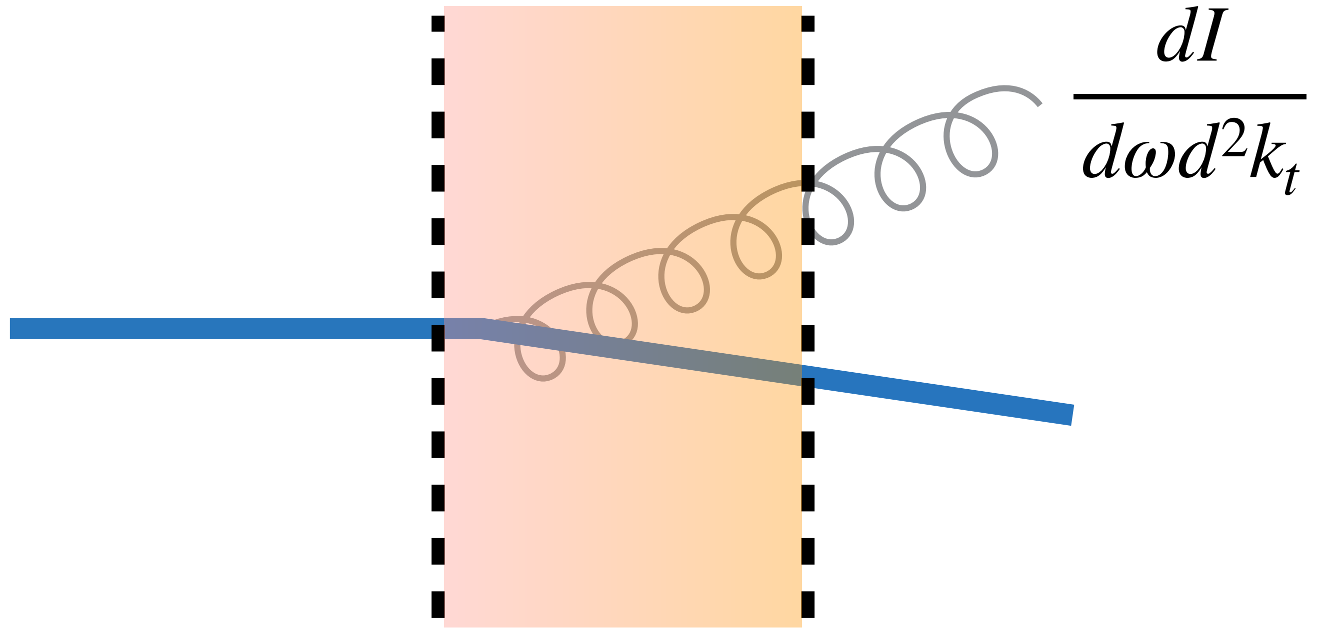
TOWARDS THE FULLY DIFFERENTIAL MEDIUM-INDUCED SPECTRUM WITH THE IMPROVED OPACITY EXPANSION



Alba Soto-Ontoso

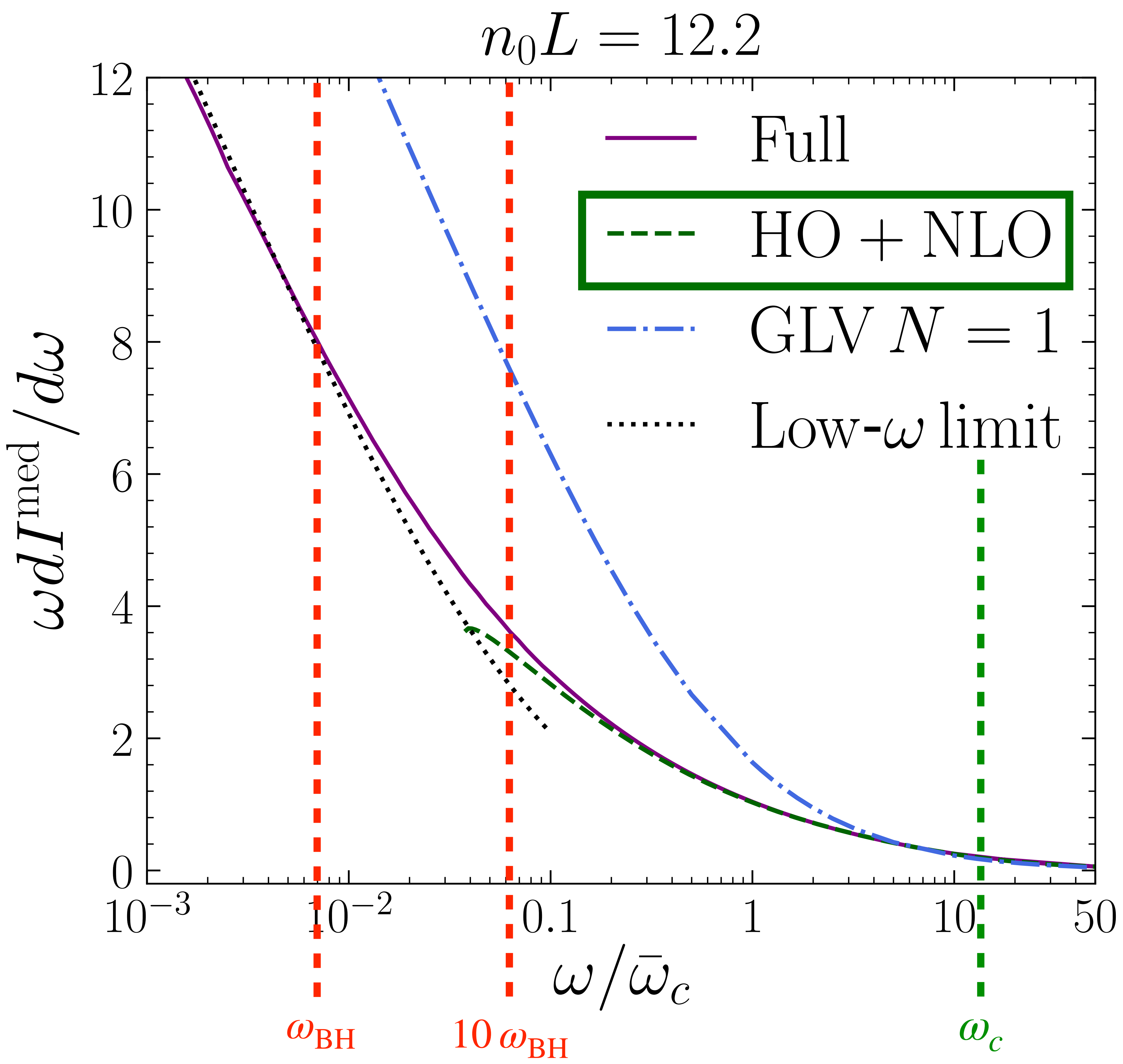
+ João Barata, Yacine Mehtar-Tani and Konrad Tywoniuk

