# $V_{us}$ FROM HADRONIC $\tau$ DECAY

*K. Maltman*

*CKM10, University of Warwick, September 7, 2010*

## OUTLINE

- *The basics*

- *A few relevant technical issues*

- *Results, assessment and the future*

# THE BASIC IDEA

- With $I^w_{ij}(s_0)$, $ij = ud, us$, the $s \le s_0$, $w(s)$-weighted integrals over flavor $ud$, $us$ V+A $\tau$ decay distributions, $[\delta J^w(s_0)]_{OPE}$ the OPE representation for

$$\delta J^w(s_0) \equiv \frac{I^w_{ud}(s_0)}{|V_{ud}|^2} - \frac{I^w_{us}(s_0)}{|V_{us}|^2}$$

$$\boxed{|V_{us}| = \sqrt{I^w_{us}(s_0) / \left[\frac{I^w_{ud}(s_0)}{|V_{ud}|^2} - [\delta J^w(s_0)]_{OPE}\right]}}$$

- Typically $[\delta J^w(s_0)]_{OPE} \sim$ few to several % of $I^w_{ud}(s_0)$ $\Rightarrow$ modest OPE errors to get accurate $|V_{us}|$ [Gamiz et al., JHEP 0301: 060]

- V,A $ij = ud, us$, $(J) = (0+1), (0)$ spectral functions from experimental differential decay distributions

$$\boxed{\begin{aligned} dR_{V/A;ij}/ds \;=\; & 12\pi^2\,|V_{ij}|^2 S_{EW}\,\Big[w_{(00)}(y_\tau)\,\rho^{(0+1)}_{V/A;ij}(s) \\ & +w_L(y_\tau)\,\rho^{(0)}_{V/A;ij}(s)\Big]\,/m_\tau^2 \end{aligned}}$$

with $R_{V/A;ij} \equiv \frac{\Gamma[\tau\to\nu_\tau\,\text{hadrons}_{V/A;ij}\,(\gamma)]}{\Gamma[\tau^-\to\nu_\tau e^-\bar{\nu}_e(\gamma)]}$, $y_\tau = s/m_\tau^2$

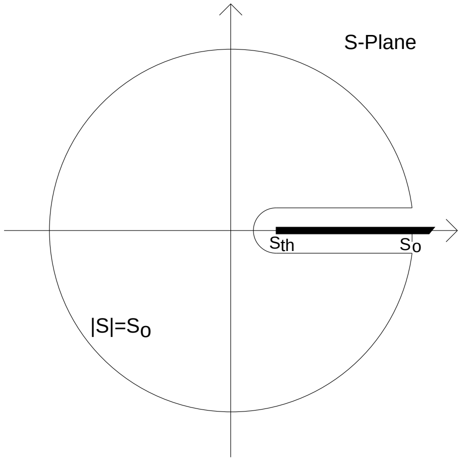
$w_{(00)}(y) = (1-y)^2(1+2y)$, $w_L(y) = -2y(1-y)^2$

- “longitudinal”: (0) part of $(0+1)/(0)$ decomposition

- $[\delta J^w(s_0)]_{OPE}$: OPE on RHS of FESR relation

$$\int_0^{s_0} w(s)\,\rho(s)\,ds \;=\; -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$

for correlators $\Pi(s) = \Pi^{(0+1)}_{ud,us;V/A}(s),\ s\Pi^{(0)}_{ud,us;V/A}(s)$



(Data on LHS, OPE on RHS)

- Bad convergence of integrated $(J) = (0)$ $D = 2$ OPE series ⇒ must treat phenomenologically. Fortunately
  - $\pi$, $K$ contributions accurately known
  - strong continuum suppression ($\propto (m_i \mp m_j)^2$)
  - small $us$ continuum contribution from $us$ scalar, PS analyses (constrained by $m_s$)
  - impact on $V_{us}$ small ($\sim$ 0.0002 or less)

- ⇒ essentially $\Delta\Pi \equiv \Pi^{(0+1)}_{ud:V+A} - \Pi^{(0+1)}_{us;V+A}$ FESRs

# A PUZZLE: THE CURRENT KINEMATIC $w_{(00)}(y)$ WEIGHT CASE RESULTS

- $s_0 = m_\tau^2$, kinematic weight $w_{(00)}(s) \Rightarrow I^w_{ud,us}$ from $B_{ud;TOT}$, $B_{us;TOT}$
- $\Rightarrow$ recent improved $us$ branching fractions sufficient for improved $|V_{us}|$ determination (for $s_0 = m_\tau^2$ AND $w_{(00)}(s)$ choice only)
- Experimentally more difficult inclusive $dR_{us;V+A}/ds$ distribution required for other $w(s)$ and/or $s_0$ [expected from BaBar, Belle, but still some time in future]

