$K^0 - \bar{K}^0$ on the Lattice

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Outline

- B_K (SM)
  - ϵ_K & B_K
  - Recent lattice results for B_K
- $\bar{K}^0 - K^0$ beyond SM
  - Preliminary results by ETMC
- Conclusions

ϵ_K & B_K

- The regeneration wave function for the $K^0 - \bar{K}^0$ system $|\psi(t)\rangle = \psi_1(t)|K^0\rangle + \psi_2(t)|\bar{K}^0\rangle$ obeys the Schrodinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix} = \begin{pmatrix} M_0 - i\Gamma_0/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_0 - i\Gamma_0/2 \end{pmatrix}\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix}$$

  where due to CPT conservation $M_0 = M_{11} = M_{22}, \; \Gamma_0 = \Gamma_{11} = \Gamma_{22}$ and due to hermiticity $M_{12}^* = M_{21}, \; \Gamma_{12}^* = \Gamma_{21}$.

- After diagonalizing the Hamiltonian we obtain $|K_{S,L}\rangle = \dfrac{1}{\sqrt{2(1+|\bar{\epsilon}|^2)}}[(1+\bar{\epsilon})|K^0\rangle \pm (1-\bar{\epsilon})|\bar{K}^0\rangle]$

  then $\langle K_S|K_L\rangle = \dfrac{2Re\bar{\epsilon}}{1+|\bar{\epsilon}|^2}$. For $\mathrm{Re}\bar{\epsilon} \neq 0 \quad \rightarrow$ CP-violation.

- It also follows that $\dfrac{1-\bar{\epsilon}}{1+\bar{\epsilon}} = \dfrac{\Delta m_K - \Delta\Gamma/2}{2M_{12} - i\Gamma_{12}}$. Defining $\phi_\epsilon = \arctan(2\Delta m/\Delta\Gamma)$ it can be derived (for $\bar{\epsilon} \ll 1$)

$$Re\;\bar{\epsilon} = sin\phi_\epsilon cos\phi_\epsilon\left[\frac{ImM_{12}}{\Delta m_K} - \frac{Im\Gamma_{12}}{2Re\Gamma_{12}}\right]$$

ϵ_K & B_K

- Experimental quantity: $\epsilon_K \equiv \frac{2\eta_{+-} + \eta_{00}}{3}$ with $\eta_{ij} = \frac{\mathcal{A}(K_L \to \pi^i \pi^j)}{\mathcal{A}(K_S \to \pi^i \pi^j)}$
- By denoting $\langle(\pi\pi)_{I=0,2}|H_W|K^0\rangle = a_{0,2} e^{i\delta_{0,2}}$ and using the approximations
  $|\frac{a_2}{a_0}| \ll 1$ and $-\frac{Im\,\Gamma_{12}}{2Re\,\Gamma_{12}} = \frac{Im\,a_0}{Re\,a_0} \equiv \xi$
  one obtains:
  $\epsilon_K = \bar{\epsilon} + i\xi$ (for both ϵ_K and $\bar{\epsilon}$ small quantities.)
  (see e.g. **L.L. Chau, Phys.Reports 1983**)

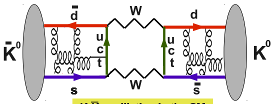
- Therefore one has:

$$Re(\epsilon_K) = Re(\bar{\epsilon}) = cos\phi_\epsilon sin\phi_\epsilon \left[\frac{Im\,M_{12}}{\Delta m_K} + \xi\right] \Longrightarrow \epsilon_K = e^{i\phi_\epsilon} sin\phi_\epsilon \left[\frac{Im\,M_{12}}{\Delta m_K} + \xi\right]$$

- From experiment:

$$\phi_\epsilon = 43.51(5)^o$$
$$\Delta m_K = 3.483(6) \times 10^{-12}\ MeV$$

(K. Nakamura et al. JPG 2010)