CERN Heavy-Ion coffee

Remote, 7th December, 2020

Recap: last week in the heavy-ion coffee…

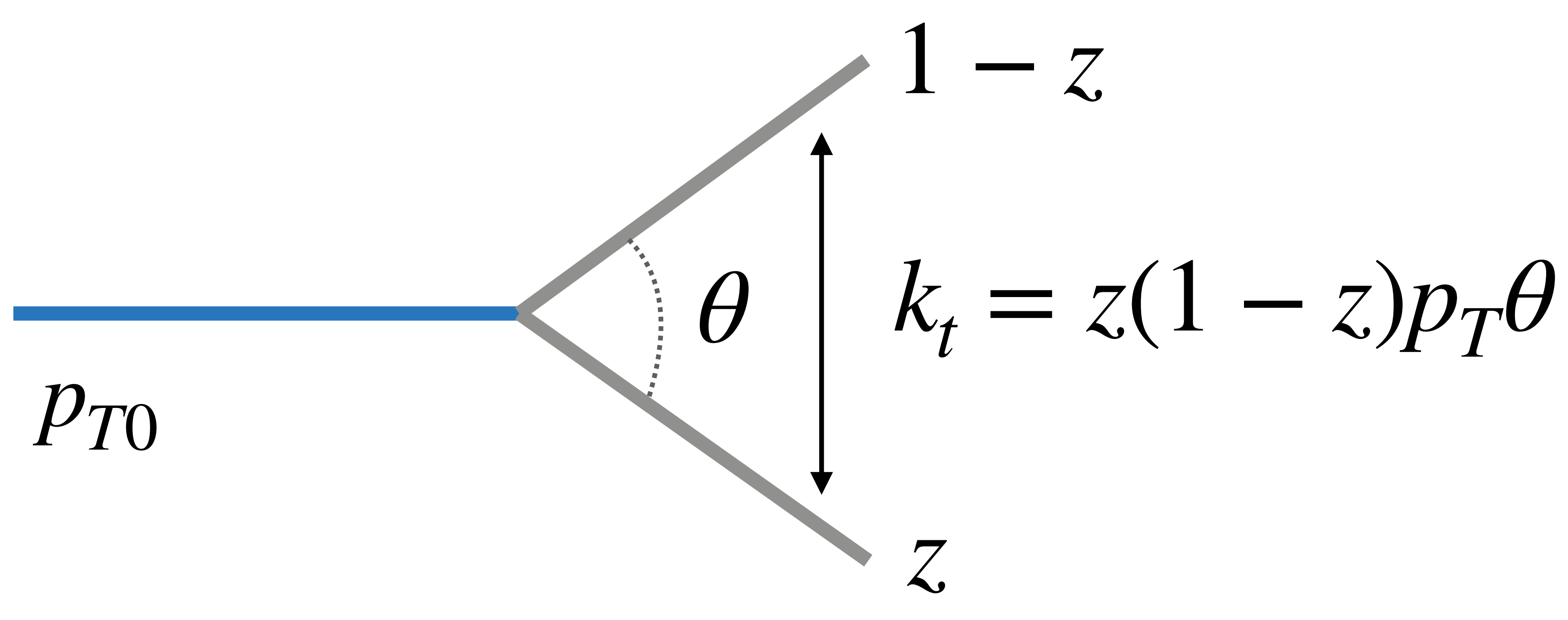
Conclusions



⇨ **This talk**

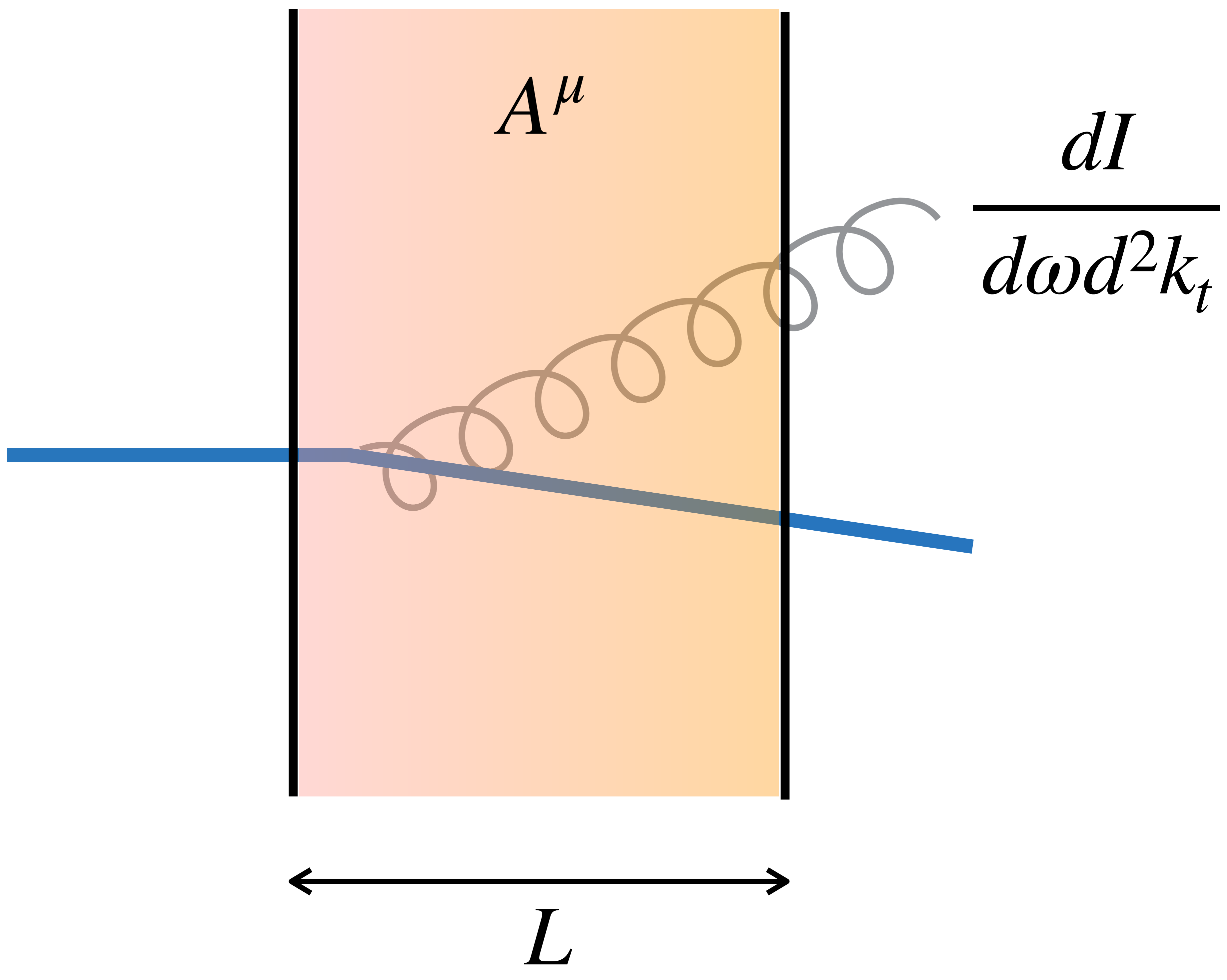
Parton branching

Vacuum



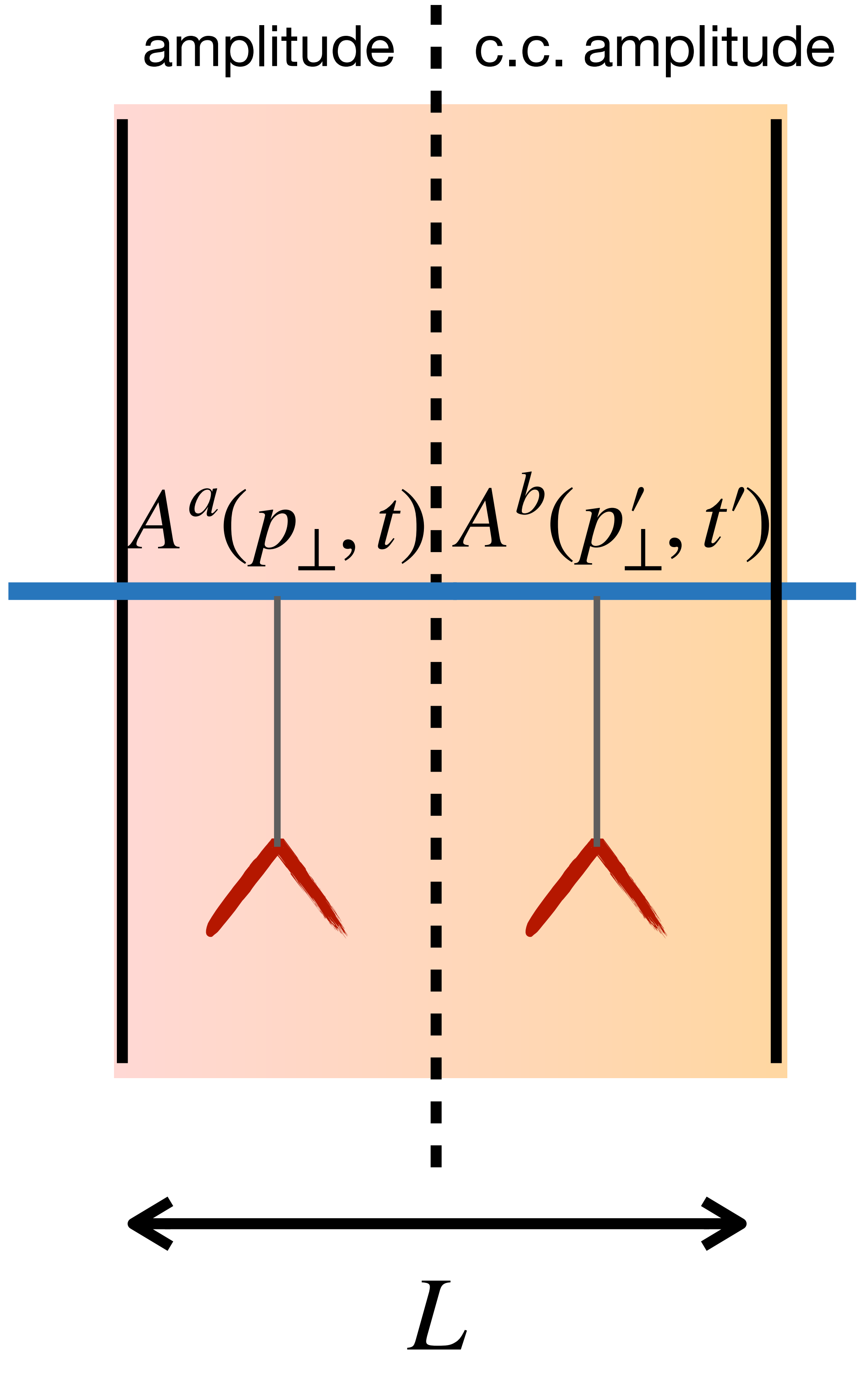
$$\mathrm{d}\Pi_{a\to bc} = \frac{C_R\alpha_s(k_t)}{\pi}\frac{\mathrm{d}\theta}{\theta}P_{a\to bc}(z)\mathrm{d}z$$

In-medium



How is the 1→2 splitting process modified by the QGP background field?

Weakly-coupled description of the QGP



- Translationally invariant:

$$\langle A^a(p_\perp, t) A^b(p'_\perp, t')\rangle = \delta^{ab}\delta(t-t')\delta(p_\perp - p'_\perp)\gamma(p_\perp)$$

- Microscopic interaction: collision rate $\gamma(p_\perp)$

Thermal medium

[Aurenche, Gelis, Zakaret JHEP'02]

$$\gamma^{\mathrm{HTL}}(p_\perp) = \frac{g^2 m_D^2 T}{p_\perp^2\,(p_\perp^2 + m_D^2)}$$

Yukawa-interaction

[Gyulassy, Wang PRL'92]

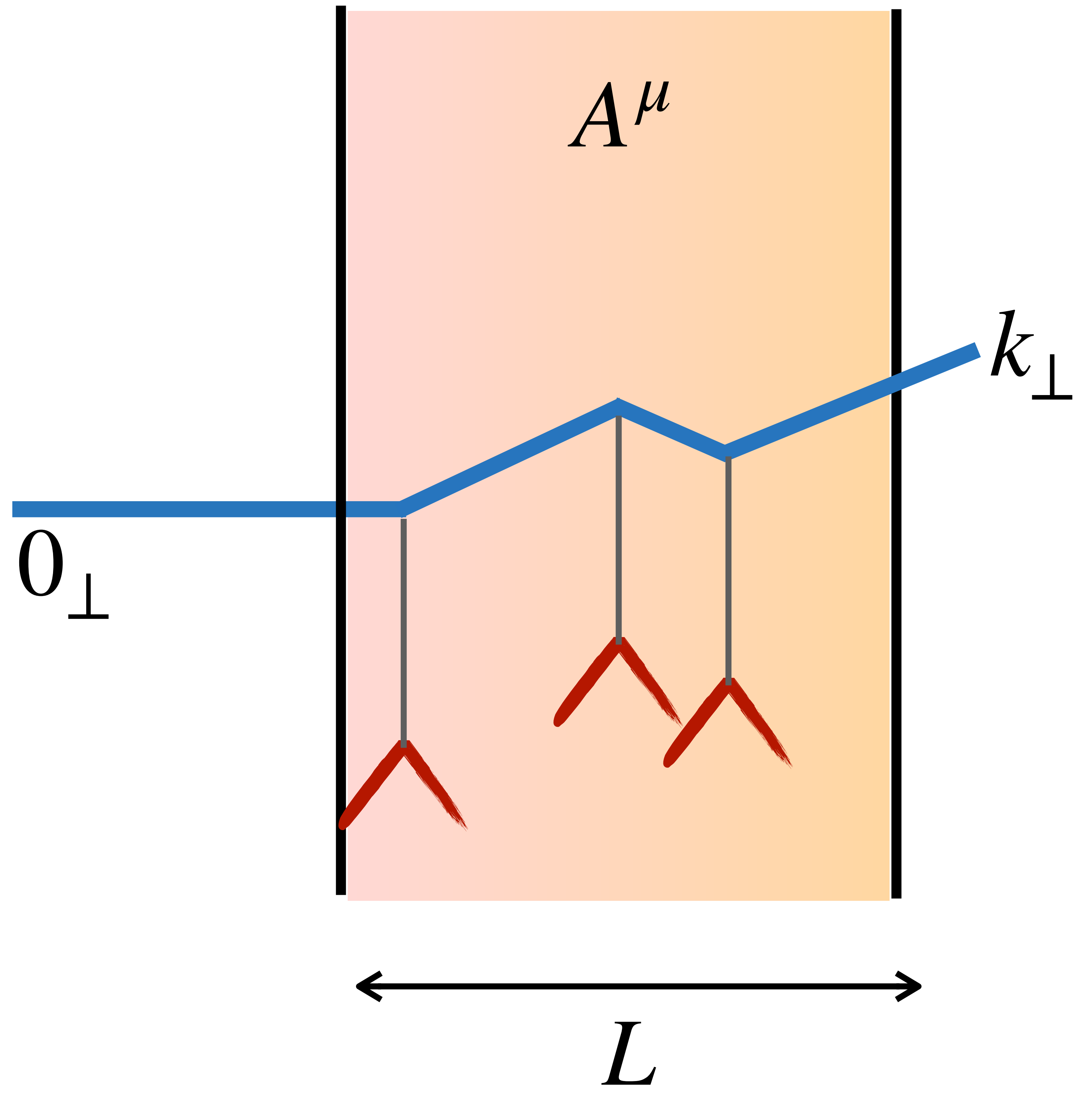
$$\gamma^{\mathrm{GW}}(p_\perp) = \frac{g^4 n}{(p_\perp^2 + \mu^2)^2}$$

Limits:

UV $\quad \gamma^{\mathrm{HTL,GW}}(p_\perp) \to 1/p_\perp^4$

IR $\quad \gamma^{\mathrm{GW}}(p_\perp) \to \mathrm{cnst}\,.\ \gamma^{\mathrm{HTL}}(p_\perp) \to 1/p_\perp^2$

Dynamics on in-medium branchings: p_t-broadening



Diffusion in transverse space follow Fokker-Planck equation

$$\frac{\partial}{\partial t}\mathscr{P}(k_\perp, t) = \frac{1}{4}\left(\frac{\partial}{\partial k_\perp}\right)^2 [\hat{q}(k_\perp^2)\mathscr{P}(k_\perp, t)]$$

where the diffusion coefficient

$$\hat{q}(Q^2) = \int_\mu^Q q^2\gamma(q) \quad \text{and} \quad \hat{q}_0 = \begin{cases} 4\pi\alpha_s^2 C_R n & \text{for GW} \\ \alpha_s C_R m_D^2 T & \text{for HTL} \end{cases}$$