- The experimental situation:
  - $I^w_{ud}(s_0)$: $\sim 0.5\%$ errors for range of $w$ and $s_0$ [ALEPH 2005 data and covariances]
  - $I^w_{us}(s_0)$: pre-2007 $us$ errors $\sim 3-4\%$ [ALEPH99 distribution, rescaled mode-by-mode for exclusive $us$ B changes] ($\Rightarrow \sim 1.5-2\%$ on $|V_{us}|$)
  - Recent improved B values for several $us$ exclusive modes [BaBar, Belle] (but not $dR_{us}/ds$)
  - Current $B_{us;TOT}$ error 2.0% [Lusiani, ICHEP10] ($\Rightarrow$ 1% on $|V_{us}|$)

- Results of conventional $w_{(00)}(s)$ analysis:
  - CIPT+Adler function/CIPT+correlator $D = 2$ OPE evaluations used previously in literature, $K_{\mu 2}$ for $K$ contribution, yield updated $|V_{us}|$ results

$$0.2166(22)_{exp}(5??)_{th} \quad (CIPT + Adler\ function)$$
$$0.2162(22)_{exp}(5??)_{th} \quad (CIPT + correlator)$$

  - $|V_{us}|$ *nominally* $3.6\sigma$ low c.f. 3-family unitarity expectations, $K_{\ell 3}$ and $\Gamma[K_{mu2}]/\Gamma[\pi_{\mu 2}]$
  - (More on ?? in nominal theory error later)

# WHAT'S GOING ON?

- Problem(s) with the $ud$ V+A data?
- Problem(s) with the $us$ V+A data? (Especially possible missing higher multiplicity modes at higher $s$)
- Underestimate of theory uncertainties/unreliable central OPE values?
- None of the above, i.e., new physics?

## INVESTIGATING THE POSSIBILITIES

- Definite problem(s) if $|V_{us}|$ not independent of *both* $s_0$ *and* $w(s)$

- Use of polynomial $w(y)=\sum_m b_m y^m$, $y=s/s_0$ to test/check higher $D$ OPE contributions ($D=2k\propto 1/s_0^{k-1}$, "absent" if $b_{k-1}=0$)

- Alternate $D=2$ OPE prescriptions differing only at higher order in $\alpha_s$ than truncation order (CIPT+correlator, CIPT+Adler function, FOPT) should give results compatible within $D=2$ truncation error estimate

- “Supplementary” $us$ V+A FESRs
  - Remove $ud$ V+A data as potential problem source
  - $|V_{us}|$ from FESRs for flavor $us$ V+A correlator combination:

$$|V_{us}| \quad = \sqrt{\frac{I^w_{us;V+A}(s_0)}{I^w_{OPE}(s_0)}}$$

  - $us$ spectral data needed identical to that for $ud$-$us$ FESRs

- OPE side of $us$ V+A FESRs
  - $*$ $O(m_s^2\alpha_s^m)$ $D=2$ coefficients almost identical to those of $ud-us$ V+A series
  - $*$ CAUTION: presence of $D=0$, $\langle\alpha_s G^2\rangle$ $D=4$ OPE contributions $\Rightarrow$ some increase in OPE error
- IF OPE OK, problem due to missing higher $s$ $us$ spectral strength $\Rightarrow$ $|V_{us}|$ *must* be larger at lower $s_0$
- $|V_{us}|$ lower at lower $s_0$ $\Rightarrow$ definite OPE problem (additional $us$ data problem not precluded)

# Problems with the $ud$ data?

- $\tau$ vs CVC+IB electroproduction expectation discrepancy for $\pi\pi$ [minor for BaBar EM, non-trivial for KLOE, Novosibirsk]

- Similar $\tau$ vs EM discrepancy for $4\pi$ [still non-trivial, even for preliminary BaBar LP07 $4\pi$ EM]

- HOWEVER, correlations in PDG global $\tau$ branching fractions fit dominantly to "nearby multiplicity" non-strange modes $\Rightarrow$ impact on $|V_{us}|$ likely small

## Problems with the $us$ data?

- $B$ for some moderately large exclusive modes not yet remeasured by B factories [Table]

- Missing modes above $s \sim 2\ GeV^2$ ($\delta V_{us} \sim 0.0004$ for each $\delta B \sim 10^{-4}$), e.g., for $w_{(00)}(s)$, $s_0 = m_\tau^2$,
    - $B[K^- \pi^0 \pi^0 \nu_\tau]$ up $3\sigma \Rightarrow \delta|V_{us}| = +0.0025$
    - $B[(K\,3\pi)^- \nu_\tau]$ up $3\sigma \Rightarrow \delta|V_{us}| = +0.0030$
    - ALEPH99 $K\,4\pi$ rough estimate $\Rightarrow \delta|V_{us}| = +0.0013$

- (See later, however, re $s_0$ stability issues etc.)

