Lattice calculation of $B \to K^{(*)}ll$ form factors

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September 7 2010

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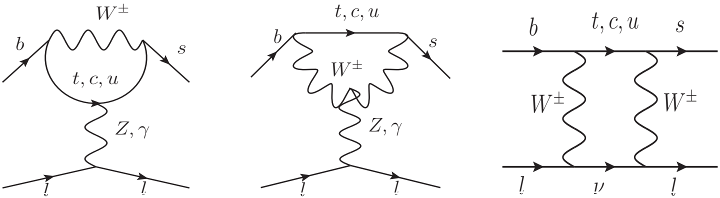
Outline

- Motivation
- Lattice setup and data
- Preliminary results

Rare B decays

- Flavor Changing Neutral Current (FCNC) transition $b \to s$ are suppressed in the Standard Model.
- Penguin and box diagrams ($b \to s\ ll$):

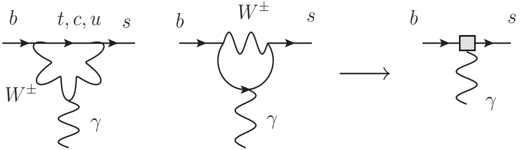


- $b \to s$ effective weak Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} V_{tb} V_{ts}^* C_i(\mu) Q_i(\mu)$$

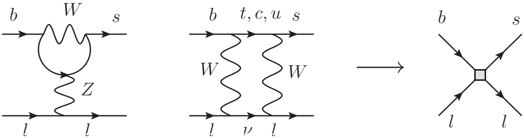
Local operators in our calculation

- $Q_7 = m_b \bar{s} \sigma^{\mu\nu}(1+\gamma_5) b F_{\mu\nu}$



Relevant for $B \to K^*\gamma$, $B_s \to \phi\gamma$, and $B \to K^{(*)} l^+ l^-$.

- $Q_9 = \bar{s}\gamma^\mu(1-\gamma_5) b \bar{l}\gamma_\mu l$, $\quad Q_{10} = \bar{s}\gamma^\mu(1-\gamma_5) b \bar{l}\gamma_\mu \gamma_5 l$.



Relevant for $B \to K^{(*)} l^+ l^-$.

Parametrization of matrix elements

$B \to K l^+ l^-$

$$\langle K(p')|\bar{s}\gamma^\mu b|B(p)\rangle = f_+(q^2)\left[p^\mu + p'^\mu - \frac{M_B^2 - M_K^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_K^2}{q^2}q^\mu, \qquad (q = p - p'),$$

$$q_\nu\langle K(p')|\bar{s}\sigma^{\mu\nu} b|B(p)\rangle = \frac{i f_T(q^2)}{M_B + M_K}\left[q^2(p^\mu + p'^\mu) - (M_B^2 - M_K^2)q^\mu\right].$$

$B \to K^*\gamma$, $\quad B_s \to \phi\gamma$, $\quad B \to K^* l^+ l^-$

$$q^\nu\langle K^*(p',\lambda)|\bar{s}\sigma_{\mu\nu} b|B(p)\rangle = 4T_1(q^2)\epsilon_{\mu\nu\rho\sigma}e_\lambda^{*\nu}p^\rho p'^\sigma, \quad (e_\lambda^\nu : \text{polarization}),$$

$$q^\nu\langle K^*(p',\lambda)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle = 2iT_2(q^2)\left[e_{\lambda\mu}^*(M_B^2 - M_{K^*}^2) - (e_\lambda^*\cdot q)(p+p')_\mu\right] + 2iT_3(q^2)(e_\lambda^*\cdot q)\left[q_\mu - \frac{q^2}{M_B^2 - M_{K^*}^2}(p+p')_\mu\right].$$

Parametrization of matrix elements

$B \to K^* l^+ l^-$

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{M_B + M_{K^*}}\epsilon^{\mu\nu\rho\sigma} e^*_{\lambda\nu} p'_\rho p_\sigma,$$

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle = 2M_{K^*} A_0(q^2)\frac{e^*_\lambda\cdot q}{q^2} q^\mu$$
$$+(M_B + M_{K^*})A_1(q^2)\left[e^{*\mu}_\lambda - \frac{e^*_\lambda\cdot q}{q^2} q^\mu\right]$$
$$-A_2(q^2)\frac{e^*_\lambda\cdot q}{M_B+M_{K^*}}\left[p^\mu + p'^\mu - \frac{M_B^2 - M_{K^*}^2}{q^2} q^\mu\right] \quad .$$

Other lattice calculations (all are quenched simulations)