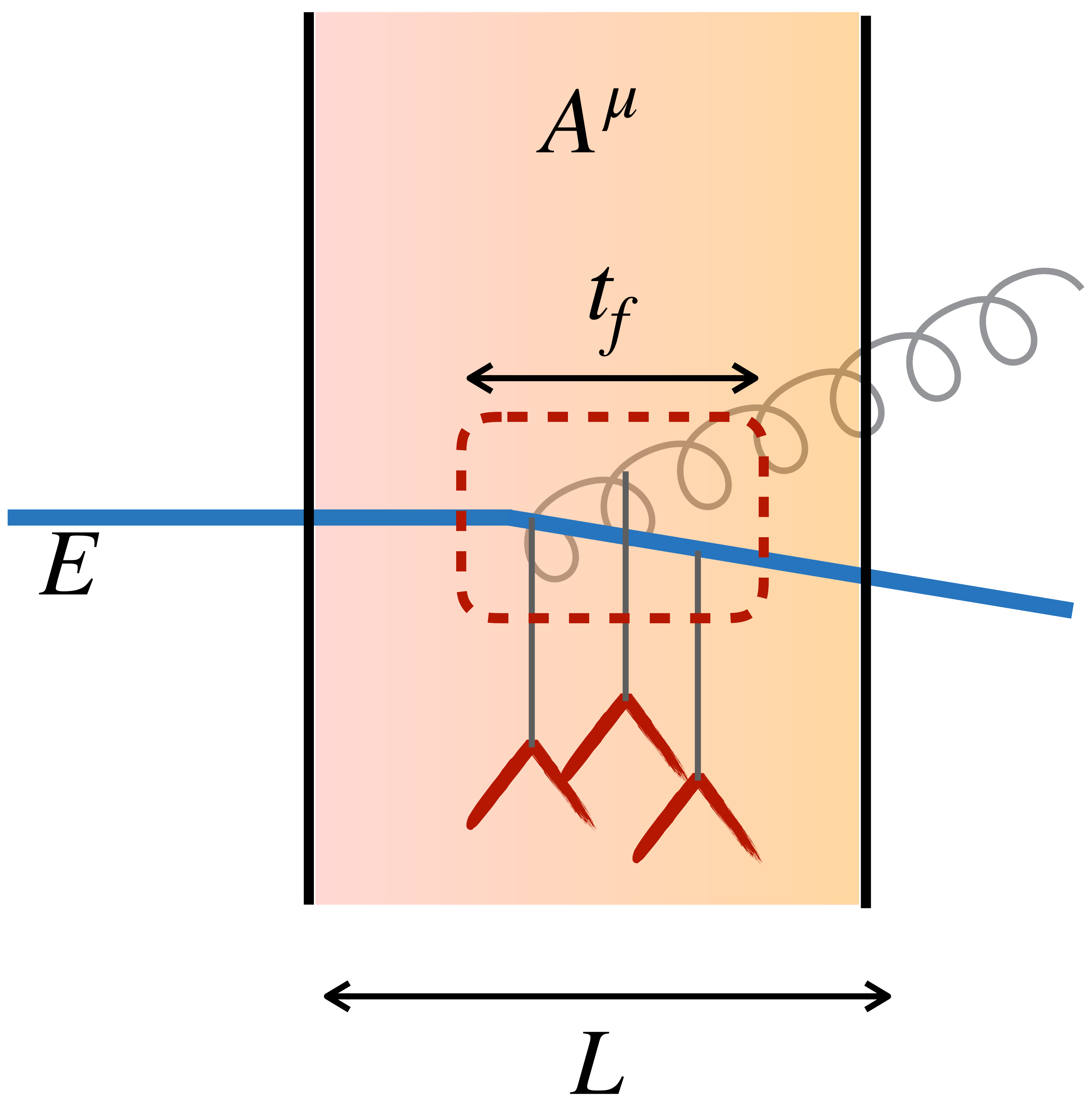
The typical transverse momentum acquired is

$$Q_s^2 \equiv \langle k_\perp^2 \rangle = \hat{q}L = \hat{q}_0 L \ln\left(\frac{Q^2}{\mu^2}\right)$$

No collinear divergence in the in-medium shower

Dynamics on in-medium branchings: LPM effect in QCD

[Landau-Pomeranchuk Migdal'53]



During the splitting time many scattering centres act coherently

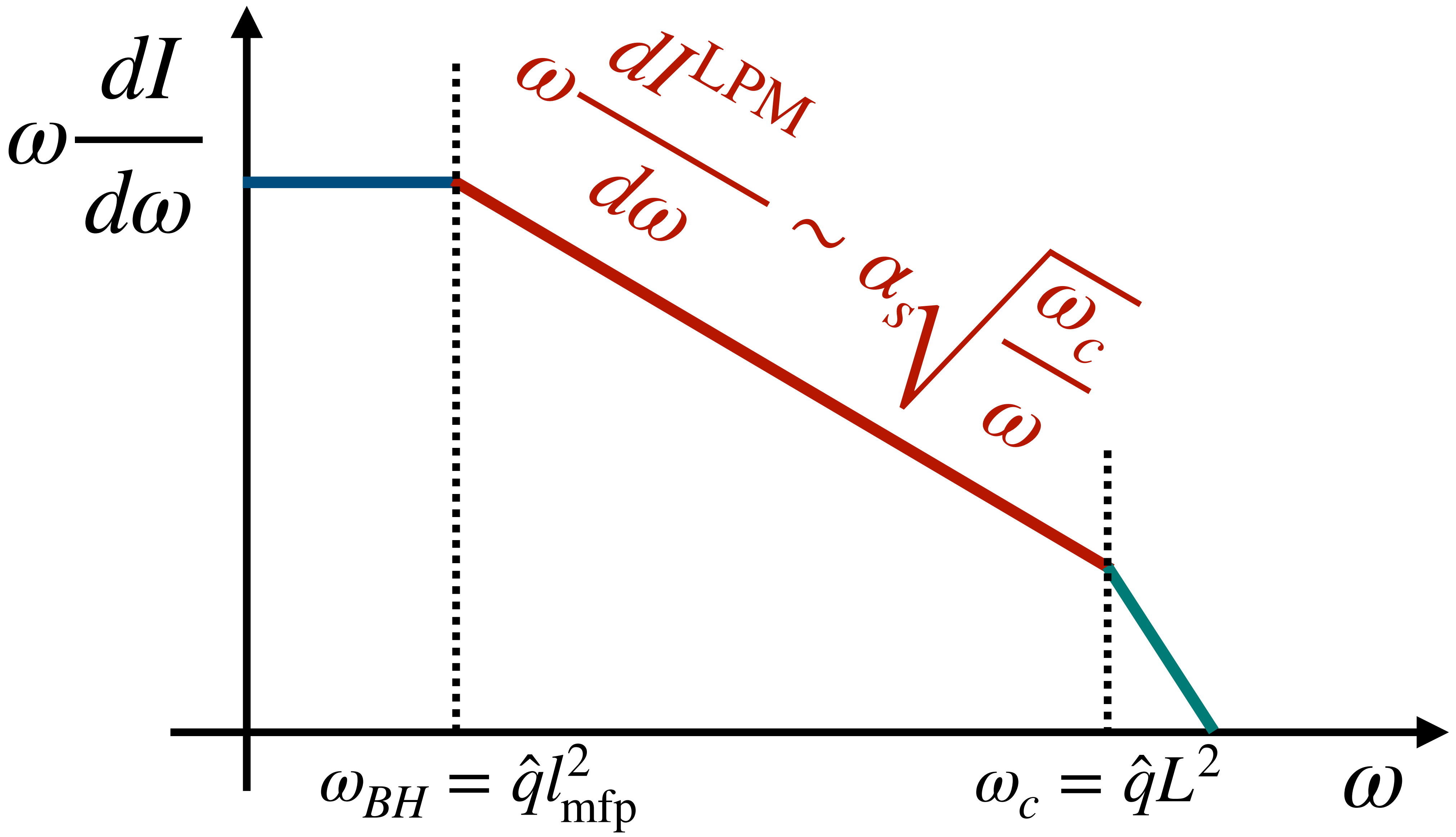
$$t_f = \omega/k_t^2 \qquad k_t = \hat{q}L \quad \Longrightarrow \quad t_{\rm br} = \sqrt{\frac{\omega}{\hat{q}}}$$

leading to the suppression of the spectrum in the UV

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{\rm br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

when $l_{\rm mfp} < t_{\rm br} < L$ and $\omega_c = \hat{q}L^2$

Dynamics on in-medium branchings: medium-induced emissions



- Bethe-Heitler regime: $t_{\rm br} < l_{\rm mfp}$ $\Longrightarrow$ $\omega \frac{dI^{\rm BH}}{d\omega} \sim \alpha_s \frac{L}{l_{\rm mfp}} \sim \alpha_s N_{\rm scatt}$
- Single hard scattering: $t_{\rm br} > L$ $\Longrightarrow$ $\omega \frac{dI^{\rm N=1}}{d\omega} \sim \alpha_s \frac{\omega_c}{\omega}$

Medium-induced radiation spectrum

[Baier, Dokshitzer, Mueller, Peigné, Schiff'96]
[Zakharov'96] [Wiedemann'00]

$$\omega \frac{dI}{d\omega} = \frac{\alpha_s C_R}{\omega^2} 2\Re \int_0^\infty dt_2 \int_0^{t_2} dt_1 \partial_{x_\perp} \cdot \partial_{y_\perp} \left[\mathcal{K}(x_\perp, t_2 | y_\perp, t_1) - \mathcal{K}_0(x_\perp, t_2 | y_\perp, t_1) \right]$$

The Green's function, $\mathcal{K}$, obeys a Schrödinger equation and $\mathcal{K} \sim \langle \mathrm{tr(UG)} \rangle$

$$\left[i \frac{\partial}{\partial t_2} + \frac{\partial^2}{2\omega} + i v(x_\perp) \right] \mathcal{K}(x_\perp, t_2, | y_\perp, t_1) = i\delta(x_\perp - y_\perp)\delta(t_2 - t_1)$$

with the imaginary potential

$$v(x_\perp) \equiv C_R \int_q \left(1 - \mathrm{e}^{iq_\perp \cdot x_\perp}\right) \gamma(q_\perp)$$

$$\rightarrow \quad v^{\mathrm{GW}}(x_\perp) = \frac{\hat{q}_0}{\mu^2} \left(1 - \mu |x_\perp| K_1(\mu |x_\perp|)\right)$$

