

Modeling final state radiation on a quantum computer

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QuantISED HEP initiative



UW EPE Seminar

October 14, 2020



Overview

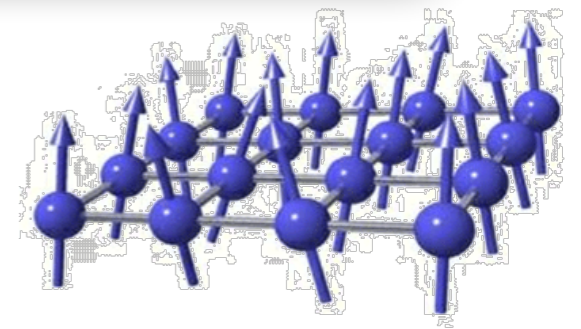
2

I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. - Feynman

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~ Outline ~

Quantum machines
("quantum computers")



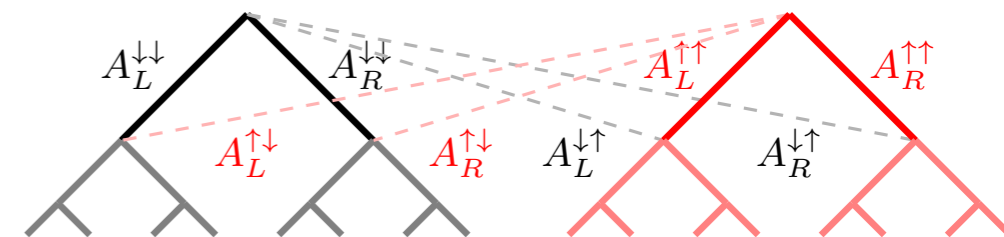
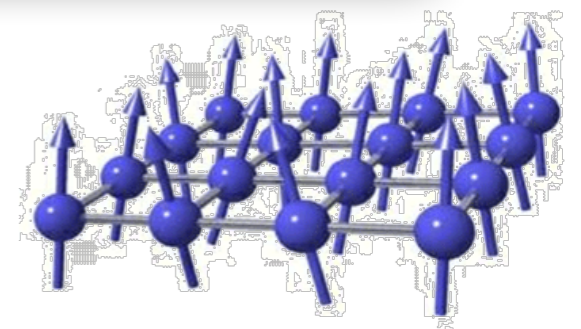
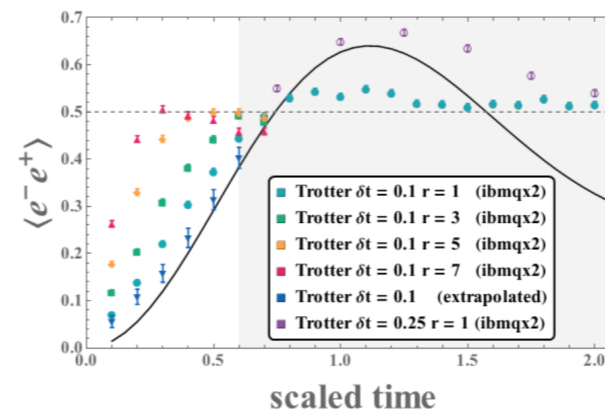
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A toy model



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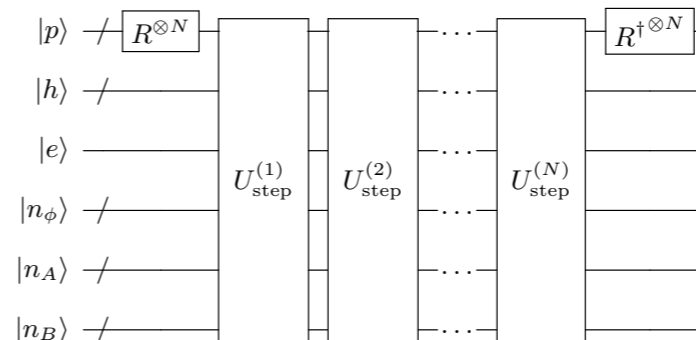
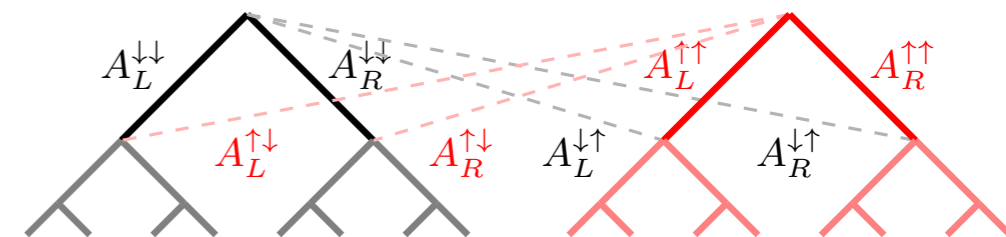
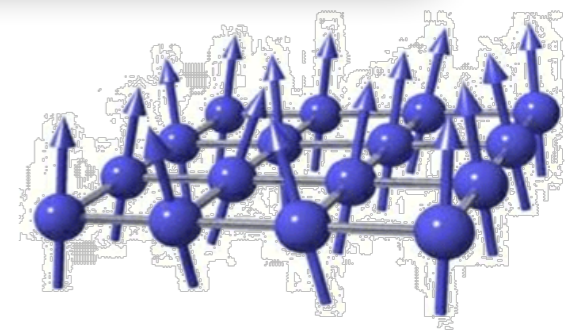
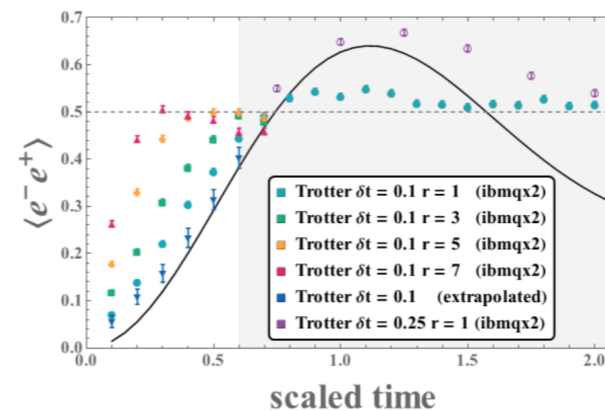
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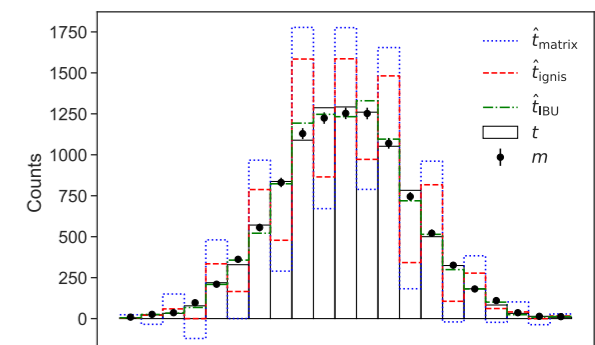
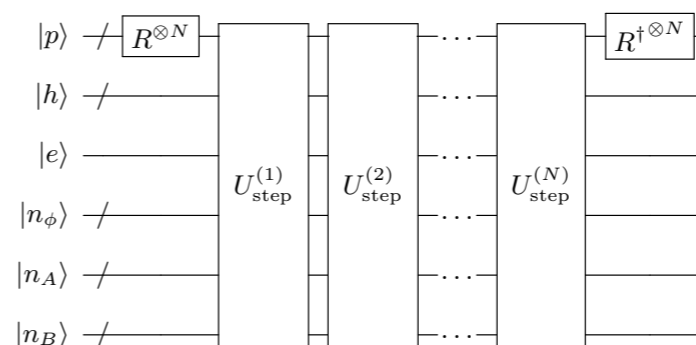
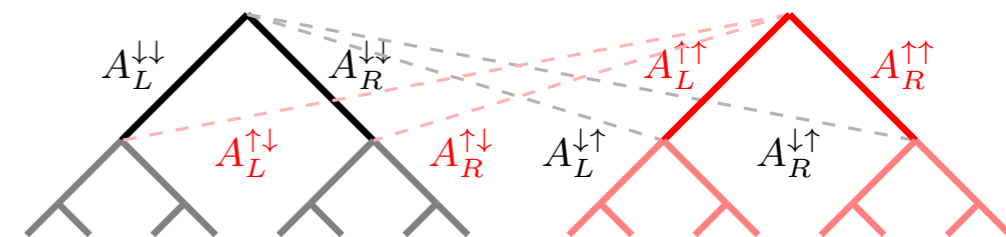
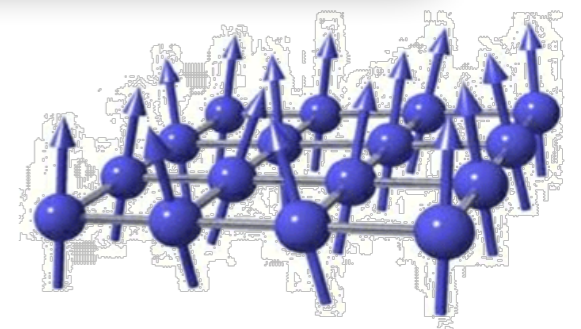
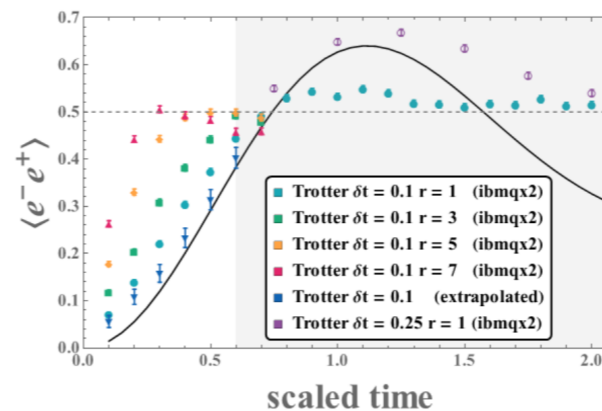
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Mitigating of noise



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Quantum machines
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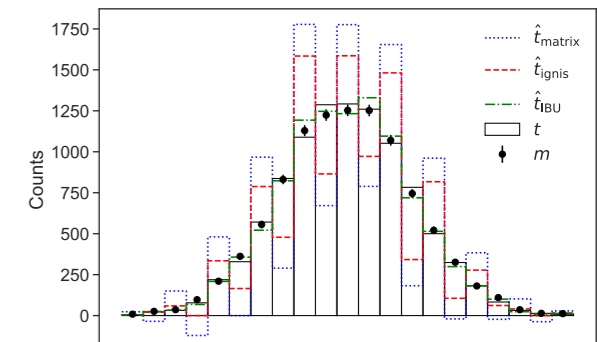
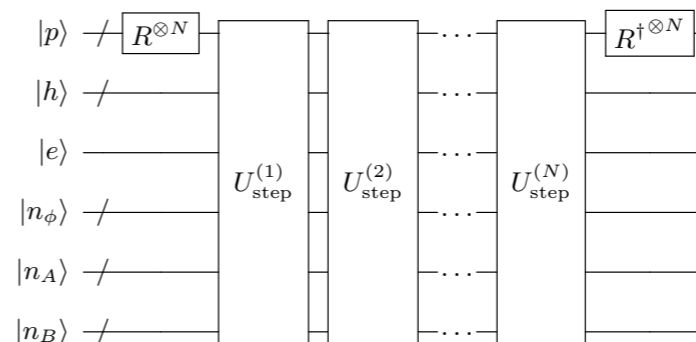
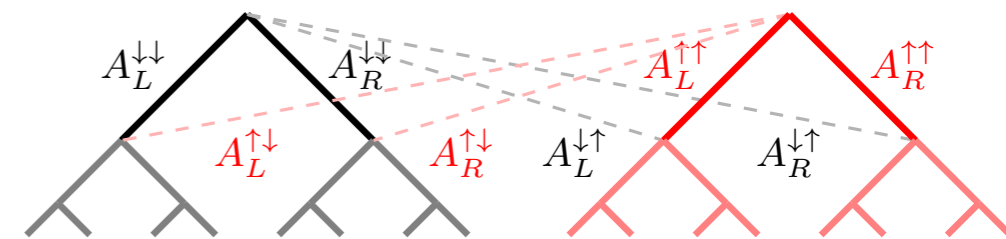
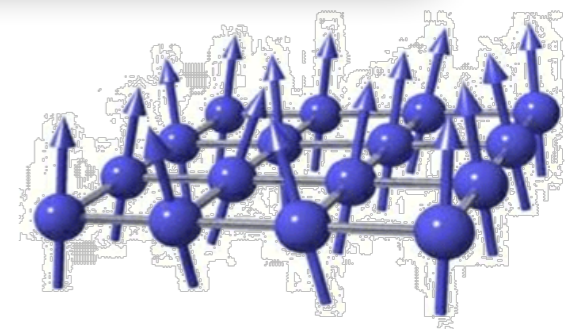
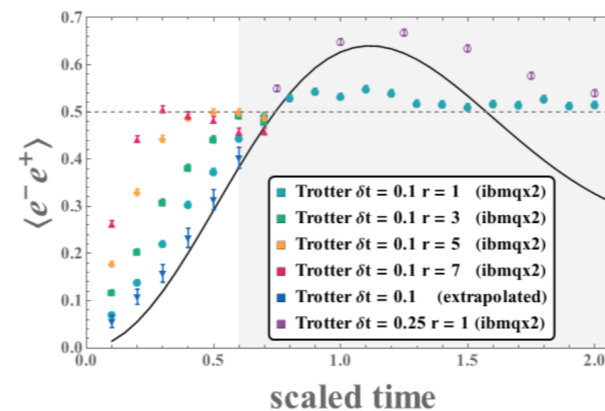
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A toy model

A simple, but real model

Mitigating of noise

The future



Quantum computers



Goal: implement our system's Hamiltonian (e.g. the SM) in a proxy system ("quantum computer") and let it evolve.

What can be a proxy system?

...any quantum system, like a collection of spins.

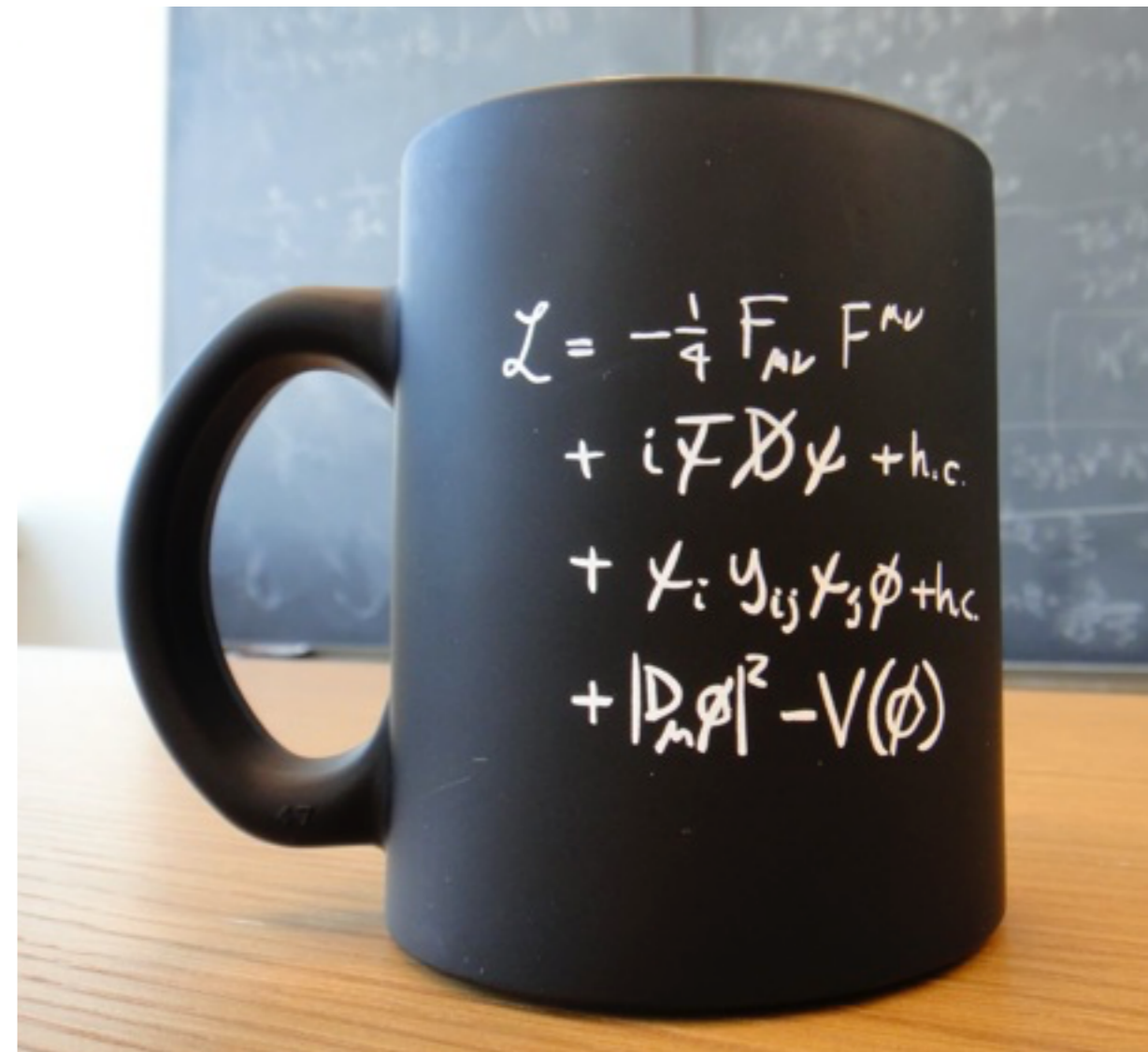
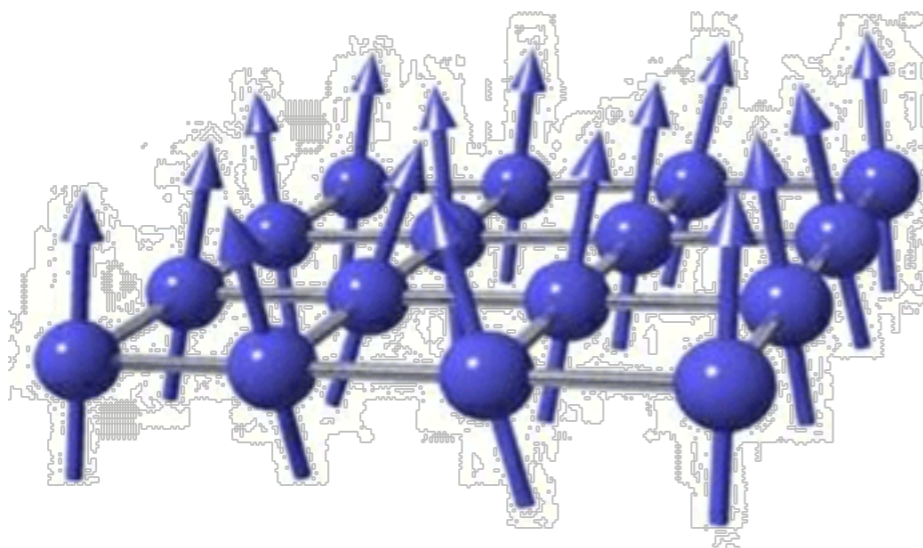


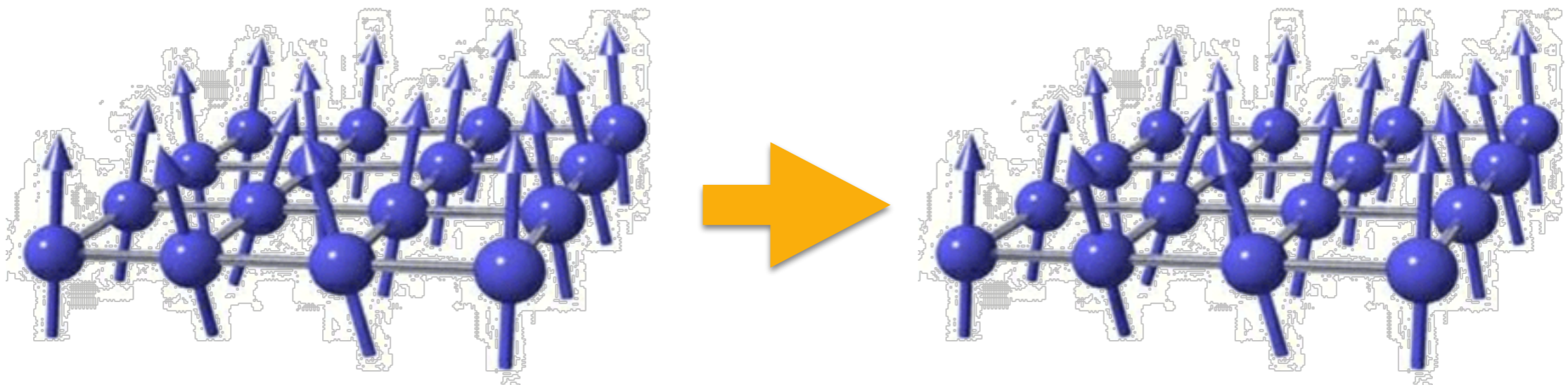
Image credit: Flip Tanedo

Analog versus Digital Quantum Circuits

10

Goal: implement our system's Hamiltonian (e.g. the SM) in a proxy system ("quantum computer") and let it evolve.

The best quantum computer is the one that looks just like the system you are trying to model!



Analog versus Digital Quantum Circuits

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The best quantum computer is the one that looks just like the system you are trying to model!

Not always possible!

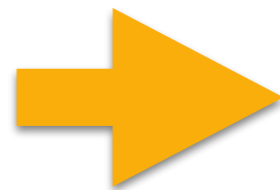
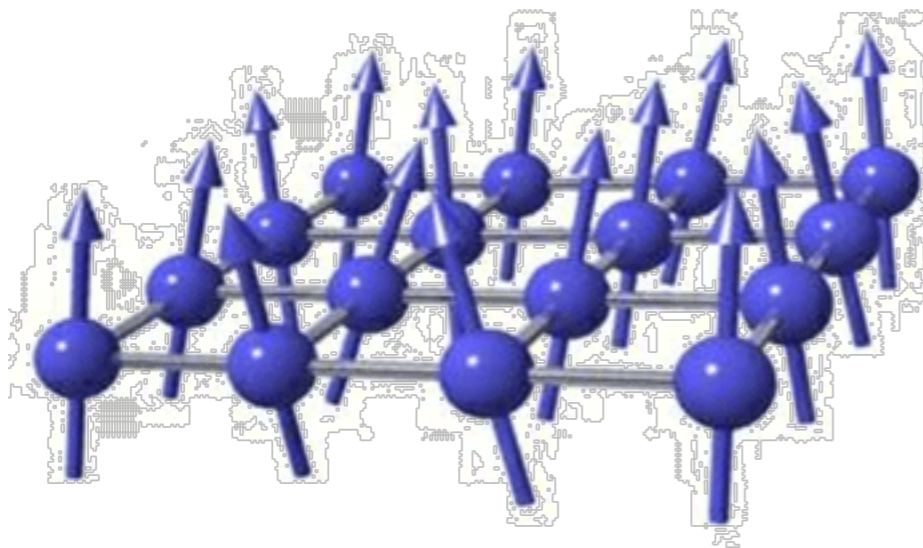


Analog versus Digital Quantum Circuits

12

Goal: implement our system's Hamiltonian (e.g. the SM) in a proxy system ("quantum computer") and let it evolve.

In this setup, the possibilities are endless; the key is efficiency.



There is no consensus on architecture, but many efforts for universal quantum computing use superconductors.

I'm not going to talk about hardware, though it is an exciting topic.

Modern Universal Quantum Computers

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I'm not going to talk about hardware, though it is an exciting topic.



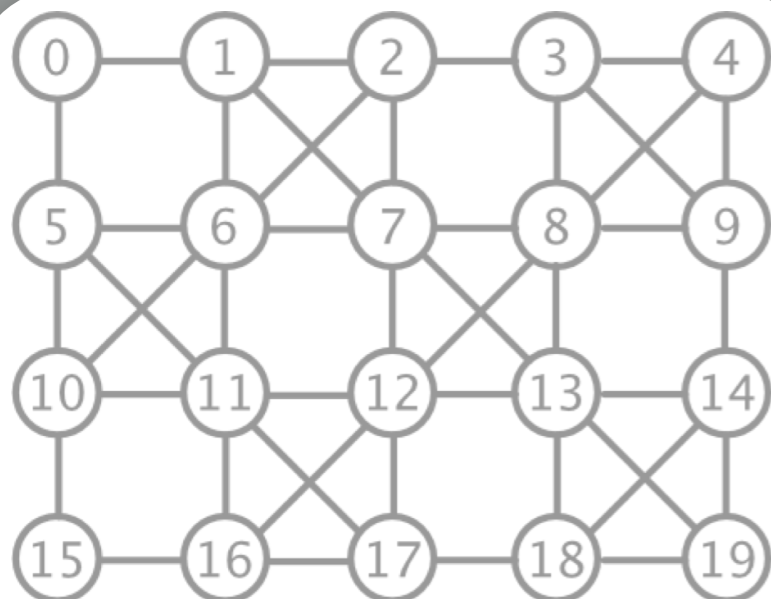
classical computing in the 1960's



quantum computing now

A **qubit** is an abstract representation of a quantum system that can be in a superposition of two states (often thought of as a spin)

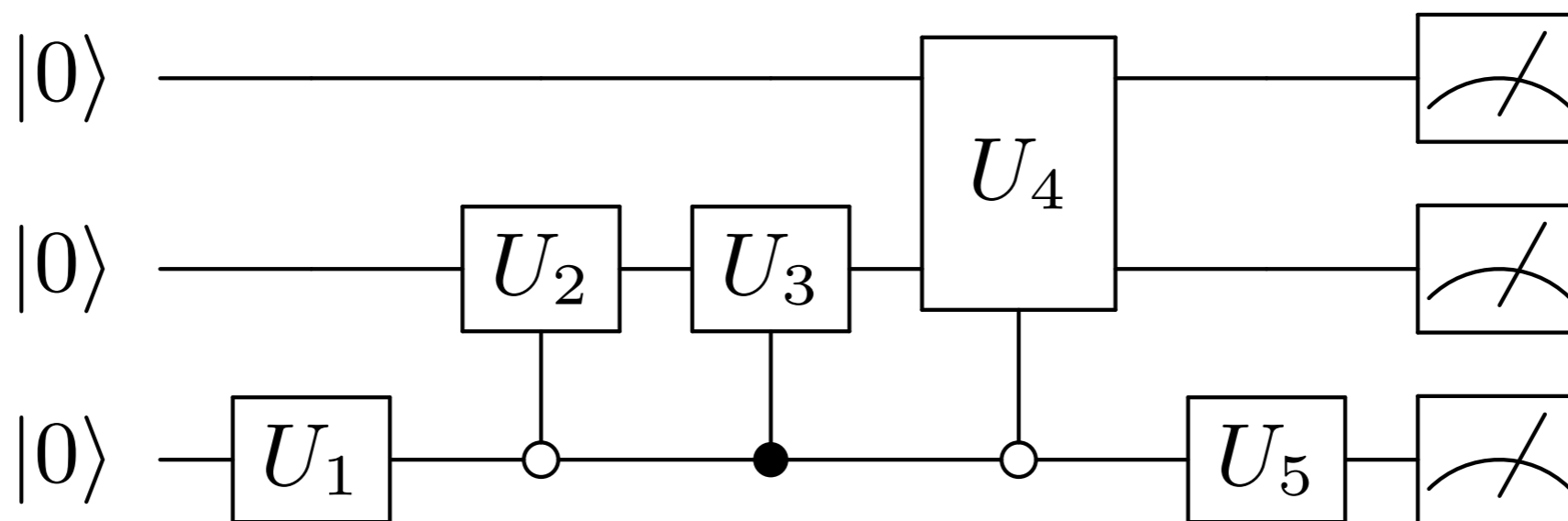
The best quantum computers have $O(10)$ **qubits** with $O(1)$ connections per qubit and can stay coherent for $O(1000)$ operations.



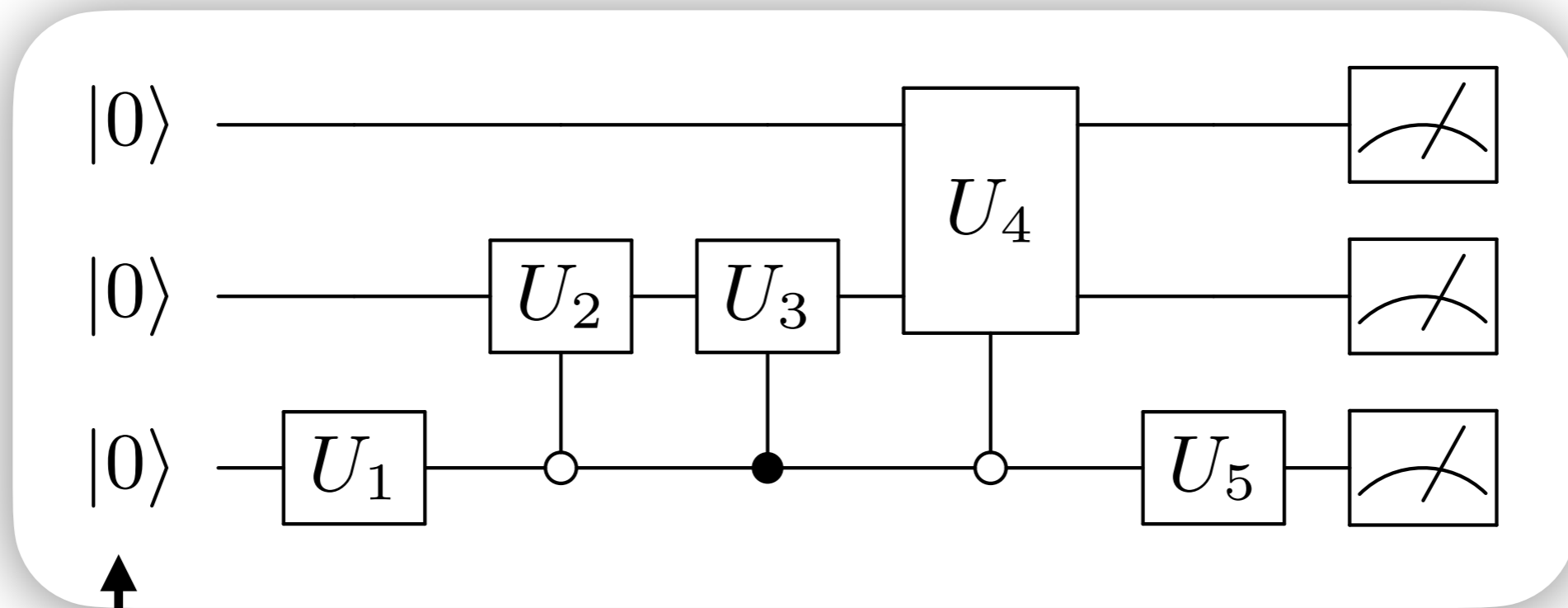
This is one of IBM's 20-qubit quantum computers. Lines represent connections.