# PRE-2007 vs Lusiani ICHEP10 $us$ $\mathcal{B}$ VALUES

| Mode | $\mathcal{B}_{2006}$ (%) | $\mathcal{B}_{ICHEP10}$ (%) |
|---|---|---|
| $K^-$ [$\tau$ decay] | 0.685(23) | 0.696(10) |
| (Alt: [$K_{\mu 2}$]) | (0.715(3)) | (0.715(3)) |
| $K^-\pi^0$ | 0.454(30) | 0.431(15) |
| $\bar{K}^0\pi^-$ | 0.878(38) | 0.827(18) |
| $K^-\pi^0\pi^0$ | 0.058(24) | 0.060(22)[**] |
| $\bar{K}^0\pi^0\pi^-$ | 0.360(40) | 0.349(15) |
| $K^-\pi^-\pi^+$ | 0.330(50) | 0.294(7) |
| $K^-\eta$ | 0.027(6) | 0.016(2) |
| $(\bar{K}\eta\pi)^-$ | 0.029(9) | 0.0141(19) |
| $(\bar{K}3\pi)^-$ | 0.141(37) | 0.165(39)[**] |
| $K\phi$ | | 0.0037(1) |
| $(\bar{K}4\pi)^-$ (est'd) | 0.011(7) | [**] |
| $(\bar{K}5\pi)^-$ (est'd) | 0.006 | [**] |
| TOTAL | 2.973(86) | 2.857(58) |
| | (3.003(83)) | (2.876(58)) |

# OPE Problems?

- Key OPE problem: slow $D = 2$ $(0+1)$ series convergence at the correlator level

- $\Delta\Pi(Q^2) \equiv \Pi^{(0+1)}_{ud;V+A} - \Pi^{(0+1)}_{us;V+A}$, $\Delta\rho(s)$: correlator and corresponding spectral function for $ud-us$ V+A FESRs

- $D = 2$ OPE series, $\bar{m}_s = m_s(Q^2)$, $\bar{a} = \alpha_s(Q^2)/\pi$, $\overline{MS}$ scheme [Baikov, Chetyrkin, Kuhn PRL95:012003]

$$\left[\Delta\Pi(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\left[1 + 2.333\bar{a} + 19.933\bar{a}^2 \right.$$
$$\left. +208.746\bar{a}^3 + (2378 \pm 200)\bar{a}^4 + \cdots\right]$$

- $a(m_\tau^2) \sim 0.1 \Rightarrow$ *very* slow convergence at spacelike point on $|s| = s_0$, even for maximum $s_0 = m_\tau^2$

- (Not surprisingly) integrated $D = 2$ $(0+1)$ series convergence typically also slow, e.g., for $s_0 = m_\tau^2$, $w_{(00)}(s)$ with CIPT+Adler function ($1^{st}$ line), CIPT+correlator ($2^{nd}$ line), FOPT ($3^{rd}$ line) $D = 2$ prescriptions, , to $O(\bar{a}^4)$:

$$\sim [1 + 0.286 + 0.103 - 0.039 - (0.197) + \cdots]$$
$$\sim [1 + 0.151 + 0.017 - 0.120 - (0.293) + \cdots]$$
$$\sim [1 + 0.405 + 0.257 + 0.154 + (0.081) + \cdots]$$

- Options for dealing with the slow $D = 2$ convergence:
  - take advantage of improved convergence in CIPT away from spacelike point via choice of weight [Here: $w_{20}$, $\widehat{w}_{10}$, $w_{10}$ of PRD62 (2000) 093020]
  - FESRs for alternate flavor-breaking correlator combinations with suppressed $D = 2$ OPE at correlator level [involves combination of EM, $\tau$ decay data]
  - $s_0$-stability checks as crucial test

