Progress in Lattice QCD Relevant for Flavor Physics

CKM 2010

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presented by :

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A major goal of Lattice QCD is to carry out theoretical calculations that are necessary and relevant to the Flavor Physics Program in Particle Physics.

The most accurate lattice calculations now typically have errors at the following level.

Kaon Physics:

$\sim 0.5\%$ ($f_+^{K\to\pi}(0)$), $\sim 0.6\%$ (f_K/f_π), $\sim 4\%$ (B_K), $\sim 15-20\%$ ($K \to \pi\pi$ ($\Delta I = 3/2$))

Charm Physics:

$\sim 1\%$ (f_{D_s}/f_D), $\sim 1-2\%$ (f_D, f_{D_s}), $\sim 3\%$ ($f_+^{D\to K}(0)$), $\sim 10\%$ ($f_+^{D\to\pi}(0)$) ($\Leftarrow$ should improve soon)

B Physics:

$\sim 2\%$ ($\mathcal{F}(1)$, f_{B_s}/f_B), $\sim 3\%$ (ξ), $\sim 4-6\%$ (f_B, f_{B_s}), $\sim 7\%$ ($f_{B_q}\sqrt{B_{B_q}}$), $\sim 10\%$ ($f_+^{B\to\pi}(q^2)$), $> 10\%$ ($B \to K(K^*)$)

— sub-percent level accuracy achieved in Kaon system
— significant improvement recently in Charm physics
— much more work necessary in B physics

OUTLINE

- **Kaon Physics (f_K/f_π, $K \to \pi$, B_K, $K \to \pi\pi$)**
  [talks by A.Juettner, A.Ramos, P.Dimopoulos in WGI]

- **Charm Physics (f_D, f_{D_s}, $D \to K(\pi)$)**
  [talk by H.Na in WGII]

- **B Physics (f_B, f_{B_s}, B_{B_q}, ξ, $B \to D^*(\pi)$, $B \to K(K^*)$)**
  [talks by N.Garron in WGIV, P.Mackenzie in WGII, Z.Liu in WGIII]

- **Quark Masses (m_b, m_c, m_s)**
  [talk by A.Hoang in WGII]

- **Summary and Future Prospects**

KAON PHYSICS

$|V_{us}|$ from K_{l3} and K_{l2} Decays

A recent global analysis by FlaviaNet (M.Antonelli et al., arXiv:1005.2323 [hep-ph]) demonstrates the precision with which K_{l3} and K_{l2} decays are now testing the Standard Model, e.g.

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0001(6)$$

Lattice QCD input crucial for determining $|V_{us}|$.

$K \to \pi, l\nu$: $\boxed{f_+^{K\to\pi}(0)} \Longrightarrow |V_{us}|$

$K \to l\nu$, $\pi \to l\nu$: $\boxed{f_K/f_\pi} \Longrightarrow |V_{us}|/|V_{ud}|$
(W.Marciano)

Semileptonic Kaon Decay: $f_+^{K\to\pi}(0)$

Collaboration (year)	$f_+(0)$	N_f	# "a"	action
RBC/UKQCD (07)	0.964(5)	2 + 1	1	domain wall
RBC/UKQCD* (10)	$0.960\left(^{+5}_{-6}\right)$	2 + 1	1	domain wall
ETMC (09)†	0.956(8)	2	3	twisted mass

* no $q^2 \to 0$ extrapolation required

† sea strange contribution accounted for up to NLO in ChPT

More results with $N_f = 2+1$ or $N_f = 2+1+1$ at several lattice spacings forthcoming.