Just like a classical computer, one can write programs for a universal quantum computer.

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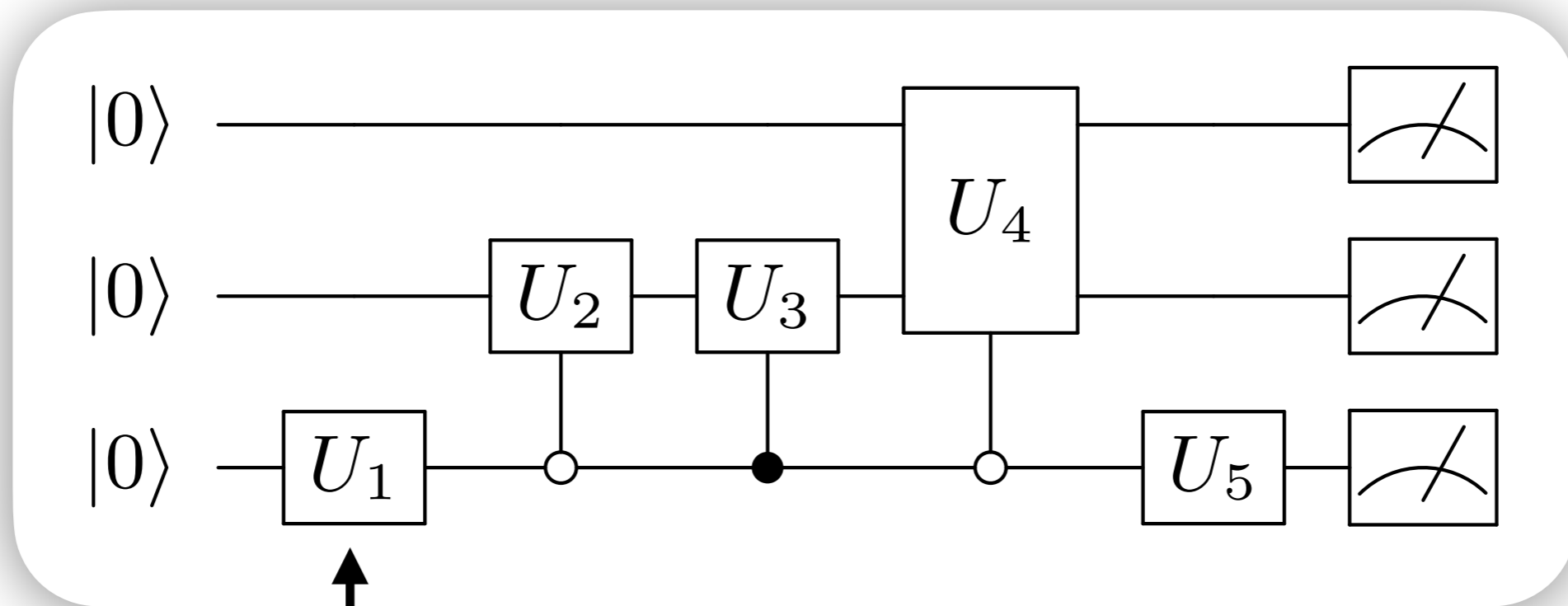


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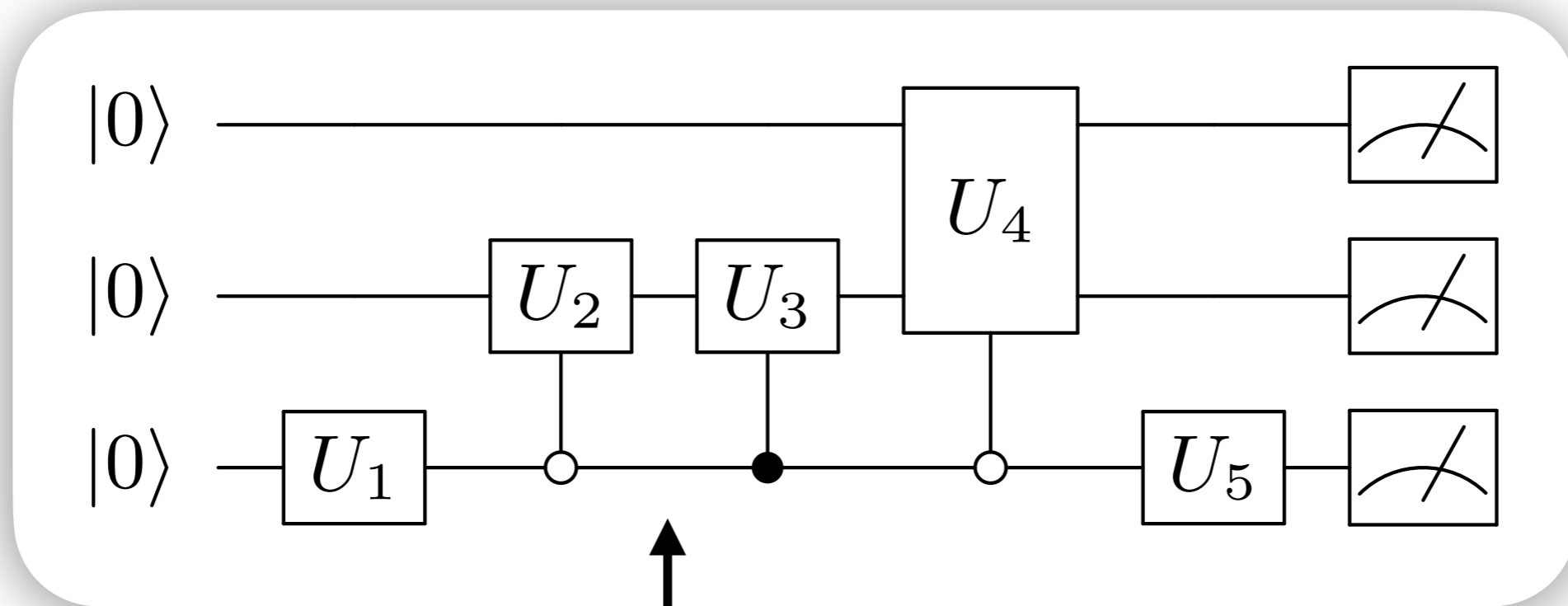
↑
Initialize in the ground state.

Just like a classical computer, one can write programs for a universal quantum computer.



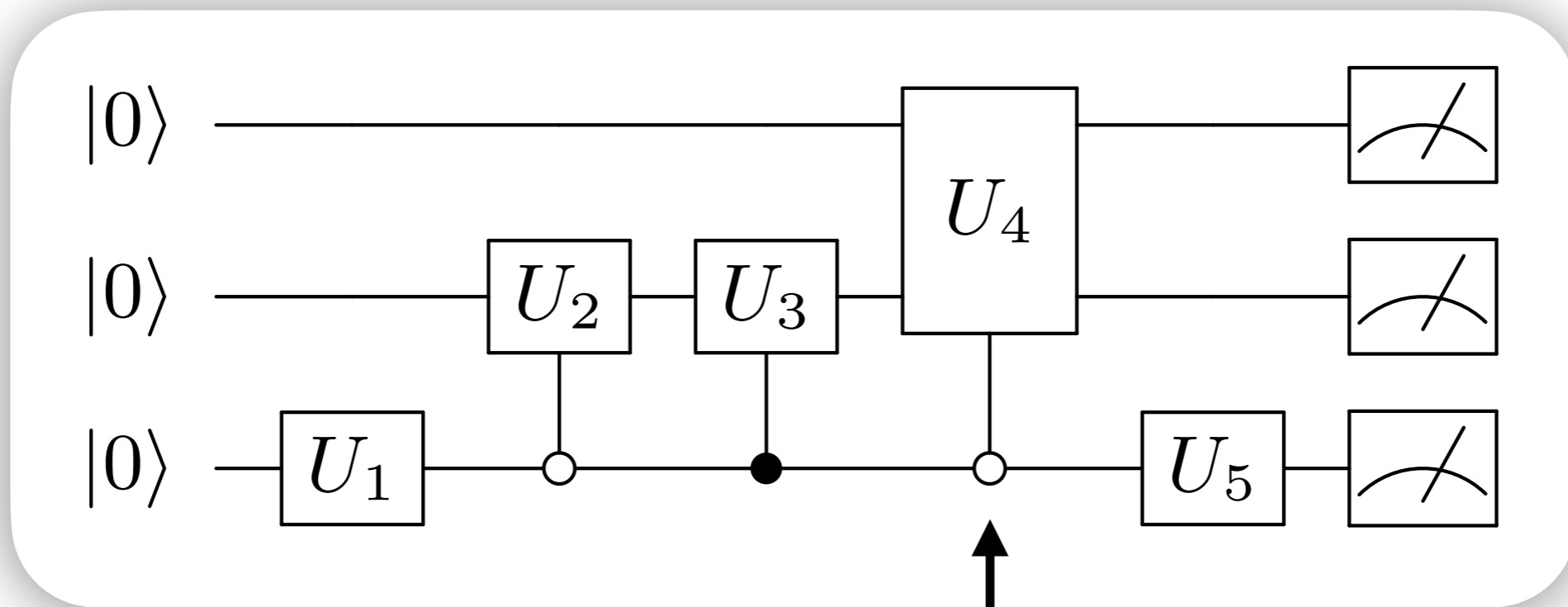
Apply unitary matrix U_1 to the third qubit

Just like a classical computer, one can write programs for a universal quantum computer.



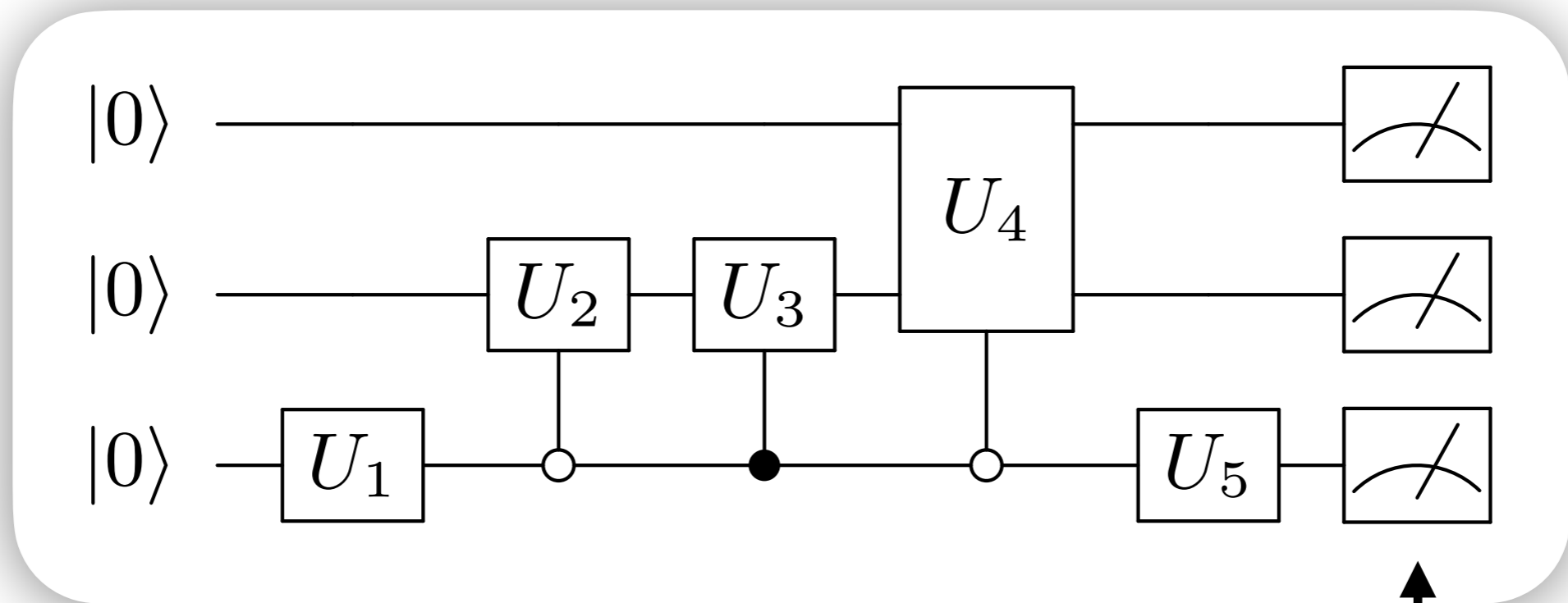
Apply unitary matrix U_2 to the second qubit when the third is 0, else apply U_3 .

Just like a classical computer, one can write programs for a universal quantum computer.



Apply unitary matrix U_4 to both the first and second qubits when the third is 0.

Just like a classical computer, one can write programs for a universal quantum computer.

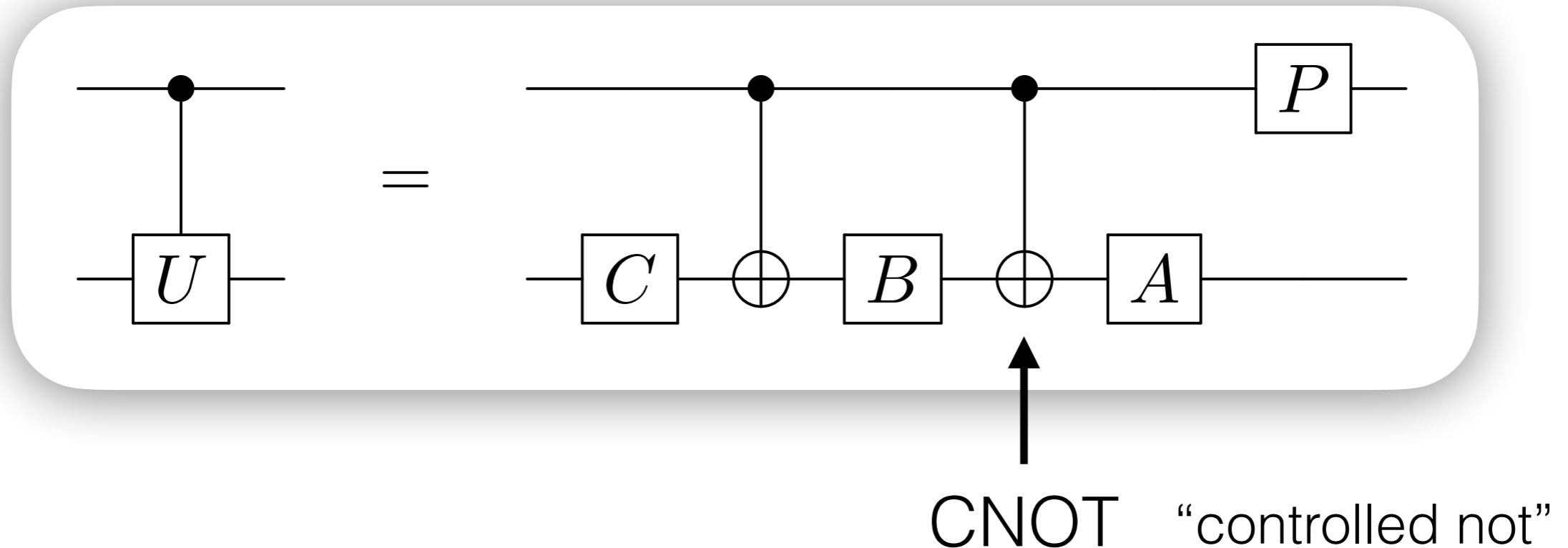


↑
Measure all
the qubits

Challenges with current computers

23

In practice: only* controlled operation that is allowed is CNOT (swap if 1 otherwise do nothing) ... need to decompose.



There is no compiler ... need to do circuit decomposition by hand (!)

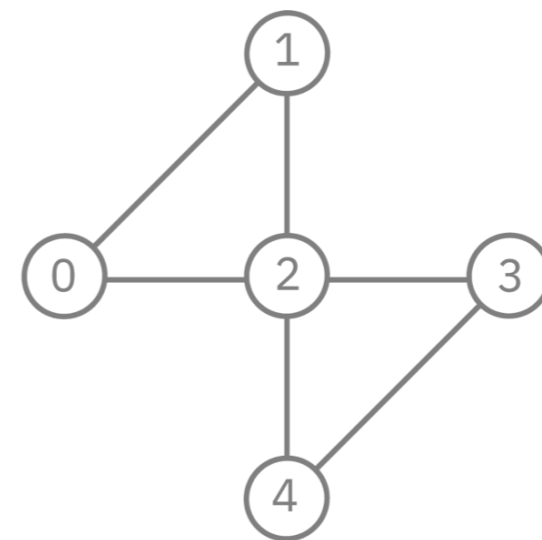
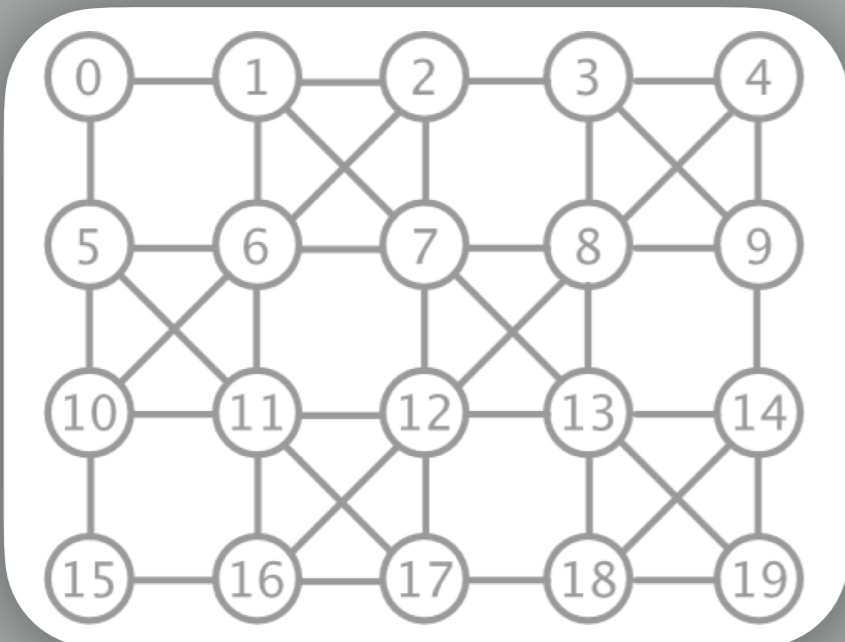
**Some computers are starting to have other basic operations, like the SWAP.*

Challenges with current computers

24

In practice: only controlled operation that is allowed is CNOT (swap if 1 otherwise do nothing) ... need to decompose.

Circuit implementation is architecture-dependent
need to know what connections are available
(can swap, but cannot copy (“clone”) qubits!)



Challenges with current computers

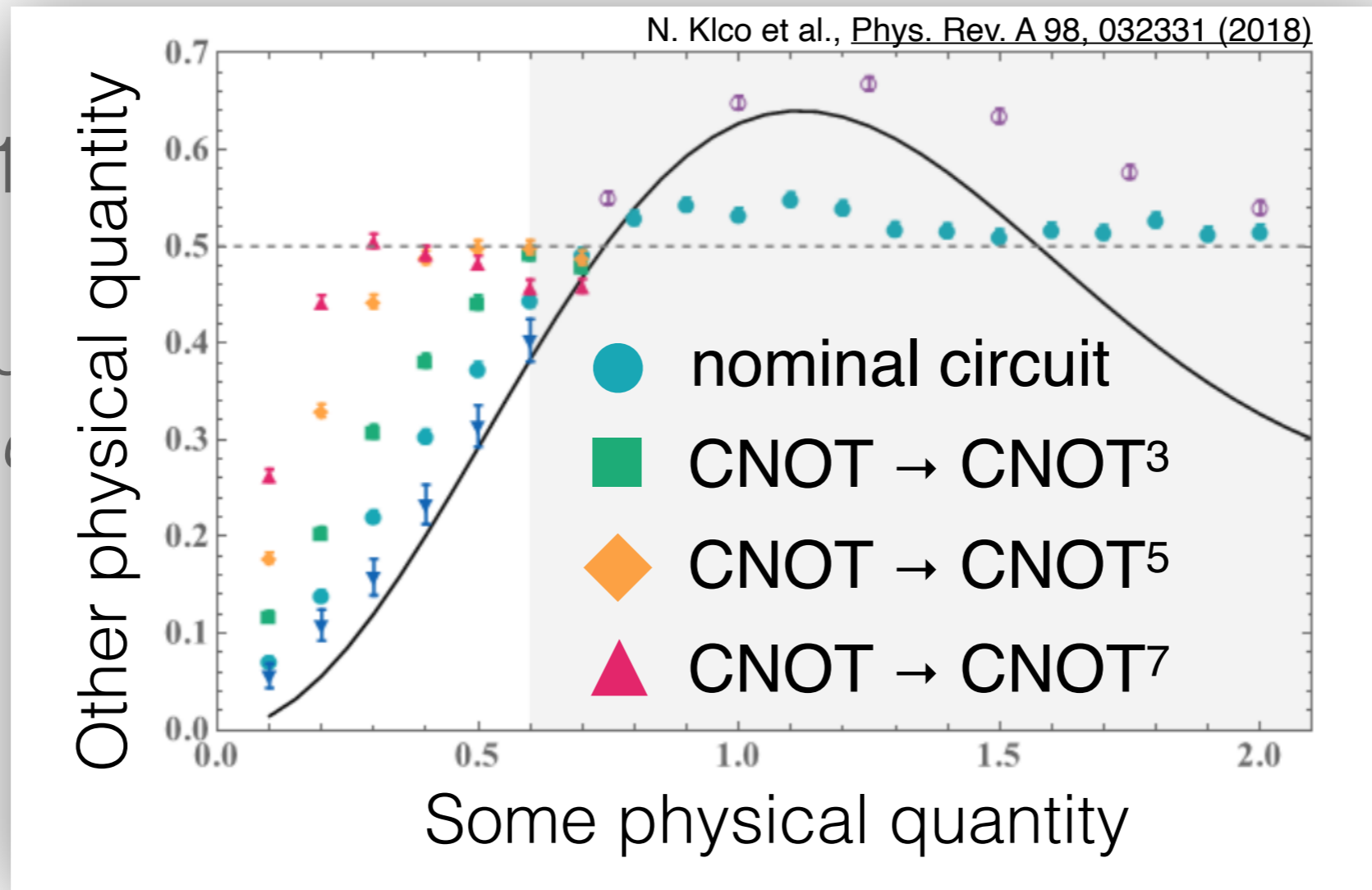
25

In practice:
(swap if 1

Circu
ne

d is CNOT
compose.

ident
ble



N.B. CNOT²
= identity

Most importantly: current quantum computers are **super noisy**. Need to minimize number of operations.

Caveats aside, there is a good reason to be excited.

There have been impressive leaps in hardware, “firmware”, & algorithms in the last years and interest has exploded.

↪ perhaps some misguided ...

Will you have a QPU in your laptop 5 years from now?

No. But you may be able to run on a QPU in 5 years that allows you to make a calculation that was not possible before (!)

Potential of quantum computers

27

Caveats aside, there is a good reason to be excited.

There have been impressive leaps in hardware, “firmware”, & algorithms in the last years and interest has exploded.

Now on to QFT!

perhaps some misguided ...

Will you have a QPU in your laptop 5 years from now?

No. But you may be able to run on a QPU in 5 years that allows you to make a calculation that was not possible before (!)

Why is this more challenging than e.g. quantum chemistry? (the “early” scientific adapter of QC)

- Continuous degrees of freedom (every spacetime point)
+ discrete and continuous quantum numbers.

Two traditional approaches:

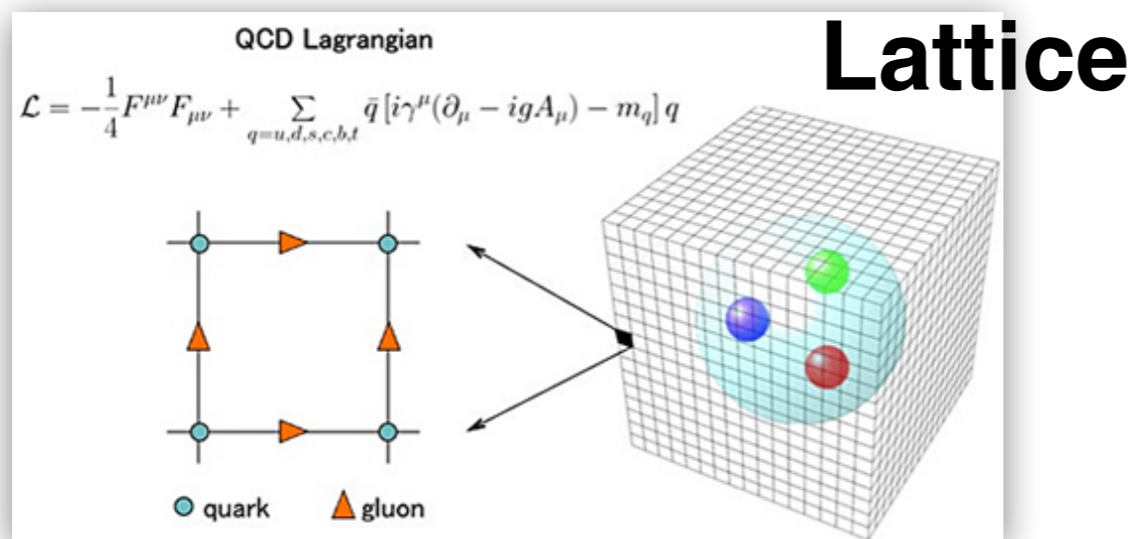


Image credit: <http://ipc-clermont.in2p3.fr/IMG/theorie/LQCD2.jpg>

Perturbation theory

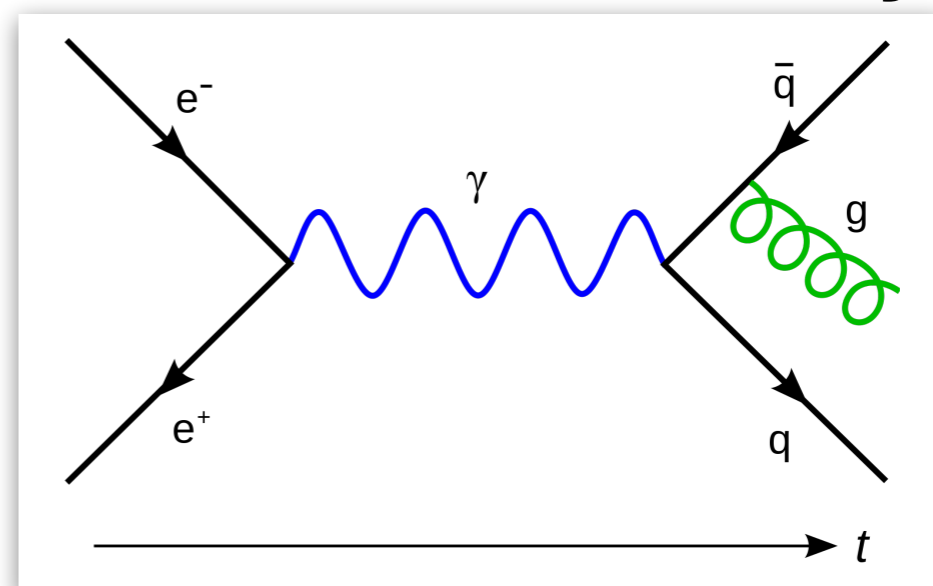


Image credit: https://en.wikipedia.org/wiki/Feynman_diagram

Pro: Full theory

Con: Dynamics are too hard

(already using super computers)

Pro: Can do high-energy dynamics

Con: An approximation

...and combinatorially many diagrams

Perturbation theory

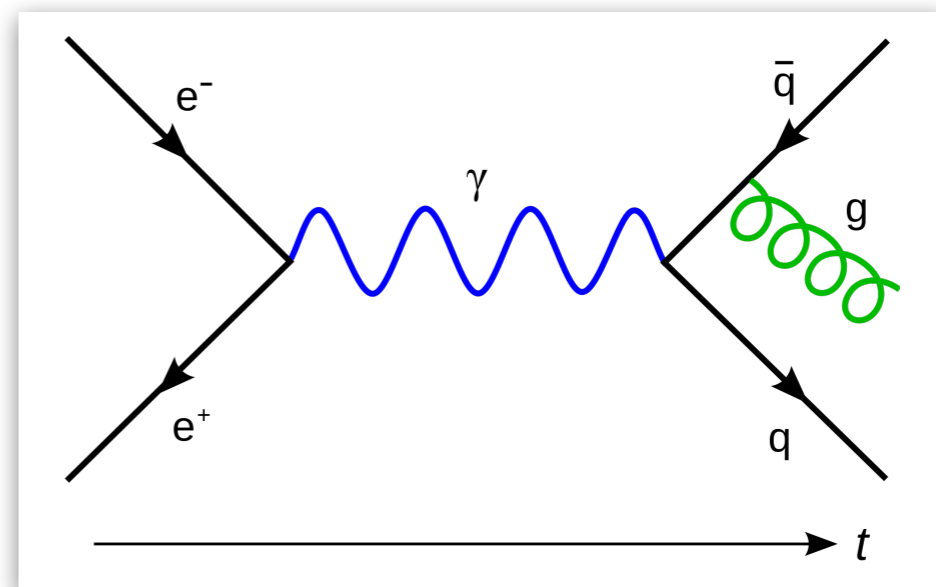


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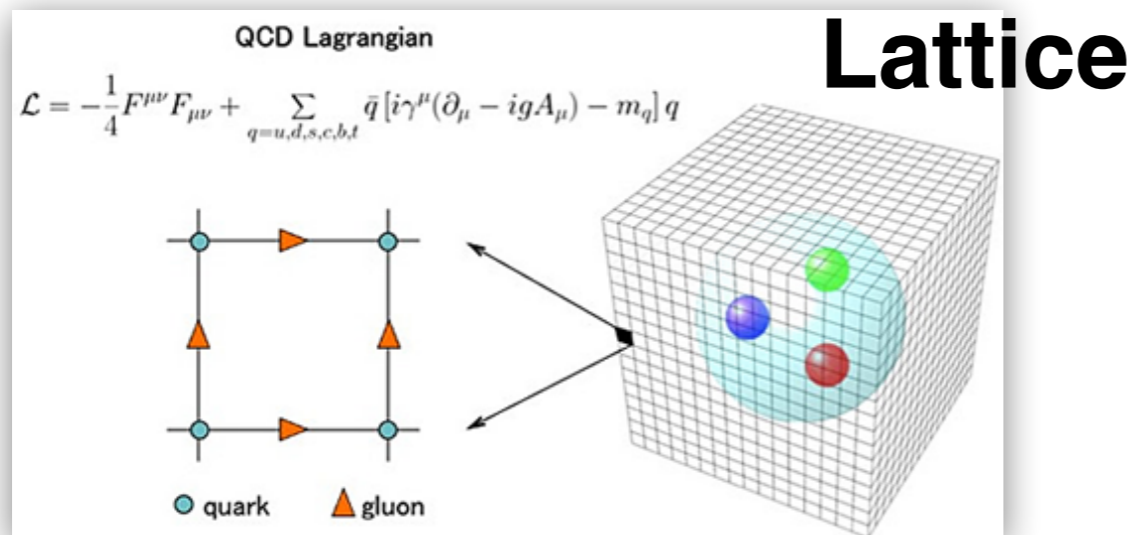