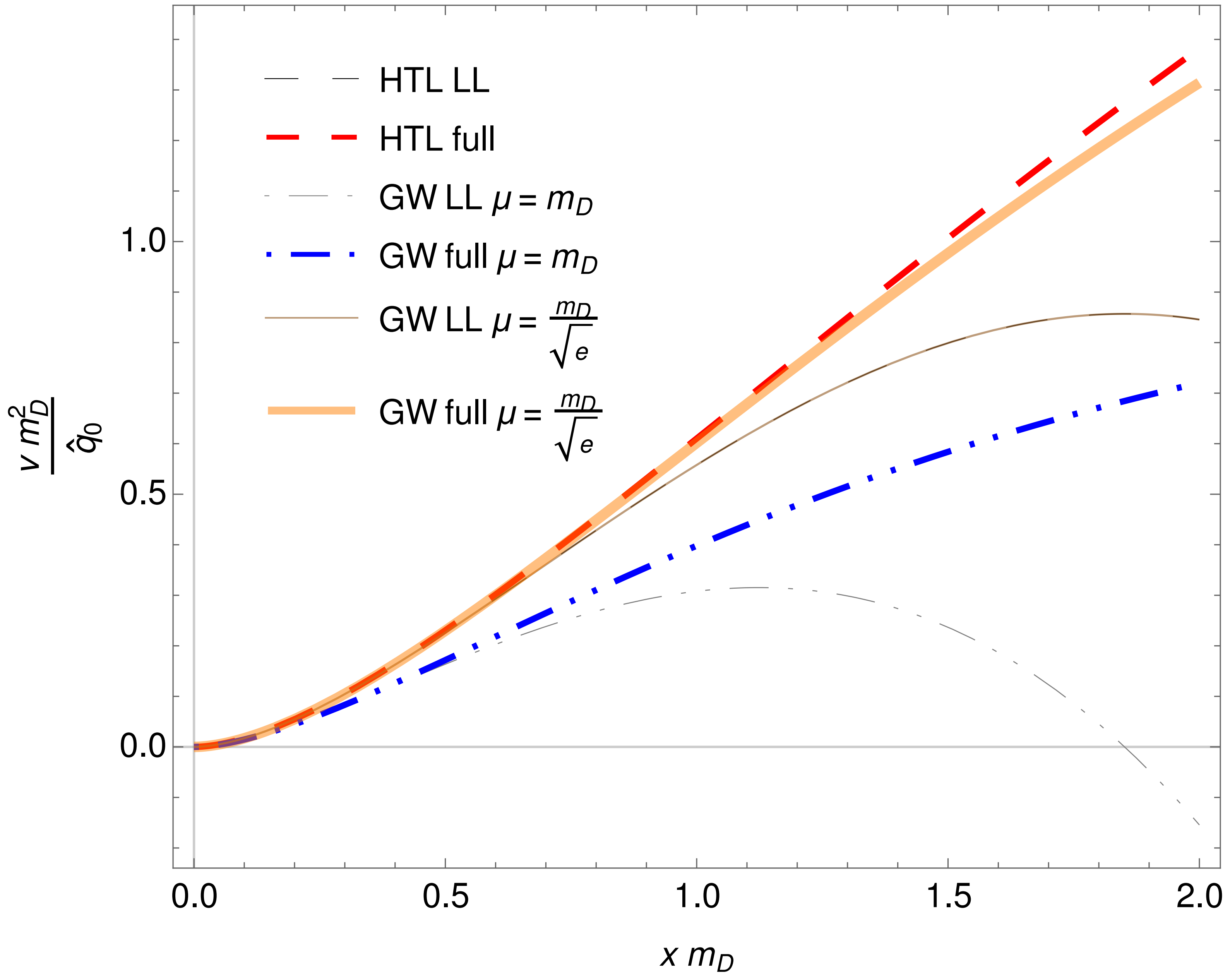
$$\rightarrow \quad v^{\mathrm{HTL}}(x_\perp) = \frac{2\hat{q}_0}{m_D^2} \left(K_0(m_D |x_\perp|) + \log\left(\frac{m_D |x_\perp|}{2}\right) + \gamma_E \right)$$

Remarks about the potential

[Barata, Mehtar-Tani'20]

When $k_\perp \sim x_\perp^{-1} \gg \mu_*$

$$v(x_\perp) = \frac{1}{4} Q_{s0}^2 x_\perp^2 \log \frac{1}{x_\perp^2 \mu_*^2} + \mathcal{O}(x_\perp^4 \mu_*^4) \quad \text{with} \quad \mu_*^2 = \begin{cases} \frac{\mu^2}{4} \mathrm{e}^{-1+2\gamma_E} & \text{for GW} \\ \frac{m_D^2}{4} \mathrm{e}^{-2+2\gamma_E} & \text{for HTL} \end{cases}$$

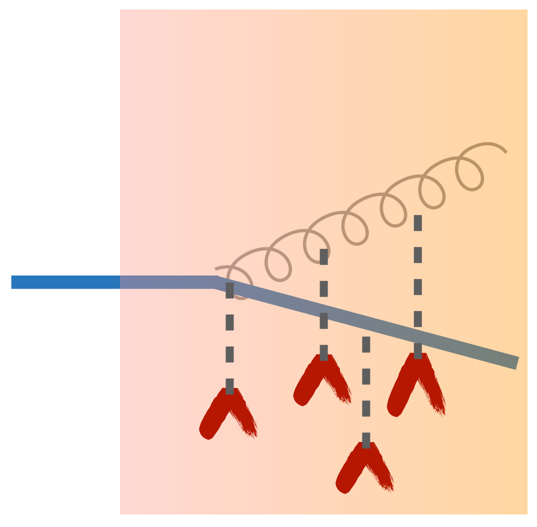


Leading order map between the two most popular medium models

Medium-induced gluon radiation spectrum: 2 analytic limits

- Multiple soft scattering approximation: HO

$$v(x_\perp) = \frac{1}{4} Q_{s0}^2 x_\perp^2 \log \frac{1}{x_\perp^2 \mu_*^2} + \mathcal{O}(x_\perp^4 \mu_*^4) \to \frac{1}{4} Q_{s0}^2 x_\perp^2$$

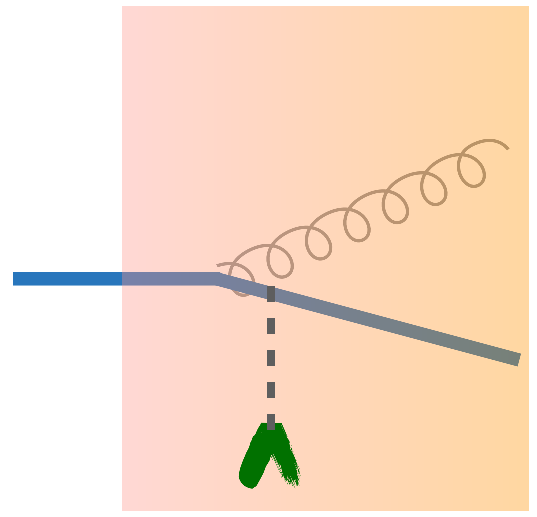


The problem reduces to that of an harmonic oscillator. Common in MCs

[Caucal, Iancu, Mueller, Soyez,18'-20']

- **Opacity expansion:** expand kernel in powers of $v(x_\perp)$

$$\mathcal{K}(t_2, t_1) = \mathcal{K}_0(t_2, t_1) + \int dt \mathcal{K}_0(t_2, t) v(t) \mathcal{K}_0(t, t_1)$$

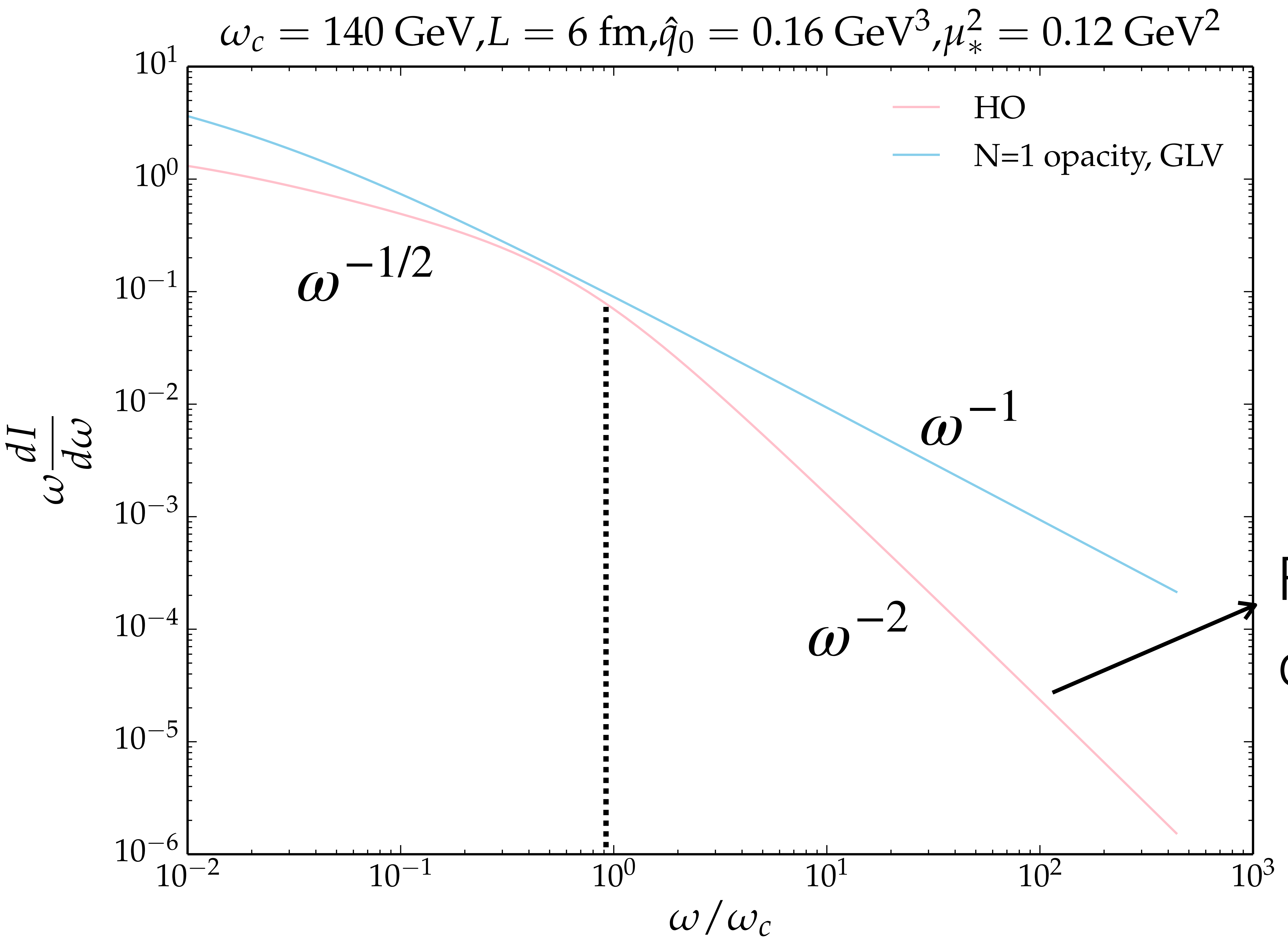


LO (N=0): vacuum radiation. NLO (N=1): single, hard scattering

GLV

[Wiedemann'00]
[Gyulassy, Levai, Vitev'01]
[Sievert, Vitev'19]

Medium-induced gluon radiation spectrum: 2 analytic limits



Fails to capture the UV due to no $1/k_t^4$

Is it possible to encompass these two limits in an analytic framework?

The ‘improved’ opacity expansion

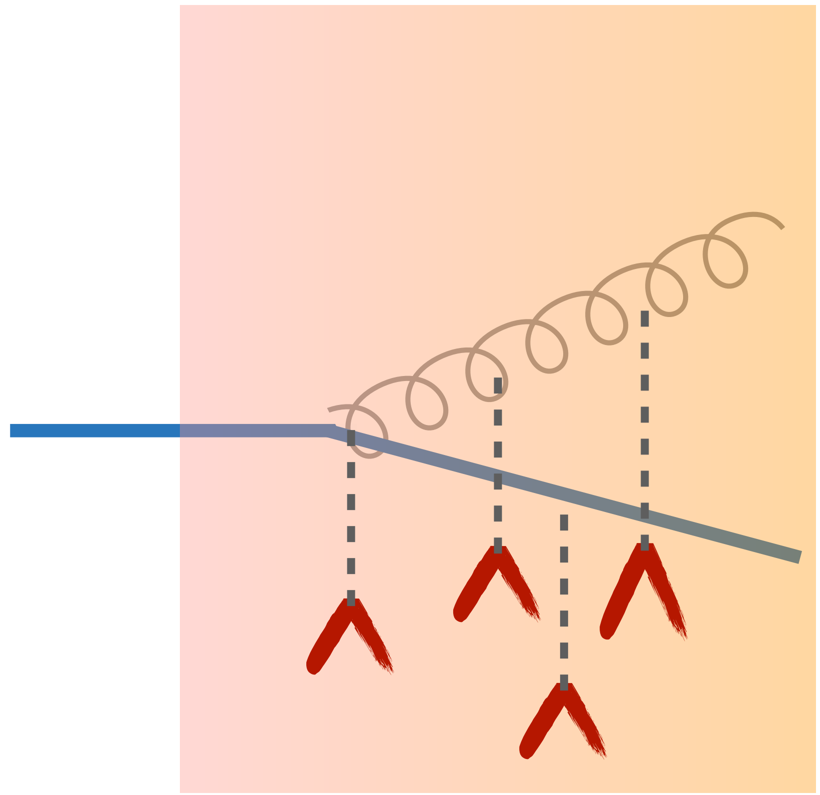
[Mehtar-Tani’19] [Mehtar-Tani,Tywoniuk’20] [Iancu,Itakura,Triantafyllopoulos’04] [Moliere’48]