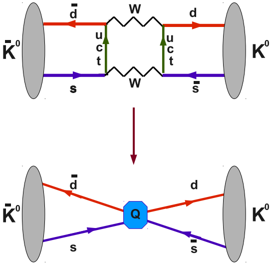
K-K̄ oscillation in the SM

ϵ_K & B_K

- $Im\ M_{12} \simeq Im\ M_{12}^{(6)} \equiv Im\ M_{12}^{SD}$
- Calculation using the effective hamiltonian

$$
\begin{aligned}
M_{12}^{(6)} &= \frac{1}{2m_K}\langle \bar{K}^0 | H_{\text{eff}}^{\Delta S=2} | K^0 \rangle^* \\
&= C(\mu)\langle \bar{K}^0 | \underbrace{(\bar{s}\gamma_\mu^L d)(\bar{s}\gamma_\mu^L d)}_{\mathcal{Q}(\mu)} | K^0 \rangle^*
\end{aligned}
$$

- ξ estimated using the $\Delta I = 1/2$ dominance and $(\epsilon'/\epsilon)_{\text{exp}}$.
- estimate $Im\ M_{12}^{LD}$ using ChPT



ϵ_K & B_K

- Considering the (LD) corrections of $Im\ M_{12}$ and ξ to ϵ_K

$$\epsilon_K = \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}} \left[\frac{Im\ M_{12}^{(6)}}{\Delta m_K} \right]$$

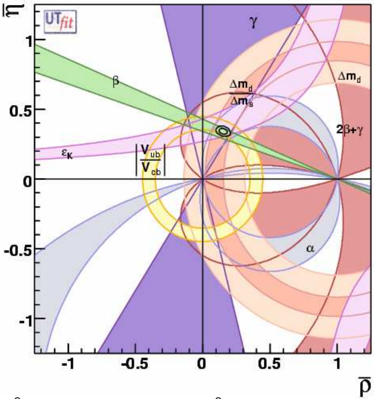
with $\kappa_\epsilon = 0.94(2)$
(Buras & Guadagnoli, Phys.Rev.D 2008; Buras, Guadagnoli & Isidori, Phys.Lett.B 2010)

see also **(A. Lenz, U. Nierste et al. 1008.1593)**.

- Until 2008, due to high uncertainties in the hadronic element calculation, the approximate formula was satisfactory by considering $\phi_\epsilon = 45^o$ and $\kappa_\epsilon = 1$.

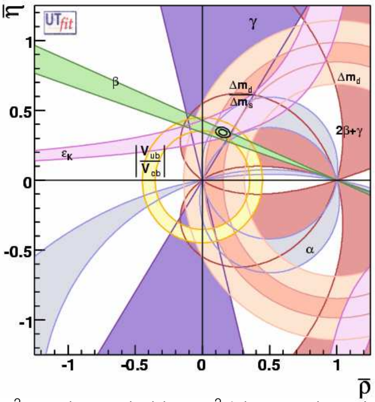
(*more details in the next talk by D. Guadagnoli*)

ϵ_K & B_K



- $|\epsilon_K| = \kappa_\epsilon\, C_\epsilon\, \hat{B}_K\, A^2 \bar{\eta}\;\times\left(-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\bar{\rho})\right)$
  with $A \equiv \dfrac{|V_{cb}|}{\lambda^2}$, $\lambda = |V_{us}|$; η_i and S_0 pert. calculable quantities.
- Bag parameter $\hat{B}_K$ quantifies the matrix element normalized with the VIA estimate.
- ϵ_K measured experimentally to sub percent precision: $|\epsilon_K| = 2.228(11)\times 10^{-3}$.

ϵ_K & B_K



- $|\epsilon_K| = \kappa_\epsilon \, C_\epsilon \, \hat{B}_K \, A^2 \bar{\eta} \;\times (-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\bar{\rho}))$ with $A \equiv \dfrac{|V_{cb}|}{\lambda^2}$, $\lambda = |V_{us}|$; η_i and S_0 pert. calculable quantities.
- Given inputs $|\epsilon_K|$, $|V_{cb}|$ and $\hat{B}_K$, design the hyperbola $\bar{\eta}(c_1 - \bar{\rho}) = c_2$.
- Since the apex is overconstrained, from a SM fit analysis we can obtain the prediction $\hat{B}_K = 0.87(8)$ **(V. Lubicz, LAT2009, 1004.3473)**