Work with Staggered quarks underway (Fermilab/MILC)

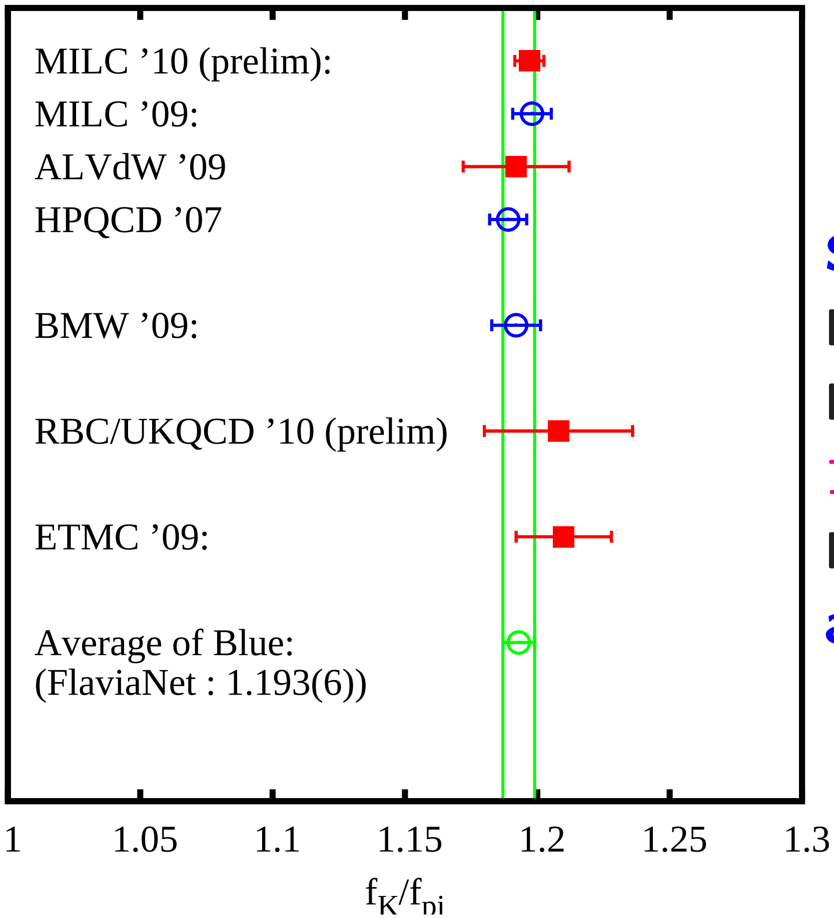
Kaon Leptonic Decay: f_K/f_π

	f_K/f_π	N_f	# "a"	a_{min}fm
MILC (10)	$1.197(2)(^{+3}_{-7})$	2 + 1	3	0.045
MILC (09)	$1.198(2)(^{+6}_{-8})$	2 + 1	3	0.045
ALVdW (09)	1.192(12)(16)	2 + 1	2	0.09
HPQCD (07)	1.189(7)	2 + 1	3	0.09
BMW (09)	1.192(7)(6)	2 + 1	3	0.07
RBC/UKQCD (10)	1.208(8)(23)(14)	2 + 1	2	0.085
RBC/UKQCD (09)	1.225(12)(14)	2 + 1	2	0.085
ETMC (09)	1.210(6)(15)(9)	2	3	0.07
ETMC (10)	1.224(13)	2 + 1 + 1	3	0.06

(numbers in red are preliminary)

Other 2+1+1 calculations underway by MILC using Highly Improved Staggered Quarks (HISQ dynamical configs)

f_K/f_π (cont'd)



Several recent averages:

FlaviaNet : 1.193(6)