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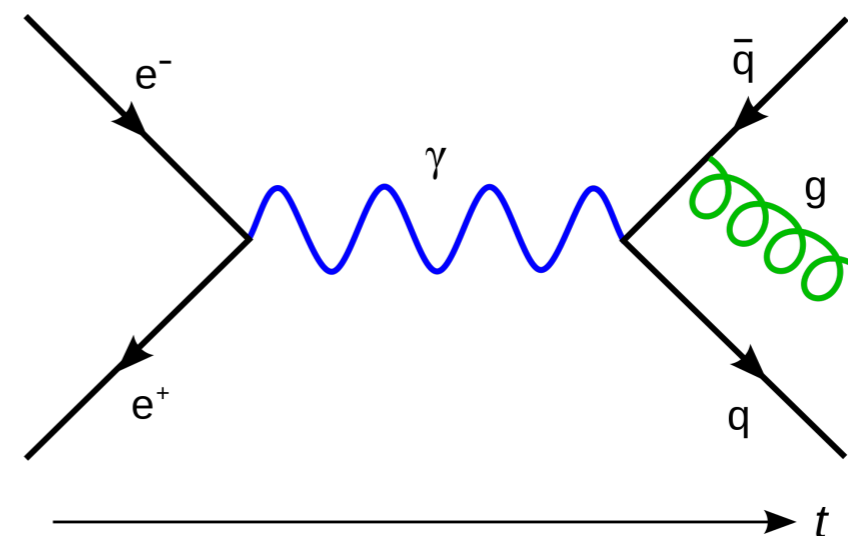


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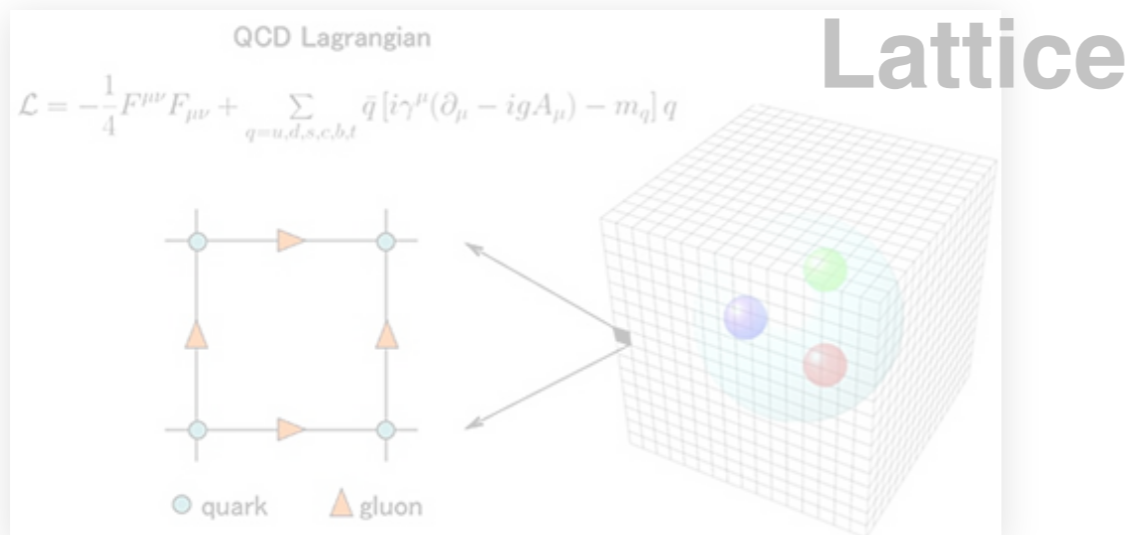


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“Quantum effects” in high energy scattering

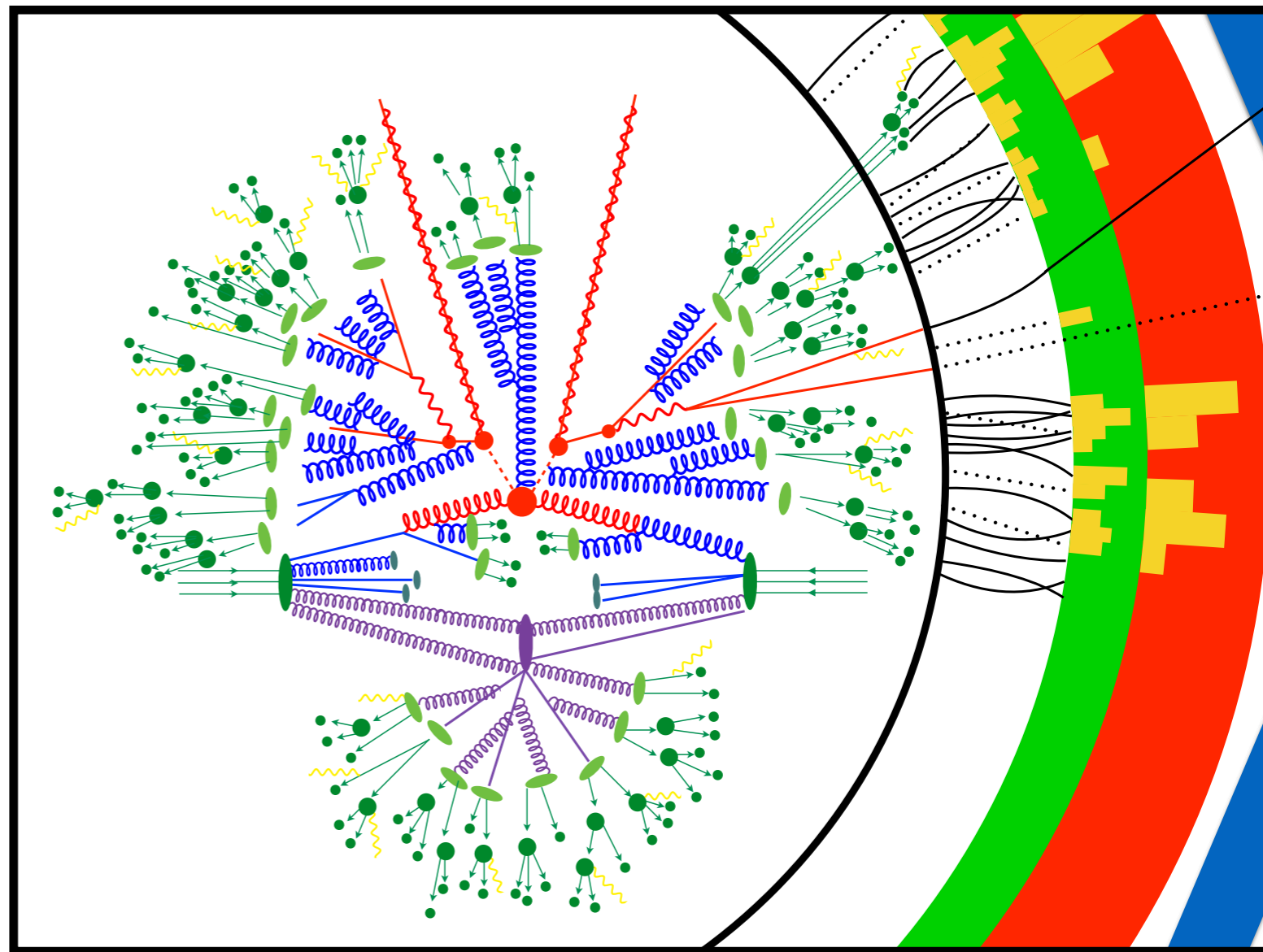
31

Let me take a step back and describe where we are in modeling inference & entanglement for high energy scattering experiments.

Image inspired by JHEP 02 (2009) 007

Simulation at the Large Hadron Collider:
length scales from
 10^{-20} m to 1 m (!)

...only possible because
of the Markov Property:
physics at different
scales **factorizes**



“Quantum effects” in high energy scattering

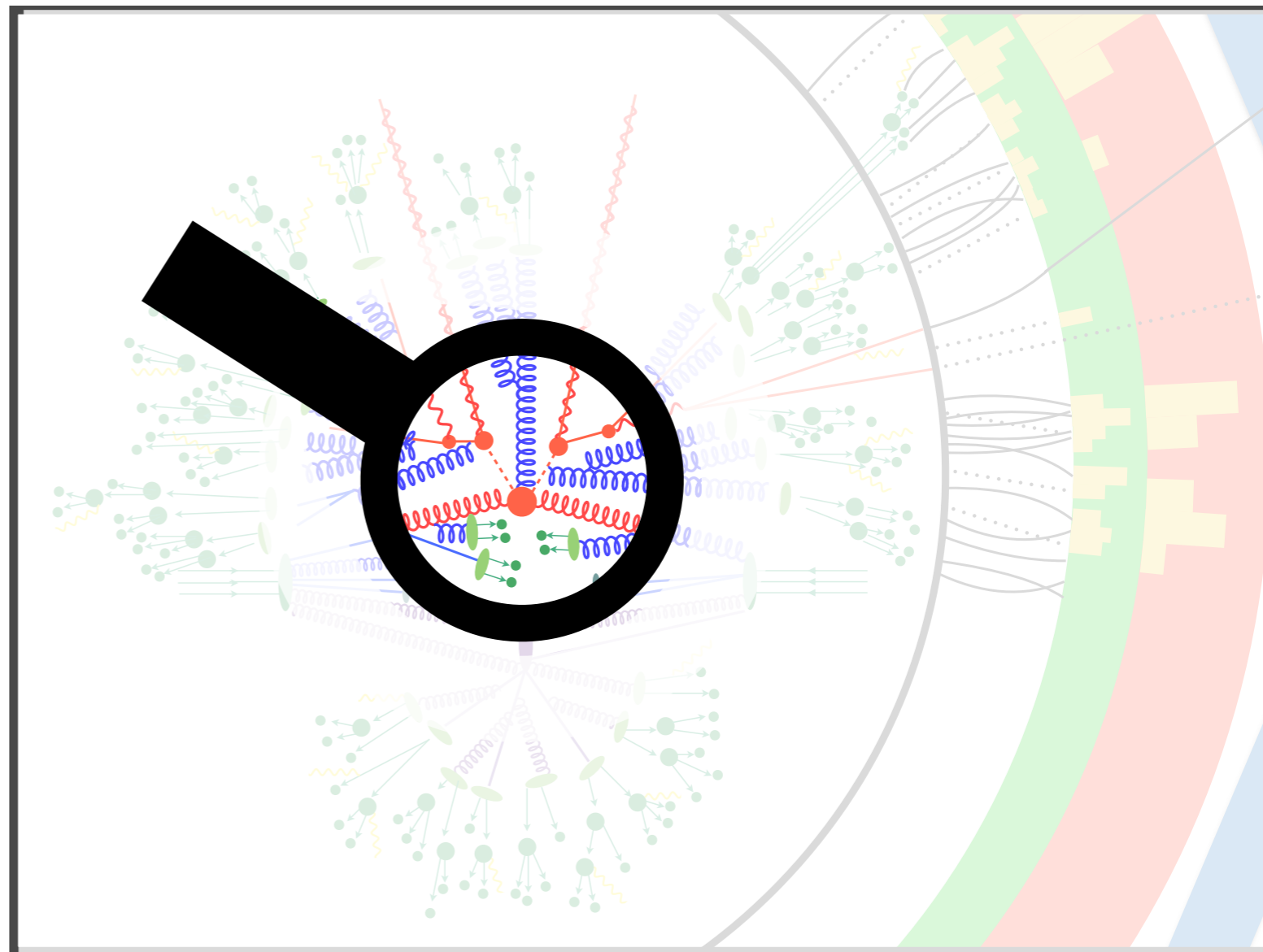
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Image inspired by JHEP 02 (2009) 007

Step 1: “Hard scatter”

very hard for lattice methods because high energy = fine grid



“Quantum effects” in high energy scattering

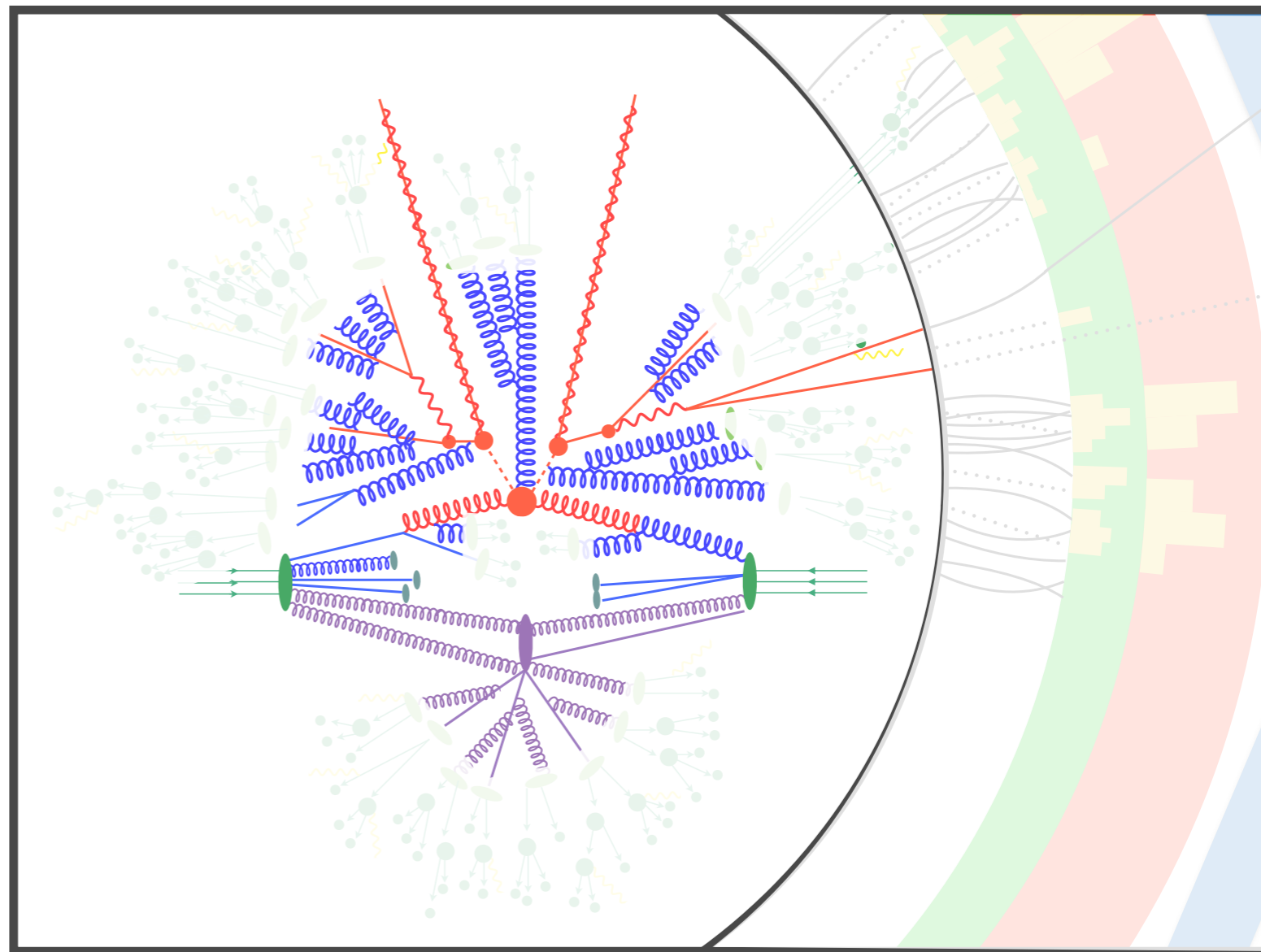
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Step 1: “Hard scatter”

Step 2: “Matching”



“Quantum effects” in high energy scattering

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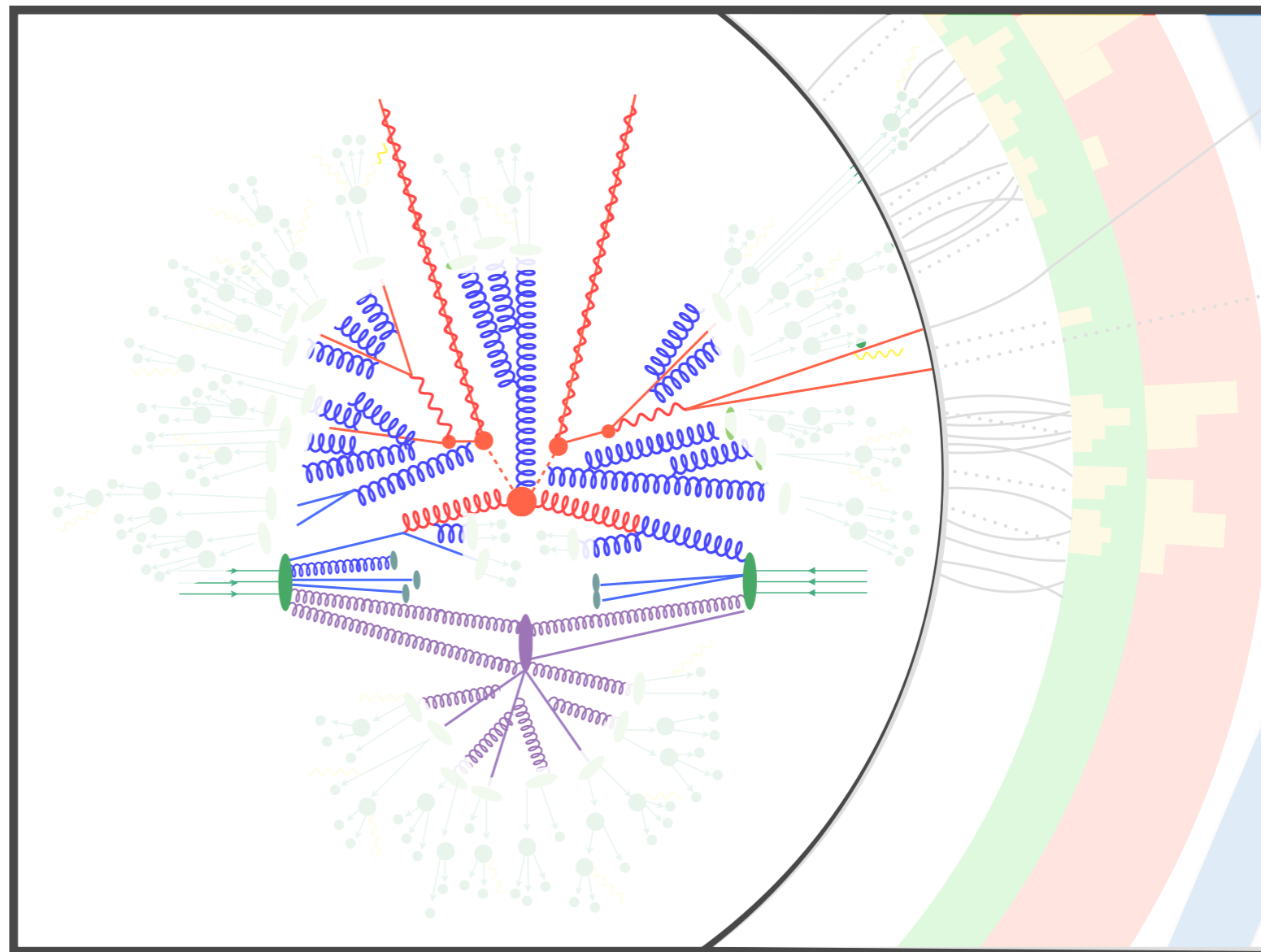
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Step 1: “Hard scatter”

Step 2: “Matching”

Step 3: “Parton Shower”



“Quantum effects” in high energy scattering

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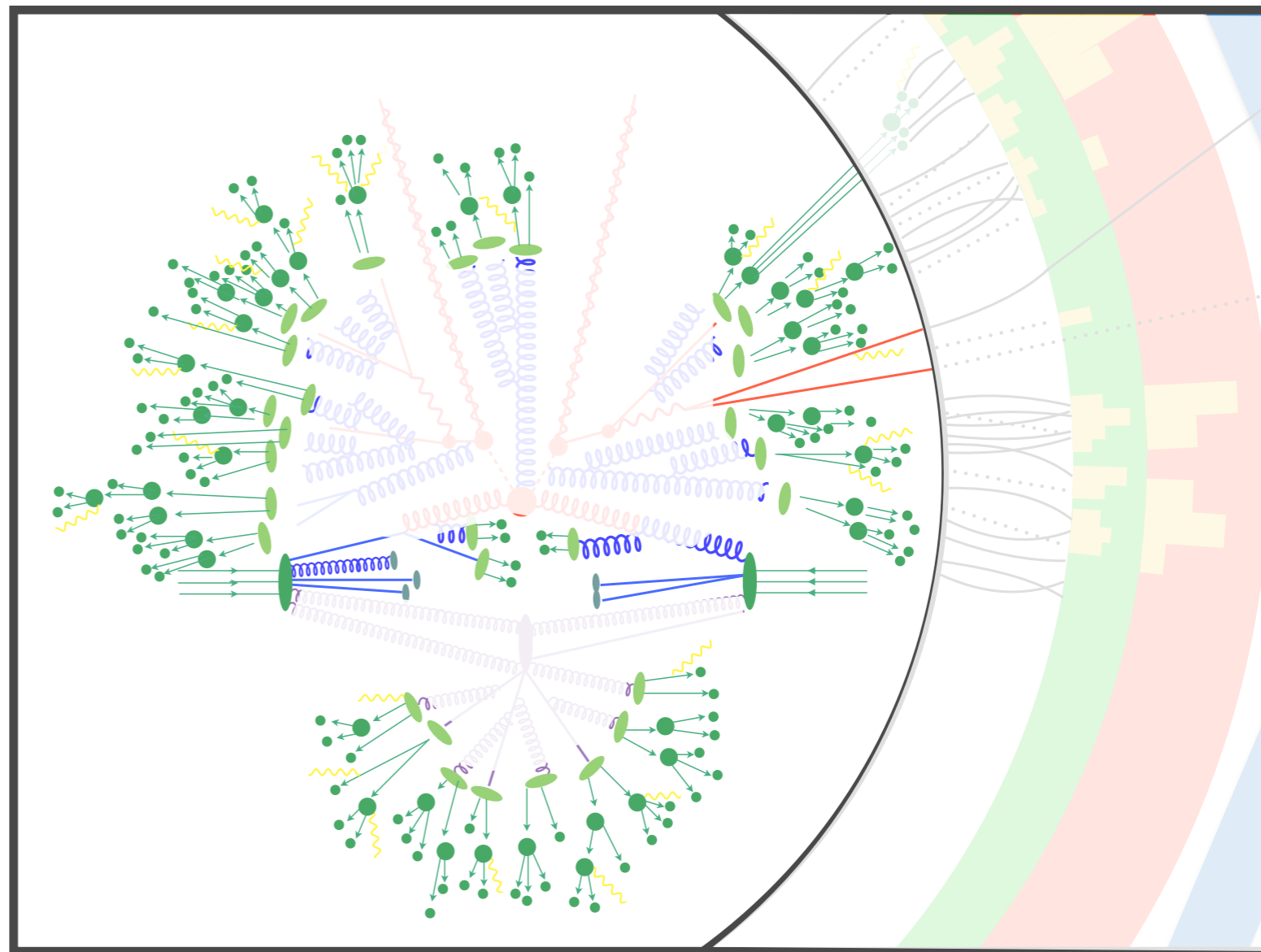
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Step 1: “Hard scatter”

Step 2: “Matching”

Step 3: “Parton Shower”

Step 4: “Hadronization”



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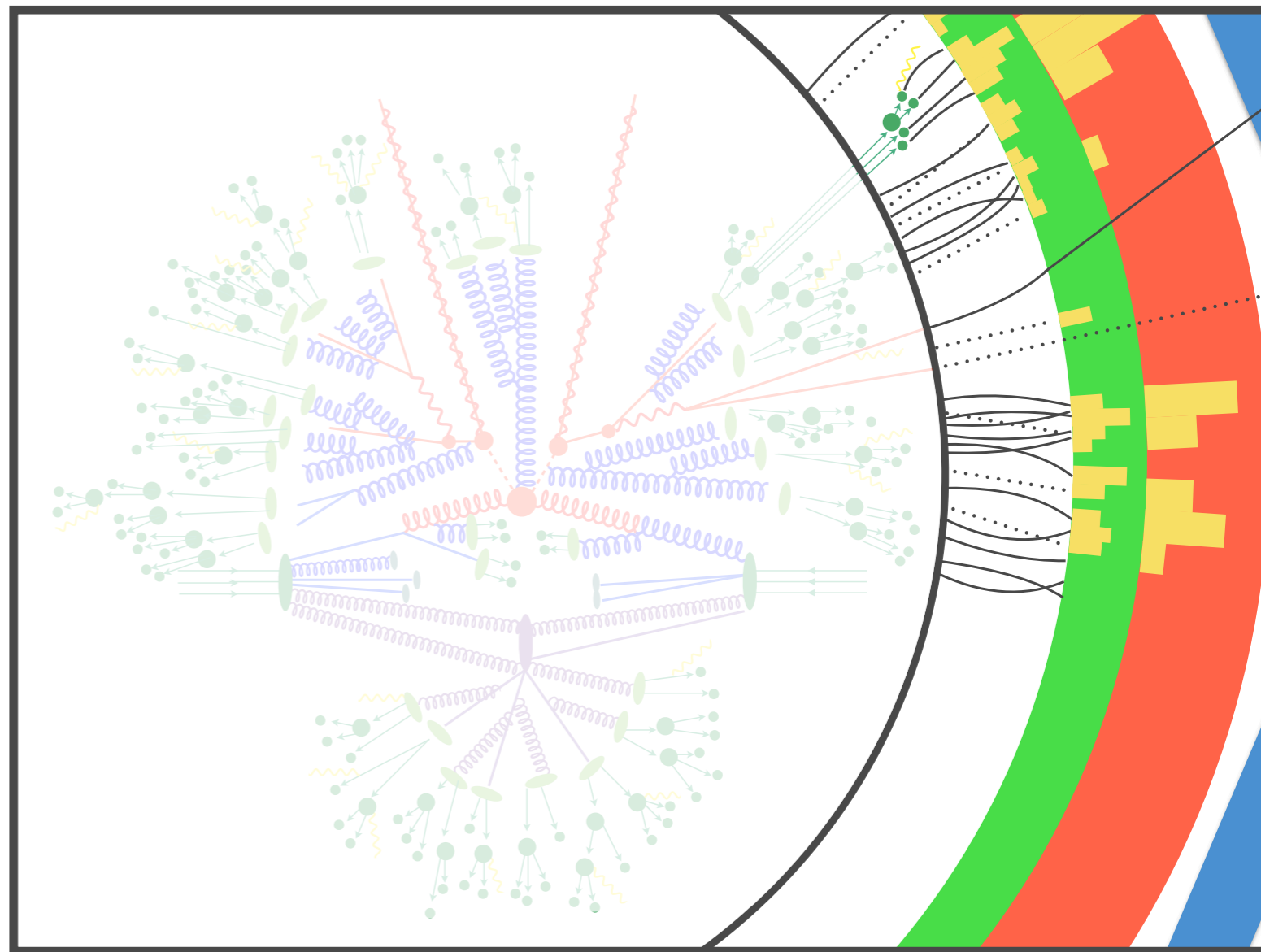
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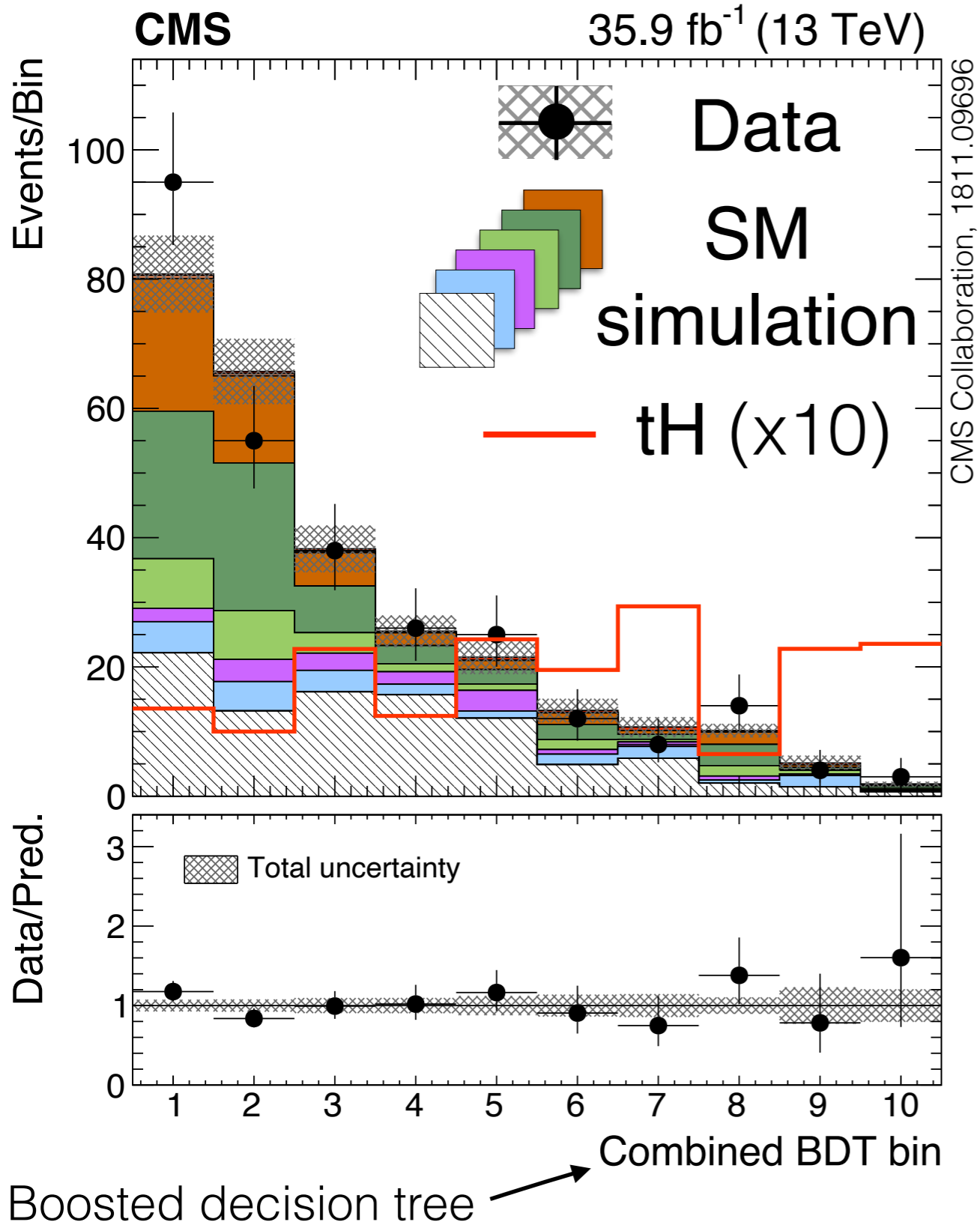
Step 3: “Parton Shower”

Step 4: “Hadronization”

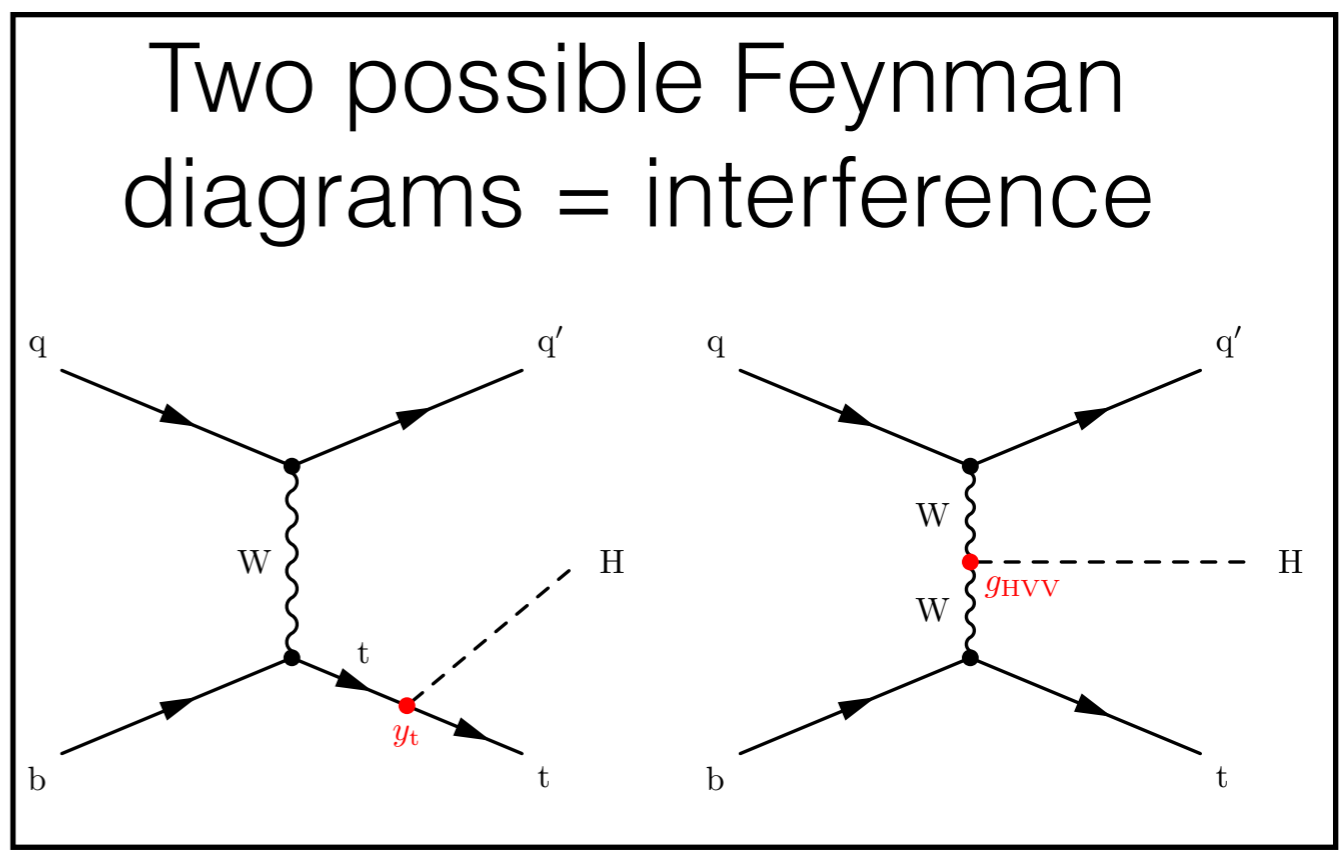
Step 5: Detector sim.