Key idea: **expand around the harmonic oscillator** by doing

$$v(x_\perp) = \frac{1}{4} Q_{s0}^2 x_\perp^2 \log \frac{1}{x_\perp^2 \mu_*^2} \Longrightarrow v(x_\perp) = \frac{1}{4} Q_{s0}^2 x_\perp^2 \left(\log \frac{Q^2}{\mu_*^2} + \frac{1}{Q^2 x_\perp^2} \right)$$

$$v(x_\perp) = v_{\rm HO} + \delta v$$

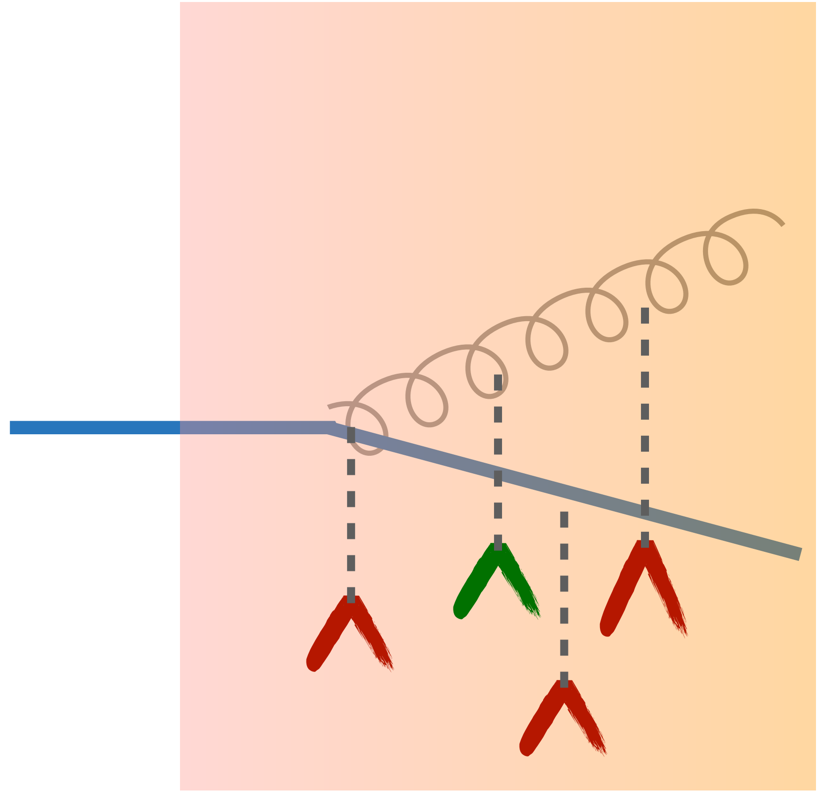
$$\mathcal{K}^{\rm LO} = \mathcal{K}_{\rm HO}$$



LO

$$\mathcal{K}(t_2, t_1) = \mathcal{K}_{\rm HO}(t_2, t_1) - \int dt \mathcal{K}_{\rm HO}(t_2, t) \delta v \mathcal{K}(t, t_1)$$

$$\mathcal{K}^{\rm NLO} \propto - \mathcal{K}_{\rm HO} \delta v \mathcal{K}_{\rm HO}$$



NLO

Harmonic oscillator + hard-scatterings as perturbation

The 'improved' opacity expansion: FAQ

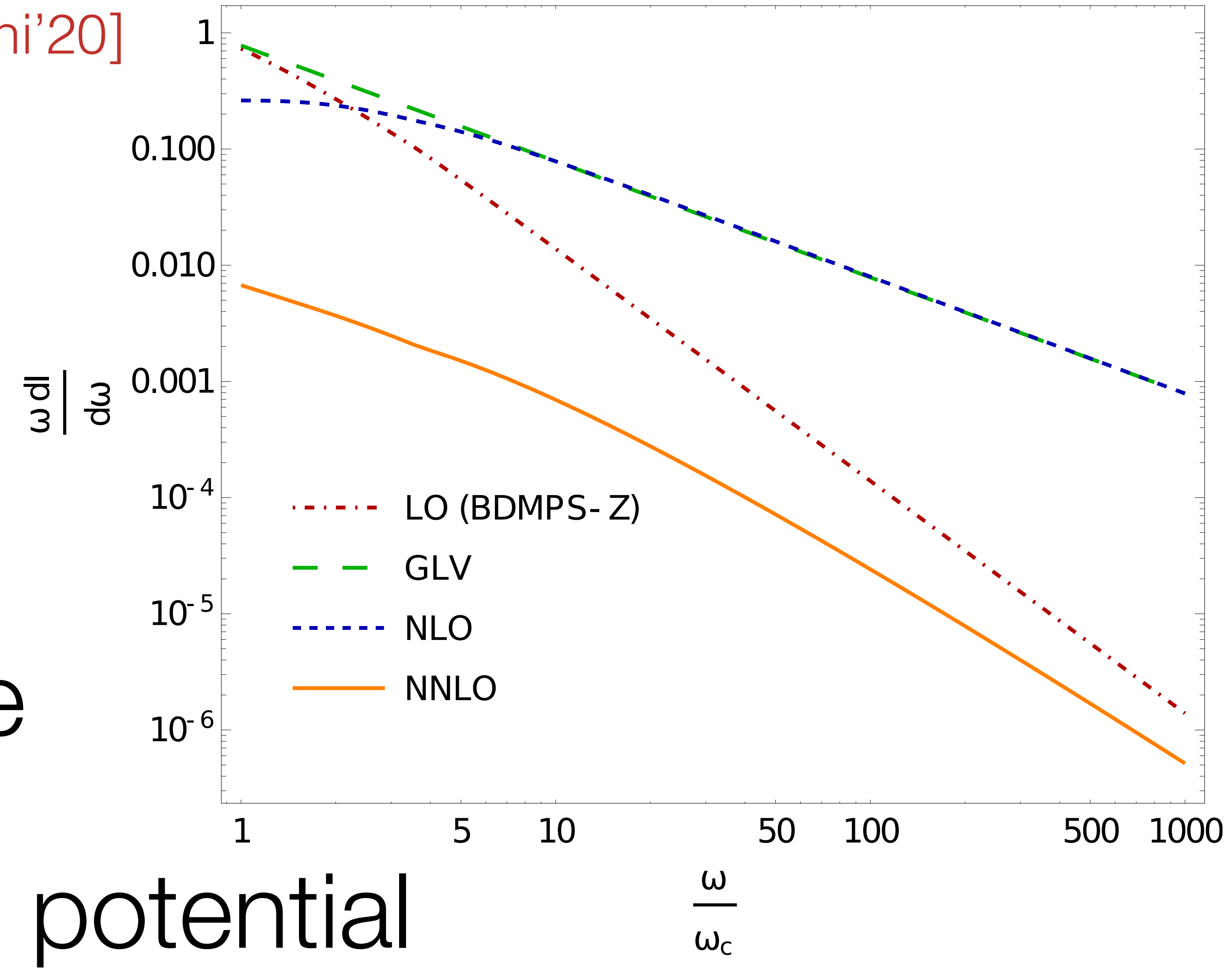
- **What is Q^2?** Related to the typical transverse momentum acquired during the splitting

If $Q^2 \equiv \hat{q}_0 L$ expansion is divergent at small frequencies $\Longrightarrow$ $Q^2 \equiv Q^2(\omega) = \sqrt{\omega \hat{q}}$

Notice that when all-orders are taken into account, result does not depend on Q^2

- **Higher orders?** NNLO computed analytically in [Barata, Mehtar-Tani'20]

$$\omega \frac{dI^{\mathrm{N}^m\mathrm{LO}}}{d\omega} = (-1)^m \frac{\bar{\alpha}\pi}{\omega^2} 2\mathrm{Re}\Big[\int_0^\infty dt_2 \int_0^{t_2} dt_1 \int_{\boldsymbol{z}_1}\int_{\boldsymbol{z}_2}\cdots\int_{\boldsymbol{z}_m}\int_{t_1}^{t_2} ds_m \int_{t_1}^{s_m} ds_{m-1}\cdots\int_{t_1}^{s_2} ds_1$$
$$\times\, \boldsymbol{\partial_x}\cdot\boldsymbol{\partial_y}\, \mathcal{K}_{\mathrm{HO}}(\boldsymbol{x},t_2;\boldsymbol{z}_m,s_m)\delta v(\boldsymbol{z}_m,s_m)\mathcal{K}_{\mathrm{HO}}(\boldsymbol{z}_m,s_m;\boldsymbol{z}_{m-1},s_{m-1})\delta v(\boldsymbol{z}_{m-1},s_{m-1})$$
$$\times\, \mathcal{K}_{\mathrm{HO}}(\boldsymbol{z}_{m-1},s_{m-1};\boldsymbol{z}_{m-2},s_{m-2})\cdots\times\mathcal{K}_{\mathrm{HO}}(\boldsymbol{z}_1,s_1;\boldsymbol{y},t_1)\Big]_{\boldsymbol{x}=\boldsymbol{y}=0}.$$



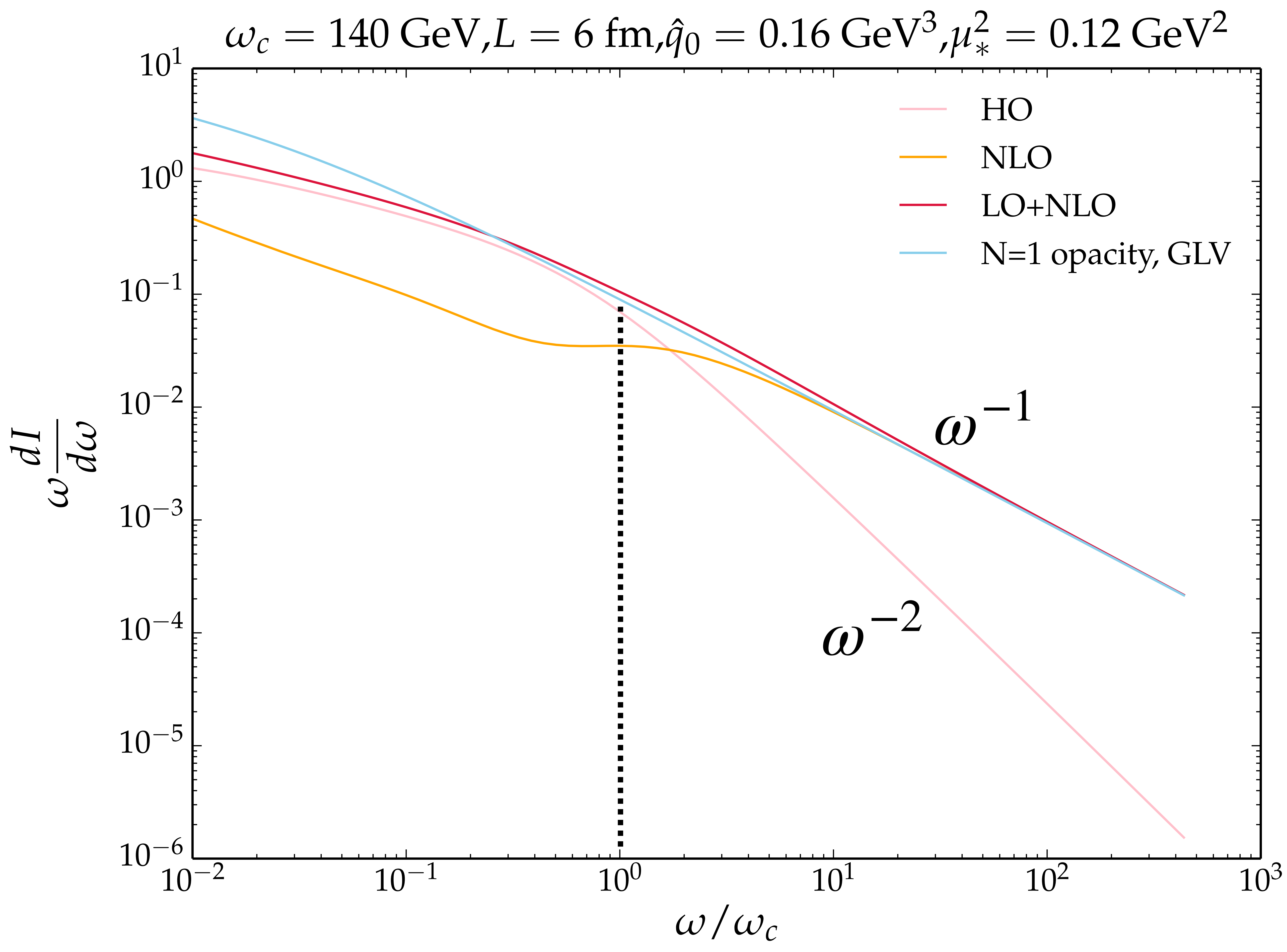
Fast numerical evaluation of the series + quick convergence

- **Bethe-Heitler regime?** Need to go beyond the leading-log potential [Mehtar-Tani,Tywoniuk'20]

$v(x_\perp) = v_{\mathrm{HO}} + \delta v_{\mathrm{full}}$ && new $Q^2(\omega)$ as current one breaks down when $Q^2 \sim \mu_*$

Region not relevan for phenomenology.

