upon the MiNLO procedure
  [Monni, Nason, Re, Wiesemann, Zanderighi, 2019] [Hamilton, Nason, Oleari, Zanderighi, 2012]

MiNLO method:

Based on appropriate choice of scales + inclusion of Sudakov form factors in NLO multijet computation

MiNLO for F+J is finite even when 1st jet is unresolved

Inclusive NLO accuracy is achieved by including B_2 term in the Sudakov and setting scales (in R, V and Sudakov expansion) to q_T

[Hamilton, Nason, Oleari, Zanderighi, 2012]

↓

MiNNLO method extends inclusive accuracy to NNLO while keeping NLO accuracy for F+J

- Derivation based on the connection between MiNLO and q_T-resummation
- Starting from the factorization formula

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times \left[H C_1 C_2\right]_{c\bar{c};a_1 a_2} \times f_{a_1} f_{a_2}$$

the following identity can be derived for the differential NNLO cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \frac{\mathrm{d}}{\mathrm{d}p_{\mathrm{T}}}\left\{\exp[-\tilde{S}(p_{\mathrm{T}})]\mathcal{L}(p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}})$$

Singular piece written as a total derivative

Finite reminder

$\tilde{S}$: Sudakov exponent (in p_T-space formulation)

L: PDFs x virtuals(F) x collinear functions

$$\mathcal{L} \sim [\tilde{H}_F \tilde{C}_1 \tilde{C}_2] \otimes f_1 \otimes f_2$$

- The luminosity *L* includes all the ingredients needed up to NNLO: $C^{(1)}, C^{(2)}, H^{(1)}, H^{(2)}$
- The above expression is then recast in the following way:

$$\frac{\mathrm{d}}{\mathrm{d}p_{\mathrm{T}}}\left\{\exp[-\tilde{S}(p_{\mathrm{T}})]\mathcal{L}(p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}}) = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{D(p_{\mathrm{T}}) + \frac{R_f(p_{\mathrm{T}})}{\exp[-\tilde{S}(p_{\mathrm{T}})]}\right\}$$

$$D(p_{\mathrm{T}}) \equiv -\frac{\mathrm{d}\tilde{S}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{\mathrm{d}\mathcal{L}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}$$

- The resulting expression can be expanded in α_S keeping terms needed to achieve NNLO accuracy after integrating over p_T

- Careful with power counting! NNLO accuracy defined in terms of $\alpha_S(Q)$ expansion

Hard scale

$$\int_{\Lambda}^{Q} \mathrm{d}p_{\mathrm{T}} \frac{1}{p_{\mathrm{T}}} \alpha_{\mathrm{S}}^{m}(p_{\mathrm{T}}) \ln^{n} \frac{Q}{p_{\mathrm{T}}} \exp(-\tilde{S}(p_{\mathrm{T}})) \approx \mathcal{O}\left(\alpha_{\mathrm{S}}^{m-\frac{n+1}{2}}(Q)\right)$$

- The expanded result is then embedded in the $\overline{B}$ function in the POWHEG method

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{FJ}}} = \bar{B}(\Phi_{\mathrm{FJ}}) \times \left\{ \Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \int \mathrm{d}\Phi_{\mathrm{rad}} \Delta_{\mathrm{pwg}}(p_{\mathrm{T,rad}}) \frac{R(\Phi_{\mathrm{FJ}}, \Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})} \right\}$$

Generation of 1st emission

Generation of 2nd emission with usual POWHEG method

$$\bar{B}(\Phi_{\mathrm{FJ}}) \equiv \exp[-\tilde{S}(p_{\mathrm{T}})] \left\{ \frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi} \left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(1)} \left(1 + \frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi} [\tilde{S}(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^{2} \left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(2)} + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^{3} [D(p_{\mathrm{T}})]^{(3)} F^{\mathrm{corr}}(\Phi_{\mathrm{FJ}}) \right\}$$