B_K calculation

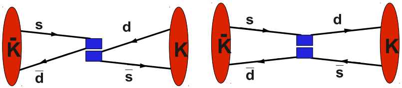
- Form the ratio between 3- and 2-point correlation functions

$$\left\langle \frac{\langle \mathcal{P}(t_{source})\, \mathcal{Q}_{\Delta S=2}(t)\, \mathcal{P}(t_{sink})\rangle}{(8/3)\langle \mathcal{P}(t_{source})\mathcal{A}(t)\rangle\, \langle \mathcal{A}(t)\mathcal{P}(t_{sink})\rangle} \right\rangle$$

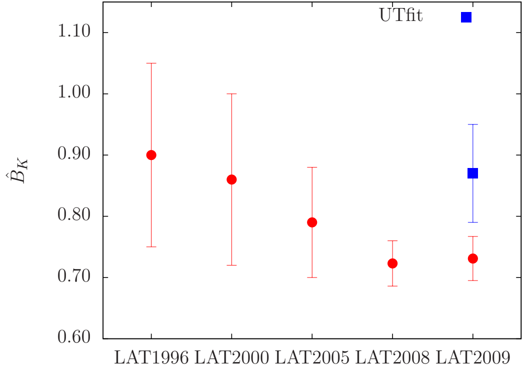
- For $t_{source} \ll t \ll t_{sink}$ obtain

$$B_K = \frac{\langle \bar{K}^0 | \mathcal{Q}_{\Delta S=2} | K^0 \rangle}{(8/3)\langle \bar{K}^0 | A | 0 \rangle \langle 0 | A | K^0 \rangle} = \frac{\langle \bar{K}^0 | \mathcal{Q}_{\Delta S=2} | K^0 \rangle}{(8/3) m_K^2 f_K^2}$$

- $\mathcal{Q}_{\Delta S=2} \longrightarrow (\gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5) \longleftarrow$ parity even part of $(V-A) \otimes (V-A)$
- Connected and disconnected trace diagrams for the 4-fermion correlator.



evolution of B_K lattice estimate



- 1996 → 2009: $\sigma_{B_K} \sim 18\% \to 4\%$.
- $\leq$ 2005 most (precise) results from the quenched approximation.
- $\geq$ 2008 average on simulations with $N_f = 2,\ 2+1$ dynamical quarks.
- *Tension* at the level of $\sim 1.6\,\sigma$ between lattice average and combined UT fit estimates.

B_K from dynamical quark simulations

B_K in the Continuum Limit from dynamical quark simulations

- **ETMC** OS/Mtm Mixed action with $N_f = 2$
(V. Bertone, P.D., R. Frezzotti et al. PoS LAT2009, 0910.4838;
P.D., R. Frezzotti, V. Gimenez et al. PoS LATTICE2008, 0810.2443;
P.D., R. Frezzotti, V. Gimenez et al. LAT2010;
ETMC in preparation.)
- **BNL + SNU + WU** HYP-smeared on MILC Asqtad fermions with $N_f = 2 + 1$
(T. Bae, Y-C. Jang, C. Jung et al. 1008.5179.)
- **ALV** DWF on MILC Asqtad fermions with $N_f = 2 + 1$
(C. Aubin, J. Laiho and R. Van de Water Phys.Rev.D 2010, 0905.3947)
- **RBC-UKQCD** DWF with $N_f = 2 + 1$
(D.J. Antonio, P.A. Boyle, T. Blumh et al. Phys.Rev.Lett. 2008, hep-ph/0702042;
C. Allton, D.J. Antonio, Y. Aoki et al. Phys.Rev.D 2008, 0804.0473;
C. Kelly, P.A. Boyle and C.T. Sachrajda PoS LAT2009, 0911.1309;
C. Kelly LAT2010)