# SOME ILLUSTRATIVE RESULTS

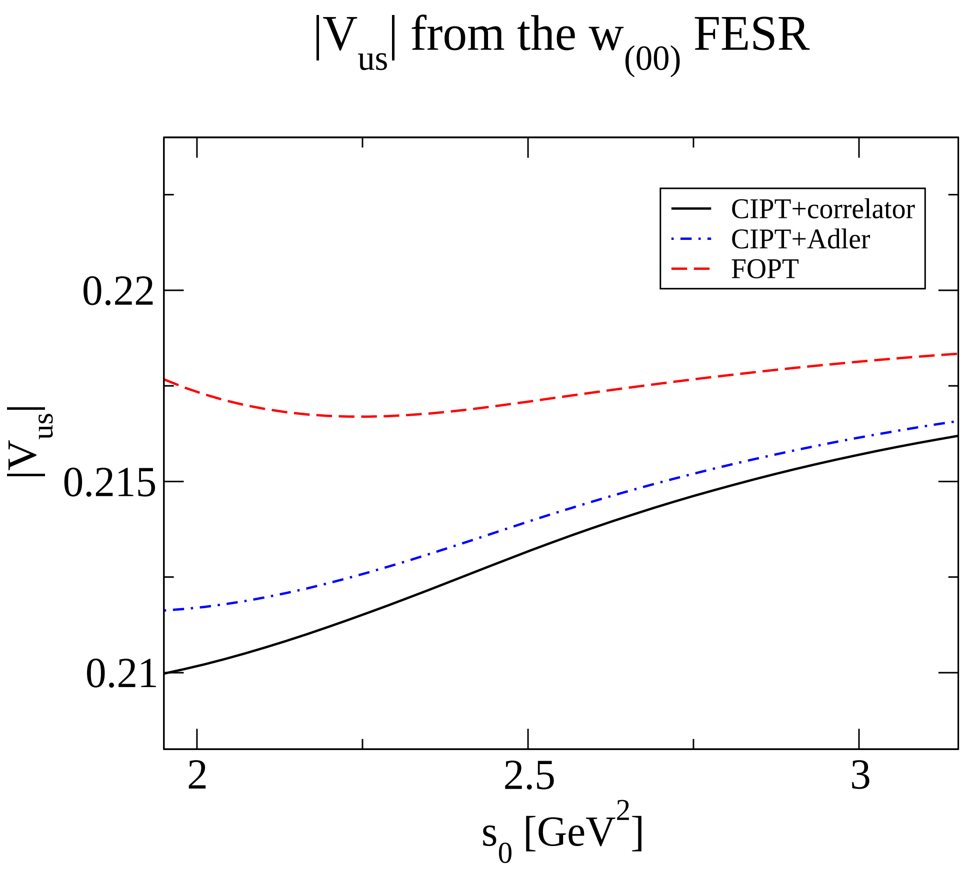
- $ud$ V+A spectral integrals, errors from ALEPH 2005 data, covariances

- $us$ V+A spectral integrals results using $K_{\mu 2}$ input, mode-by-mode rescaled ALEPH 1999 $us$ distribution to handle $w \neq w_{(00)}$, $s_0 \neq m_\tau^2$ cases

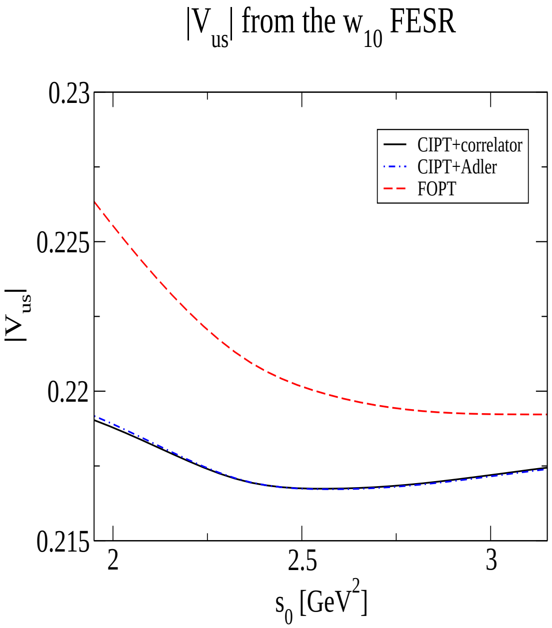
- Rescaling necessary as updated distributions currently available only for $K^-\pi^+\pi^-$, $K^-K^+K^-$ [BaBar]

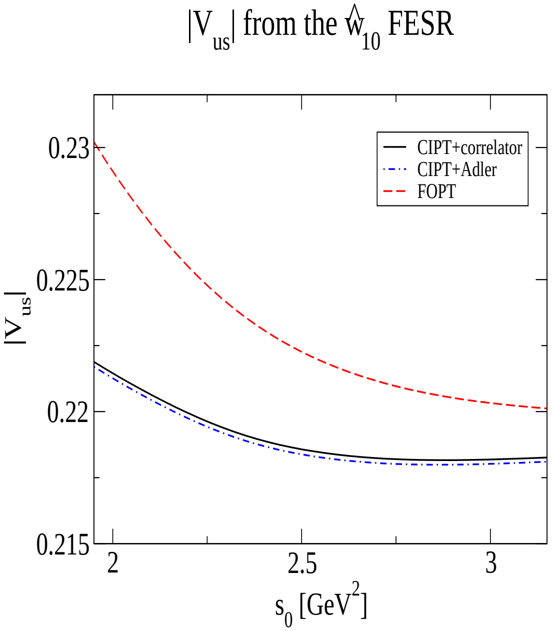
- However, test of rescaling for weighted $K^-\pi^+\pi^-$ integrals (BaBar vs rescaled ALEPH99) shows rescaling very reliable for central values, despite large rescaling