Already in MiNLO

Extra term to achieve NNLO accuracy

Third order expansion of

Factor spreading the D(3) term over the F+J phase space

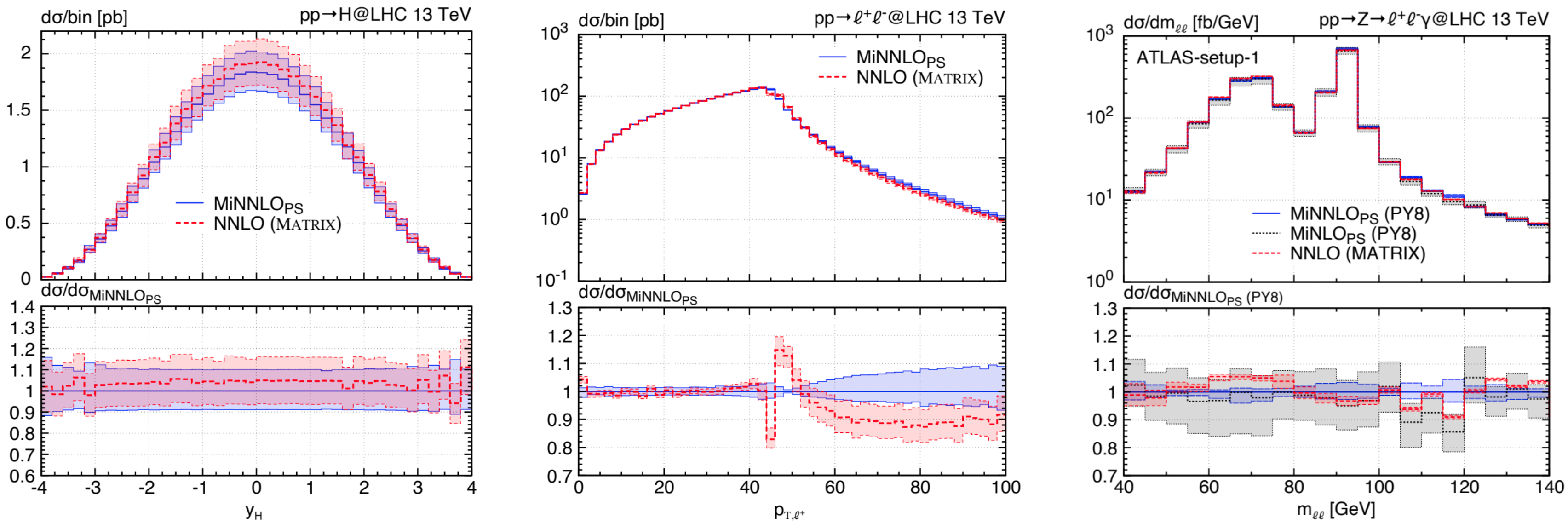
$$D(p_{\mathrm{T}}) \equiv -\frac{\mathrm{d}\tilde{S}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}} \mathcal{L}(p_{\mathrm{T}}) + \frac{\mathrm{d}\mathcal{L}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}$$

See also [Monni, Re, Wiesemann, 2020] for alternative definition of extra term

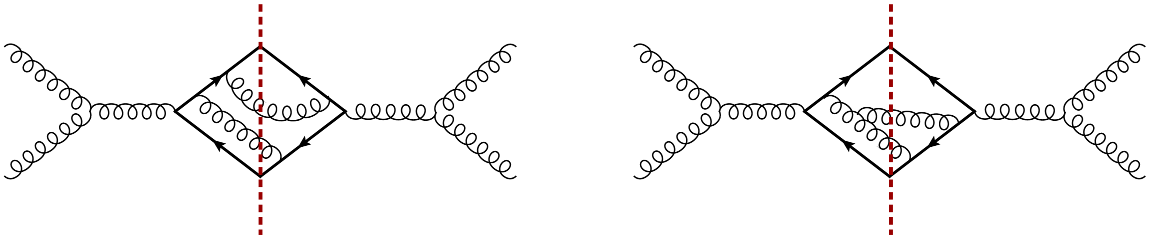
MiNNLO method

- Applied so far to the production of colour singlets: H, Drell-Yan, Zγ

[Monni, Nason, Re, Wiesemann, Zanderighi, 2019], [Monni, Re, Wiesemann, 2020], [Lombardi, Wiesemann, Zanderighi, 2020]



- Extension to $t\bar{t}$ production more complicated due to final state radiation!



