# $g_A$ on the lattice

CKM 2010

Warwick, 06-10.09.2010

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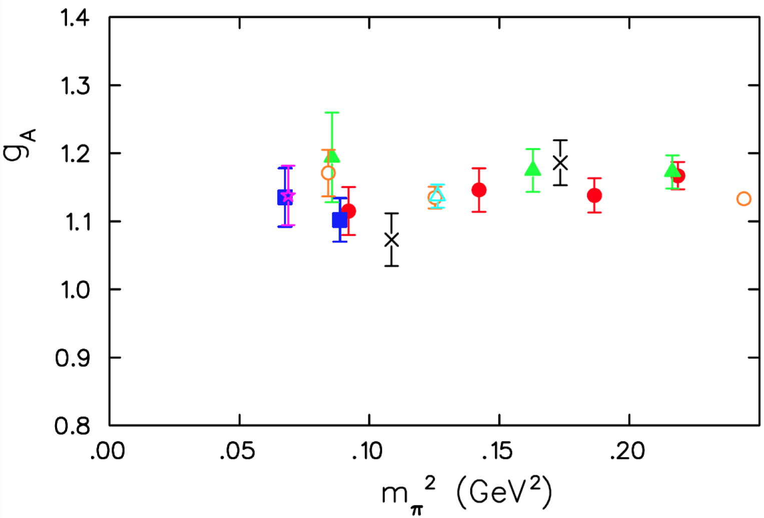
CERN Theory Division

- why are baryons harder than mesons?
- a critical look at current calculations

# Overview

*Plot taken from Dina Alexandrou's talk at Lattice 2010*




Results from 2 and 3-flavour computations *ETMC PoS LAT 2009; Dinter at Lattice 2010; Yamazaki et al., PRD 97, 14505 (2009); LHPC arXiv:1001.3620; QCDSF, Pleiter at Lattice 2010; CLS, Knippschild at Lattice 2010, RBC+UKQCD, Ohta, Lattice 2010*

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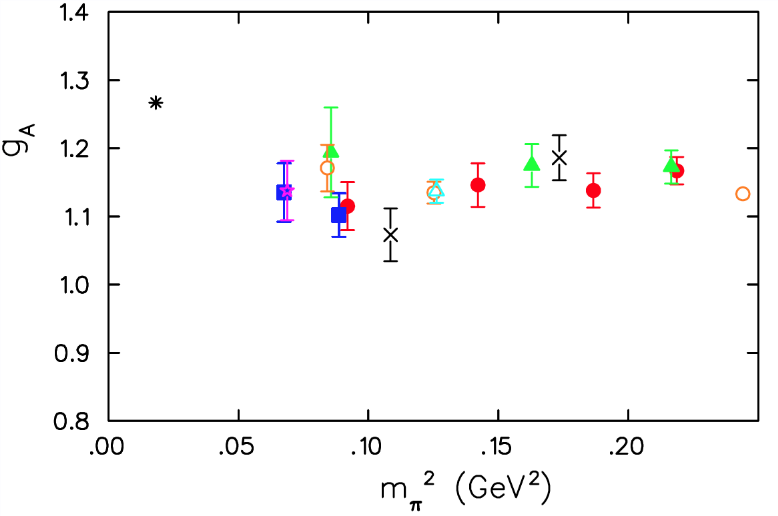


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  - $m_q$
  - fitting/excited states
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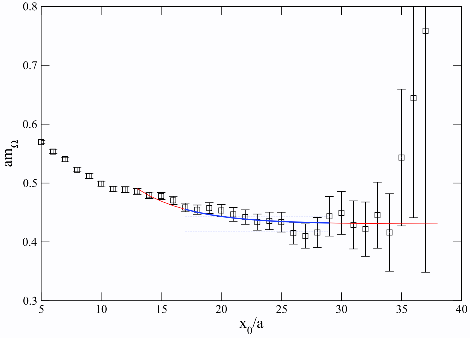