# $s_0$-STABILITY, $w_{(00)}$ FESR



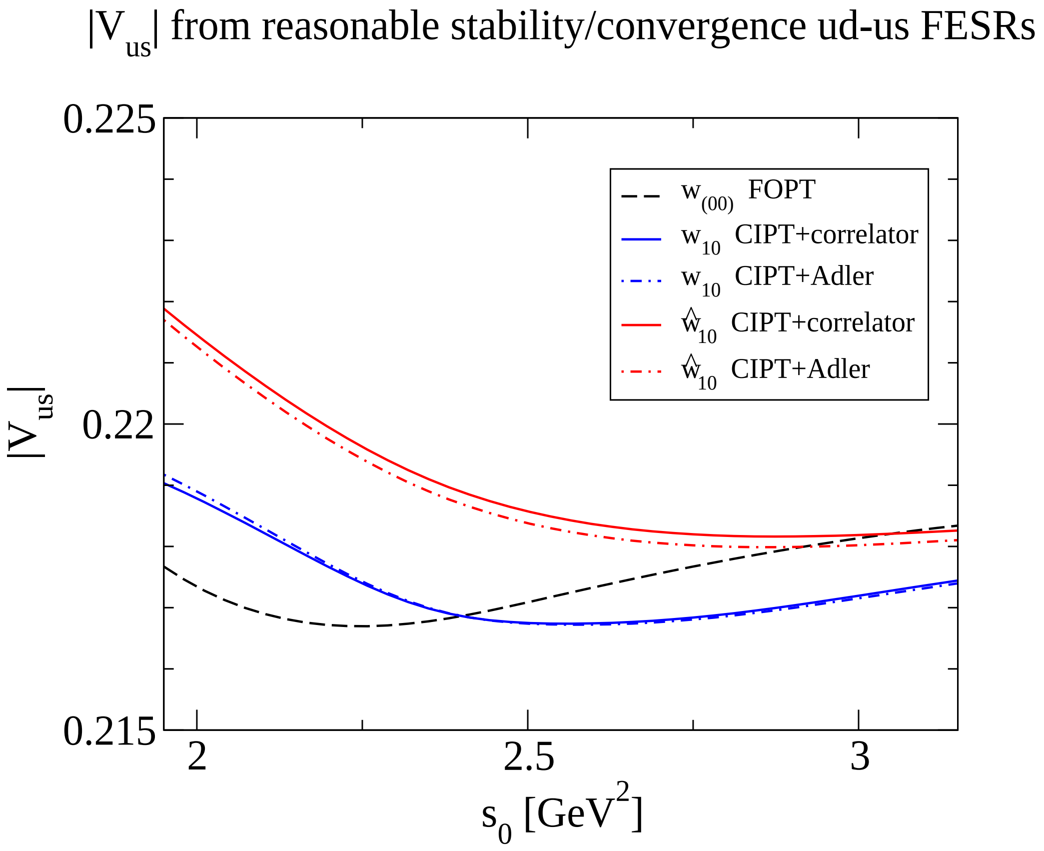
# $s_0$-STABILITY, $w_{10}$, $\hat{w}_{10}$ FESRs