Medium-induced gluon radiation spectrum with the IOE

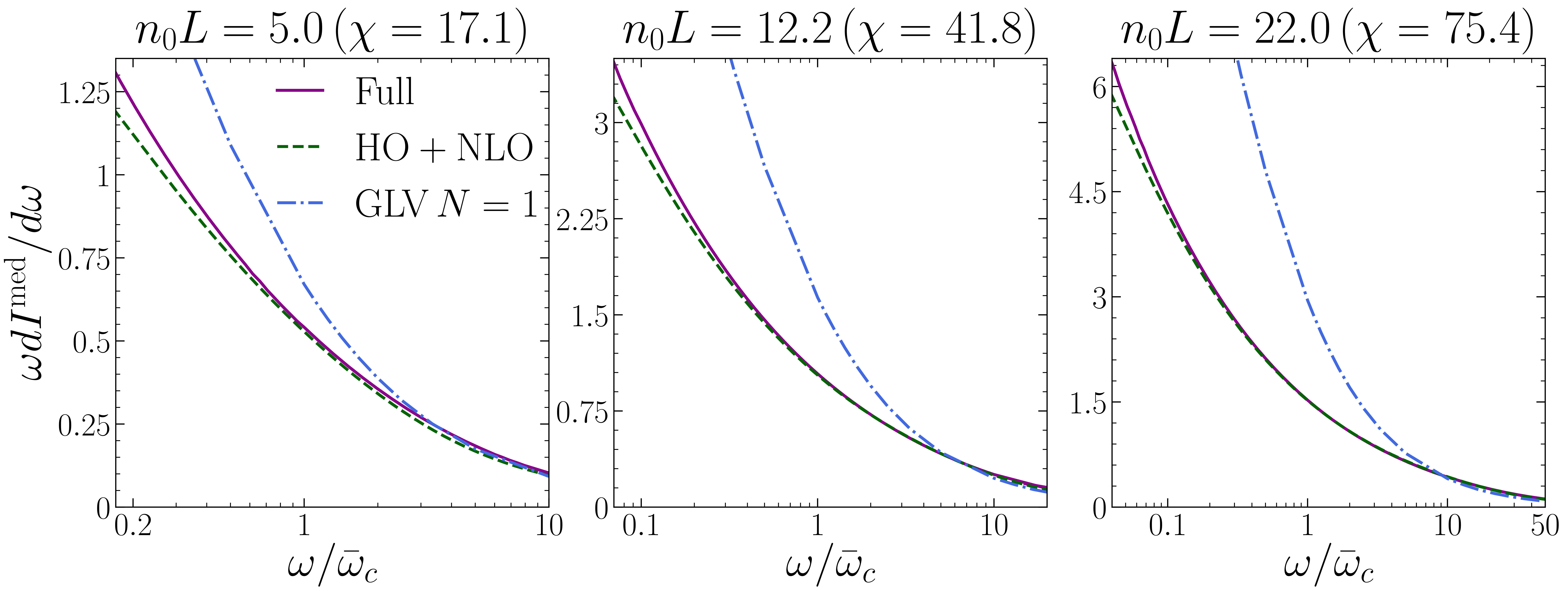


Smooth matching between the multiple soft scattering and single hard regimes

Numerical solution vs. IOE

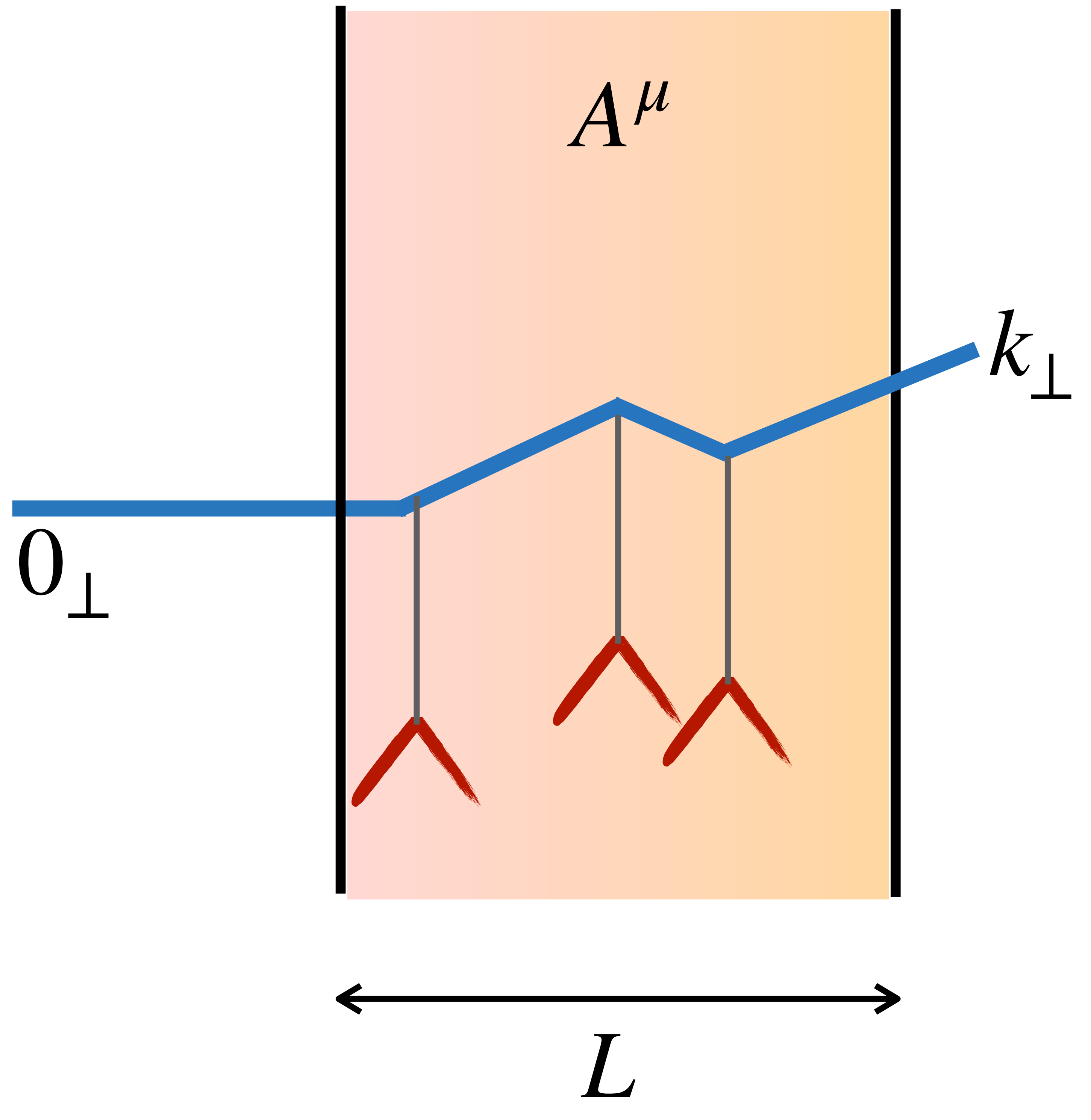
[Andrés, Dominguez, Gonzalez Martinez'20]

IOE less valid ⇦ $\chi \sim 1/\lambda$ ⇨ IOE more valid



Excellent agreement with the full solution in its regime of validity

Transverse momentum broadening (again)



Kinetic theory formulation of momentum broadening

$$\frac{\partial}{\partial L}\mathscr{P}(k_\perp, L) = C_R \int_{q_\perp} \gamma(q_\perp)\ \left[\mathscr{P}(k_\perp - q_\perp, L) - \mathscr{P}(k_\perp, L)\right]$$

Fourier transform to position space $S(x_\perp, L) = \int_{q_\perp} \mathscr{P}(k_\perp, L)\, e^{iq_\perp \cdot x_\perp}$

$$\frac{\partial}{\partial L} S(x_\perp, L) = -\, v(x_\perp) S(x_\perp, L)$$

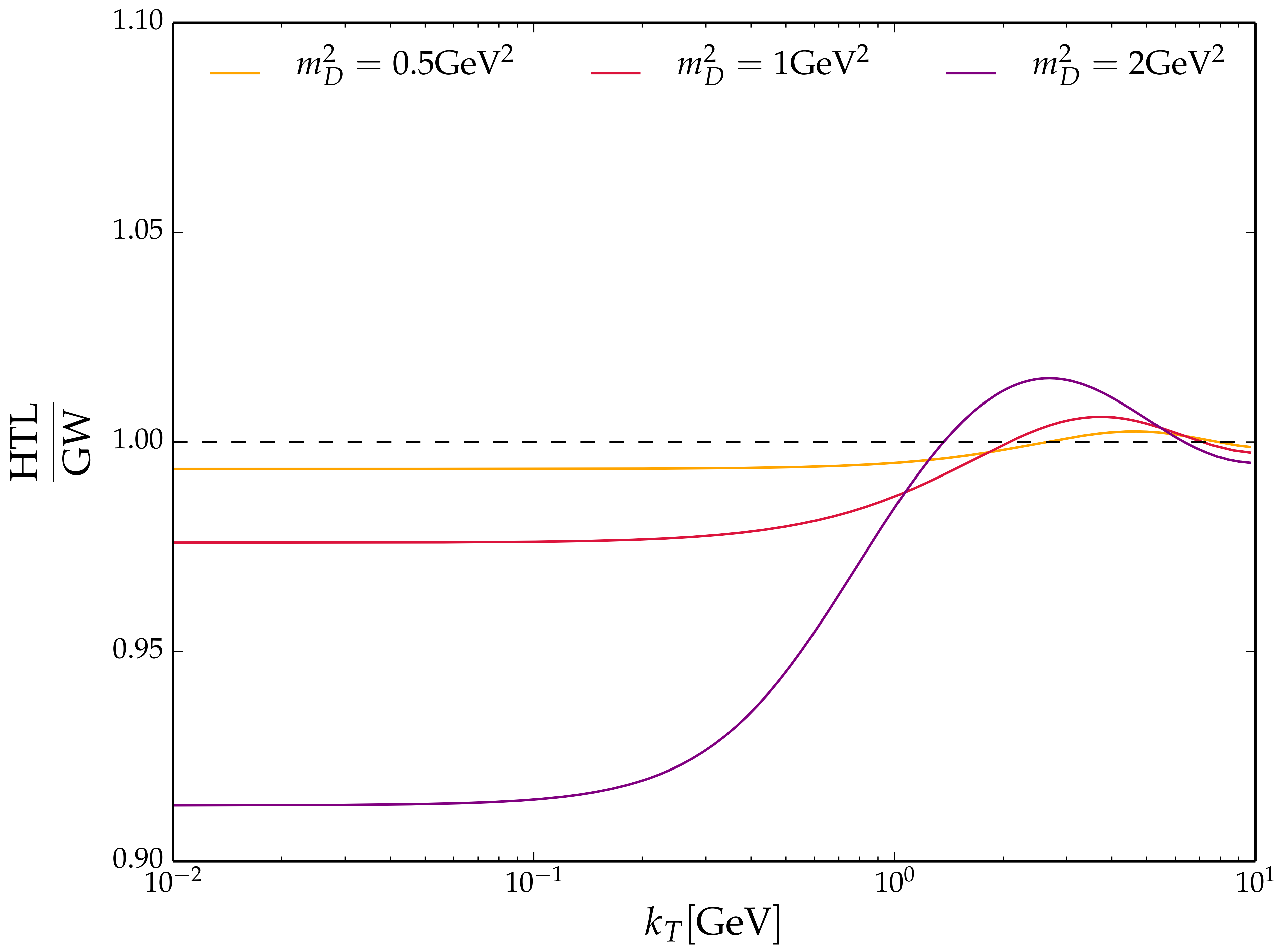
that for a homogenous and static medium leads to