B_K & Wilson fermions

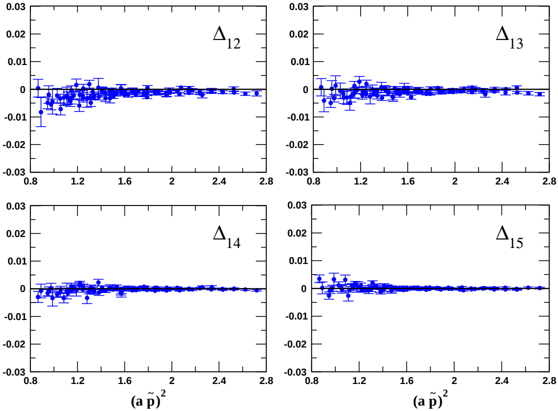
- Due to Wilson term (and loss of chirality) B_K calculation is characterised by:
  1. $O(a)$ discretisation effects
  2. Complicated renormalization pattern: mixing with operators of wrong chirality
- Cure **(2)** using
  - WI and 4-point correlation function
    (D. Becirevic, P. Boucaud, V. Gimenez et al. Eur.Phys.J. 2004)
  - combinations of Wilson quarks with various combinations of twisted angle
    (ALPHA coll, P. D., J. Heitger, F. Palombi et al. NPB 2006)
- Cure **(1)** using
  - Symanzik program and include dim-7 counterterms $\longrightarrow$ large uncertainties
  - Mtm QCD
- Cure both **(1)** and **(2)** employing
  - Mtm QCD & Mixed action with OS valence quarks
    (R. Frezzotti and G.C. Rossi JHEP 2004)

ETMC

- **OS/MtmQCD** (*Mixed Action*) $N_f = 2$ dynamical quarks.
- **O**sterwalder **S**eiler valence quarks are considered as one component tm-quarks with twisted angle $+\pi/2$ or $-\pi/2$. Unitarity violations up to $\mathcal{O}(a^2)$. Discrete symmetries ensure **automatic $\mathcal{O}(a)$-improvement** and **absence of mixing** with operators of wrong chirality by employing three val. quarks with $+\pi/2$ and one with $-\pi/2$ or vice versa.
- $a \approx 0.10,\ 0.09,\ 0.07$ fm
  $V \times T = 24^3 \times 48,\ \ (24^3 - 32^3) \times 48,\ \ 32^3 \times 48.$
- $m_\pi^{\min} \approx 270\ \mathrm{MeV}$; $(m_\pi L)^{\min} \approx 3.3$; $L = (2.2 - 2.9)$ fm
- **NP renormalisation** using RI-MOM method.
  (G. Martinelli, C. Pittori, C. Sachrajda et al. Nucl.Phys.B 1995) .
  Analytic subtractions of $\mathcal{O}(a^2 g^2)$ contributions.
  $\mathcal{O}_{VV+AA}$ renormalised multiplicatively.

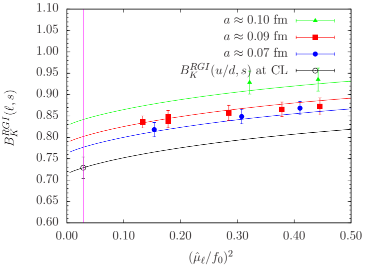
ETMC (continue)

- Absence of mixing with operators of wrong chirality. Example ...



ETMC (continue)

- Combined SU(2)-chiral and continuum extrapolation.
- Check compatibility using an analytic fit.



- Final result (error estimate using bootstrap method)

$B_K^{\overline{MS}}(2\ \text{GeV}) = 0.517(18)_{\text{stat}+\chi+\text{REN}}(11)_{\text{scale+fit syst}}$ [21] or

$\hat{B}_K = 0.729(25)_{\text{stat}+\chi+\text{REN}}(16)_{\text{scale+fit syst}}$ [30]

($\sim$ 1% stat. error on ME; $\sim$ 2% stat. error on RCs.)