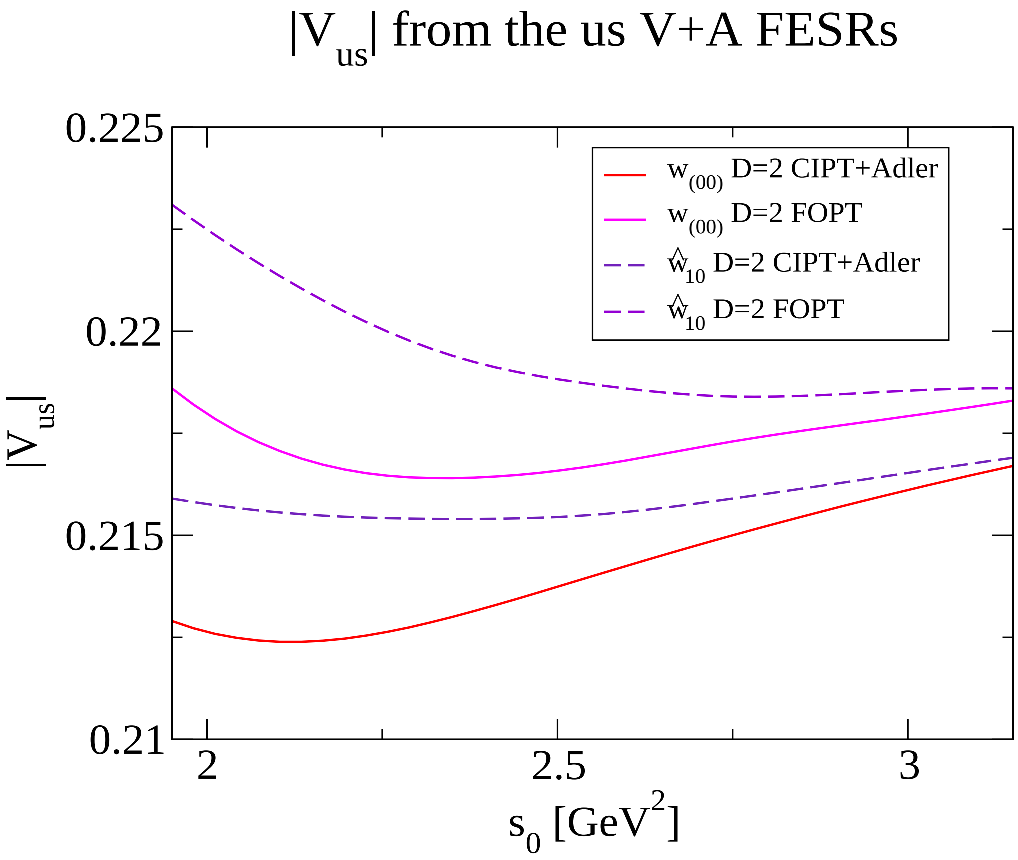


*CAUTION: VERY slow FOPT convergence*

# OK STABILITY, D=2 CONVERGENCE CASES



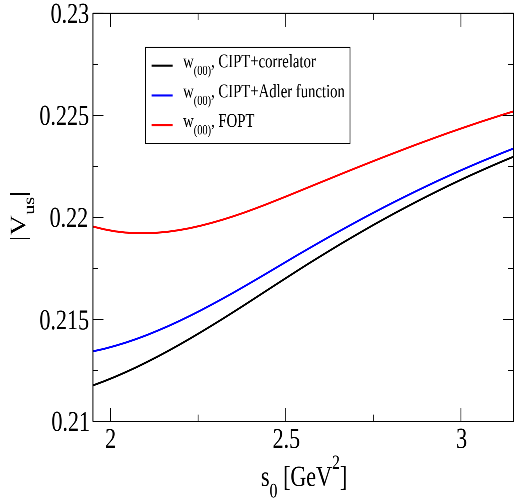
# THE *us* V+A FESRs



# Impact of $3\sigma$ $B$ increases for largest $us$ modes not yet remeasured by BaBar or Belle: $|V_{us}|$ vs $s_0$

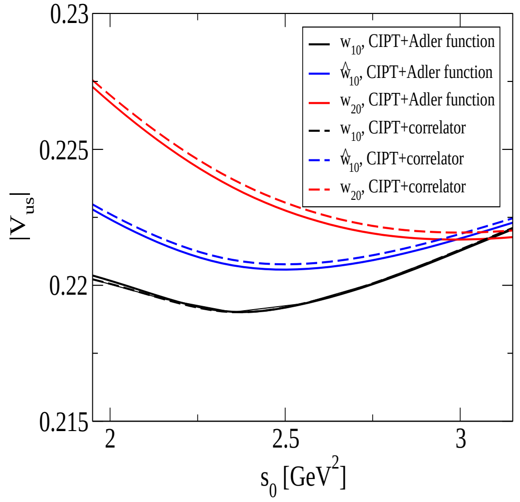
### $w_{(00)}$ FESR

$w_{(00)}$ ud-us FESR, $B[K^-2\pi^0]$, $B[K3\pi]$ up 3 $\sigma$



### $w_{10}$, $\hat{w}_{10}$, $w_{20}$ FESRs

ud-us FESRs, $B[K\ 2\pi^0]$, $B[K\ 3\pi]$ up 3 $\sigma$



# ALTERNATE EM-$\tau$ FESRs

- Slow convergence of the integrated $D = 2$ OPE series for $\Delta\Pi$ due to slow convergence at the correlator level (for scales kinematically accessible in $\tau$ decay)

- Suggests trying alternate flavor-breaking combinations with suppressed $D = 2$ OPE contributions, e.g.,

  $$\Delta\Pi^{EM,\tau} \equiv 9\Pi_{EM} - \left[5\Pi_{ud;V} - \Pi_{ud;A} + \Pi_{us;V+A}\right]$$

  (same normalization for $us$ V+A as in $\Delta\Pi$)

- $D = 2$ suppression choice also suppresses $D = 4$
  - $D = 2$

$$\left[\Delta\Pi(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\left[1 + \frac{7}{3}\bar{a} + 19.933\bar{a}^2 + 208.75\bar{a}^3 + \cdots\right]$$

$$\left[\Delta\Pi^{EM,\tau}(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\left[0 + \frac{1}{3}\bar{a} + 4.3839\bar{a}^2 + 44.943\bar{a}^3 + \cdots\right]$$

  - $D = 4$

$$\left[\Delta\Pi(Q^2)\right]_{D=4} = \frac{\langle m_s\bar{s}s\rangle - \langle m_\ell\bar{\ell}\ell\rangle}{Q^4}\left[-2 - 2\bar{a} - \frac{26}{3}\bar{a}^2\right]$$

$$\left[\Delta\Pi^{EM,\tau}(Q^2)\right]_{D=4} = \frac{\langle m_s\bar{s}s\rangle - \langle m_\ell\bar{\ell}\ell\rangle}{Q^4}\left[0 + \frac{8}{3}\bar{a} + \frac{59}{3}\bar{a}^2\right]$$