“Quantum effects” for the hard scatter



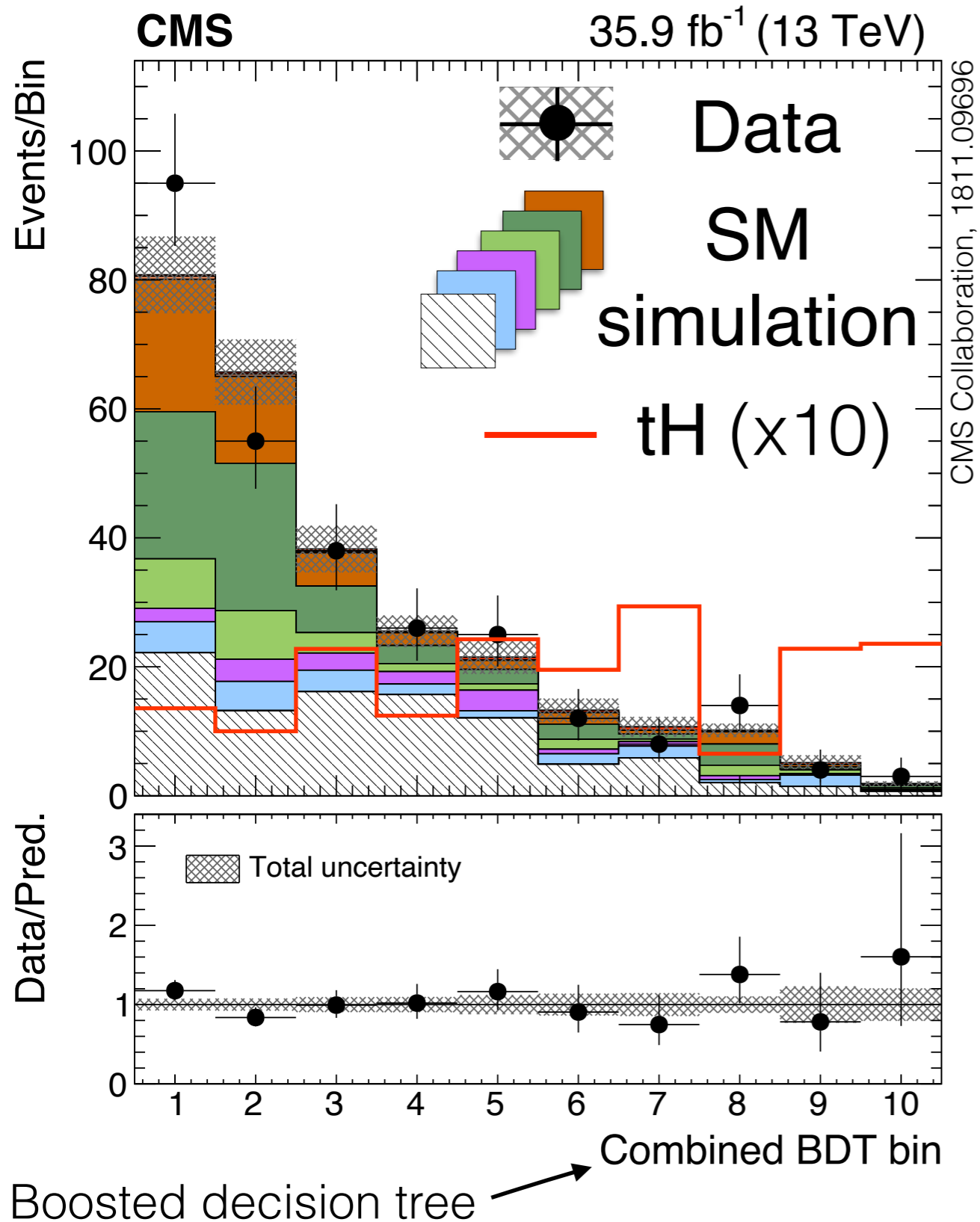
proton proton → Higgs boson + top quark



This is a huge effect, but we are confident enough in our hard scatter simulations to use them for machine learning!

“Quantum effects” for the hard scatter

38



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~SOLVED WITH CLASSICAL COMPUTERS

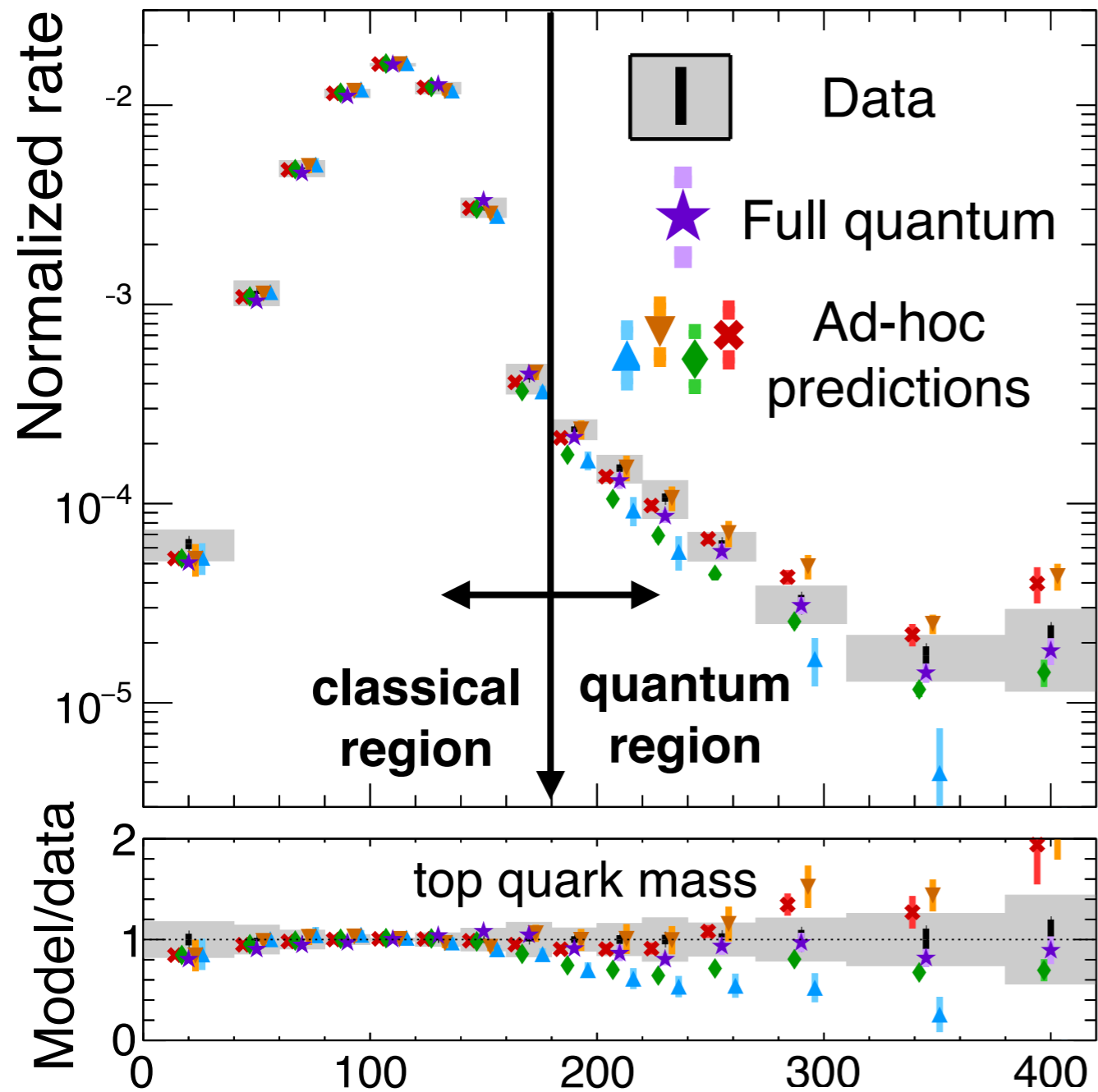
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“Quantum effects” for matching

[Phys. Rev. Lett. 121 \(2018\) 152002](#)

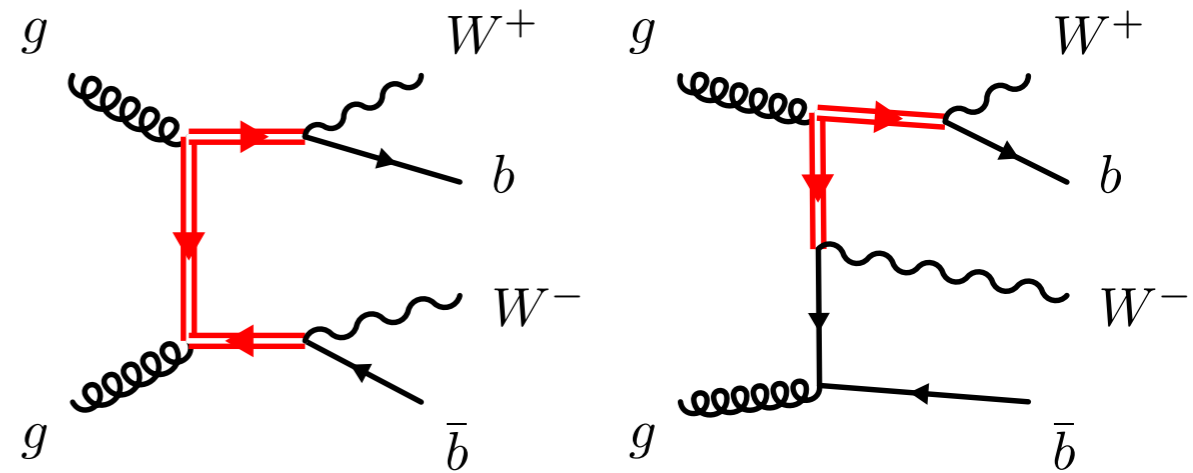
Analysis Team: T. Eifert, C. Herwig, **BPN**

ATLAS $\sqrt{s}=13$ TeV, 36.1 fb $^{-1}$ $pp \rightarrow \ell^+ \ell^- b \bar{b} + X$



proton proton \rightarrow two W bosons and two b-quarks

Multiple possible diagrams = interference



More complicated, but recent innovations are bringing to the same status as previous slide.

Invariant mass of a lepton and hadrons [GeV/c^2]

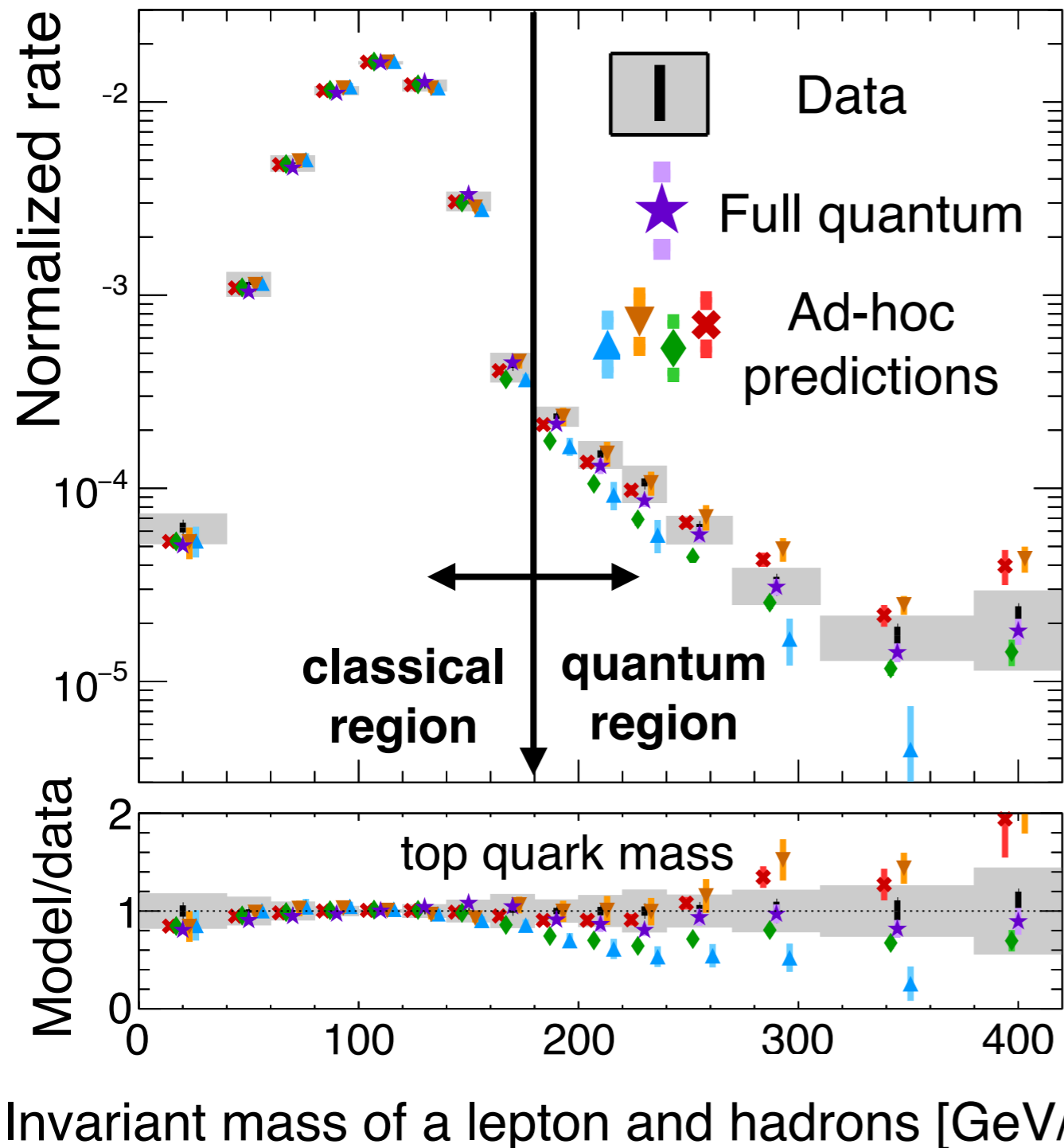
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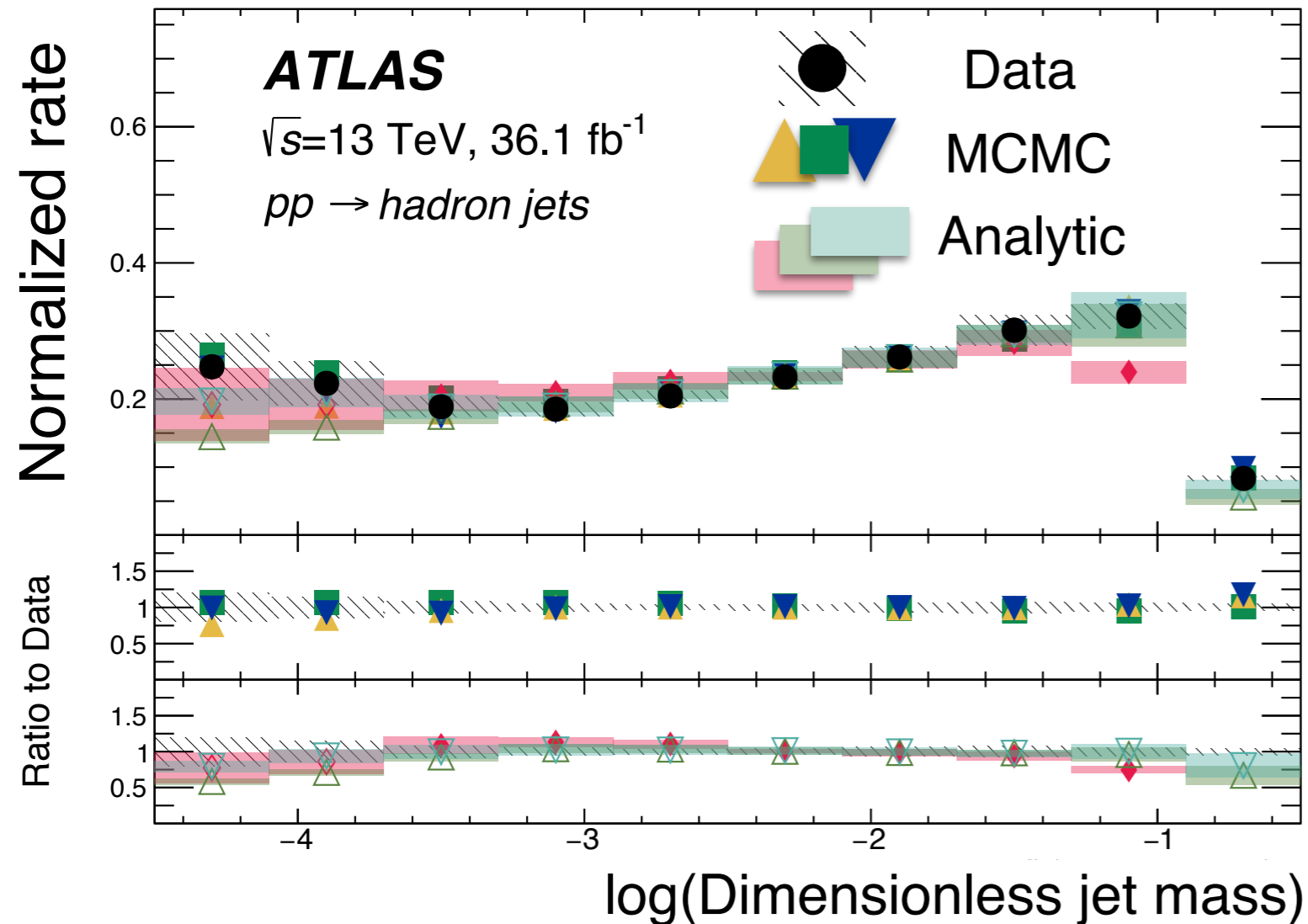
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“Quantum effects” for the parton shower

41

[Phys. Rev. Lett. 121 \(2018\) 152002](#)

Analysis Team: **BPN**, J. Roloff, M. Swiatlowski



proton proton \rightarrow two jets of hadrons (easily hundreds of particles!)

This is a complex many-body quantum system!

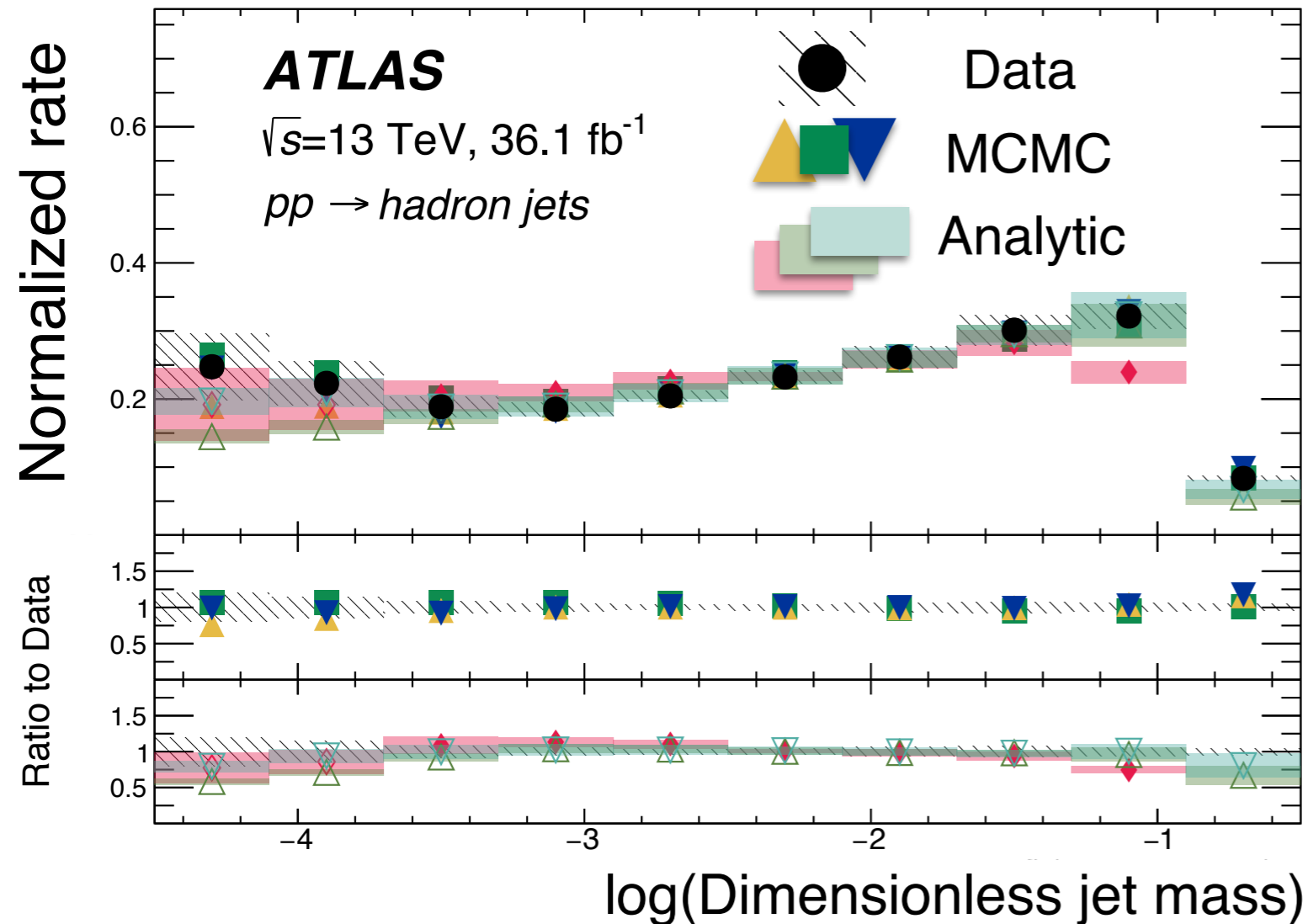
Go-to solution: Markov Chan Monte Carlo. This ignores most “quantum” effects; full effects can be (painstakingly) included for some specific observables on a case-by-case basis.

“Quantum effects” for the parton shower

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[Phys. Rev. Lett. 121 \(2018\) 152002](#)

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proton proton \rightarrow two jets of hadrons (easily hundreds of particles!)

NOT SOLVED IN GENERAL!

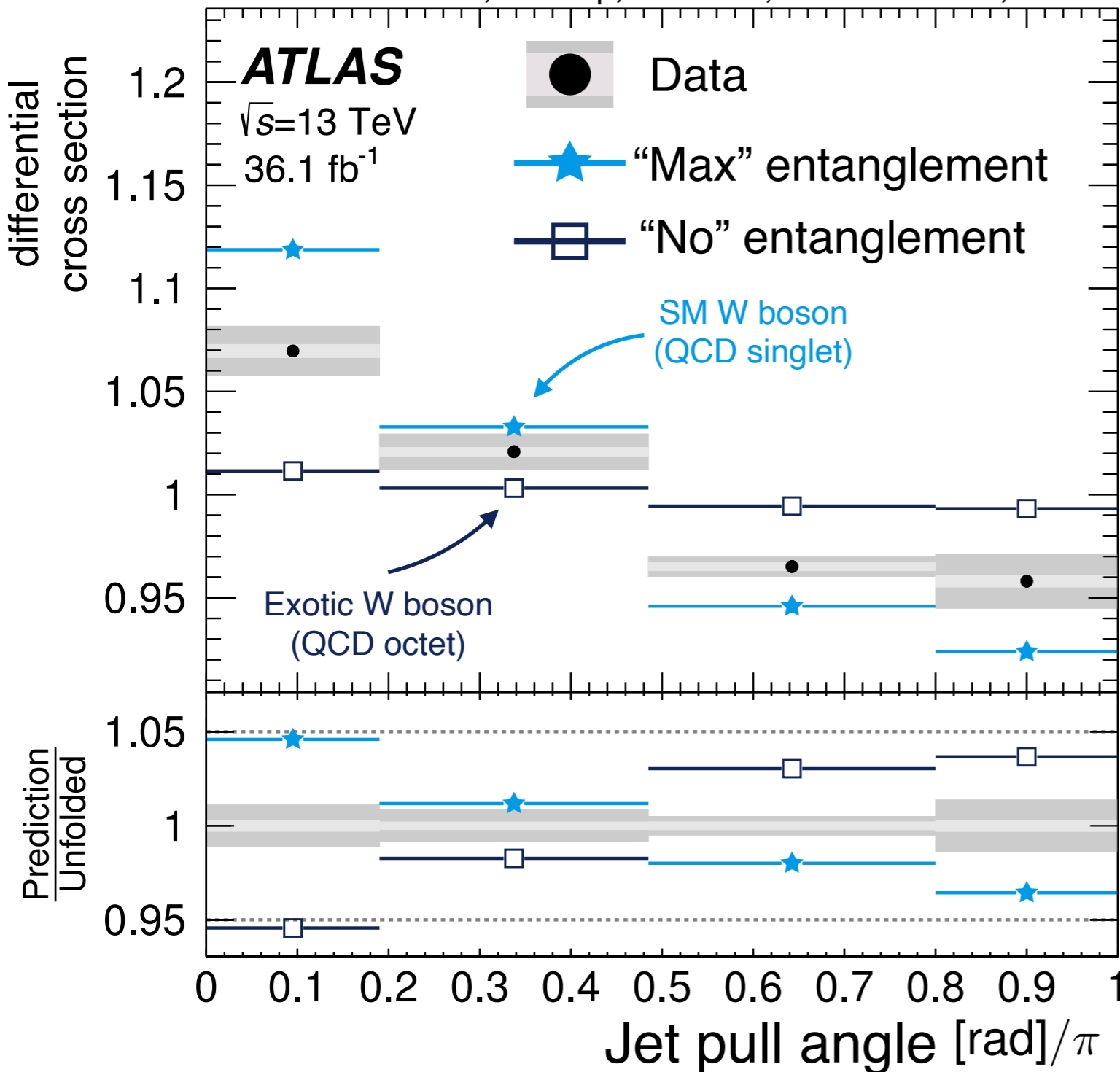
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“Quantum effects” for the parton shower

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Eur. Phys. J. C 78 (2018) 847, *Phys. Lett. B* (2015) 475

Team: BPN, T. Neep, Y. Peters, M. Swiatlowski, F. Wilk



W boson \rightarrow two jets of hadrons

To drive the point home
here is an observable where we can't distinguish between entanglement turned “on” and “off”.

How much the radiation from one jet “leans” toward the other.

How might we solve this?

Final state radiation is a complex many-body quantum system.

Perhaps quantum tools can be used to incorporate quantum degrees of freedom!