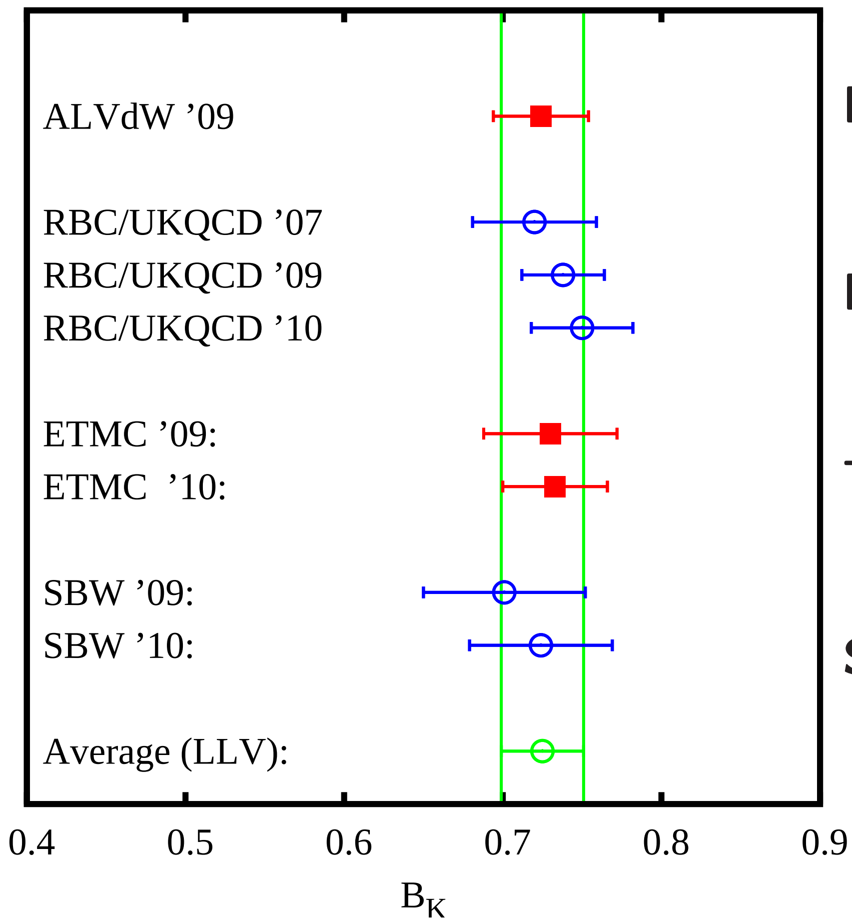
Laiho-Lunghi-VandW (LLV) 1.1925(56)

Lubicz (Lat09) 1.196(1)(10)

all consistent

The B_K Parameter

Collaboration (year)	$\hat{B}_K$	Action(s)
HPQCD/UKQCD (2006)	0.83(2)(18)	Staggered
RBC/UKQCD (2007)	0.720(13)(37)	Domain Wall
ALV (2009)	0.724(8)(29)	D.Wall on Stagg.
Average (LLV)	0.725(26)	
ETMC (2009) $N_f = 2$	0.730(30)(30)	Twisted Mass
ETMC (2010) $N_f = 2$	0.733(29)(16)	Twisted Mass
$N_f = 2+1+1$ underway		
RBC/UKQCD (2009)	0.738(8)(25)	Domain Wall
RBC/UKQCD (2010)	0.750(10)(30)	Domain Wall
SBW (2009)	0.701(19)(47)	Staggered
SBW (2010)	0.724(12)(43)	Staggered



Domain wall on stagg.

Domain Wall

Twisted Mass ($N_f = 2$)

Staggered

Excellent agreement between results from different lattice quark actions. Errors at $4 \sim 6$% level.

Tension B_K versus SM

(copied from slides by S.Sharpe, "Latt. meets Exper. 2010")

Lattice averages

$\hat{B}_K = 0.725(26)$ **[LLV]**

$\hat{B}_K = 0.731(7)(35)$ **[Lubicz]**

Unitarity Triangle Fit: [LLV]

$$(\hat{B}_K)_{fit} \begin{cases} 1.09 \pm 0.12 & |V_{cb}|_{excl} \\ 0.903 \pm 0.086 & |V_{cb}|_{incl} \\ 0.98 \pm 0.10 & |V_{cb}|_{excl+incl} \end{cases}$$

$\Longrightarrow 2-3\sigma$ **tension**

Errors dominated by those in V_{cb} not those in B_K
($\epsilon_K \propto |V_{cb}|^4$)

Precision Kaon Physics and B Physics intertwined.

$K \to \pi\pi$ in $\Delta I = 3/2$ and $\Delta I = 1/2$ Channels

Difficult to simulate on Euclidean Lattice (Maiani-Testa No-Go Theorem)

Two approaches currently being pursued:

RBC/UKQCD uses "direct" Lellouch-Luescher finite volume method. Requires finite momentum, large volumes, physical light quarks.