- FESRs based on $\Delta\Pi^{EM,\tau} \Rightarrow$ (suppressing $s_0$-dependence of the OPE and spectral integrals)

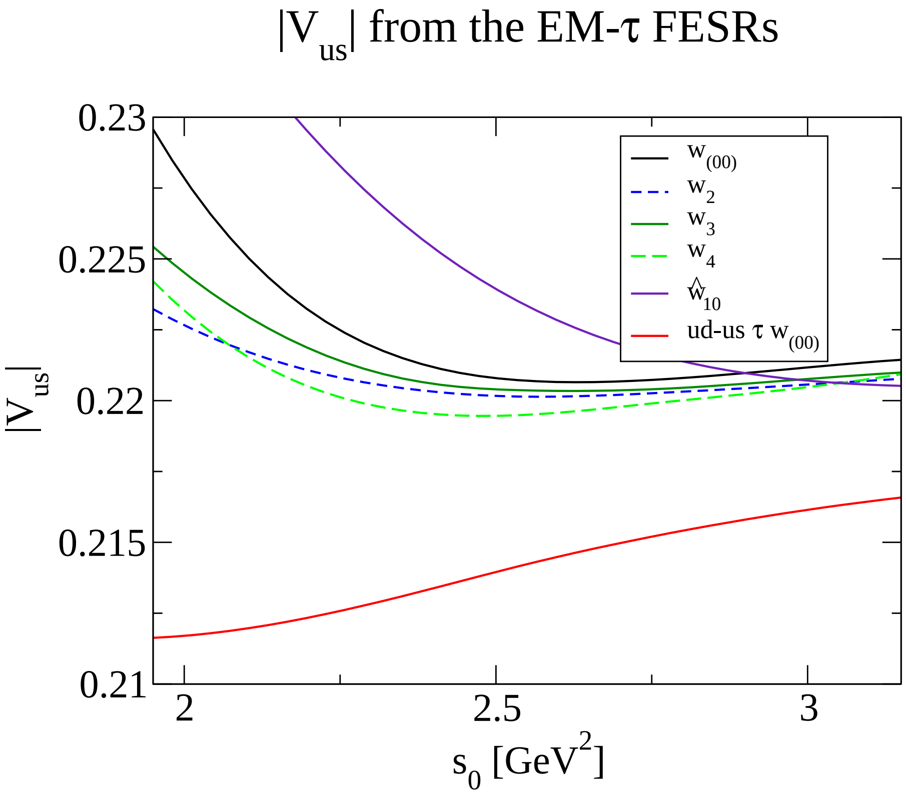
$$|V_{us}| \quad = \sqrt{\frac{I^w_{us;V+A}}{\frac{3}{2}I^w_{EM,I=0} - \frac{1}{2}I^w_{ud;V} + I^w_{ud;A} - I^w_{OPE}}}$$

(with $I^w_{EM,I=0}$ normalized as for a charged current correlator)

- Strong suppression of $D = 2,4$ contributions $\Rightarrow$ $w(y)$ usable even without improved $D = 2$ convergence, hence e.g. $w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N$

- Advantages of $w_N$ FESR choice:
  - single integrated $D > 4$ contribution ($D = 2N + 2$) (up to $O(\alpha_s^2)$ corrections)
  - $D = 2N + 2$ suppressed by relevant $w_N$ coefficient, $1/(N-1)$
  - $1/s_0^N$ dependence provides handle on integrated $D = 2N + 2$ contributions
- NOTE: $D > 4$ typically NOT suppressed at correlator level: E.g. in VSA, $D = 6$ a factor of $9/2$ larger for $\Delta\Pi^{EM,\tau}$ than for $\Delta\Pi$ $\Rightarrow$ small relevant coefficient values useful
- However, can fit $D > 4$ strengths to data via $s_0$-dependence, especially when only one such contribution present

# MIXED $\tau$-EM vs. the $w_{(00)}$ $ud-us$ FESR



# CURRENT RESULTS/OBSERVATIONS

- The $ud - us$ V+A FESRs:
  - Clear $s_0$-stability problem for $w_{(00)}$, CIPT $D = 2$; $us$ V+A results ⇒ significant OPE component
  - Better convergence, stability with FOPT for $w_{(00)}$, $|V_{us}| = 0.2183(5)_{ud}(22)_{us}(??)_{th}$

    ($\sim 0.0020$ higher c.f. CIPT)
  - Best of improved CIPT convergence weights, $\hat{w}_{10}$, yields $|V_{us}| = 0.2182(5)_{ud}(22)_{us}(??)_{th}$
  - OPE uncertainties ($s_0$-instability, $w(y)$-dependence) clearly much larger than $\delta V_{us} \sim 0.0005$ at present

  - Upward $B$ shifts for as-yet-unremeasured $us$ modes could still shift $|V_{us}|$ significantly, but N.B. re stability issues