Lines: quarks; curls: gluons; colors: quantum numbers

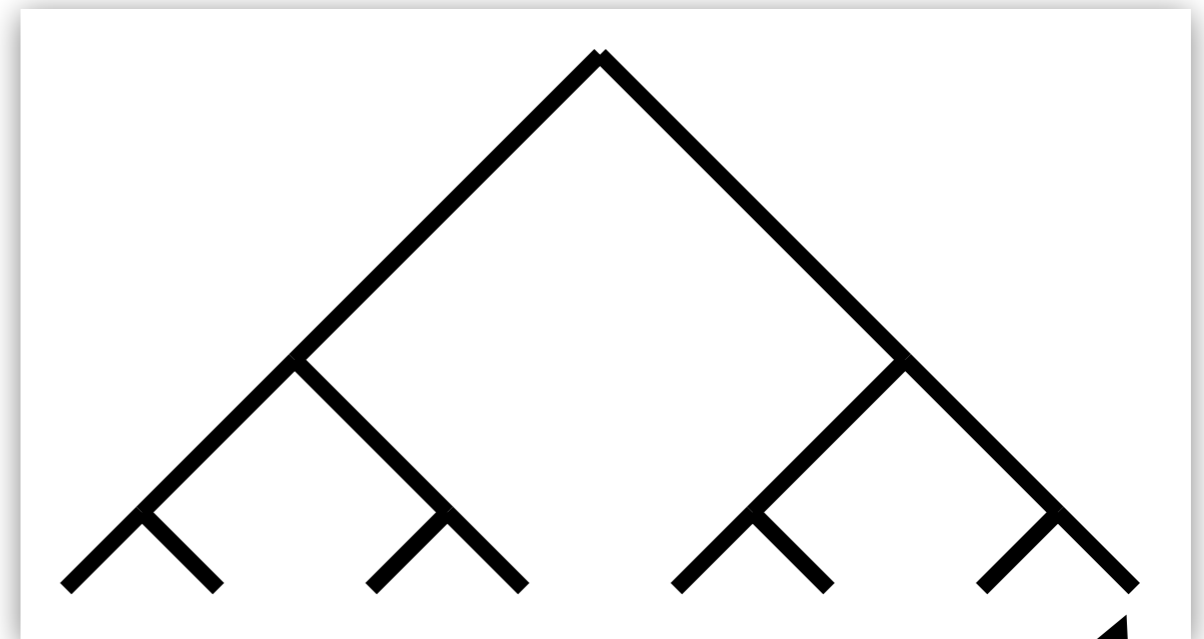
Whet your appetite

45

Let's think of a parton shower like a tree.

Discretize "time".

At each "time", a particle can radiate (go left) or not radiate (go right).



leaf

N.B. not a quantum random walk:
leaf = history is observable!

Markov chain of amplitudes: $A_{\text{leaf}} = \prod_{n=1}^N A_{\lambda_n}(n)$

Solved by a classical MCMC

$$\lambda \in \{L, R\}^N$$

Whet your appetite

46

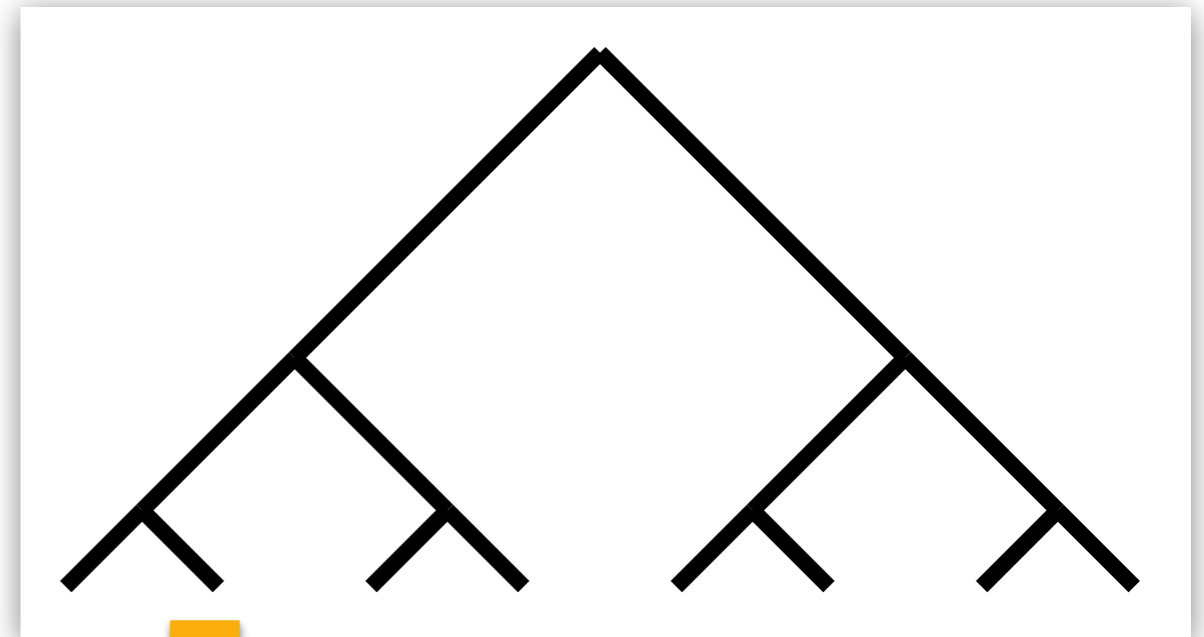
Let's think of a parton shower like a tree.

Discretize "time".

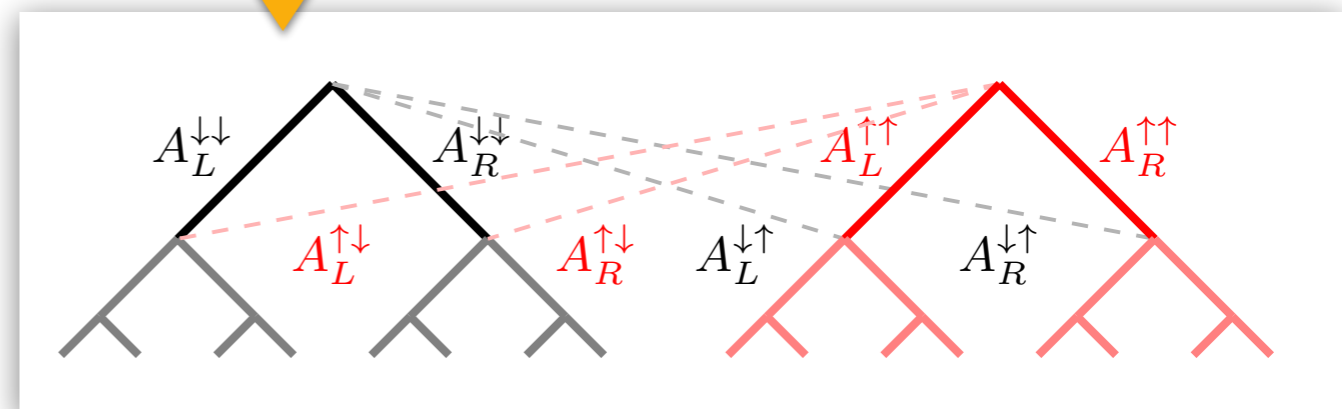
At each "time", a particle **can flip spin (flip trees)** and radiate (go left) or not radiate (go right).

$$A_{s_0, s_N} = \sum_{\substack{\vec{s}' \in \{\downarrow, \uparrow\}^N \\ s'_0 = s_0, s'_N = s_N}} \prod_{n=1}^N A_{\lambda_n}^{s'_{n-1}, s'_n}(n)$$

Classical MCMC fails!



+quantum number

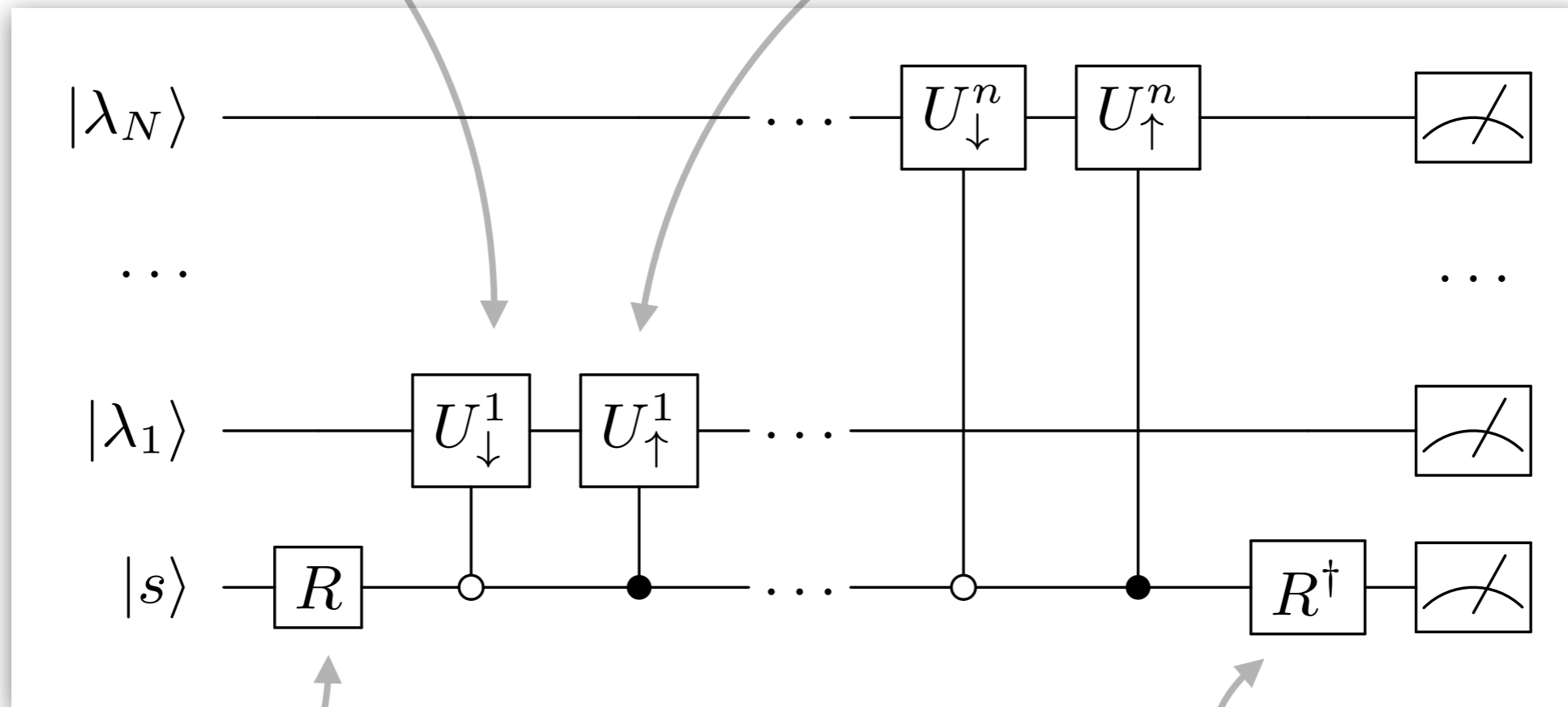


Interference from summing over intermediate spins!

Quantum solution to interfering trees

go left, given spin down

go left, given spin up



N.B. this only works if there exists such an $R \rightarrow$ subspace of full interfering tree problem.

D. Provasoli, BPN, W. de Jong, C. Bauer,
Quantum Science and Tech. 5 (2020) 035004

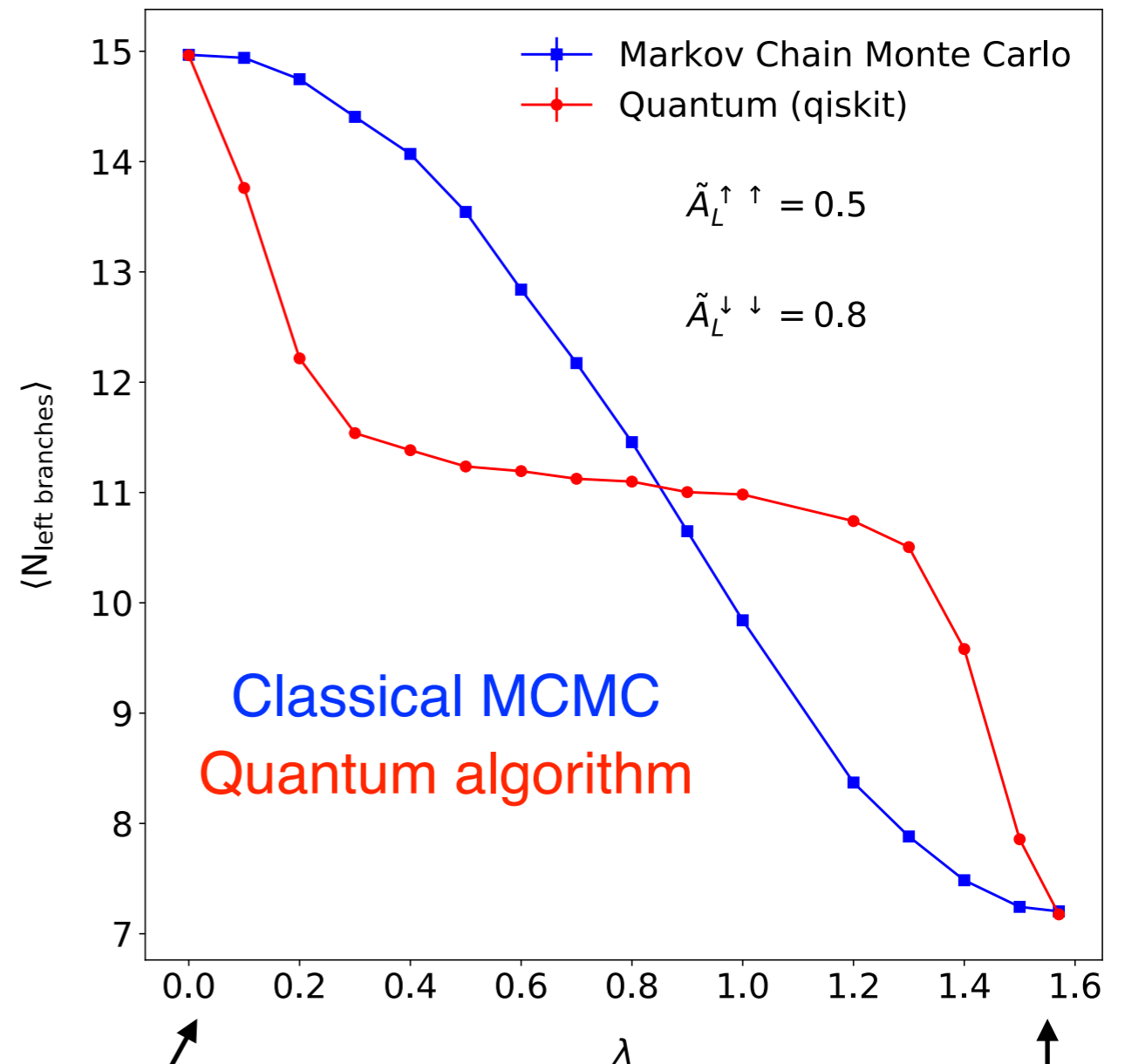
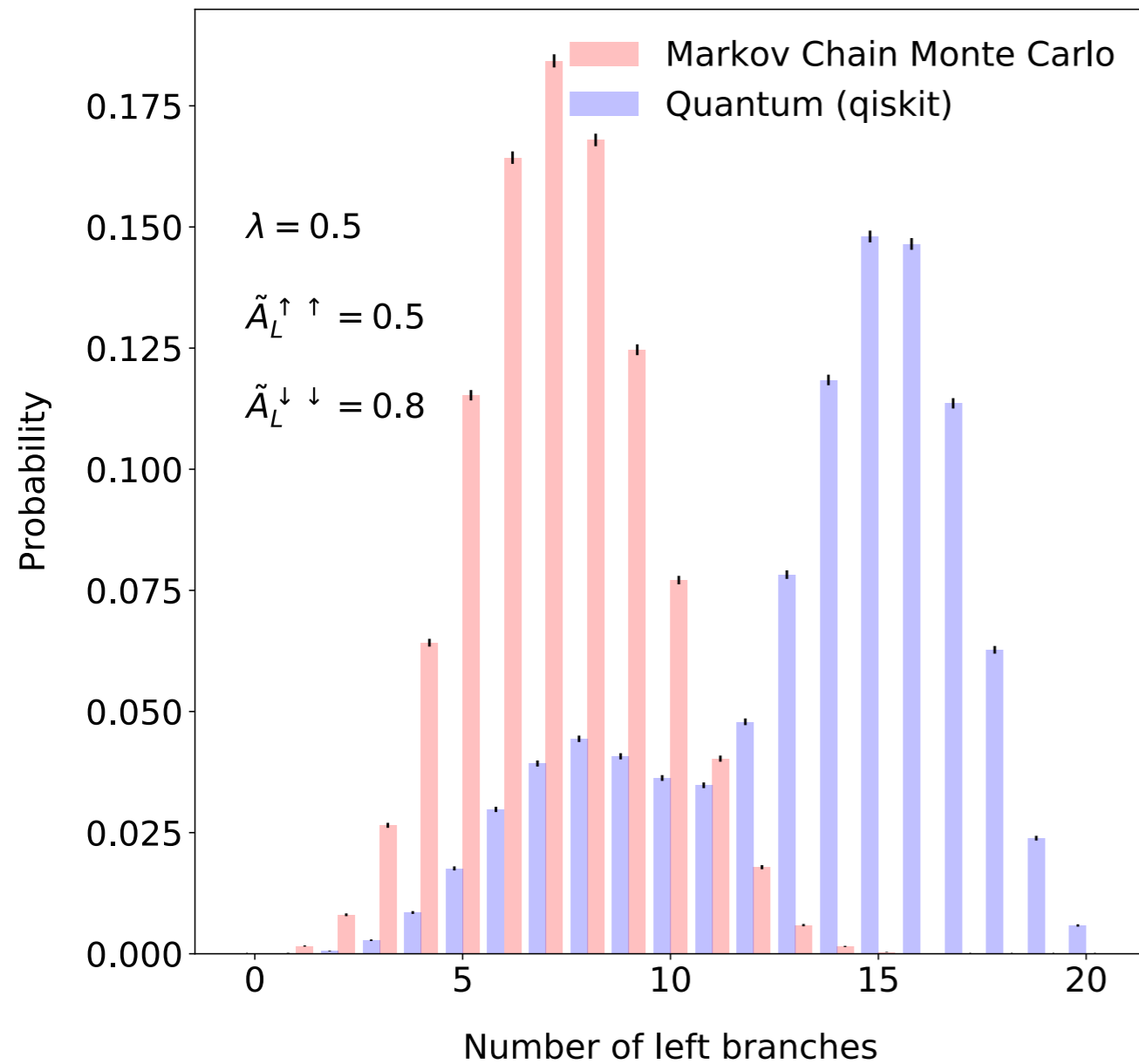
rotate into non-interacting basis

rotate back into physical basis

Linear-time quantum circuit with one qubit / step + 1 qubit for the spin.

Results with a quantum simulator

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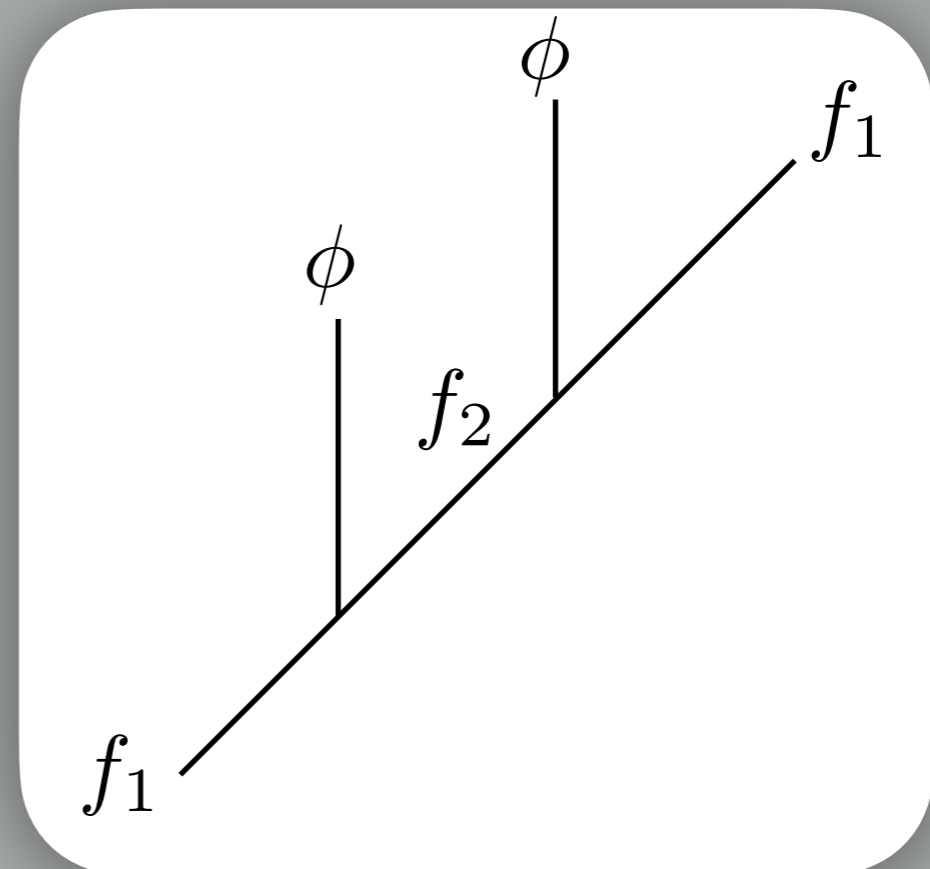
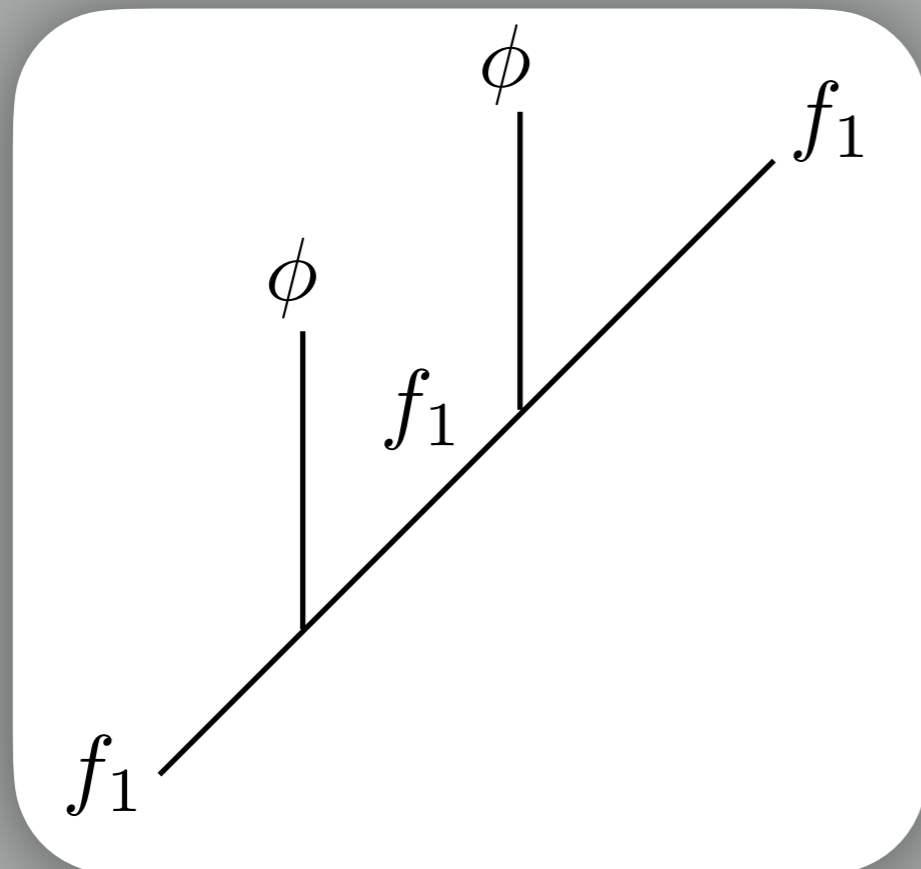
$\lambda =$ rotation angle

no flipping ← no interference → flip every step

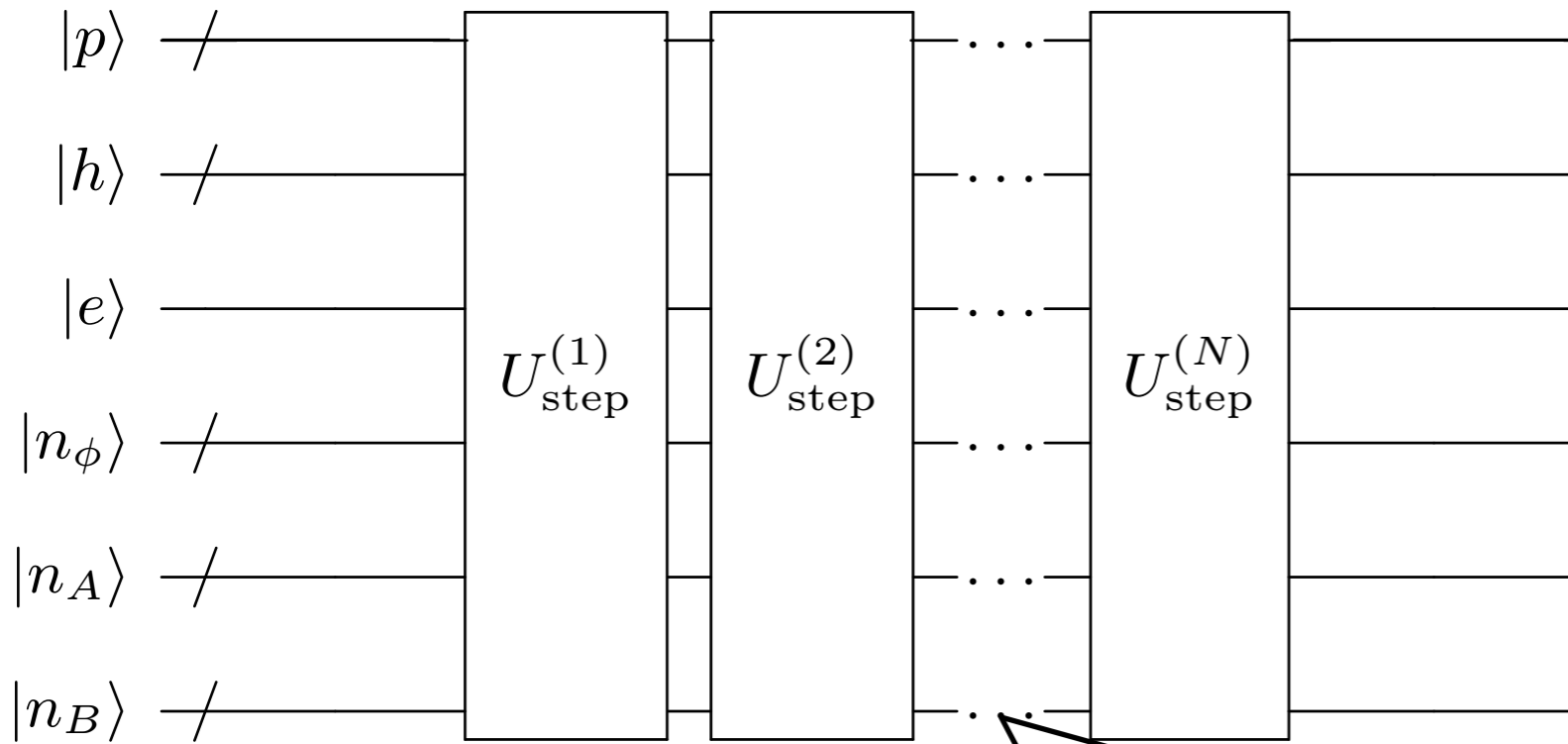
A more realistic model

$$\mathcal{L} = \bar{f}_1 i(\not{\partial} + m_1) f_1 + \bar{f}_2 (i\not{\partial} + m_2) f_2 + (\partial_\mu \phi)^2 \\ + g_1 \bar{f}_1 f_1 \phi + g_2 \bar{f}_2 f_2 \phi + g_{12} [\bar{f}_1 f_2 + \bar{f}_2 f_1] \phi$$

(like the SM Higgs when $g_{12} \sim m/v$ and $g_1 = g_2 = 0$)



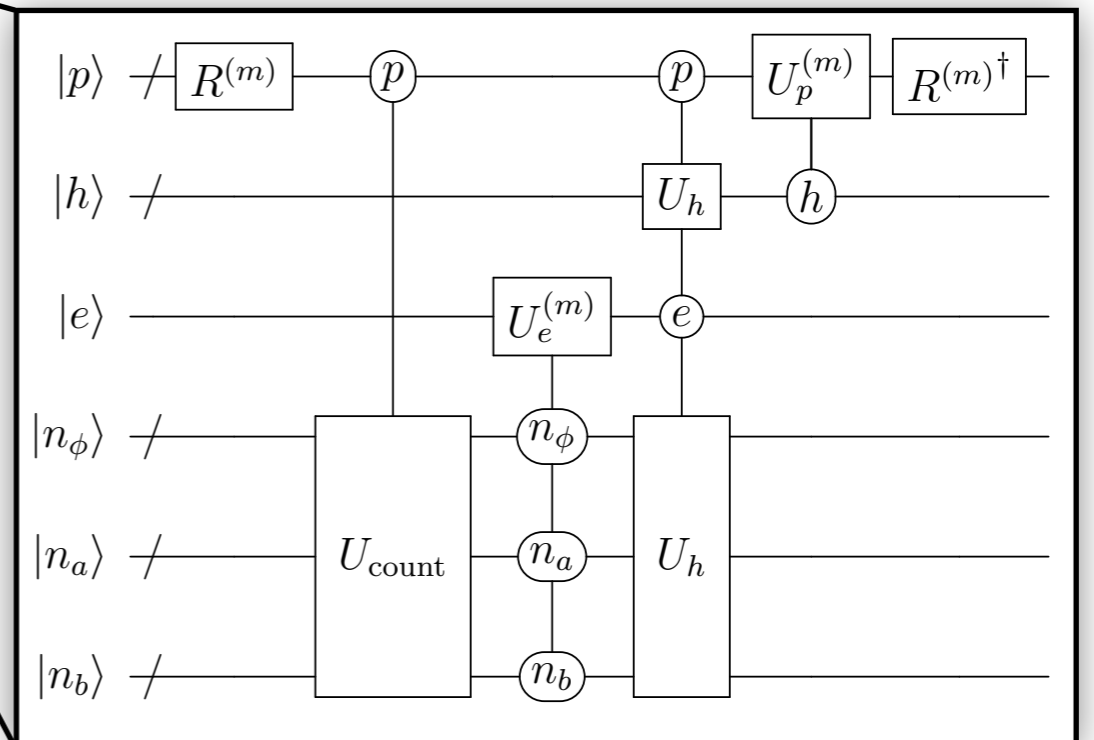
The quantum circuit



Circuit is complicated because the number of fermions is not constant:

$$\phi \rightarrow f \bar{f}$$

Register	Purpose	# of qubits
$ p\rangle$	Particle state	$3(N + 1)$
$ h\rangle$	Emission history	$N \log_2(N + 1)$
$ e\rangle$	Did emission happen?	1
$ n_\phi\rangle$	Number of bosons	$\log_2(N + 1)$
$ n_a\rangle$	Number of f_a	$\log_2(N + 1)$
$ n_b\rangle$	Number of f_b	$\log_2(N + 1)$