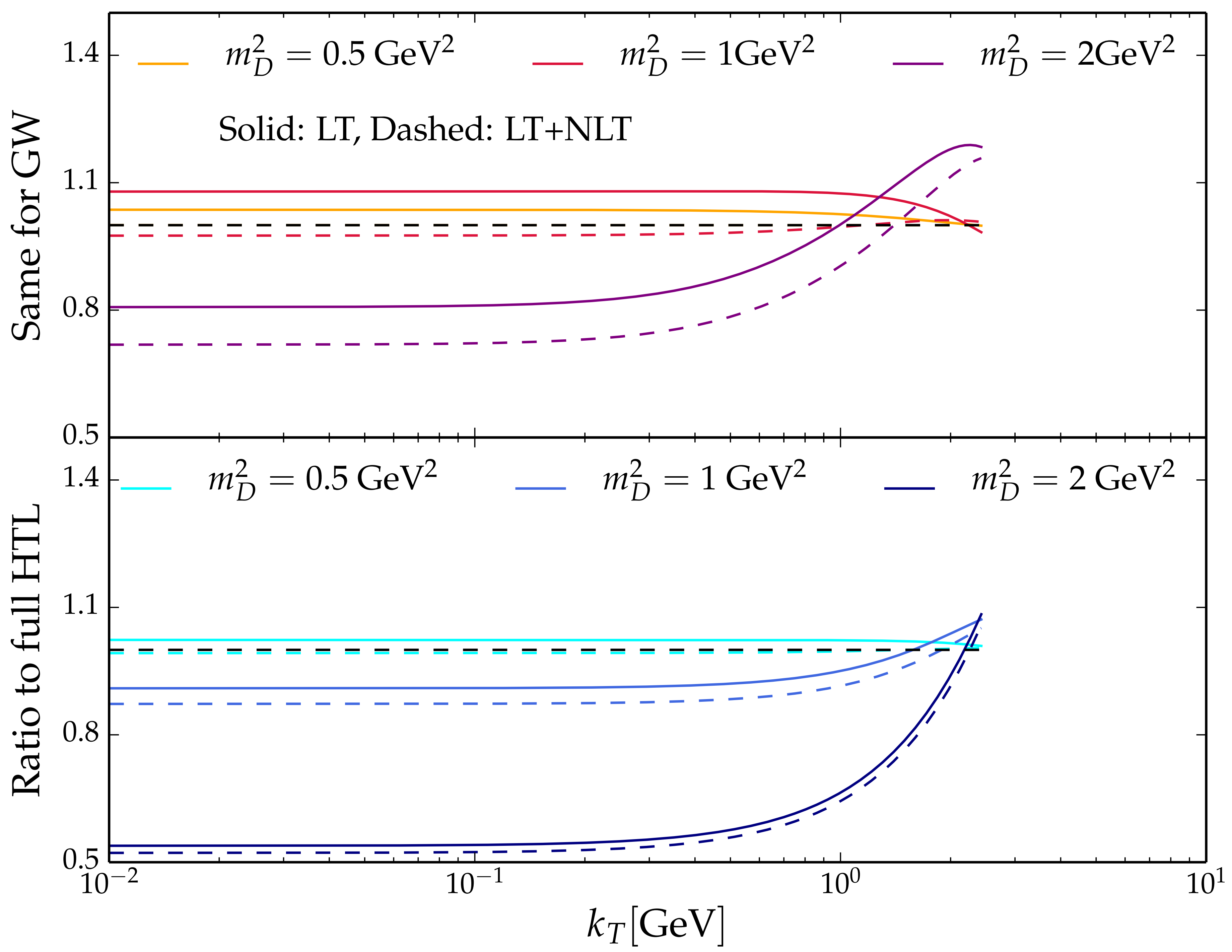
$$S(x_\perp, L) = e^{-v(x_\perp)\, L} \quad \Longrightarrow \quad \mathscr{P}(k_\perp, L) = \int_{x_\perp} S(x_\perp, L) e^{-ix_\perp \cdot k_\perp}$$

Analytic solution for GW/HTL potential does not exist

Sensitivity to IR modelling with a twist expansion [Barata, Mehtar-Tani, ASO, Tywoniuk'20]

Reminder: $v(x_\perp) = \frac{1}{4} Q_{s0}^2 x_\perp^2 \log \frac{1}{x_\perp^2 \mu_*^2} + \mathcal{O}(x_\perp^4 \mu_*^4)$ with $\mu_*^2 = \begin{cases} \frac{\mu^2}{4} e^{-1+2\gamma_E} & \text{for GW} \\ \frac{m_D^2}{4} e^{-2+2\gamma_E} & \text{for HTL} \end{cases}$





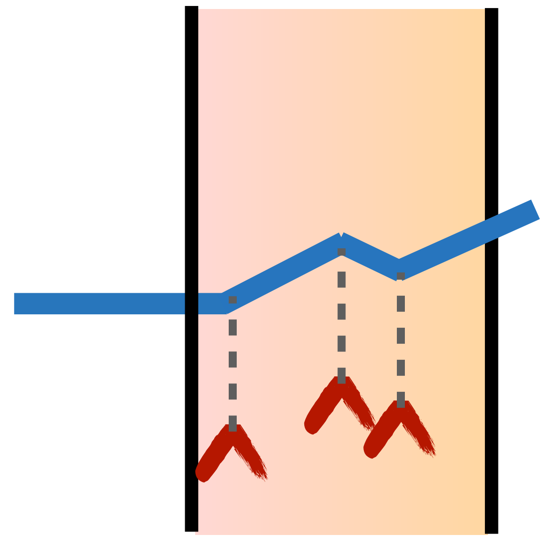
Non-universal power corrections $\mathcal{O}(x_\perp^4 \mu_*^4)$ <10% in realistic conditions

Transverse momentum broadening (again): 2 limits

$$S^{\rm LT}(x_\perp) = \exp\left[-\frac{1}{4}Q_{s0}^2\, x_\perp^2 \log\frac{1}{x_\perp^2\mu_*^2}\right] + \mathcal{O}(x_\perp^2\mu_*^2)$$

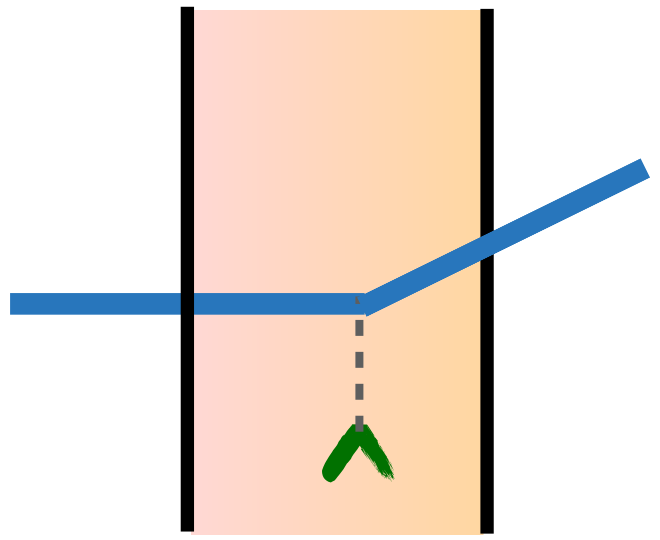
- Multiple soft scattering approximation: gaussian broadening

$$S^{\rm LT}(x_\perp) \to \exp\left[-\frac{1}{4}Q_{s0}^2\, x_\perp^2\right] \Longrightarrow \mathscr{P}^{\rm MS}(k_\perp, L) = \frac{4\pi}{Q_s^2}{\rm e}^{-\frac{k_\perp^2}{Q_s^2}}$$



- Single hard scattering:

$$S^{\rm LT}(x_\perp) \to 1 - \frac{1}{4}Q_{s0}^2 x_\perp^2 \log\frac{1}{x_\perp^2\mu_*^2} + \mathcal{O}\left(x_\perp^4 Q_{s0}^4\right) \Longrightarrow \mathscr{P}^{\rm SH}(k_\perp, L) = \frac{4\pi Q_{s,0}^2}{k_\perp^4}$$



Is it possible to encompass these two limits analytically?

Transverse momentum broadening with the IOE [Barata, Mehtar-Tani, ASO, Tywoniuk'20]

Reminder: $v(x_\perp) = \frac{1}{4} Q_{s0}^2 x_\perp^2 \left(\log \frac{Q^2}{\mu_*^2} + \frac{1}{Q^2 x_\perp^2} \right) = v^{\mathrm{MS}} + \delta v$

[Moliere'48]

Theorie der Streuung schneller geladener Teilchen II

Mehrfach- und Vielfachstreuung [1]

$$f^{(1)}(\vartheta) = 2\, e^{-\vartheta^2} \{ (\vartheta^2 - 1) [\overline{Ei}(\vartheta^2) - \log \vartheta^2] + 2 - e^{\vartheta^2} \} . \quad (7, 3\,c)$$