- EM-$\tau$ FESR results:
  - Good $s_0$-stability, $w(y)$ independence
  - For $w_{(00)}$, $s_0 = m_\tau^2$, including variation with weight-choice in theory error (totally dominant)

$$|V_{us}| = 0.2214\,(22)_{us;V+A}(5)_{ud;V,A}(28)_{EM}(6)_{th}$$

  - Theory errors much better BUT experimental errors much worse c.f. $ud{-}us$ V+A (EM-$\tau$ spectral integral differences, with independent errors)

# FUTURE PROSPECTS/DIRECTIONS

- The $ud - us$ V+A FESRs:
  - Many $us$ B errors already reduced, others still needed
  - Ingredients for full remeasurement of actual $us$ spectral distribution in place and work in progress
  - Some obvious targets for near term BaBar, Belle attention ($K^-\pi^0\pi^0$, $K3\pi$, $K4\pi$ , $\cdots$)
  - Updates on $ud$ $2\pi$, $4\pi$ $\tau$ decay modes desirable
  - Better understanding of $D = 2$ OPE truncation error needed to significantly reduce theory error

- The flavor-breaking EM-$\tau$ FESR:
  - $us$ V+A error reductions as per the $ud - us$ V+A FESRs
  - Much improved $s_0$-stability, $w(y)$-independence compatible with OPE as significant error source for $ud - us$ V+A FESRs
  - Need resolution of EM vs $\tau$ $\pi\pi$ and $4\pi$ issues
  - Significantly reduced $I = 0$ EM cross-section errors needed to make competitive with other methods

# SUPPLEMENTARY PAGES

- Details on the handling of potential $D > 6$ OPE contributions

- Rough scale of longitudinal subtraction, $(0+1)$ OPE relative to $ud$ spectral integrals

- Details on the integrated $D=2$ for improved-CIPT-convergence Kambor-Maltman weights

- Impact of 3$\sigma$ increases of $B[K^-2\pi^0]$, $B[K3pi]$ on $|V_{us}|$ from the $us$ V+A FESR

# HIGHER $D$ OPE CONTRIBUTIONS

- rough estimates for $D = 6$ condensates, $D > 6$ combinations unknown, usually assumed negligible

- $w(y) = \sum_m c_m y^m$, $y = s/s_0$ $\Rightarrow$ integrated $D = 2k + 2$ OPE $\propto c_k/s_0^k$ (up to logs) $\Rightarrow$ avoid large $c_k$, $k \geq 2$

- neglect of non-negligible higher $D$ terms $\Rightarrow$ $s_0$-instability of output $\Rightarrow$ *need to study output as function of* $s_0$

# RELATIVE SCALES IN THE $ud - us$ V+A FESR

*E.g., $ud - us$ V+A, $s_0 = m_\tau^2$ contributions:*

- $R_{ud;V+A} = 3.478(16)$

- Longitudinal subtraction $\left[\delta R_\tau^{(0)}\right]_L = 0.1544(37)$

  (0.1204 from $K$, $\pi$ poles, 0.0340 from continuum)

- $\left[\delta R_\tau^{(0+1)}\right]_{OPE} = 0.0612(15)$ (Gamiz et al. 2008)

  [90% of uncertainty from $m_s^2$ $D = 2$ scale]

# CONVERGENCE OF $w_{10}$, $\hat{w}_{10}$ and $w_{20}$-WEIGHTED $D=2$ OPE SERIES FOR VARIOUS $D=2$ PRESCRIPTIONS, $s_0 = m_\tau^2$

- First lines: CIPT + Adler function; second lines: CIPT + correlator; third lines: FOPT

- $\hat{w}_{10}$:

$$\sim [1 + 0.391 + 0.278 + 0.215 + (0.167) + \cdots]$$
$$\sim [1 + 0.241 + 0.185 + 0.150 + (0.109) + \cdots]$$
$$\sim [1 + 0.514 + 0.432 + 0.400 + (0.411) + \cdots]$$

- $w_{10}$:

$$\begin{aligned} &\sim [1 + 0.371 + 0.246 + 0.173 + (0.115) + \cdots] \\ &\sim [1 + 0.226 + 0.160 + 0.114 + (0.062) + \cdots] \\ &\sim [1 + 0.487 + 0.387 + 0.332 + (0.325) + \cdots] \end{aligned}$$

- $w_{20}$:

$$\begin{aligned} &\sim [1 + 0.412 + 0.307 + 0.246 + (0.198) + \cdots] \\ &\sim [1 + 0.255 + 0.205 + 0.172 + (0.126) + \cdots] \\ &\sim [1 + 0.558 + 0.502 + 0.490 + (0.535) + \cdots] \end{aligned}$$

# Impact of 3$\sigma$ increases of $B[K^-2\pi^0]$, $B[K3\pi]$ on $|V_{us}|$ from the $us$ V+A FESR