The quantum circuit - U_e ($e = \text{emit}$)

$$U_e^{(m)} = \begin{pmatrix} \sqrt{\Delta^{(m)}(\theta_m)} & -\sqrt{1 - \Delta^{(m)}(\theta_m)} \\ \sqrt{1 - \Delta^{(m)}(\theta_m)} & \sqrt{\Delta^{(m)}(\theta_m)} \end{pmatrix}$$

$$\Delta_i(\theta_m, \theta_{m+1}) = e^{-\Delta\theta P_i(\theta_m)}$$

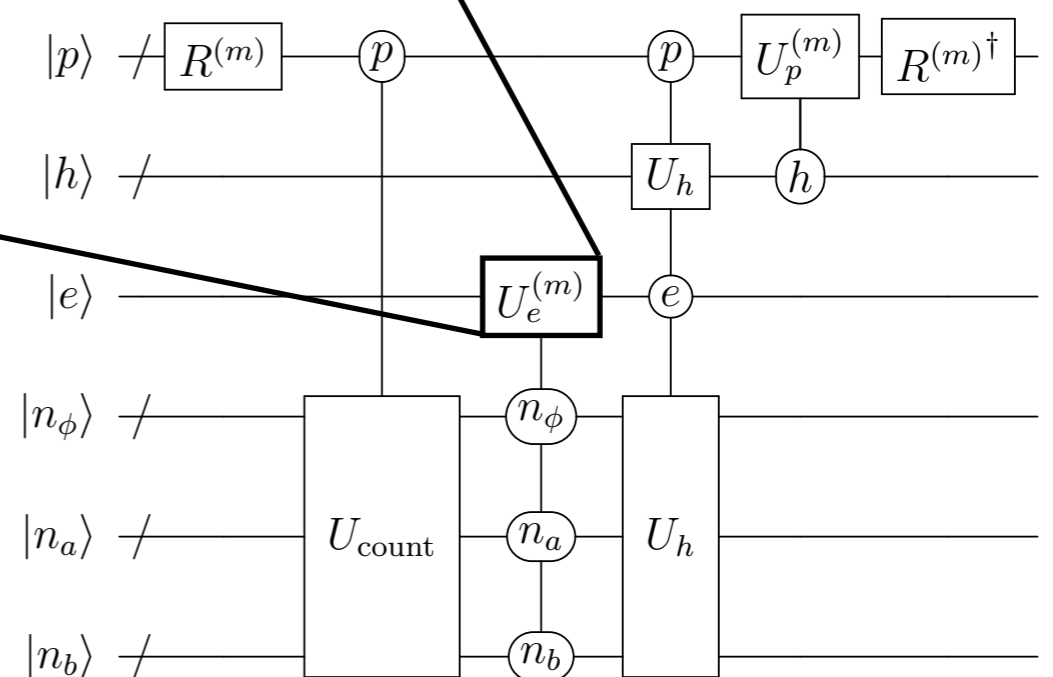
(Sudakov factor)

$$\Delta^{(m)}(\theta_m) = \Delta_\phi^{n_\phi}(\theta_m) \Delta_{f_1}^{n_{f_1}}(\theta_m) \Delta_{f_2}^{n_{f_2}}(\theta_m)$$

Circuit is complicated because the number of fermions is not constant:

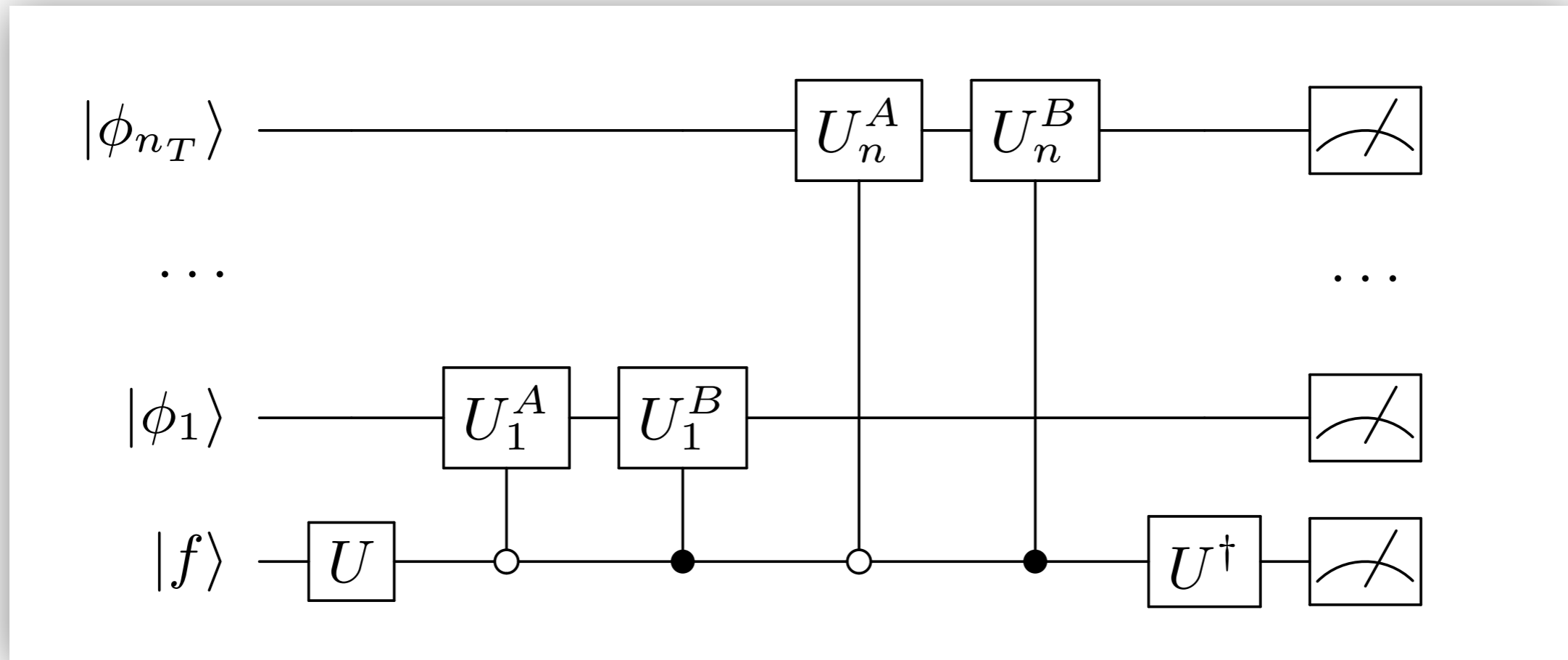
$$\phi \rightarrow f \bar{f}$$

This is just one part of the circuit that calculates the no emission amplitudes



The circuit without scalar splitting

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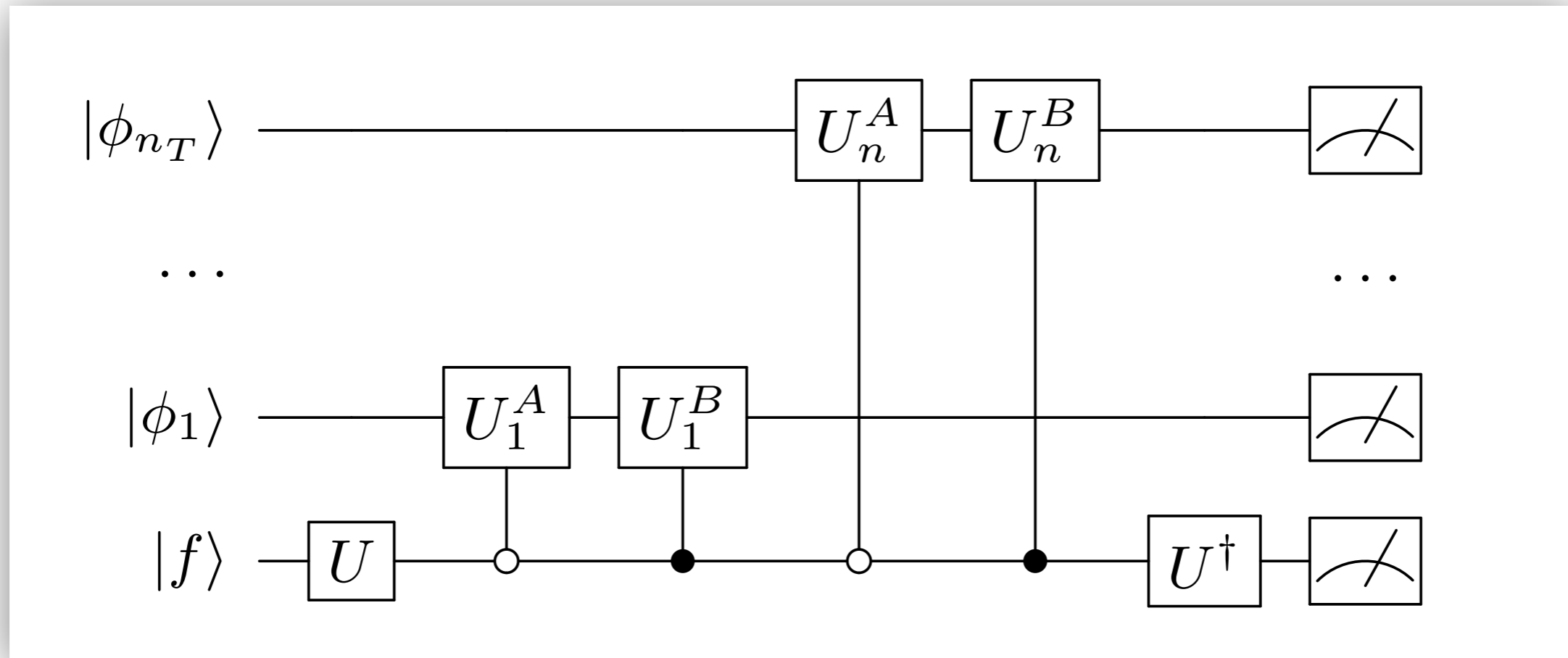


In words: rotate to the basis where there is no interference, “emit” scalars (at the **amplitude** level), and then rotate back to the physical basis at the end.

This is exactly the interfering trees circuit !

The circuit without scalar splitting

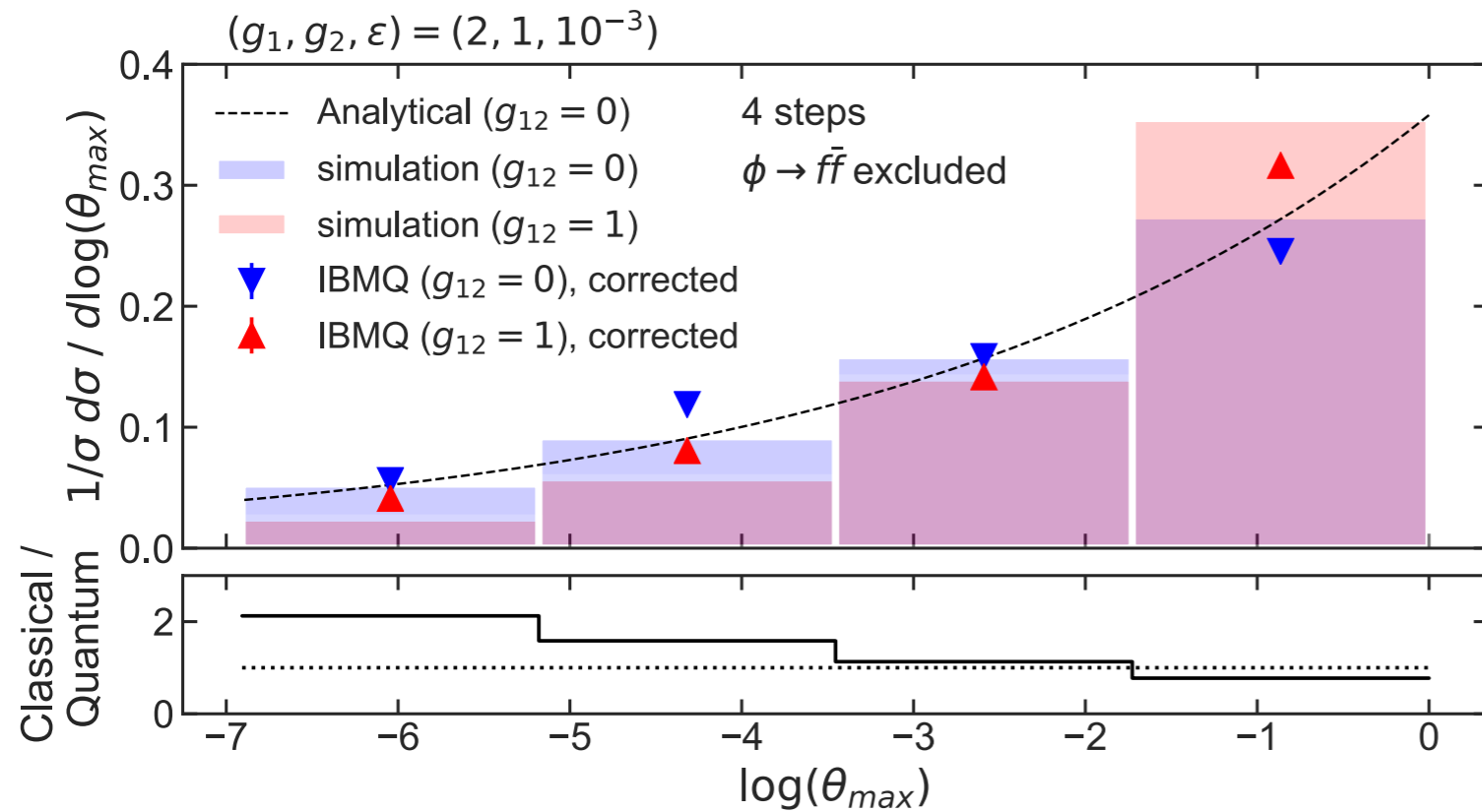
53



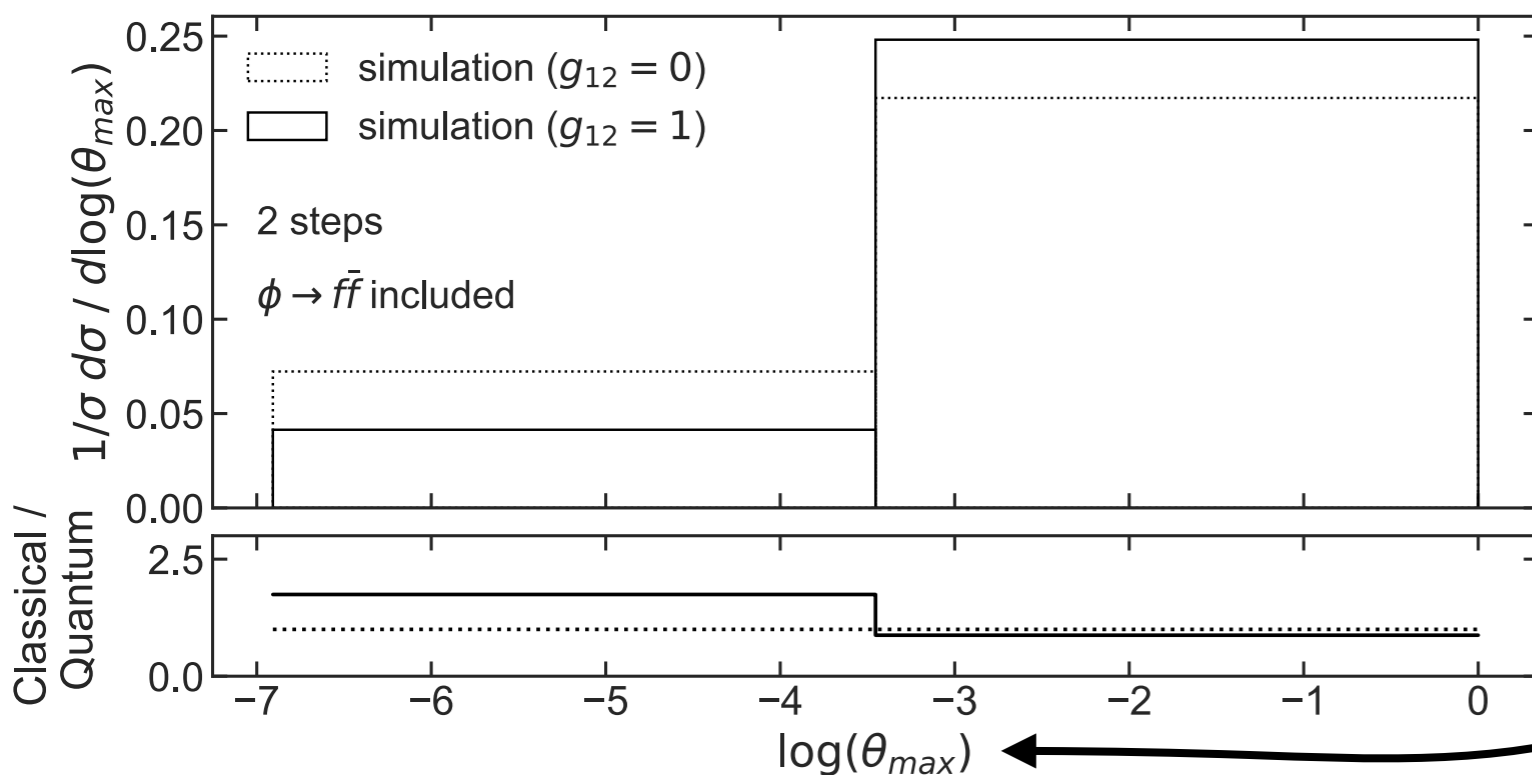
Note: $|\phi_i\rangle$ is not touched after timestep i and so one can **reuse qubits** ... only need 2 total qubits (!)

Fine print: (1) re-measurement is not a feature of most current quantum computers and (2) this led us to a classical algorithm that can capture the full interference effects (but is not standard MCMC).

Numerical results



The predictions / simulations are realized on a real quantum computer!

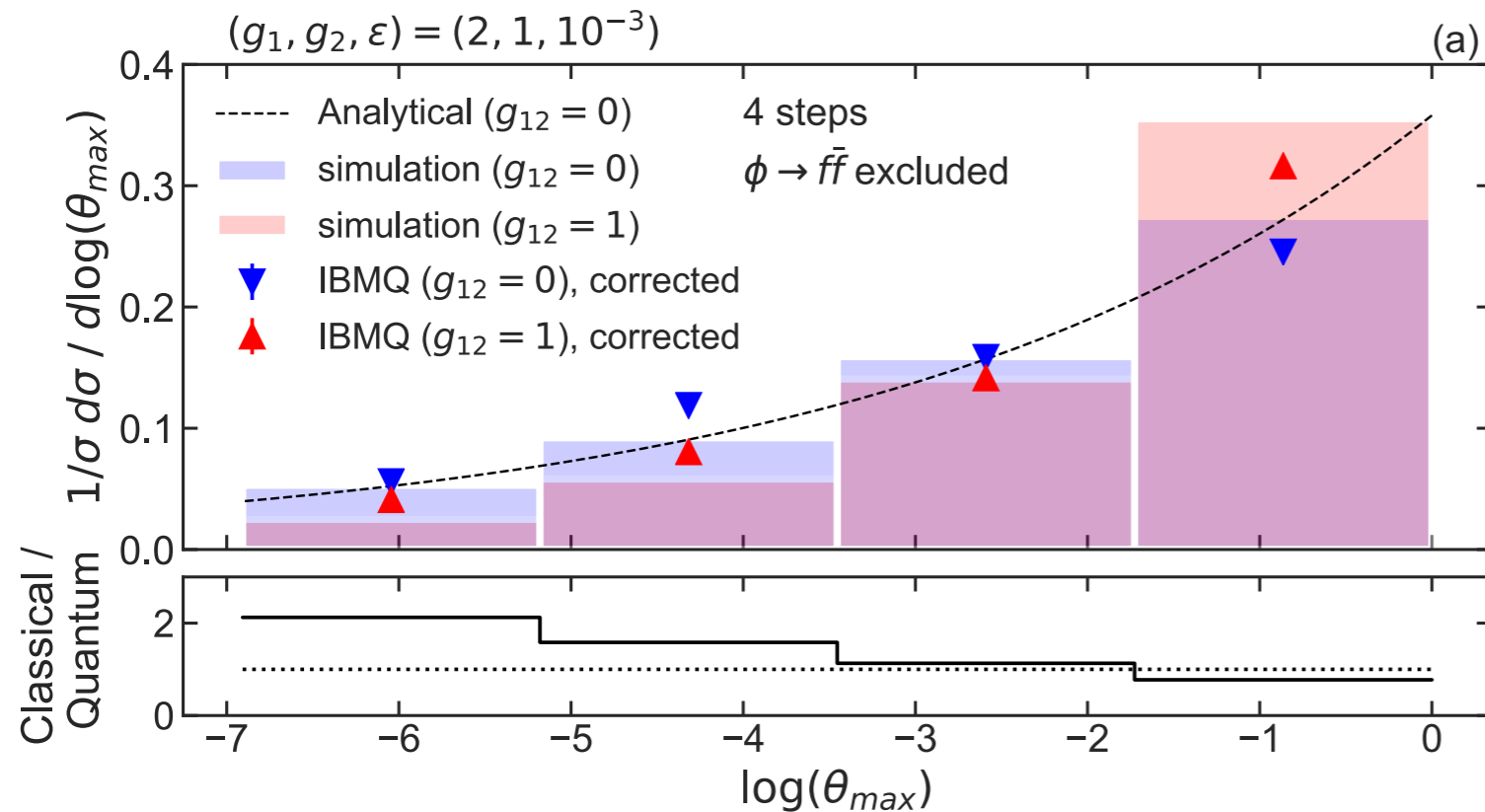


Classical: exponential
Naive quantum: 5th order
Optimized quantum: 3rd

angle of maximum emission

The fine print

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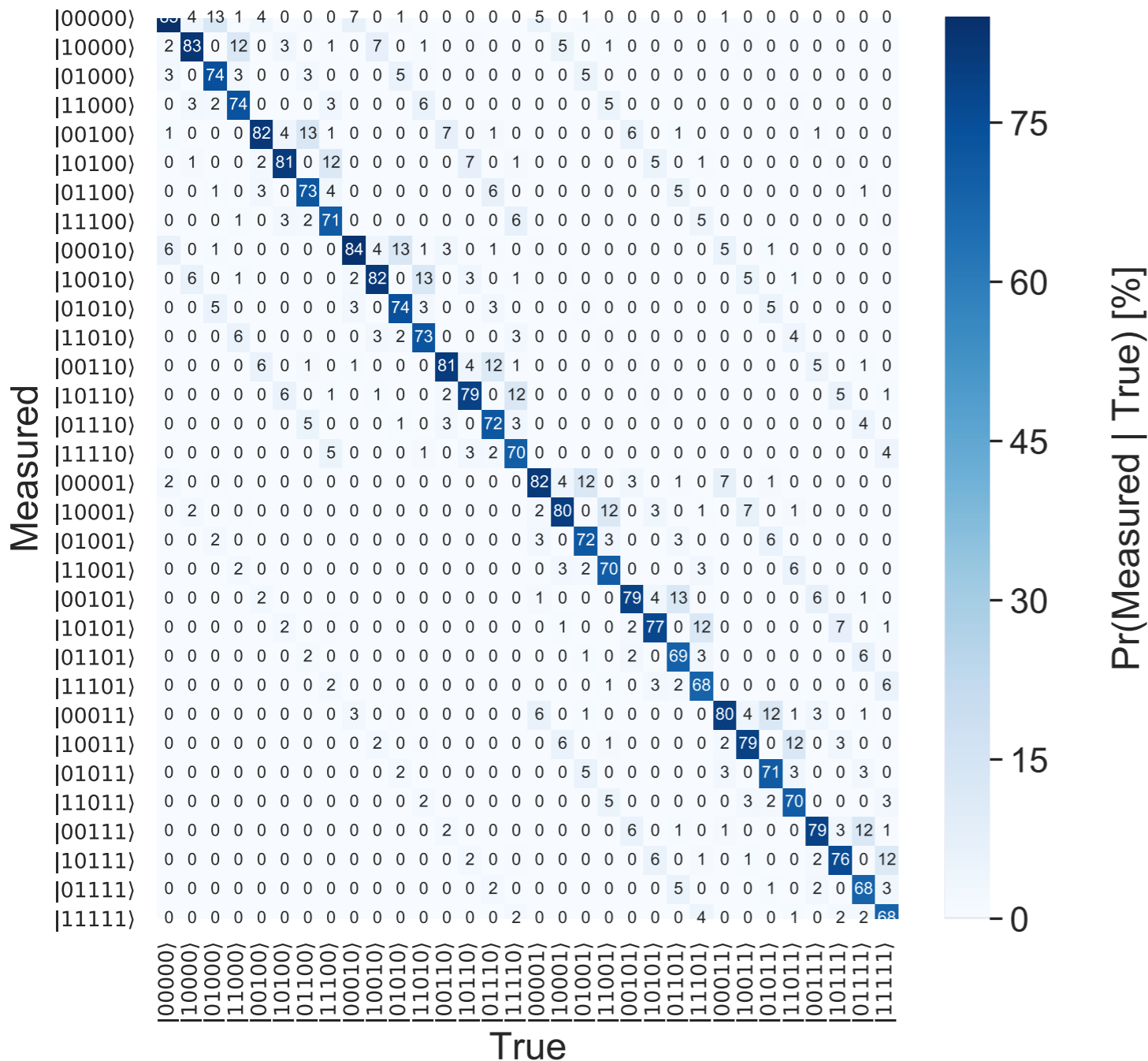
Results “out of the box” do not look this good. We optimized the nodes on the quantum computer and performed **readout error** and **gate error** corrections.

In the remaining slides, I’ll give you a taste of ongoing work in improving these corrections.

Readout error corrections



Qiskit Simulator
IBM Q Johannesburg Readout Errors



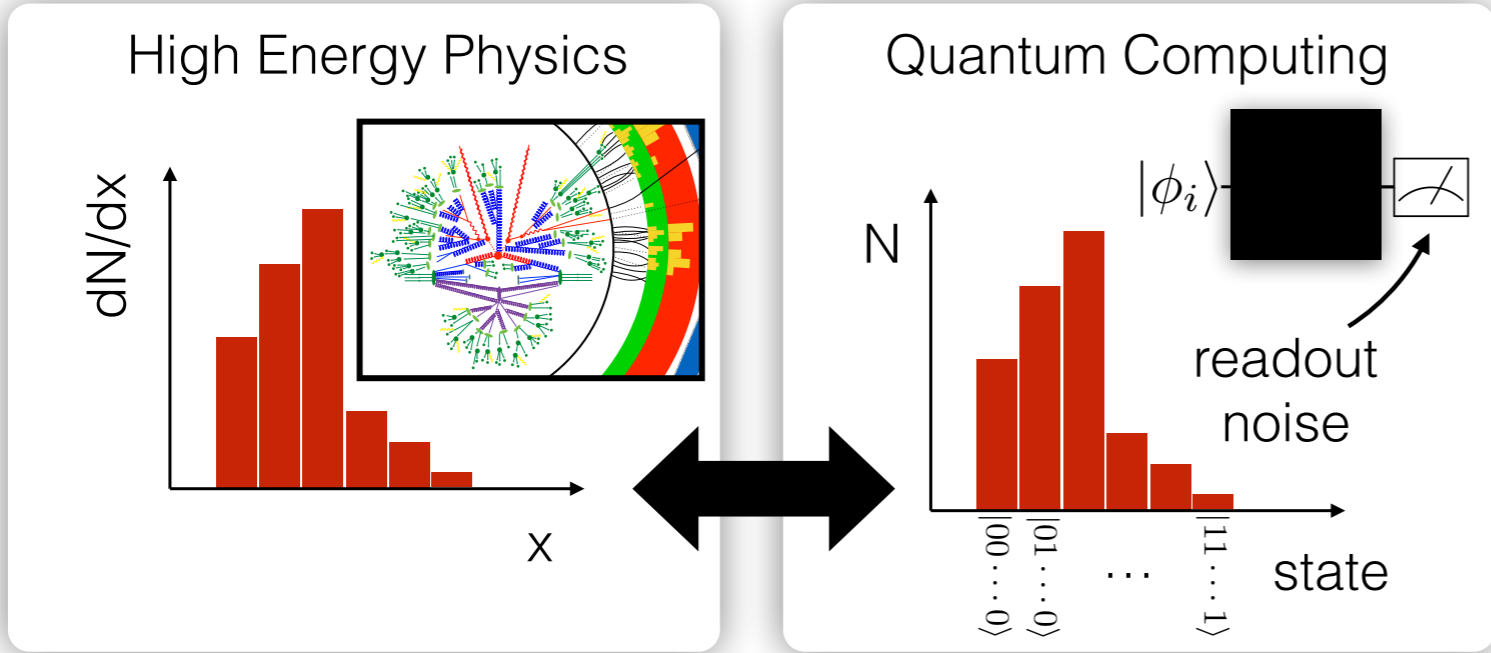
On a quantum computer, the state may be 1 but readout as a 0, etc.

For n qubits, there is a $2^n \times 2^n$ transition matrix.