- As before, the starting point can be the q_T-resummation formula

but...

...the more complicated colour structure doesn't allow to directly follow the derivation used for colour singlet

q_T-resummation for $t\bar{t}$

Following [Catani, Grazzini, Torre, 2014], see also [Li, Li, Shao, Yang, Zhu, 2012, 2013]

Bold: operator in colour space

$|M\rangle$: vector in colour space

Let's compare the colour singlet case with the $t\bar{t}$ formula:

Colour singlet: $$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times \left[H C_1 C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1} f_{a_2}$$

$t\bar{t}$ production: $$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times \left[\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) C_1 C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1} f_{a_2}$$

Effects coming from soft emissions from the FS contained in operator **Δ**

In the colour singlet case, H is given by the (IR-subtracted) all-orders matrix element for $c\bar{c} \to F$ → $$H = \mathrm{Tr}(\mathbf{H}) \sim \langle \mathcal{M} | \mathcal{M} \rangle$$

In the $t\bar{t}$ case, the presence of the operator **Δ** leads to non-trivial colour correlations → $$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) \sim \langle \mathcal{M} | \boldsymbol{\Delta} | \mathcal{M} \rangle$$

$$\boldsymbol{\Delta} \sim \exp\left\{-\int_{b_0^2/b^2}^{M} \frac{dq^2}{q^2}\, \boldsymbol{\Gamma}(\alpha_s(q))\right\}^{\dagger} \mathbf{D}(\alpha_s(b_0/b), \phi) \exp\left\{-\int_{b_0^2/b^2}^{M} \frac{dq^2}{q^2}\, \boldsymbol{\Gamma}(\alpha_s(q))\right\}$$

Exponential of soft anomalous dimension matrix

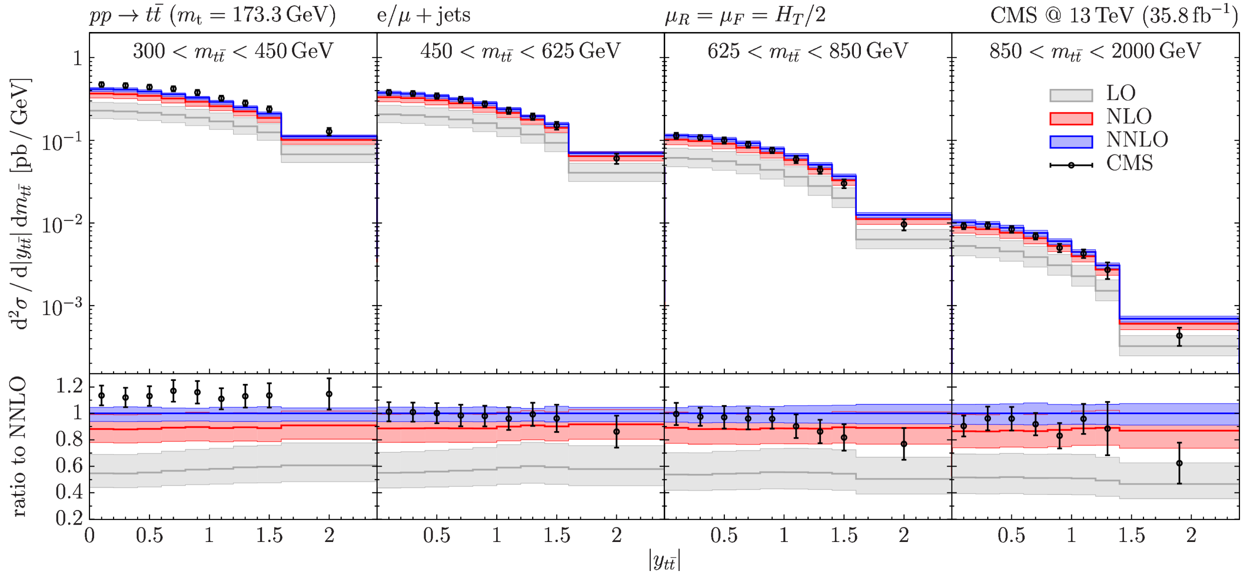
Operator leading to azimuthal correlations