BNL + SNU + WU

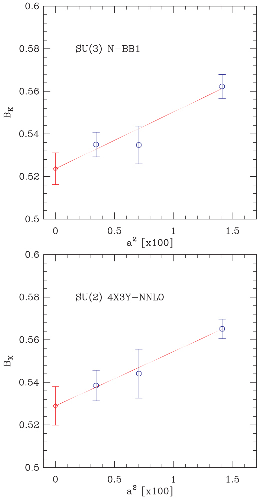
- **HYP-smeared stag./Asqtad MILC** (*Mixed Action*) $\mathbf{N_f = 2 + 1}$ dyn. quarks.
- $a \approx 0.12,\ 0.09,\ 0.06$ fm ($a = 0.045$ fm in progress)
  $V \times T = (20^3 - 28^3) \times 64,\ 28^3 \times 96,\ 48^3 \times 144.$
- $m_\pi^{\min} \approx 200$ MeV; $(m_\pi L)^{\min} \approx 2.5$; $L = (2.4 - 2.9)$ fm
- Computationally cheap; Taste breaking exists but reduced with HYP-smeared stag. valence quarks.
- Perturbative renormalisation at 1-loop for the 4-f operator → the major systematic uncertainty in the final result.
- SU(2) and SU(3) StaggeredχPT + inclusion of NNLO analytic terms.

BNL + SNU + WU (continue)

- SU(2) SChPT fit more straightforward than the SU(3) one: less fit parameters (three vs. seven fit param.); no Bayesian constraints.
- Mild dependence of B_{K} on the light sea quark mass.
- Compatible B_K CL estimates from both chiral fit choices; smooth CL compatible with small $O(a^2)$ discretization effects.
- Final result (from SU(2) chiral fit)

$$B_{\mathrm{K}}^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 0.529(9)_{\mathrm{stat}}(32)_{\mathrm{syst}}$$ or

$$\hat{B}_{\mathrm{K}} = 0.724(12)_{\mathrm{stat}}(43)_{\mathrm{syst}}$$

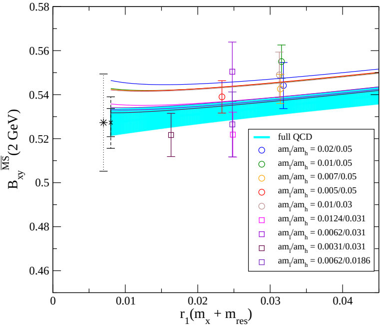


ALV

- **DWF/Asqtad MILC** (*Mixed Action*) $\mathbf{N_f = 2 + 1}$ dynamical quarks.
- $a \approx 0.12,\ 0.09$ fm
  $V \times T = (20^3 - 24^3) \times 64,\ \ (28^3 - 40^3) \times 96$ & $L_s = 16$.
- $m_\pi^{\min} \approx 240\ \mathrm{MeV}$; $(m_\pi L)^{\min} \approx 3.5$; $L = (2.5 - 3.6)$ fm
- **NP renormalisation**; RI-MOM method and 1-loop mean field improved lat-PT to estimate the systematics. (Very mild dependence of Z_{B_K} on m_s-sea quark whose value is kept finite.)
- Suppressed chiral operator mixing; no taste mixing; $\mathcal{O}_{VV+AA}$ renormalised multiplicatively.

ALV (continue)

- SU(3) Mixed Action χPT at NLO + analytic terms for the s-quark region.
- Combined chiral and continuum fit.



uncertainty	B_K
statistics	1.2%
chiral & continuum extrapolation	1.9%
scale and quark mass uncertainties	0.8%
finite volume errors	0.6%
renormalization factor	3.4%
total	4.2%

- Final result

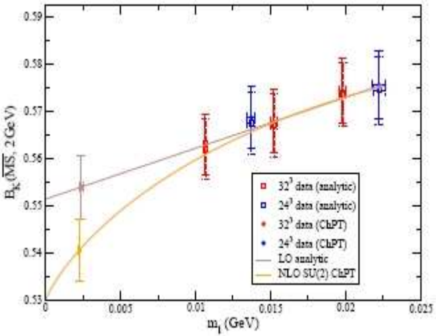
$$B_K^{\overline{MS}}(2\text{ GeV}) = 0.527(6)_{stat}(20)_{syst} \;\; or \;\; \hat{B}_K = 0.724(8)_{stat}(28)_{syst}$$