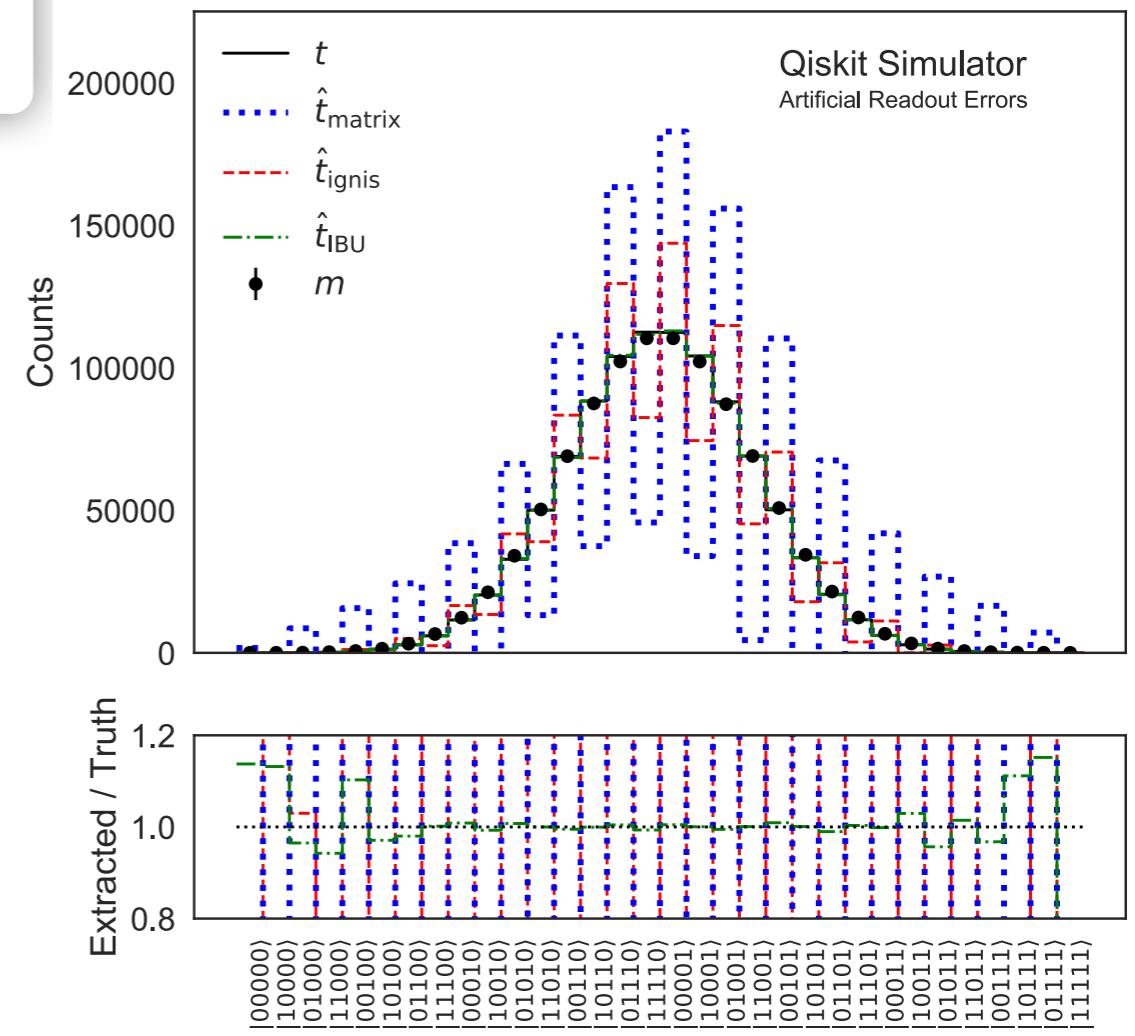
HEP has proposed many solutions to this problem!

...and we call them **unfolding**

Readout error corrections



Naïve inversion
 IBM standard
 HEP standard

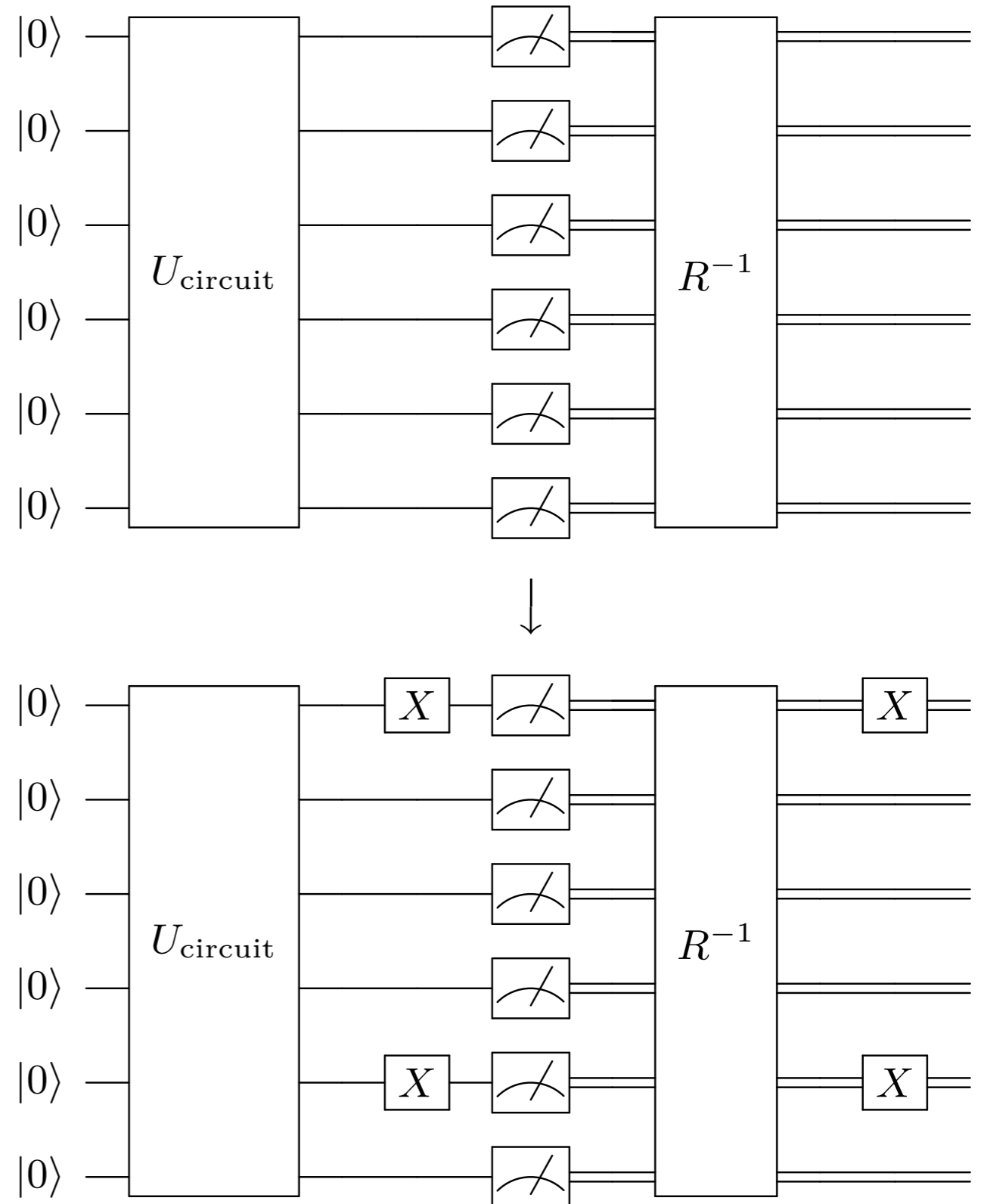


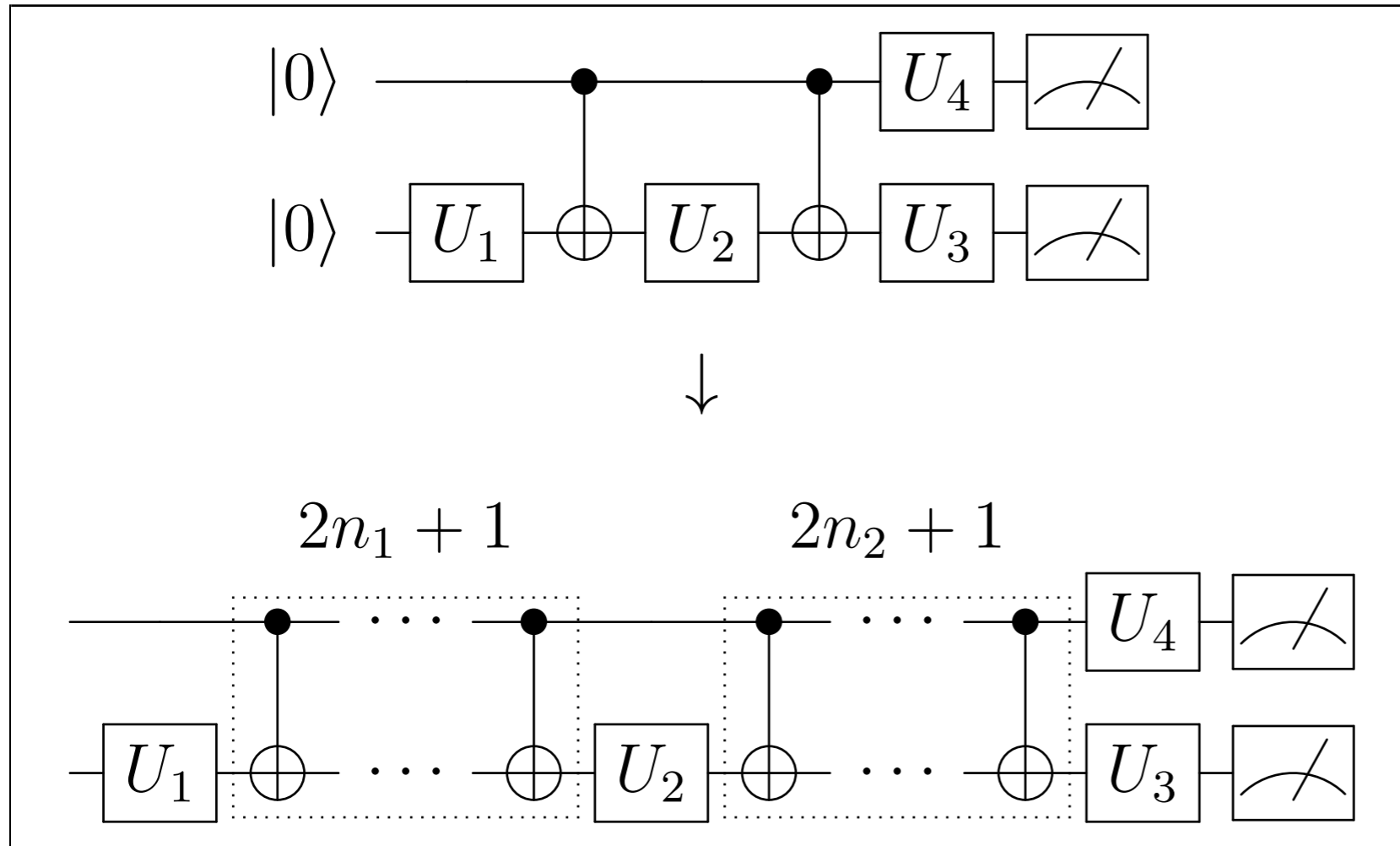
I have proposed to use HEP unfolding techniques to correct quantum computer readout errors.

→ Circumvent known pathologies with more naïve methods (!)

We are still actively developing methods to reduce readout errors.

For example, note that $\Pr(1 \rightarrow 0) > \Pr(0 \rightarrow 1)$. One can apply a simple “rebalancing” in order to improve precision.

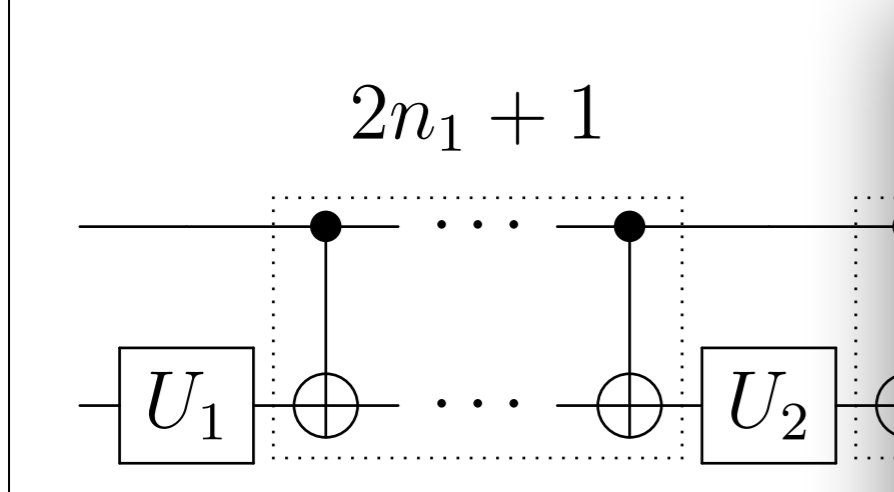
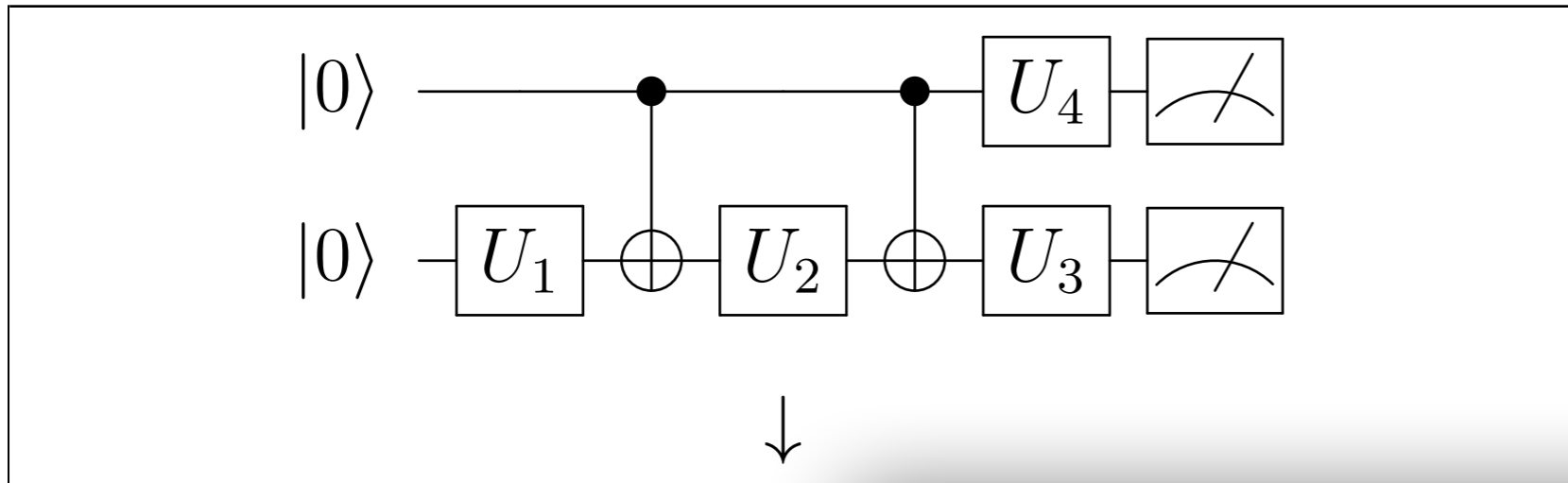




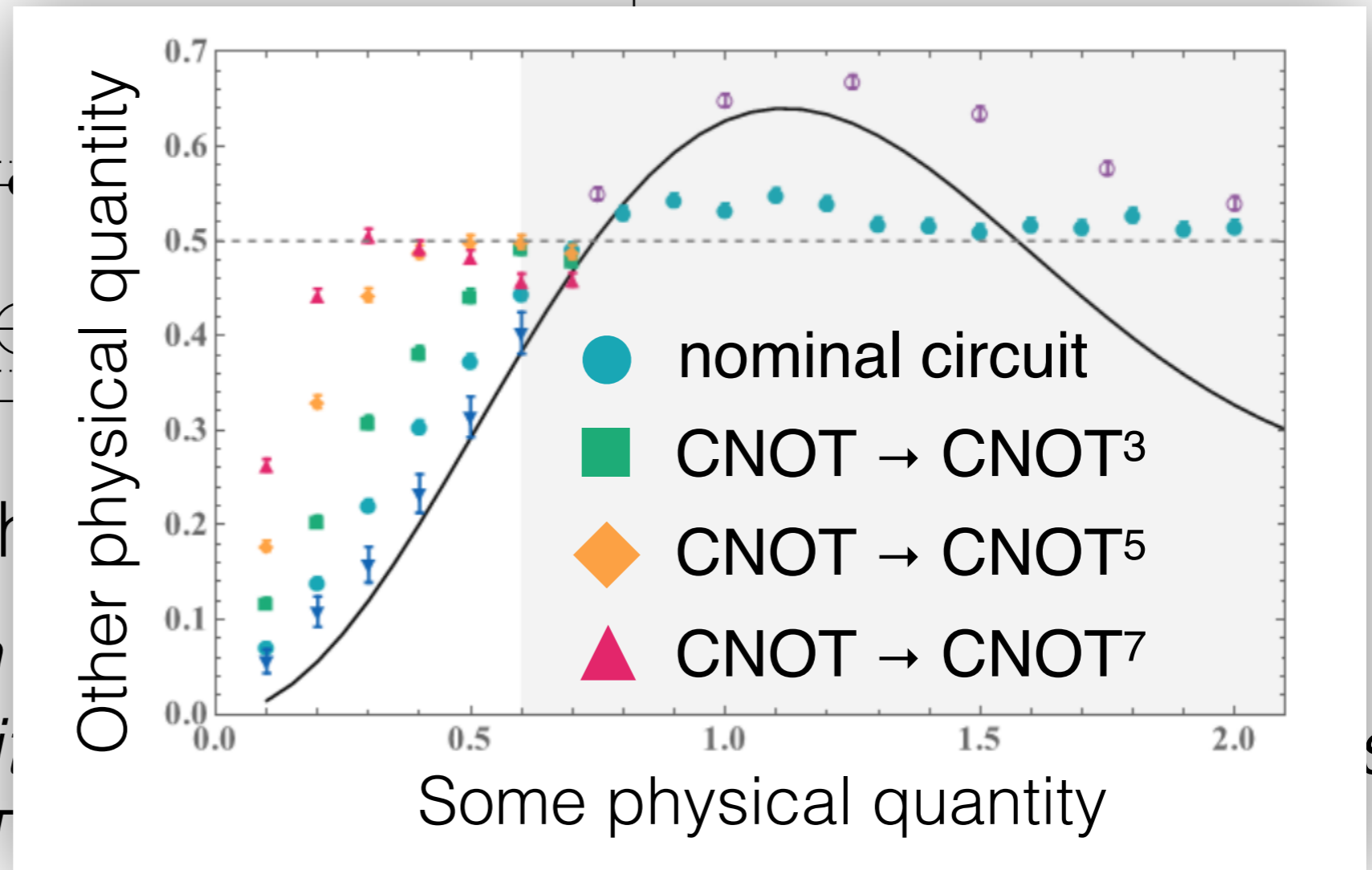
One common technique is **Zero Noise Extrapolation**

Idea: replace each CNOT by $2n+1$ CNOTs. This doesn't change the answer without noise, but systematically increases the noise. Then, extrapolate to zero noise.

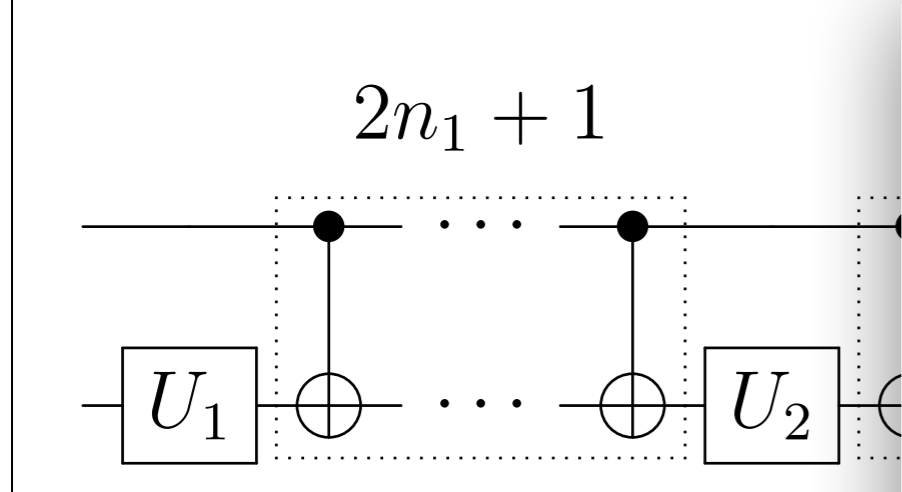
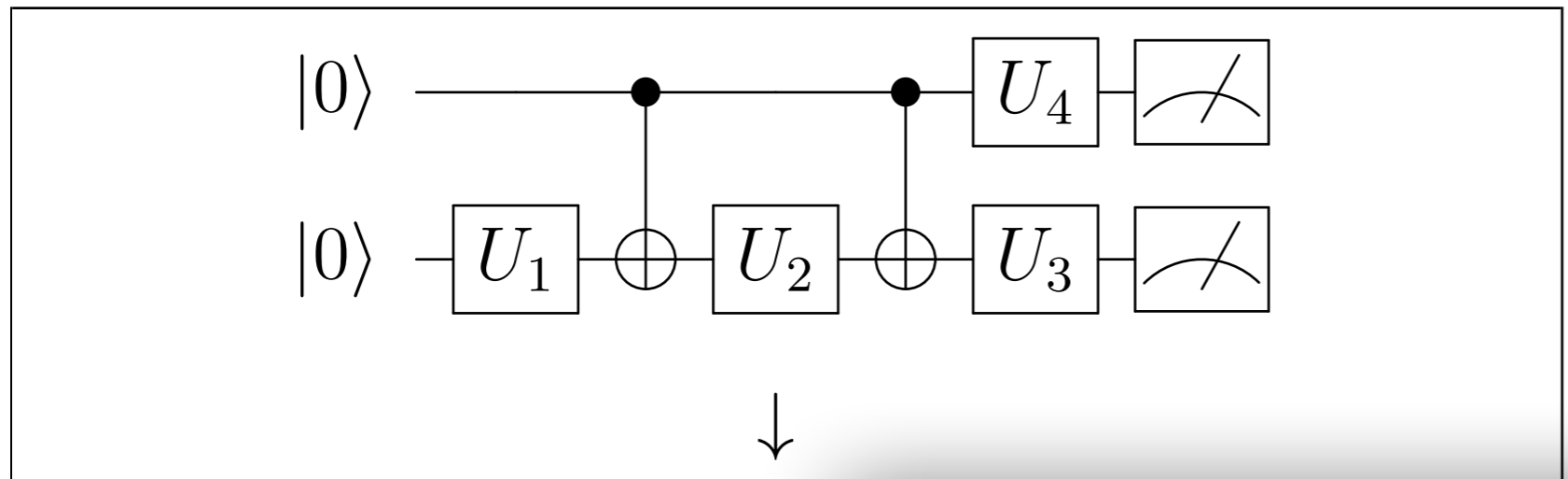
Gate error mitigation



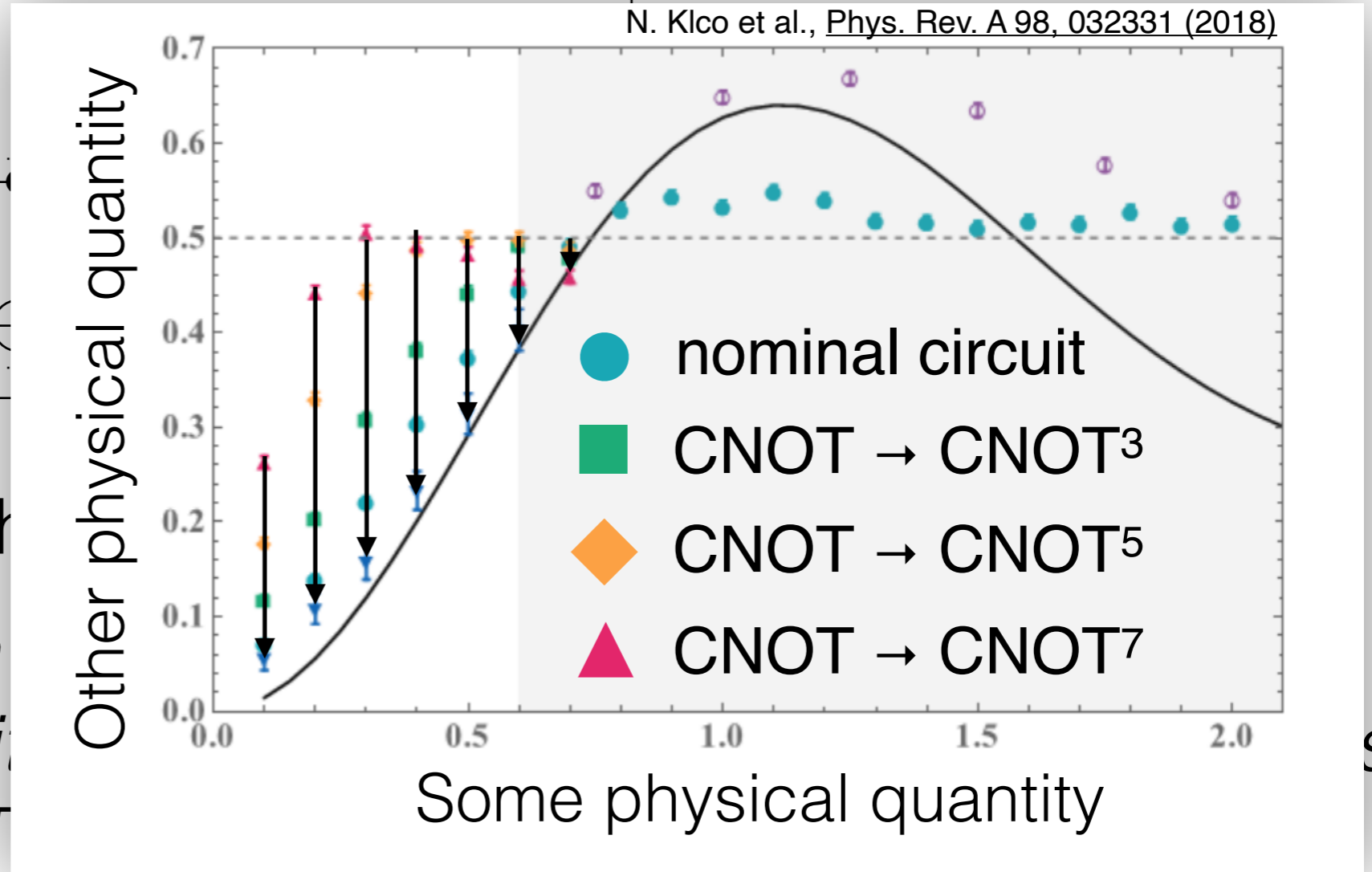
One common technique
Idea: replace each gate with a noisy version and change the answer with the noise. This

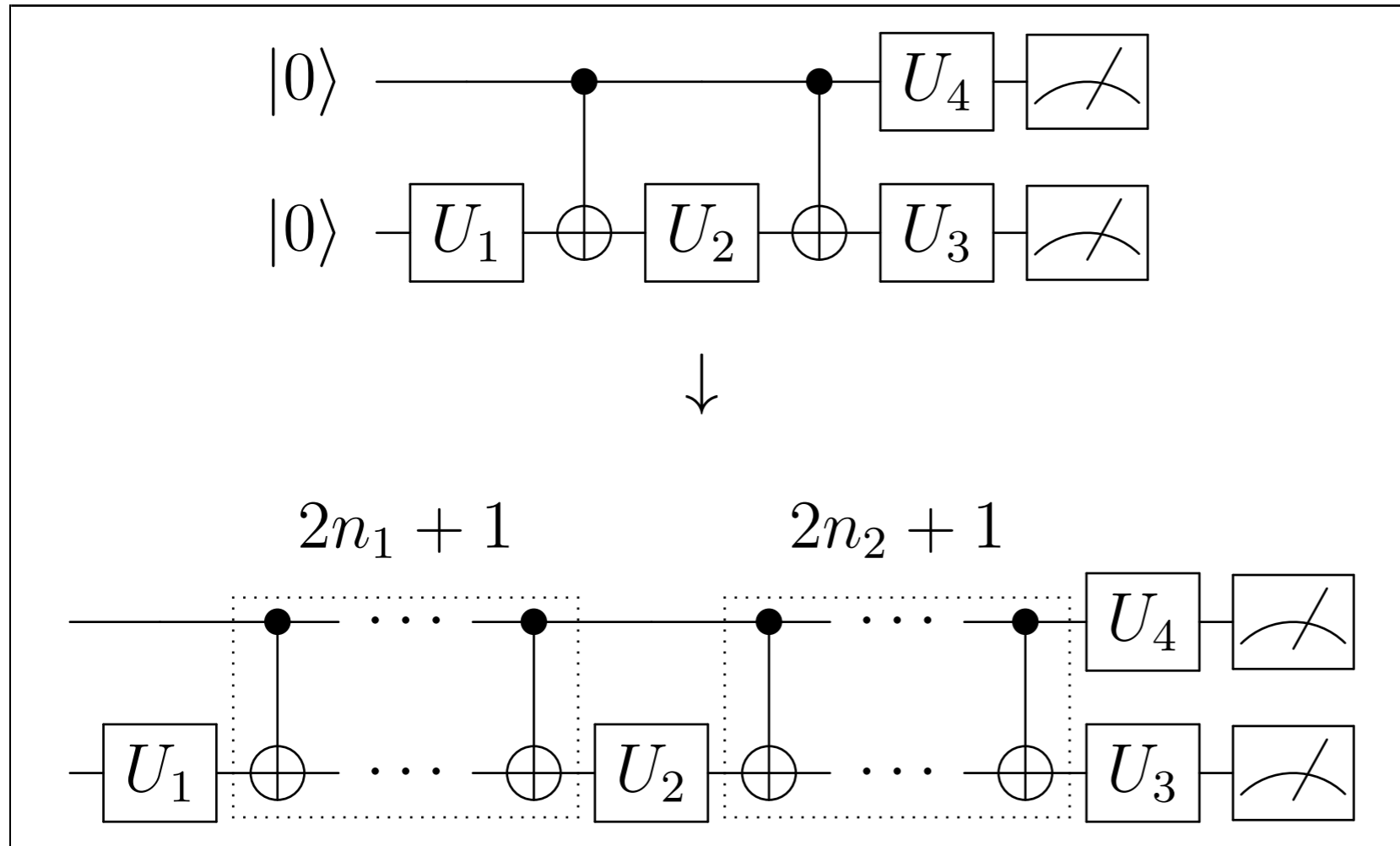


Gate error mitigation



One common technique for error mitigation is to change the answer with the noise. The idea is to replace each CNOT gate with a CNOT gate raised to a power of $2n_1 + 1$.