[Catani, Grazzini, Torre, 2014]
[Catani, Grazzini, Sargsyan, 2017, 2018]

$$d\sigma^{(\text{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times \left[\text{Tr}(\mathbf{H}\boldsymbol{\Delta})C_1 C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1} f_{a_2}$$

$$\boldsymbol{\Delta} \sim \exp\left\{-\int_{b_0^2/b^2}^{M} \frac{dq^2}{q^2}\,\boldsymbol{\Gamma}(\alpha_s(q))\right\}^{\dagger} \mathbf{D}(\alpha_s(b_0/b),\phi)\,\exp\left\{-\int_{b_0^2/b^2}^{M} \frac{dq^2}{q^2}\,\boldsymbol{\Gamma}(\alpha_s(q))\right\}$$

- All the ingredients for NNLL+NLO resummation known (while $\mathbf{D}^{(2)}$ missing for NNLL+NNLO)
- The operator **D** satisfies $\langle \mathbf{D} \rangle_{\Phi,\text{av}} = 1$
- Even for q_T azimuthally-averaged cross sections, **D** contributes in the gluon channel due to the interference with the collinear coefficient functions (starting at NNLO)
- $\mathbf{D}^{(2)}$ contributes with a constant term at $O(\alpha_s^4)$ that vanishes upon azimuthal average
- Fixed-order expansion of the Φ-averaged factorization formula known up to order α_s^4 and applied to extend q_T-subtraction to heavy-quark production at NNLO



[Catani, Devoto, Grazzini, Kallweit, JM, 2019]

Extending MiNNLO for $t\bar{t}$

- What we stated before:

...the more complicated colour structure doesn't allow
to directly follow the derivation used for colour singlet

means more specifically that it is not straightforward to describe the NNLO XS as

$$\frac{d\sigma}{d\Phi_F dp_T} = \frac{d}{dp_T}\left\{ \exp[-\tilde{S}(p_T)]\mathcal{L}(p_T) \right\} + R_f(p_T) \qquad (1)$$

However, in eq. (1) we are only interested in the fixed order accuracy

⬇

We can modify the $t\bar{t}$ factorization formula as long as we keep
NNLO accuracy (and LL in view of the matching with the shower)

⬇

We can take it into a shape that resembles the colourless final state case

⬇

Connection to MiNNLO derivation becomes simpler

Extending MiNNLO for $t\bar{t}$

E.g.: one could perform the following approximation (schematically)

$$\langle \mathcal{M} | \exp(\mathbf{\Gamma}) | \mathcal{M} \rangle \approx \exp\left(\underbrace{\frac{\langle \mathcal{M}^{(0)} | \mathbf{\Gamma} | \mathcal{M}^{(0)} \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle}}_{\text{Exponential of just a number}} \right) \underbrace{\langle \mathcal{M} | \mathcal{M} \rangle}_{\text{Colourless-like } H}$$