RBC-UKQCD

- **DWF** $\mathbf{N_f = 2 + 1}$ dynamical quarks.
- $a \approx 0.11,\ 0.08$ fm using $V \times T = 24^3 \times 64,\ 32^3 \times 64$ and $L_s = 16$.
- $m_\pi^{\rm min} \approx 290\ \mathrm{MeV}$; $(m_\pi L)^{\rm min} \approx 4.1$.
- **NP renormalisation**; RI-MOM method with non exceptional momentum renormalisation conditions. **(C. Sturm, Y. Aoki, N. Christ et al. Phys.Rev.D 2009)**
- Suppressed chiral operator mixing at the level of $\mathcal{O}(m_{\rm res}^2)$;
  (T. Blum et al. Phys.Rev.D 2002; Y. Aoki, P.A. Boyle, N. Christ et al. Phys.Rev.D 2008).
  O_{VV+AA} renormalised multiplicatively.

RBC-UKQCD (continue)

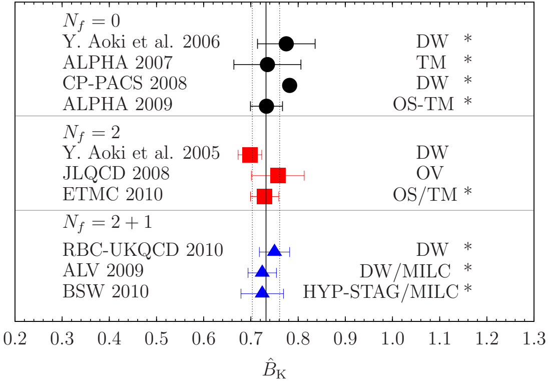
- Combined SU(2)-chiral and continuum extrapolation.
- Use also LO analytic fit function (absence of curvature); take the average.



- **Preliminary** result (C. Kelly LAT2010)

$$B_K^{\overline{MS}}(2\text{ GeV}) = 0.546(7)_{\text{stat}}(16)_\chi(3)_{\text{FV}}(14)_{\text{REN}}$$

$\hat{B}_K$



- $*$ $\longrightarrow$ result already in the **CL**.
- Average: $\hat{B}_K^{(N_f=2)}$(ETMC) $= 0.729(30)$; $\hat{B}_K^{(N_f=2+1)} = 0.732(06)(26)$
- No dependence on the strange quark (with the present precision)!
- Difference of less than $\sim 2\,\sigma$ with the most precise quenched result.

$K^0 - \bar{K}^0$ oscillation Beyond SM

Lattice calculation with $N_f = 2$ dyn. quarks by ETMC

Full basis of $\Delta S = 2$ operators

The effective Hamiltonian takes the form:

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu)\mathcal{O}_i(\mu) + \sum_{i=1}^{3} \tilde{C}_i(\mu)\tilde{\mathcal{O}}_i(\mu)$$

- In SM case only $\mathcal{O}_1$ contributes.

$$\begin{aligned}
\mathcal{O}_1 &= [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1-\gamma_5)d^b] & \tilde{\mathcal{O}}_1 &= [\bar{s}^a\gamma_\mu(1+\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b] \\
\mathcal{O}_2 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b] & \tilde{\mathcal{O}}_2 &= [\bar{s}^a(1+\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] \\
\mathcal{O}_3 &= [\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1-\gamma_5)d^a] & \tilde{\mathcal{O}}_3 &= [\bar{s}^a(1+\gamma_5)d^b][\bar{s}^b(1+\gamma_5)d^a] \\
\mathcal{O}_4 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] & & \\
\mathcal{O}_5 &= [\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1-\gamma_5)d^a] & &
\end{aligned}$$

(Gabrielli et al. 1996; Bagger et al. 1997; Ciuchini et al. 1997,1998)

- Parity-even parts of $\mathcal{O}_i$ and $\tilde{\mathcal{O}}_i$ coincide.