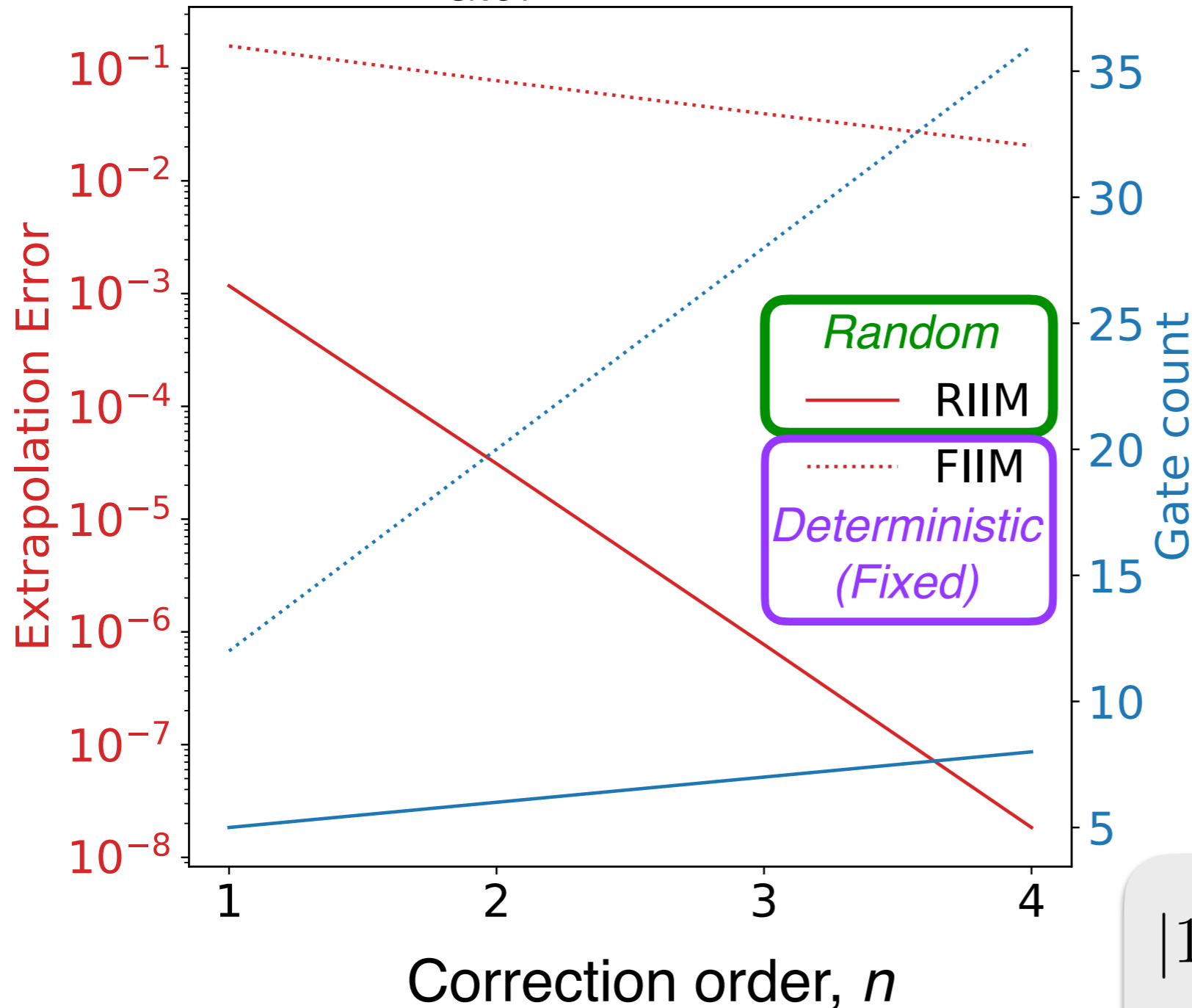


One common technique is **Zero Noise Extrapolation**

*New idea: promote n_i to a **random variable**. Instead of replacing every CNOT deterministically, randomly replace.*

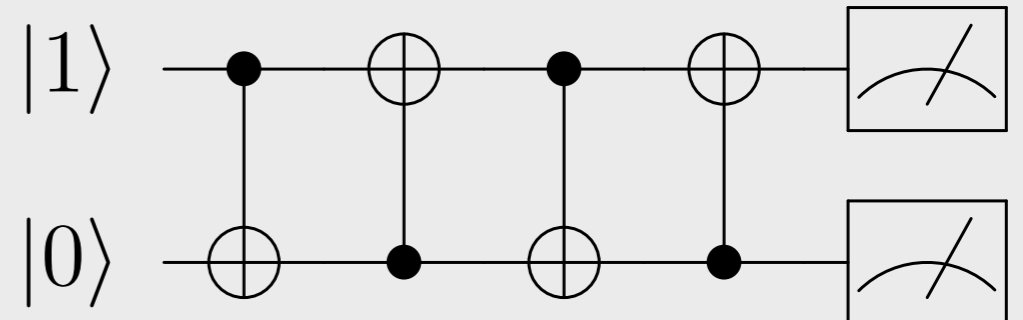
Gate error mitigation

$N_{CNOT} = 4, \epsilon = 1.0\%$



Circuit with N noisy gates:
 traditional method
 needs $(n+1) \times N$
 additional gates

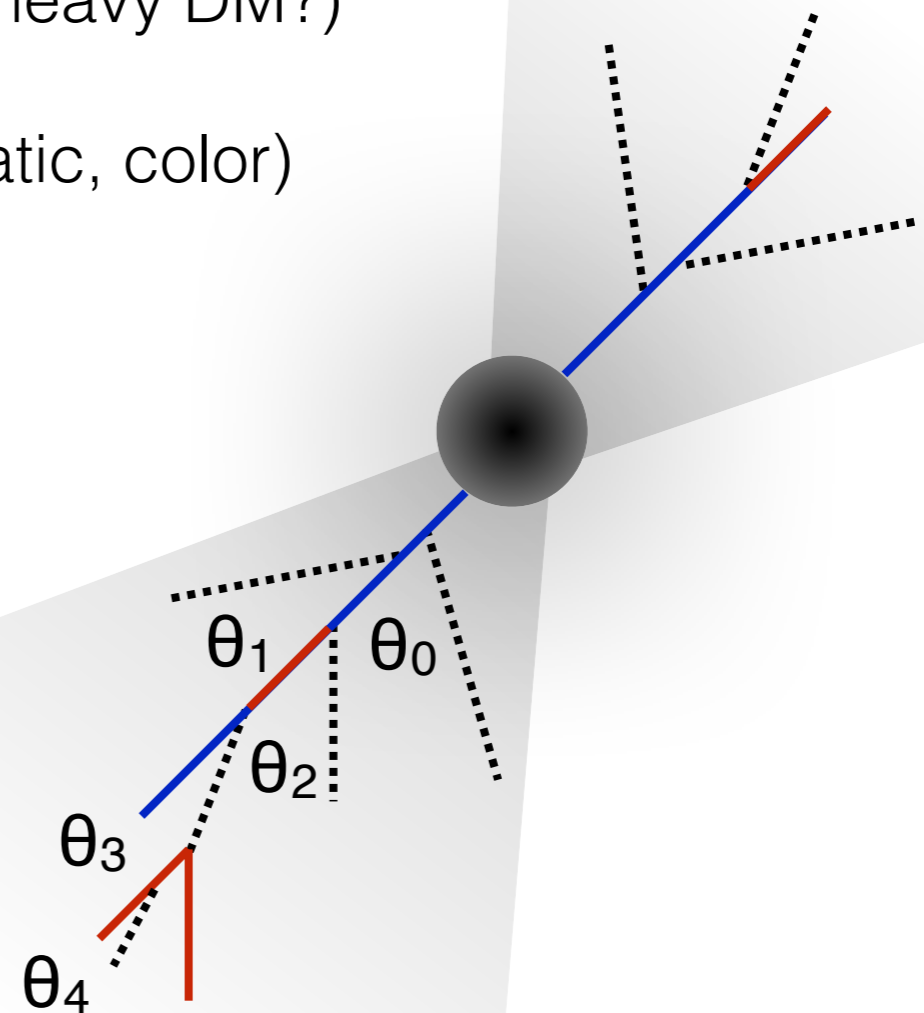
Random method
 only needs $n+1$
 additional gates (!)





- **QFT**

- Extend shower model
 - Electroweak radiation in SM (full SU(2))
 - Phenomenology with scalar model (heavy DM?)
- Towards QCD
 - Other source of interference (kinematic, color)
 - Soft radiation
 - Hybrid lattice methods

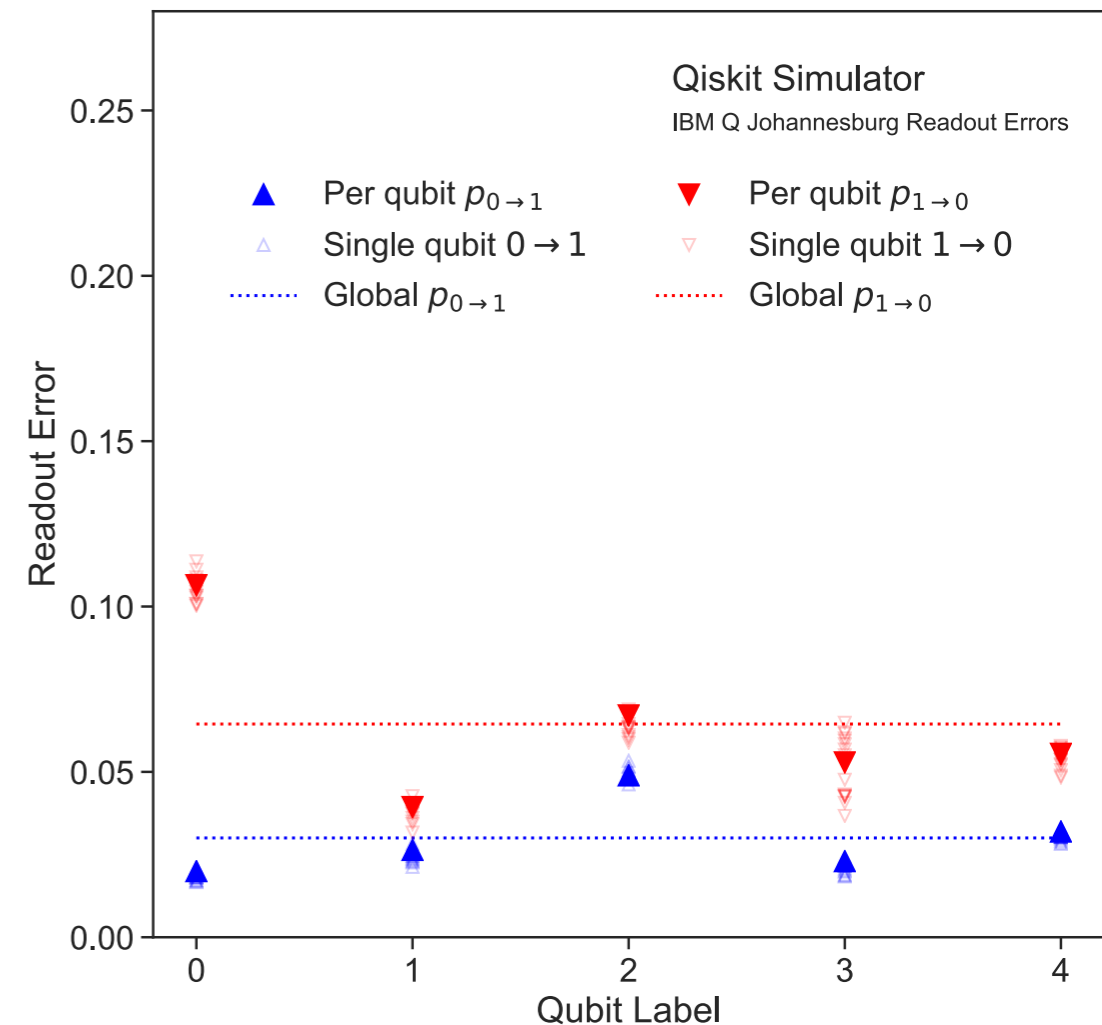


- **QFT**

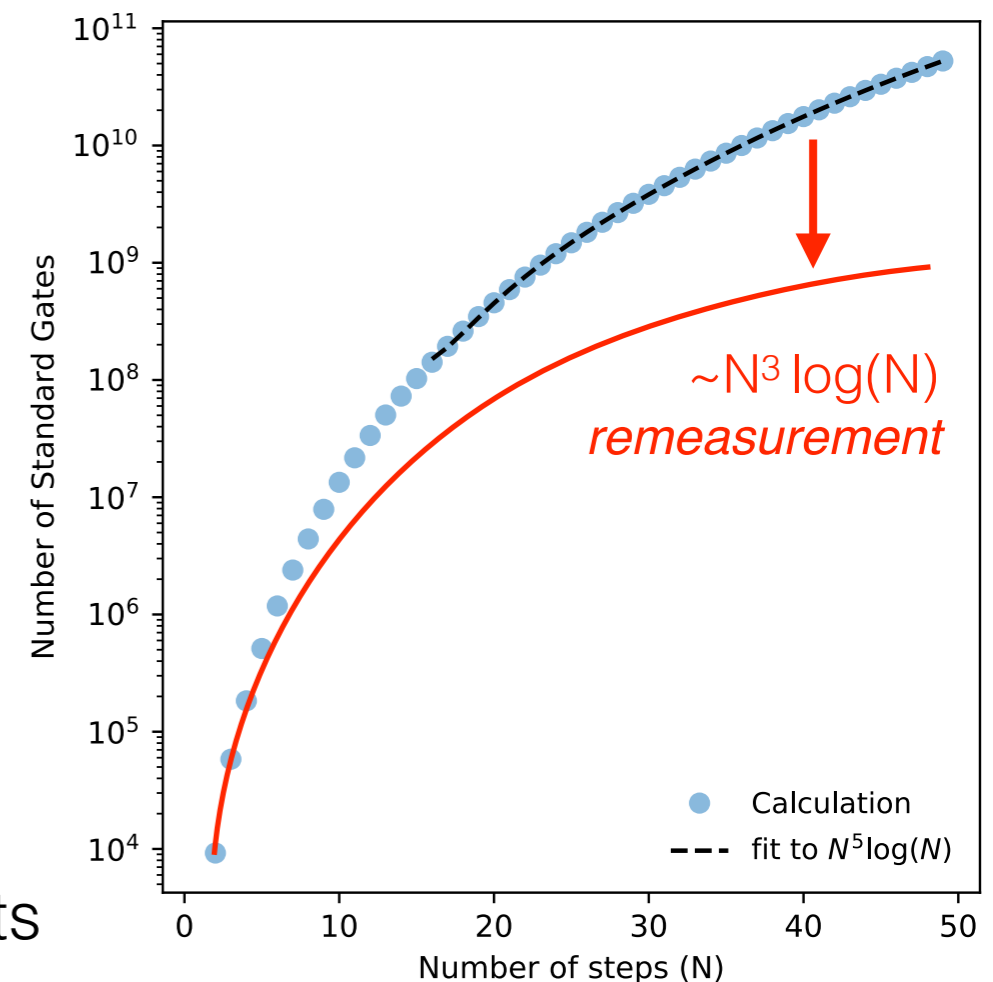
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- **Error Mitigation**

- Readout errors
 - Subexponential unfolding
- Gate errors
 - Parallel zero noise extrapolation
 - Interplay of active & passive mitigation



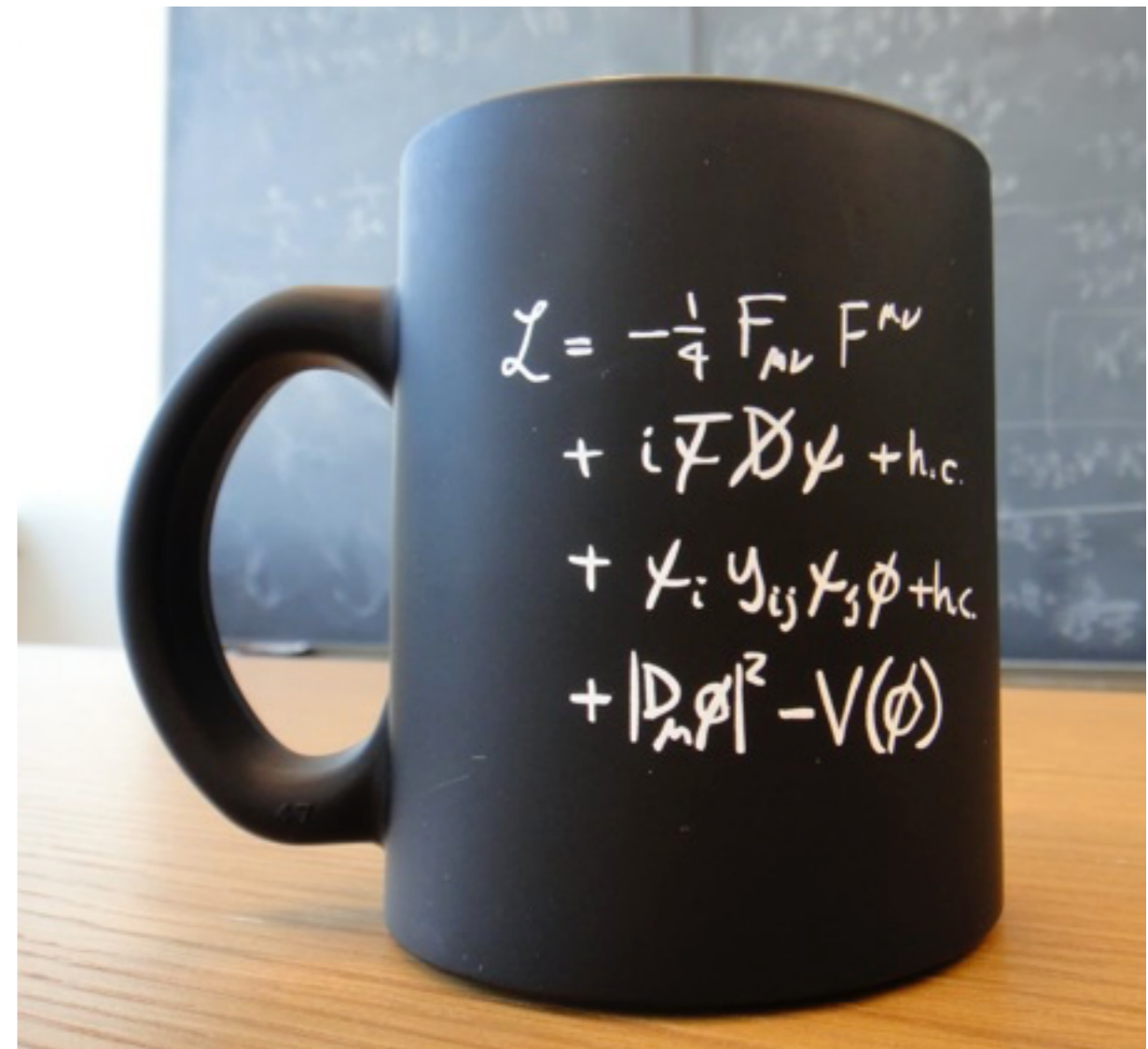
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- **Error Mitigation**
 - Readout errors
 - Subexponential unfolding
 - Gate errors
 - Parallel zero noise extrapolation
 - Interplay of active & passive mitigation
- **Software-hardware interface**
 - Custom operations
 - Resetting qubits, repeated operations, qudits
 - QFT-tailored hardware
 - Optimal lattice

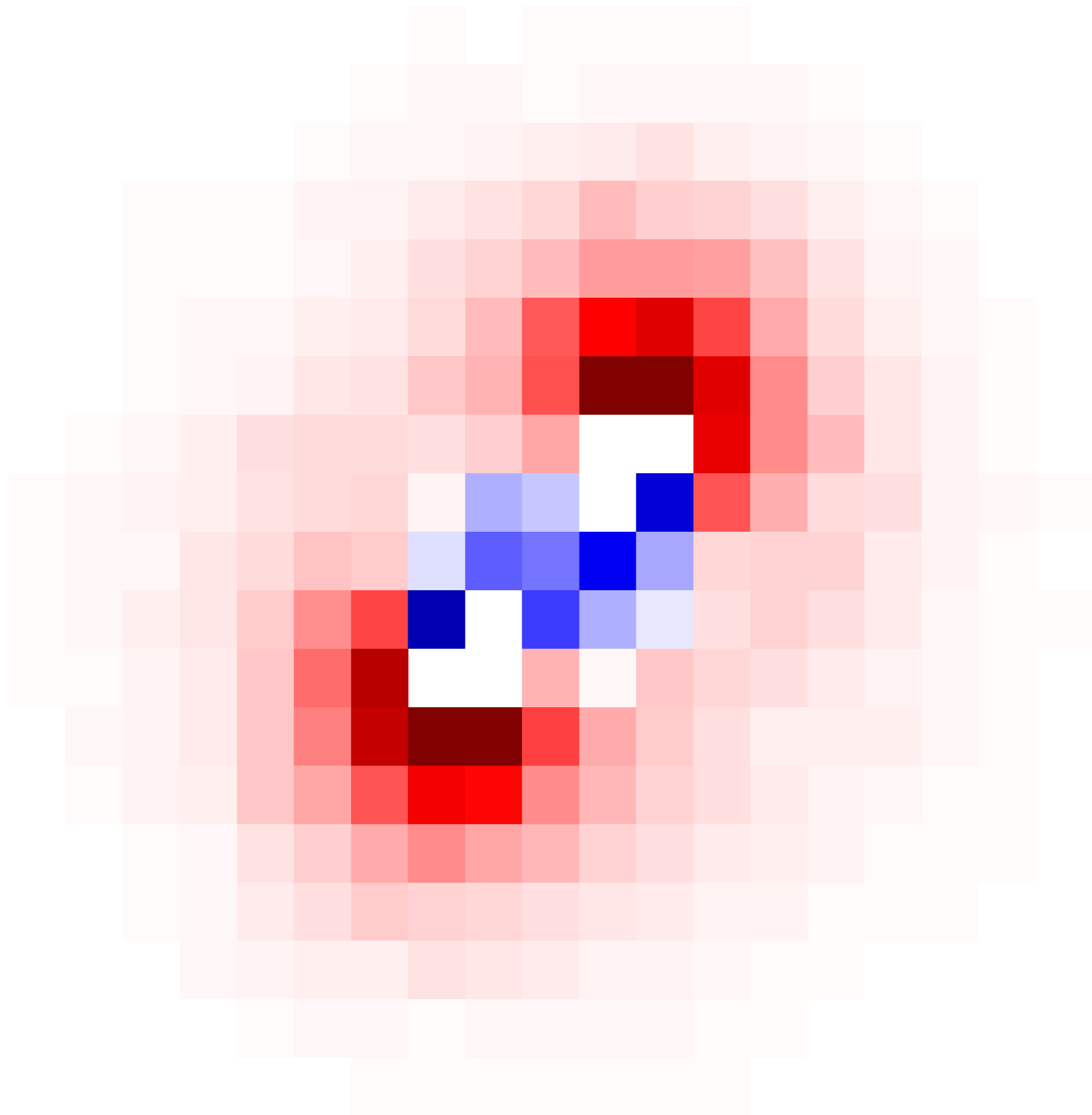


There is a long road ahead, but quantum algorithms are very promising for modeling high energy scattering processes.

At the same time, we can use our experience in experimental/theoretical HEP to contribute to quantum computing **in general**.

The field of QIS is rapidly advancing and there are growing connections between experiment, theory, instrumentation, and computing.





Fin.

Backup



Pro: Full theory

Con: Dynamics are too hard

(already using supercomputers)

Pro: Can do high-energy dynamics

Con: An approximation

Perturbation theory

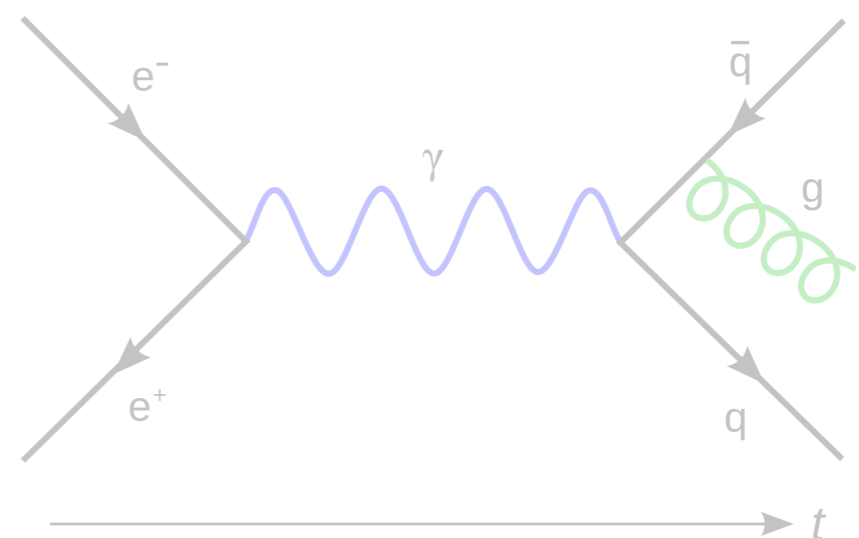


Image credit: https://en.wikipedia.org/wiki/Feynman_diagram

Lattice

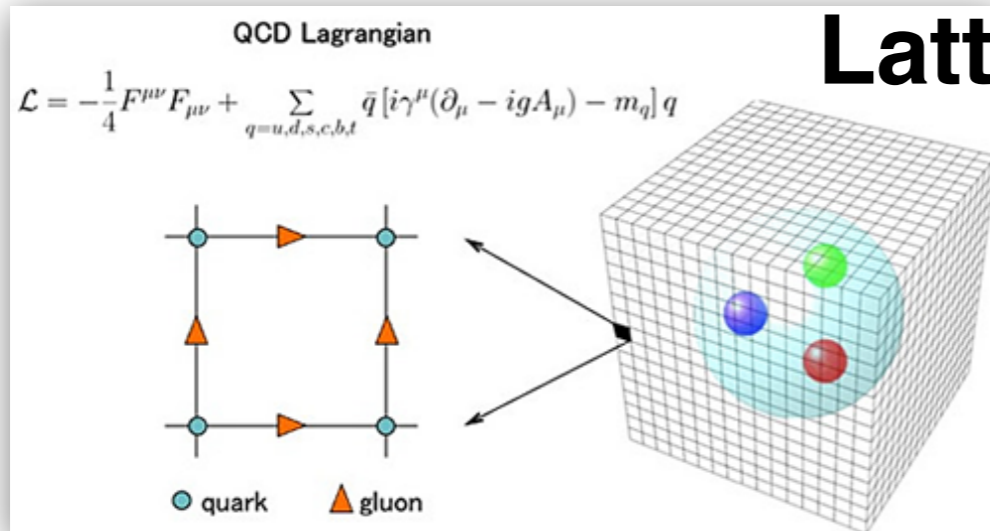


Image credit: <http://ipc-clermont.in2p3.fr/IMG/theorie/LQCD2.jpg>

Pioneering work by Preskill & collaborators
(Science 336 (2012) 1130).

Number of quantum gates to reach precision ϵ in $d+1$ dimensions

$$\sim \begin{cases} \left(\frac{1}{\epsilon}\right)^{1.5+o(1)}, & d = 1, \\ \left(\frac{1}{\epsilon}\right)^{2.376+o(1)}, & d = 2, \\ \left(\frac{1}{\epsilon}\right)^{3.564+o(1)}, & d = 3. \end{cases}$$

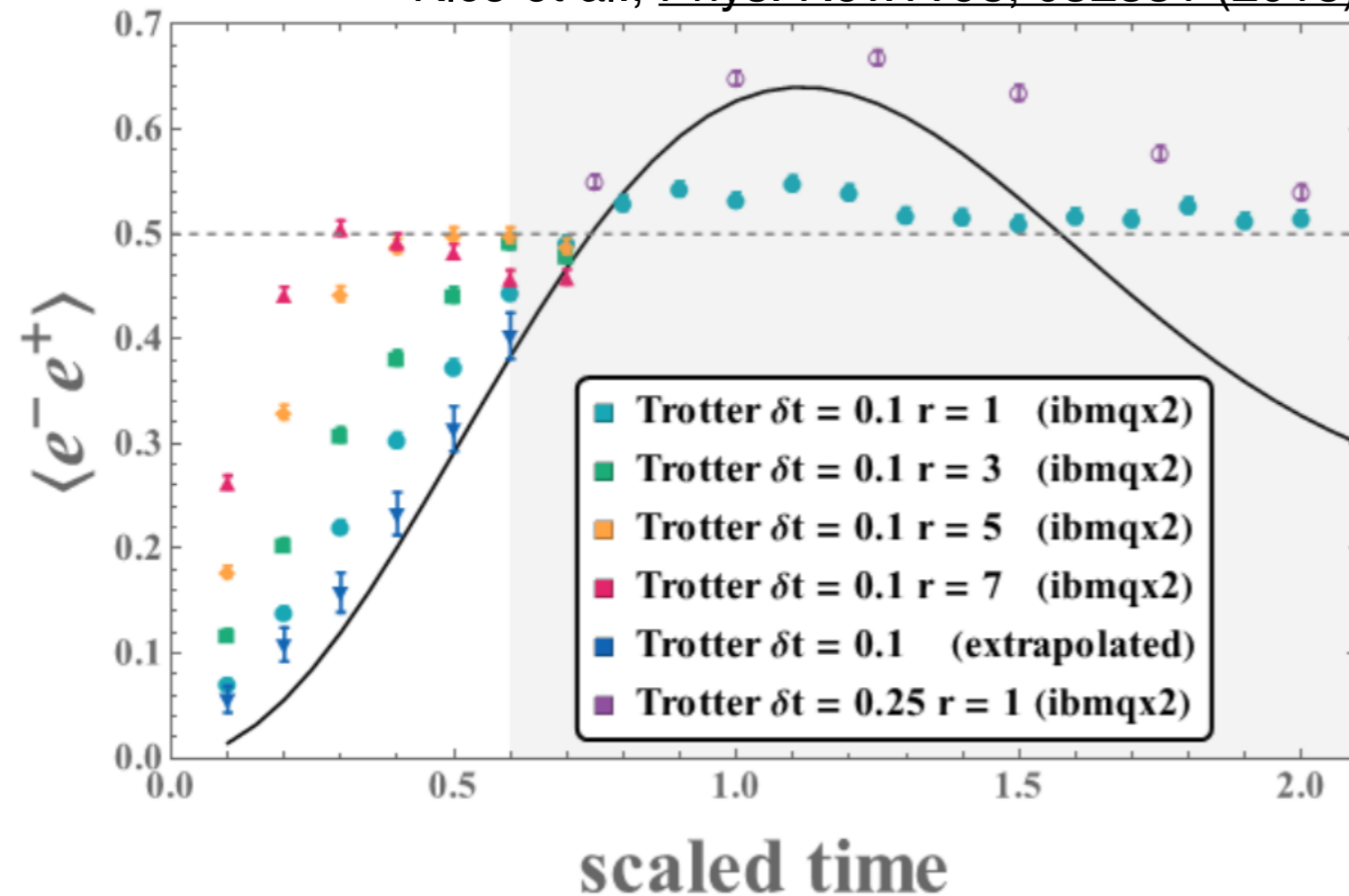
$$H = \sum_{\mathbf{x} \in \Omega} a^d \left[\frac{1}{2} \pi(\mathbf{x})^2 + \frac{1}{2} (\nabla_a \phi)^2(\mathbf{x}) + \frac{1}{2} m_0^2 \phi(\mathbf{x})^2 + \frac{\lambda_0}{4!} \phi(\mathbf{x})^4 \right]$$

This (and subsequent) work is more about formal scaling properties - actual number of qubits is too large to make practical calculations yet.

Lattice QFT with a Quantum Computer

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Klco et al., *Phys. Rev. A* 98, 032331 (2018)



This is the 1+1 Schwinger model; calculating the probability of finding an e^+e^- pair from the vacuum.

r = “noise ratio”; for $r > 1$, add extra CNOTs that correspond to the identity operation.

Recent progress by simplifying the problem has led to actual calculations of dynamics on a quantum computer!

Lattice QFT with a Quantum Computer