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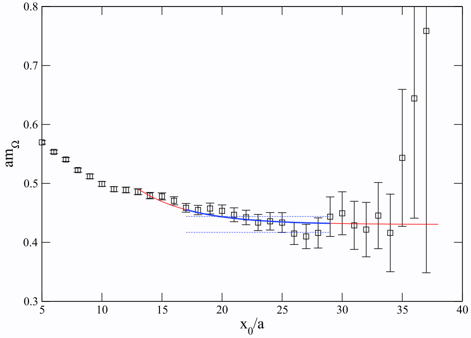
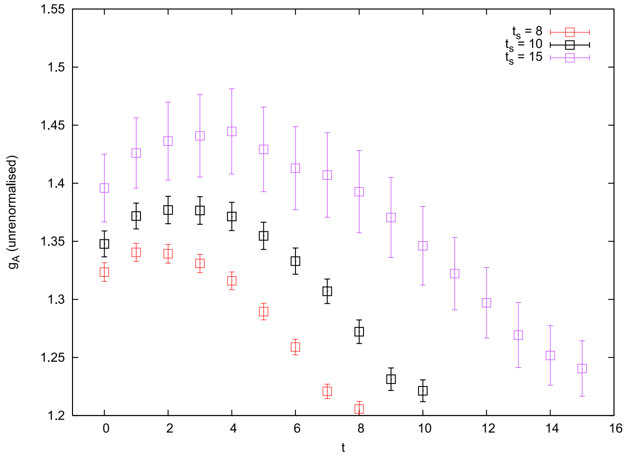



- the deterioration of signal makes extraction of ground state difficult
- less severe in ratios of n-pt functions

# Excited states

$$R(\vec{0},t,t_s) = \frac{C_{\mu,3}(\vec{0},t,t_s)}{C_2(\vec{0},t_s)}$$

*plot taken from Bastian Knippschild's talk at Confinement 2010*



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the summation method:*

- standard plateau-method:

$$R(\vec{q},t,t_s) = R_G + \mathcal{O}\left(e^{-\Delta t}\right) + \mathcal{O}\left(e^{-\Delta'(t_s - t)}\right)$$

- sum the ratio in t up to $t_s$
- after some calculation one gets:

$$\sum_{t=0}^{t_s} R(\vec{q},t,t_s) = R_G \cdot t_s + c(\Delta,\Delta') + \mathcal{O}\left(e^{-\Delta t_s}\right) + \mathcal{O}\left(e^{-\Delta' t_s}\right)$$

- linear behavior in $t_s$
- higher state corrections are much smaller for the summation method than for the standard method

how to extract $R_G$:

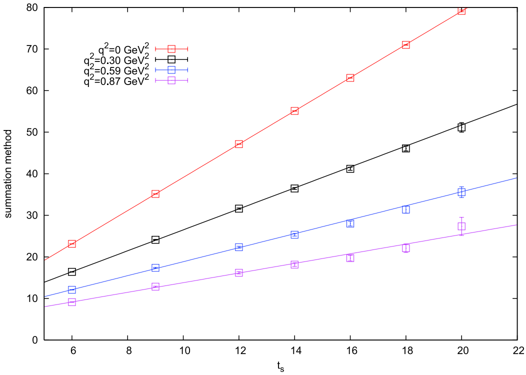
- do inversions for several $t_s$
- fit a straight line and extract the slope

*(see e.g.: Maiani et al. 1987)

# Excited states

*plot taken from Bastian Knippschild's talk at Confinement 2010*

summation method at work:
connected isoscalar $V_0$ for different momenta
lattice data: $64x32^3$, $m_\pi = 550MeV$, smeared-local-operator