D.Coumbe, J.Laiho, M.Lightman, R.VandWater evade no-go theorem by going first to unphysical kinematic point ($2M_\pi = M_K$). Then use ChPT to get back to physical pions.

Both groups making progress for $\Delta I = 3/2$ case (15 - 20% errors). $\Delta I = 1/2$ harder.

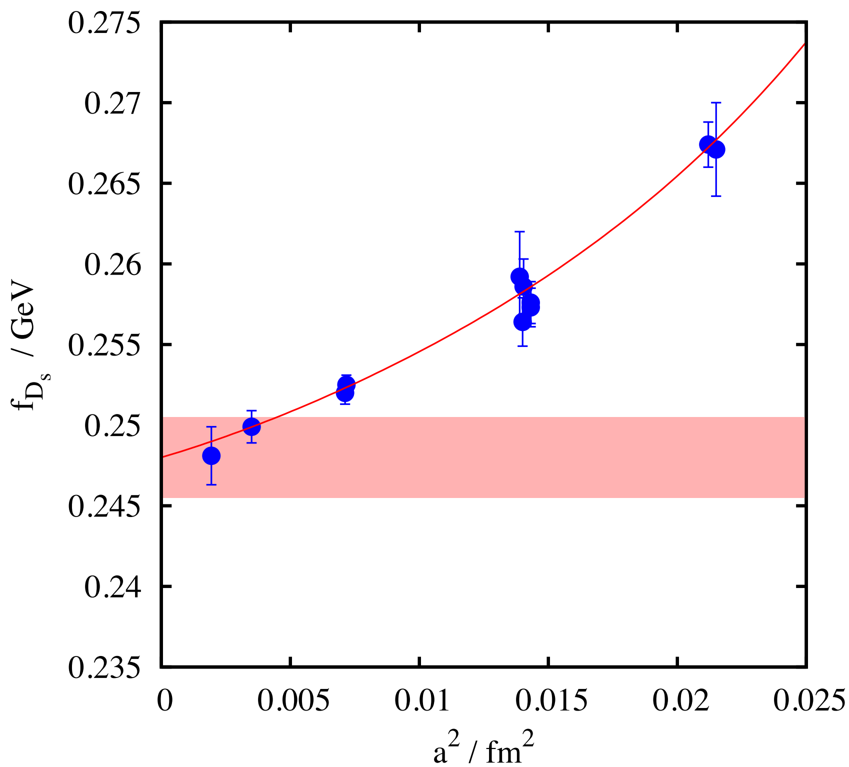
This represents the next set of challenges in Kaon Physics on the lattice

CHARM PHYSICS

Results for Charmed Meson Decay Constants

Collaboration	f_D (MeV)	f_{D_s} (MeV)	f_{D_s}/f_D
Fermi/MILC '05	201 $\pm$ 17	249 $\pm$ 16	1.24 $\pm$ 0.07
Fermi/MILC '10 (preliminary)	220 $\pm$ 9 $\pm$ 5	261 $\pm$ 8 $\pm$ 5	1.19 $\pm$ 0.01 $\pm$ 0.02
HPQCD '07	207 $\pm$ 4	241 $\pm$ 3	1.164 $\pm$ 0.011
HPQCD '10		248.0 $\pm$ 2.5	
HPQCD '10	206.3 $\pm$ 4.3		
ETM '09 $N_f = 2$	197 $\pm$ 9	244 $\pm$ 8	1.24 $\pm$ 0.03
ETM '10 $N_f = 2+1+1$ v. preliminary	204(3)(...)	250(3)(...)	1.230(6)(..)