Physical basis and Lattice basis

Through Fierz transformation

$$\begin{aligned}
\mathcal{O}_1 &= (O^{VV} + O^{AA}) \\
\mathcal{O}_2 &= (O^{SS} + O^{PP}) \\
\mathcal{O}_3 &= (O^{SS} + O^{PP} - O^{TT})(-\frac{1}{2}) \\
\mathcal{O}_4 &= (O^{SS} - O^{PP}) \\
\mathcal{O}_5 &= (O^{VV} - O^{AA})(-\frac{1}{2})
\end{aligned}$$

- OS/TM mixed action setup brings to a continuum-like renormalisation pattern

$$\begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \\ \mathcal{O}_5 \end{pmatrix}_{\mathrm{REN}} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \\ \mathcal{O}_5 \end{pmatrix}$$

B_i

$$\langle \bar{K}^0 | \mathcal{O}_1(\mu) | K^0 \rangle = B_1(\mu)\ (8/3) m_K^2 f_K^2 = B_K(\mu)\ (8/3) m_K^2 f_K^2$$

$$\langle \bar{K}^0 | \mathcal{O}_2(\mu) | K^0 \rangle = B_2(\mu) \left[\frac{m_K^2 f_K}{m_s(\mu) + m_d(\mu)} \right]^2 (-5/3)$$

$$\langle \bar{K}^0 | \mathcal{O}_3(\mu) | K^0 \rangle = B_3(\mu) \left[\frac{m_K^2 f_K}{m_s(\mu) + m_d(\mu)} \right]^2 (1/3)$$

$$\langle \bar{K}^0 | \mathcal{O}_4(\mu) | K^0 \rangle = B_4(\mu) \left[\frac{m_K^2 f_K}{m_s(\mu) + m_d(\mu)} \right]^2 (2)$$

$$\langle \bar{K}^0 | \mathcal{O}_5(\mu) | K^0 \rangle = B_5(\mu) \left[\frac{m_K^2 f_K}{m_s(\mu) + m_d(\mu)} \right]^2 (2/3)$$

R_i

- Avoid systematic uncertainties due to the quark mass calculation: construct appropriate ratios

$$R_i = \left(\frac{f_K^2}{m_K^2}\right)_{\rm exp}\left[\left(\frac{m_K}{f_K}\right)_{\rm tm}\left(\frac{m_K}{f_K}\right)_{\rm OS}\frac{\langle \bar{K}^0|\mathcal{O}_i(\mu)|K^0\rangle}{\langle \bar{K}^0|\mathcal{O}_1(\mu)|K^0\rangle}\right] \quad i = 2,\ldots,5$$

- Then

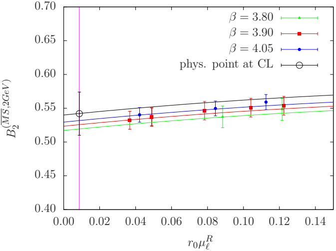
$$\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle \propto C_1(\mu)\langle \bar{K}^0|\mathcal{O}_1(\mu)|K^0\rangle\left(1 + \sum_{2,\ldots,5}\frac{C_i(\mu)}{C_1(\mu)}R_i\right)$$

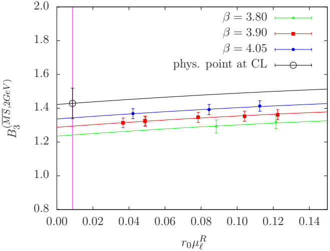
(Donini et al. 2000; Babich et al. 2006)

- Up to now, quenched results published:

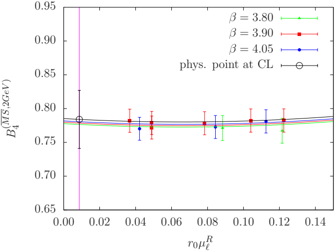
(Allton et al. 1998; Donini et al. 2000; Babich et al. 2006; Nakamura et al. 2006.) and a preliminary analysis with $N_f = 2+1$ DWF (RBC-UKQCD) **(J. Wennekers, PoS LAT2008)**.