The transverse momentum broadening probability distribution reads

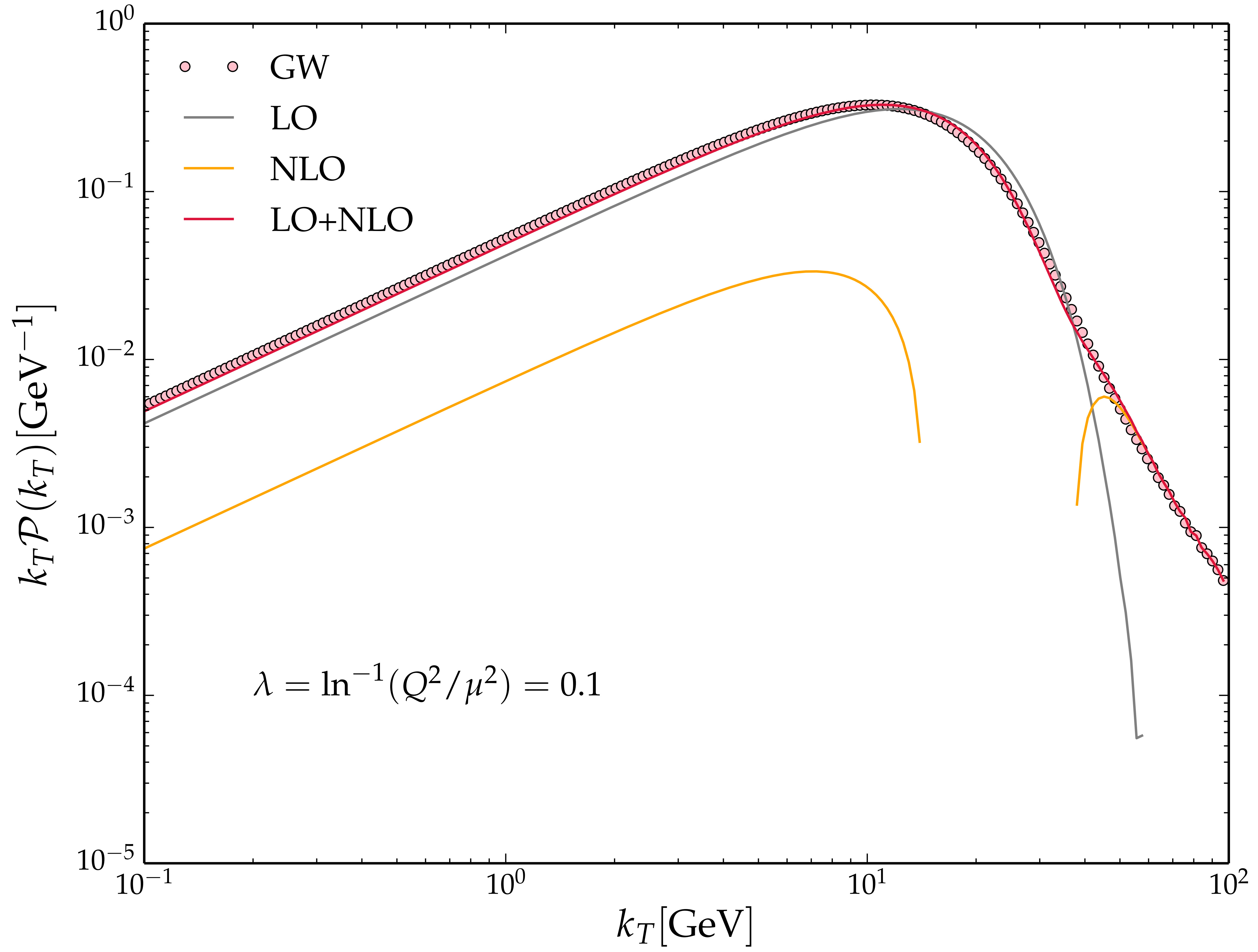
$$\mathscr{P}^{\mathrm{LT}}(k_\perp, L) = \sum_n \int_{x_\perp} \mathrm{e}^{-i x_\perp \cdot k_\perp} \mathrm{e}^{-\frac{1}{4} x_\perp^2 Q_s^2} \frac{(-1)^n Q_{s0}^{2n}}{4^n n!} x_\perp^{2n} \log^n \frac{1}{x_\perp^2 Q^2}$$

$$\equiv \mathscr{P}^{\mathrm{LO}} + \mathscr{P}^{\mathrm{NLO}} + \mathscr{P}^{\mathrm{NNLO}} + \cdots$$

The first two orders can be computed analytically leading to

$$\mathscr{P}^{\mathrm{LO+NLO}}(k_\perp, L) = \frac{4\pi}{Q_s^2} \mathrm{e}^{-x} \left\{ 1 - \lambda \left(e^x - 2 + (1-x) \left(\mathrm{Ei}(x) - \log(4x\, a) \right) \right) \right\}, \quad x \equiv \frac{k_\perp^2}{Q_s^2}$$

Transverse momentum broadening with the IOE: test case

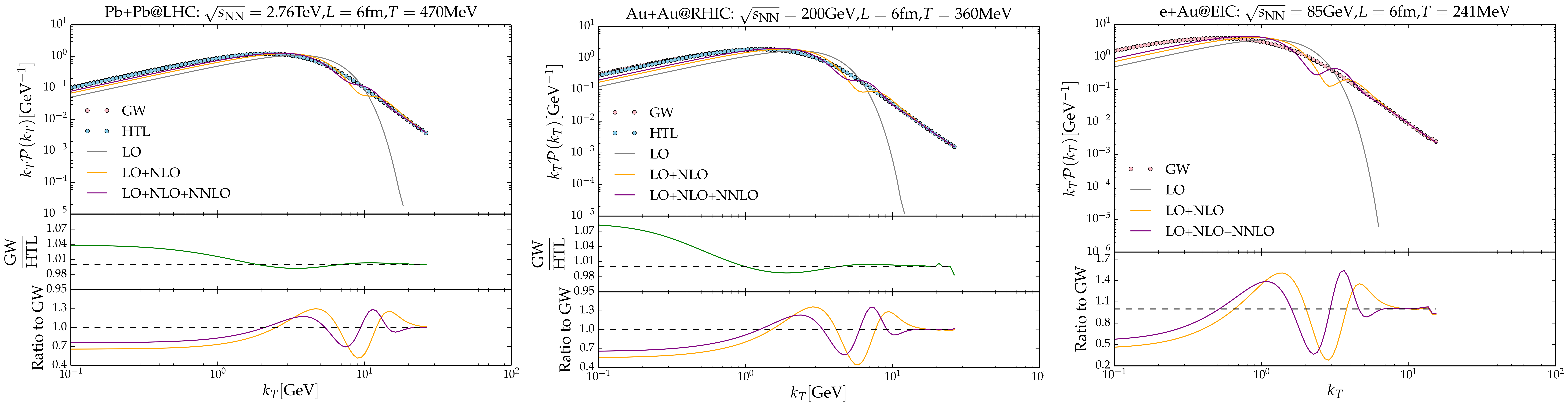


LO+NLO reproduces the exact numerical solution accurately

Transverse momentum broadening with the IOE: phenomenology

Is $\lambda = \dfrac{1}{\log(Q^2/\mu_*^2)} \ll 1$ in realistic experimental conditions?

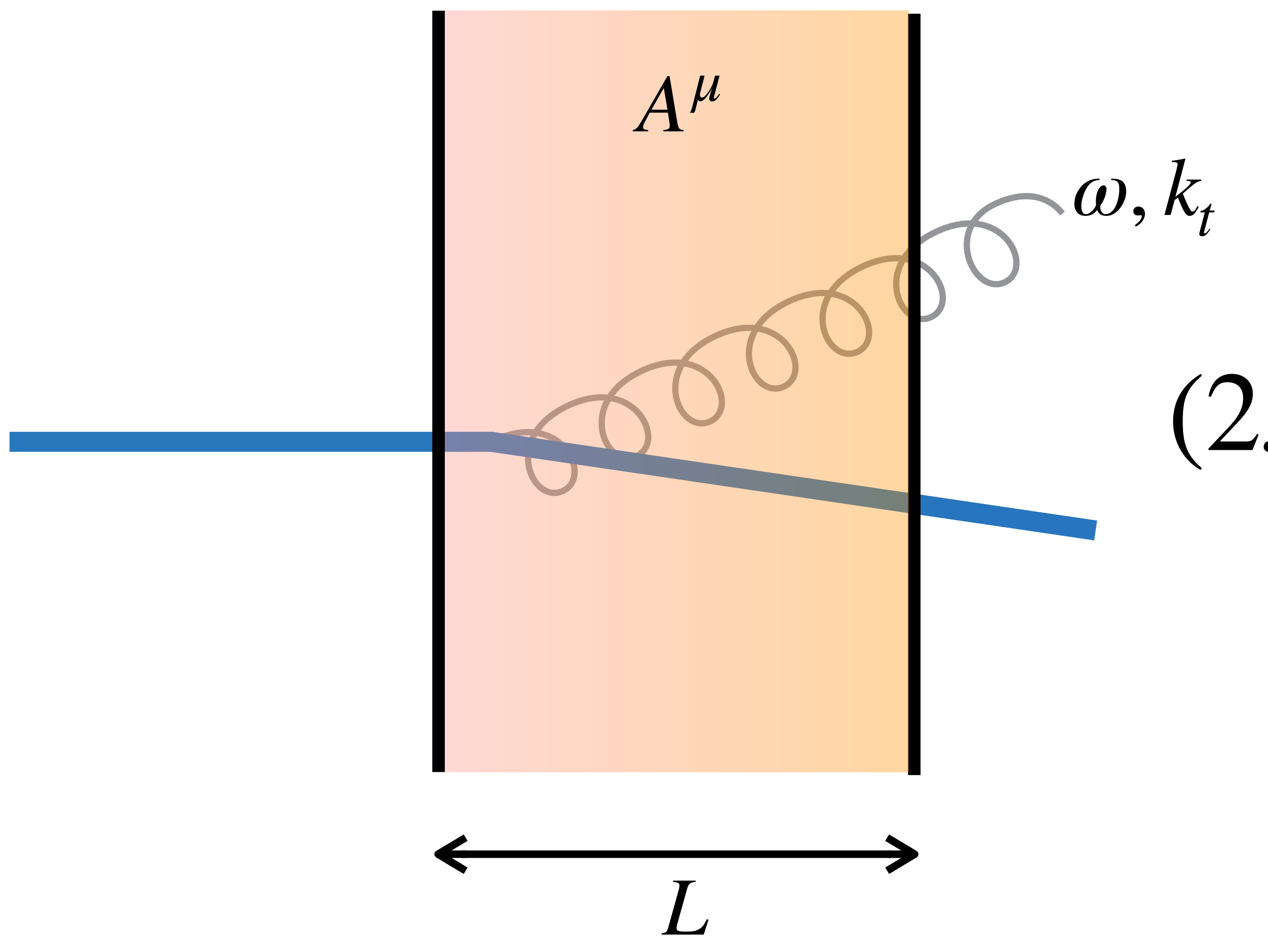
LHC | **RHIC** | **EIC**



$\lambda \sim 0.19$ | $\lambda \sim 0.21$ | $\lambda \sim 0.25$

Fully differential medium-induced spectrum

[Barata, Mehtar-Tani, ASO, Tywoniuk in progress]



[Baier, Dokshitzer, Mueller, Peigné, Schiff'96]
[Zakharov'96]

$$(2\pi)^2\omega\frac{dI}{d\omega dk_\perp} = \frac{2\bar{\alpha}\pi}{\omega^2}\Re\left[\int_0^\infty dt_2\int_0^{t_2}dt_1\int_{x_\perp} e^{ik_\perp\cdot x_\perp}\mathscr{P}(x_\perp,t_2)\right.$$
$$\left.\times\,\partial_{y_\perp}\cdot\partial_{x_\perp}\left(\mathscr{K}(x_\perp,t_2;y_\perp,t_1)\right)_{y_\perp=0}\right] \text{ - vacuum}$$

In the IOE, two expansions: one for the broadening, another for the kernel

$$(2\pi)^2\omega\frac{dI}{d\omega dk_\perp} = \frac{2\bar{\alpha}\pi}{\omega^2}\Re\left[\int_0^\infty dt_2\int_0^{t_2}dt_1\int_{x_\perp} e^{ik_\perp\cdot x_\perp}\mathscr{P}^{\rm LO+NLO}(k_\perp,t_2)\right.$$
$$\left.\times\,\partial_{y_\perp}\cdot\partial_{x_\perp}\left(\mathscr{K}_{HO}(x_\perp,t_2;y_\perp,t_1)_{y_\perp=0}+\delta\mathscr{K}(x_\perp,t_2;y_\perp,t_1)\right)\right] \text{ - vacuum}$$

Wrap-up

Introduced the **improved opacity expansion** that extends Molière's theory of multiple scattering (1948) to modern jet quenching theory

Semi-analytic approach to account for multiple-soft and rare-hard scatterings at LO+NLO. Systematically improvable order-by-order in perturbation theory

Orthogonal and complementary approach to the recent numerical efforts to solve exactly the medium induced spectrum [Caron-Huot, Gale'10] [Feal, Vazquez'18] [Andres, Apolinario, Dominguez'20]

To-do list:

- Finalise transverse momentum dependent spectrum and compare to exact solution
- Implement in jet quenching Monte Carlo codes [e.g. JetMed by Caucal,Iancu, Soyez'20]
- Impact on phenomenological observables, e.g. R_{AA} [Mehtar-Tani, Pablos, Tywoniuk in preparation]