- D. Becirevic, V. Lubicz and F. Mescia, Nucl. Phys. B **769**, 31 (2007)
  - Quenched calculation of $T(0)(= T_1(0) = T_2(0))$ for $B \to K^*\gamma$.
  - Two lattice spacings.
  - Unphysical heavy quark mass: $M_B > M_H \geq M_D$. Extrapolation: $1/M_H \to 1/M_B$ using heavy quark scaling laws.
  - Extrapolating to M_B at $q^2 = 0$: $T(0; \mu = m_b) = 0.24(3)^{+0.04}_{-0.01}$ (should be divided by 2 if using our normalization).
  - Checked by extrapolating to M_B at fixed $q^2(\neq 0)$ and then to $q^2 = 0$.
- L. Del Debbio, J. M. Flynn, L. Lellouch and J. Nieves [UKQCD Collaboration], Phys. Lett. B **416**, 392 (1998)
- A. Abada *et al.* [APE Collaboration], Phys. Lett. B **365**, 275 (1996)
- T. Bhattacharya and R. Gupta, Nucl. Phys. Proc. Suppl. **42**, 935 (1995)
- K. C. Bowler *et al.* [UKQCD Collaboration], Phys. Rev. Lett. **72**, 1398 (1994)
- C. W. Bernard, P. Hsieh and A. Soni, Phys. Rev. Lett. **72**, 1402 (1994)

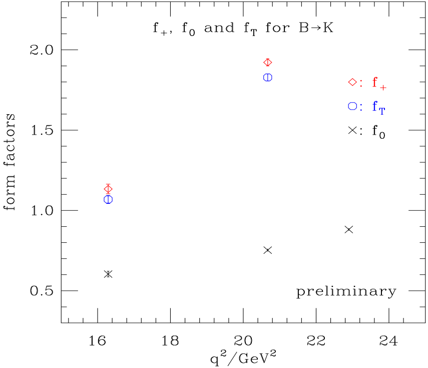
Lattice setup and data

- $2+1$ flavor simulation (u, d, s sea quarks) with Staggered fermions (MILC configurations).
- Using (moving-)NRQCD for the b quark and Staggered fermions for light quarks.
- The bare b quark mass is determined from the Υ masses ($am_b = 2.8$ on coarse lattices, 1.95 on the fine lattice).
- The pion mass is in the range of [300, 500] MeV.

	a(fm)	am_{sea}	Volume	$N_{conf} \times N_{src}$	am_{val}
coarse	$\sim$0.12	0.007/0.05	$20^3 \times 64$	2109×8	0.007/0.04
		0.02/0.05	$20^3 \times 64$	2052×8	0.02/0.04
fine	$\sim$0.09	0.0062/0.031	$28^3 \times 96$	1910×8	0.0062/0.031

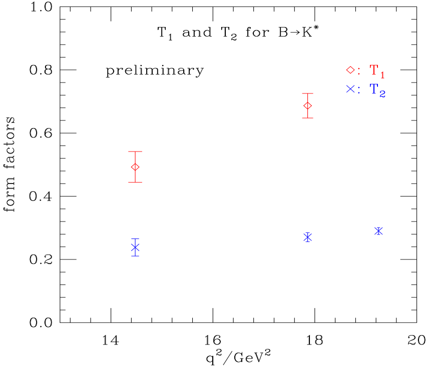
- Renormalisations for the vector and tensor (at the scale $\mu = m_b$) currents were calculated in Eike H. Müller et al., PoS **LAT2009**, 241 (2009) [arXiv:0909.5126 [hep-lat]].

Preliminary results of form factors f_0, f_+ and f_T



- $a \approx 0.12$ fm and $m_\pi \approx 300$ MeV. Only statistical errors.
- Errors increase from 1% to 3% as q^2 decreases.

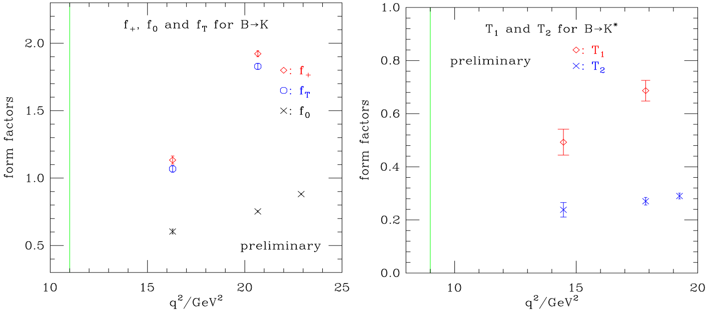
Preliminary results of form factors T_1 and T_2



- $a \approx 0.12$ fm and $m_\pi \approx 300$ MeV. Only statistical errors.
- Errors increase from 4% to 12% as q^2 decreases.

The march to small q^2

- By giving the B meson a non-zero momentum in the opposite direction, we can reduce the final meson's momentum at a given q^2 to reduce discretisation errors and noise to signal ratios in correlators.
- The formalism for achieving this is called moving NRQCD.