HPQCD's f_{D_s}



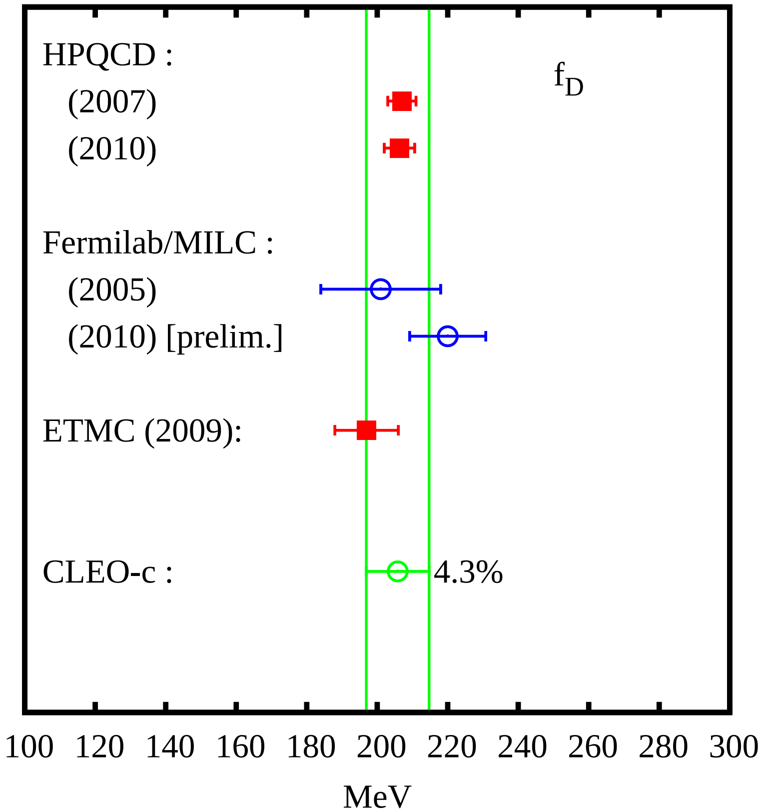
2007 3 lattice spacings (0.15 - 0.09fm). Largest source of uncertainty was the "**scale**" (from Υ 2S - 1S)

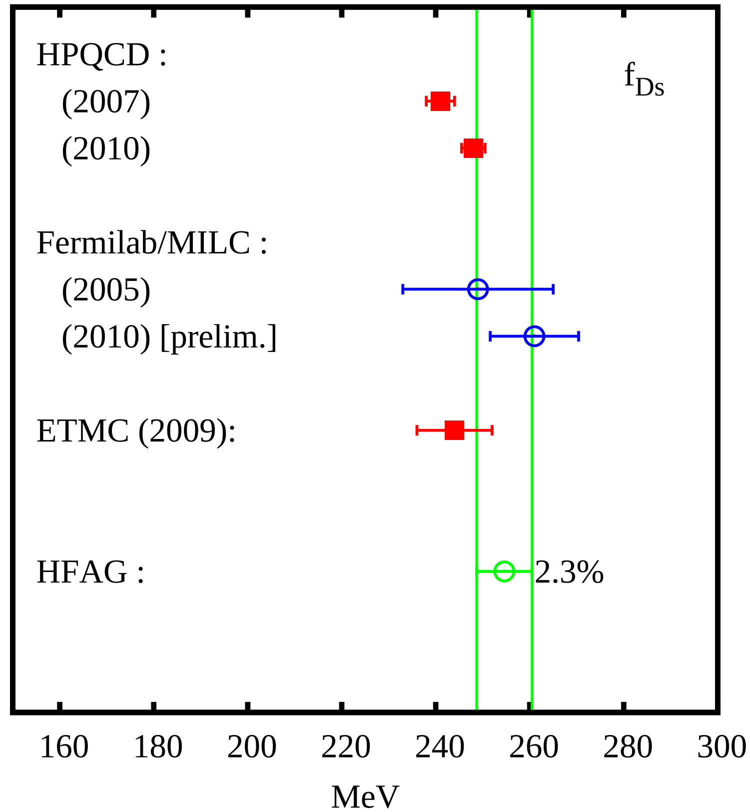
2009 new scale setting using 3 input quantities simultaneously (2S - 1S, $M_{D_s} - \frac{1}{2}M_{\eta_c}$, f_{η_s}). **Increase by $\sim 2.5\%$** (unexpectedly large shift)

2010 new f_{D_s} from 5 lattice spacings (0.15 - 0.045fm) with new scale

$\Longrightarrow$ $\boxed{f_{D_s} = 248.0 \pm 2.5\text{MeV}}$

Charmed Meson Decay Constants (cont'd)





No descrepancy between theory and experiment !