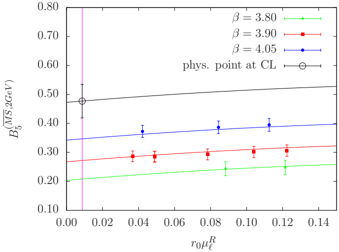
$B_{i=2,\dots,5}$





(CL taken using combined fits over three latt. spacings employing quadratic functions wrt μ_ℓ)





Results (preliminary)

fit function	i	$\frac{<\mathcal{O}_i>}{<\mathcal{O}_1>}$ from R–method	$\frac{<\mathcal{O}_i>}{<\mathcal{O}_1>}$ from B–method	B_i
quadratic	2	-16.4(1.0)	-17.8(1.5)	0.54(0.04)
	3	8.6(0.5)	9.4(0.7)	1.43(0.09)
	4	28.5(1.9)	30.9(2.3)	0.78(0.05)
	5	5.4(0.6)	6.3(0.8)	0.48(0.06)
linear	2	-15.4(0.5)	-17.8(1.0)	0.54(0.02)
	3	8.1(0.3)	9.4(0.5)	1.43(0.05)
	4	26.4(0.9)	30.7(1.1)	0.78(0.03)
	5	5.1(0.3)	6.3(0.5)	0.48(0.03)

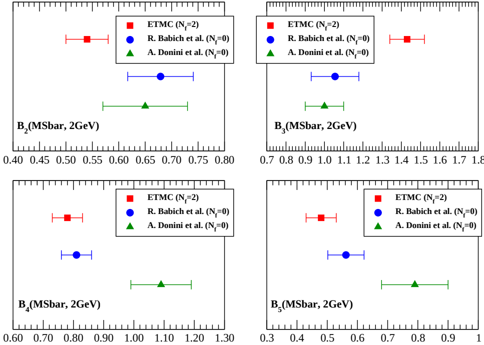
- R- and B-method results in a good agreement.
- No significant difference from the use of analytic or SU(2)-χPT fit function.
- Physics implications' study ... in progress.

Comparison between $N_f = 2$ and (older) $N_f = 0$ results

■ *ETMC (2010)*: $O(a)$-improved OS/TM-Wilson fermions; $N_f = 2$; Continuum like renormalisation pattern; NP renormalisation; CL estimate from $a = 0.1,\ 0.09,\ 0.07$ fm simulations.

● *R. Babich et al. (2006)*: overlap fermions; $N_f = 0$; Continuum like renormalisation pattern; NP renormalisation; CL estimate from $a = 0.09$ fm simulations.

▲ *A. Donini et al. (2000)*: tL improved Wilson fermions; $N_f = 0$; Mixing renormalisation pattern; NP renormalisation; CL estimate from average over $a = 0.09,\ 0.07$ fm simulations.



Comparison between $N_f = 2$ and (older) $N_f = 0$ results

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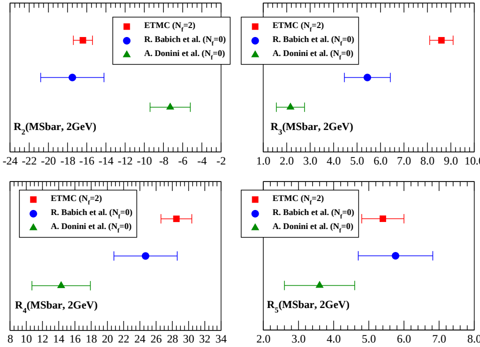

Conclusions

B_K (SM)

- $B_{\rm K}$ in CL: very good agreement from four independent calculations with $N_f = 2, 2+1$ dynamical quarks.
- Total $B_{\rm K}$ uncertainty $\simeq 4\%$.
- No significant dependence on s-quark.
- Tension at the level of 1.6 σ with the SM prediction for $B_{\rm K}$.
- Possible lattice calculation improvements:
  - safer control of the CL for ALV and RBC-UKQCD by working on a third finer lattice spacing.
  - NP renormalisation for BSW.
  - $N_f = 2+1+1$ dyn. quark calculation of ETMC (in progress).

$\bar{K}^0 - K^0$ beyond SM on the lattice

- First calculation of B_i-BSM in the CL using $N_f = 2$ dyn. quarks (ETMC); good scaling properties; total uncertainty $\sim 5-10\%$.