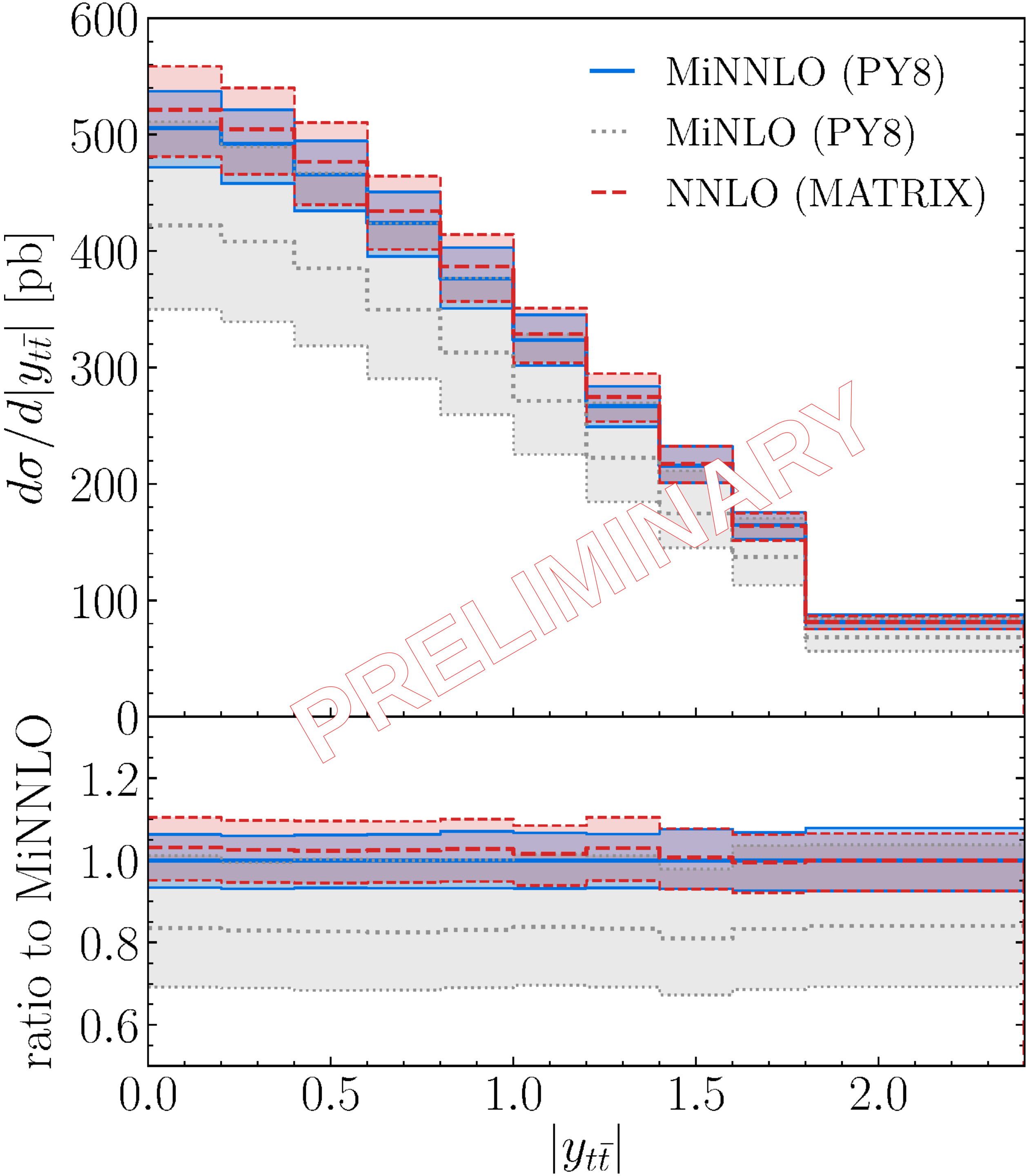
and correct for the mistake that is being made up to NNLO

Obs: there are several ways of writing this ‘effective factorization formula’, all formally NNLO accurate. But higher order terms can still have an impact in certain PS regions

⬇

Work still in progress, different options need to be studied!

Preliminary results



- Implementation based upon $t\bar{t}$+J process in POWHEG [Alioli, Moch, Uwer, 2012]
- Preliminary results for the rapidity of the top-pair system
- QCD corrections mostly flat for this distribution
- Great **agreement** between **MiNNLO** and **NNLO**
- Similar size in MiNNLO and NNLO **scale uncertainties**, strong reduction w.r.t. MiNLO
- Events showered using PY8, though this observable is not sensitive to shower effects

Obs: no direct correspondence between the scales used in MiNNLO (proportional to p_T) and the ones of the F.O. prediction. Upon integration they are of the order of $m_{t\bar{t}}$ (which is the scale used at NNLO in this comparison)

Summary

- The MiNNLO method provides a way to match NNLO QCD to parton showers
- Applied so far to the production of colourless final states
- Extension to top-quark production poses a challenge due to the more complicated colour structure
- Progress has been made based on simplification of factorization formula when higher-order terms (not relevant to NNLO accuracy) are neglected
- Work in progress, but preliminary results look promising
- Stay tuned for news on $t\bar{t}$@NNLO$_{PS}$

Thanks!