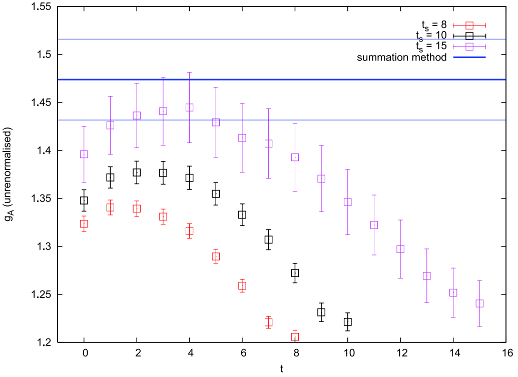


# Excited states

*plot taken from Bastian Knippschild's talk at Confinement 2010*

unrenormalised isovector axial charge $g_A$

lattice data: $64x32^3, m_\pi = 415 MeV$, smeared-local-operator



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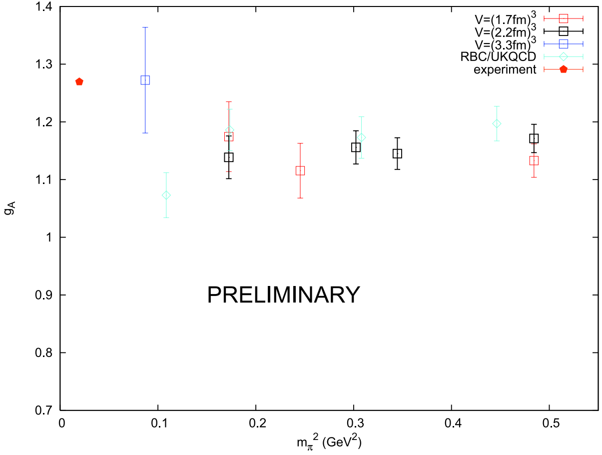
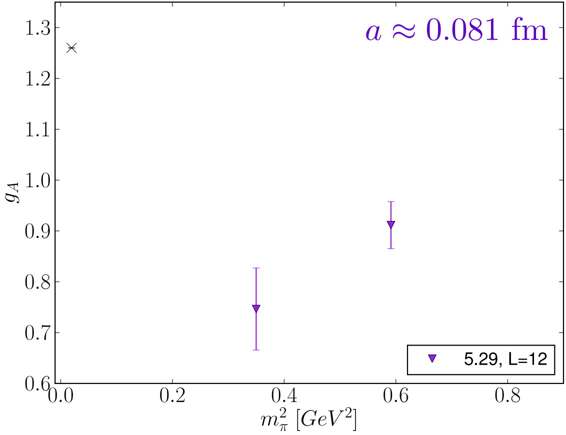
renormalised axial charge $g_A$:



# Finite Volume effects

*plot taken from James Zanotti's talk at Confinement 2010*

$L \approx 1.0\text{fm}$



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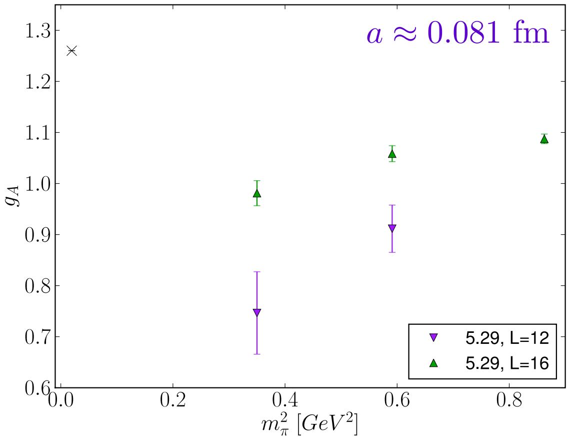
$L \approx 1.3$fm



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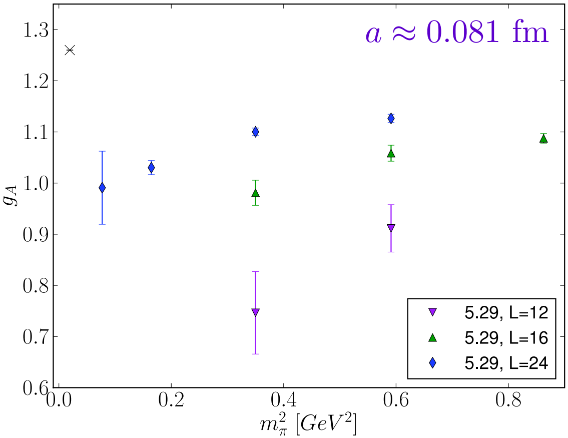
*plot taken from James Zanotti's talk at Confinement 2010*

$L \approx 1.9$fm



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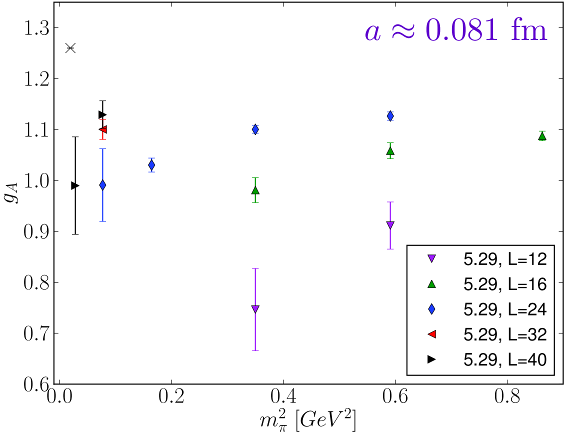
$L \approx 2.6/3.2\text{fm}$



# Summary & outlook

- nucleons on the lattice are hard, mainly due to the signal-to-noise issue
- sysetmatic effects are easily under-estimated
  - finite size effects in nucleons are much worse than for mesons
  - the exponential decay of signal-to-noise makes it hard to identify excited states - summation method and/or generalized eigenvalue problem seem to be a good thing to do
- currently I would consider $g_A$ as a candle for lattice computations rather than a prediction
- we need a fundamental understanding of the signal-to-noise ratio - ideas?