Summary

- We are doing a 2+1 flavour lattice calculation of form factors for rare $B \to K^{(*)}$ decays.
- With (moving-)NRQCD, one works directly at the physical b quark mass.
- Our calculation are most precise in the low recoil region $q^2 \approx q^2_{max}$.
- Extrapolation to the physical u/d quark mass point.
- Extrapolation to low-q^2 and $q^2 = 0$ for $B \to K^*\gamma$ (using pole dominance/z-parameterisation).

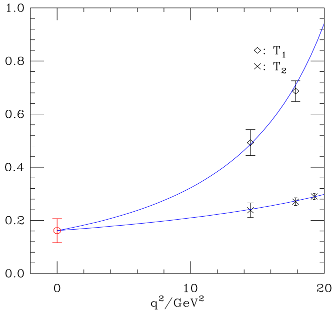
Thank you for your attention!

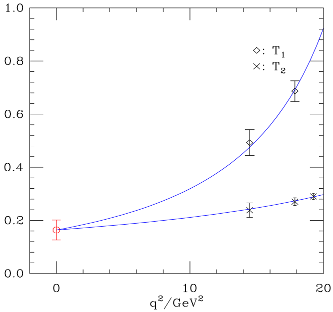
BACKUP

Extrapolation of T_1 and T_2 to $q^2 = 0$

Pole dominance [Becirevic & Kaidalov (2000), Ball & Zwicky (2005), Becirevic et al. (2007)]

$$T_1(q^2) = \frac{T(0)}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)}, \quad T_2(q^2) = \frac{T(0)}{1-\tilde{q}^2/\beta}, \quad \tilde{q}^2 = q^2/M_{B_s^*}^2.$$





$T(0) = 0.161(45)$ if $M_{B_s^*}$ is a free parameter (left graph).
$T(0) = 0.164(38)$ if $M_{B_s^*} = 5.4158$ GeV is fixed from PDG2010.

Lattice setup

- Tadpole improved $\mathcal{O}(1/m_b^2, v_{rel}^4)$ moving NRQCD action. Discretisation error starts at $\mathcal{O}(\alpha_s a^2)$ (tree-level errors begin at $\mathcal{O}(a^5)$).
- The bare b quark mass is determined from the physical Υ masses using NRQCD.

  [A. Gray *et al.*, Phys. Rev. D **72**, 094507 (2005)]
- Lüscher-Weisz gluon action. AsqTad fermion action (sea and light valence quarks).
- The local operators (currents) are expanded to $\mathcal{O}(1/m_b)$.
- Operator matching factors are calculated by tadpole-improved 1-loop lattice perturbation theory.

$$J^{cont} = (1 + \alpha_s\, c_+)\, J_+^{(0)} + \alpha_s\, c_-\ J_-^{(0)}\ + \frac{1}{m_b}\, J_+^{(1)}.$$

$\mathcal{O}(\alpha_s/m_b, \alpha_s^2, 1/m_b^2)$ ignored.

Correlators

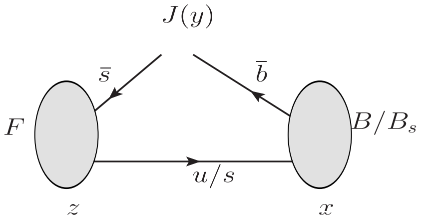
Interpolating fields:

- Light mesons: $\Phi_F = \bar{q}\Gamma s, \quad q = u, s, \quad \Gamma = \gamma_5, \gamma_i$.
- B/B_s mesons: $\Phi_B = \bar{q}\gamma_5\Psi_b, \quad q = u, s$.

2-point correlators (with a point source):

- $$C_{FF}(x_t, \vec{p}') = \sum_{\vec{x}} \langle \Phi_F(x)\Phi_F^\dagger(0)\rangle e^{-i\vec{p}'\cdot\vec{x}}$$
- $$C_{BB}(x_t, \vec{p}) = \sum_{\vec{x}} \langle \Phi_B(x)\Phi_B^\dagger(0)\rangle e^{-i\vec{p}\cdot\vec{x}}$$

3-point correlators



$$C_{FJB}(\vec{p}, \vec{p}', T, t) = \sum_{\vec{x}} \sum_{\vec{y}} \langle \Phi_B(\vec{x}, T) J(\vec{y}, t) \Phi_F^\dagger(0) \rangle e^{-i\vec{p}\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}},$$

- $q = p - p'$.
- $T = x_t - z_t$ is varied between 8 and 26 on the coarse lattice, 15 and 36 on the fine lattice. (About 1.3 to 3.2 fm.)
- $t = y_t - z_t = 0, 1, \cdots, T$. Fit both t and T.