Need more theory and experimental results with $\sim 1\%$ errors

It will be exciting to get new results from BESIII, several lattice groups, ..., in the near future.

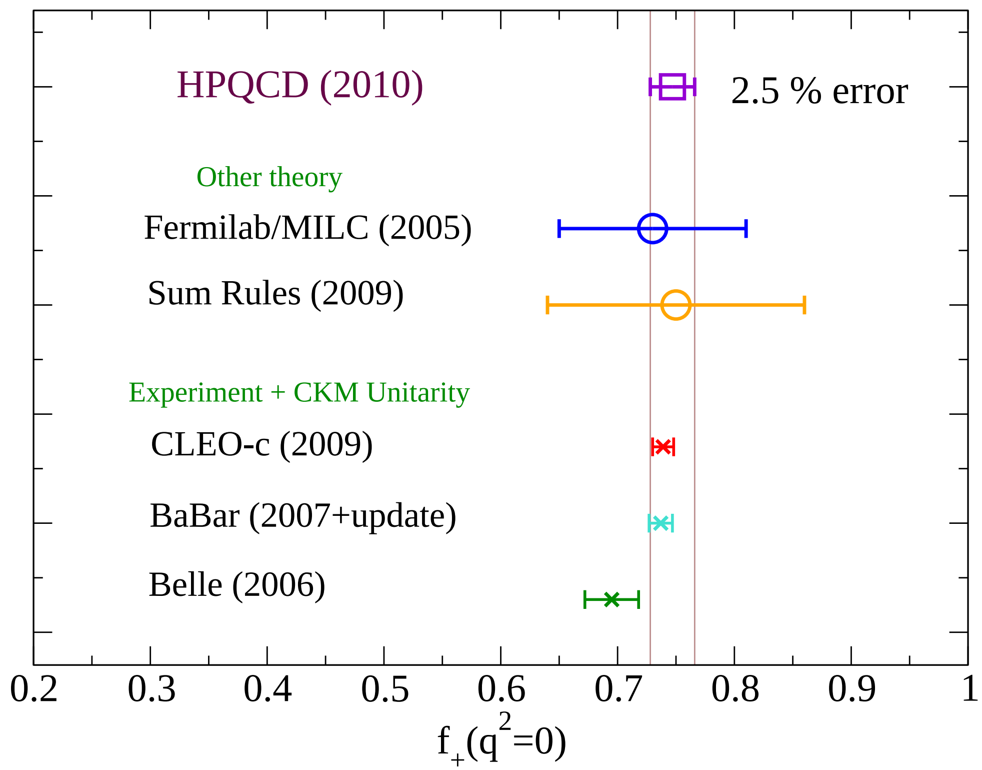
D Semileptonic Decays

$f_+^{D\to K}(0)$ with HISQ Charm
(HPQCD)

(see talk by H.Na WGII)

$$f_+^{D\to K}(0) = 0.747(19)$$

Significant reduction in errors



Source of Large Improvement in Error

— **HISQ (Highly Improved Staggered Quark) Action** :
all (am_c), $(am_c)^2$ lattice artifacts removed.
all $\alpha_s(am_c)^2$ and $(am_c)^4$ errors removed at leading order in v^2/c^2.

$\Longrightarrow$ most accurate quark action on market which works even for heavy quarks

— **can use PCVC:** $f_+^{D\to K}(0) = \frac{(m_{0c}-m_{0s})\langle K|S|D\rangle}{M_D^2-M_K^2}|_{q^2=0}$
No operator matching necessary

— **new chiral/continuum extrapolation method**

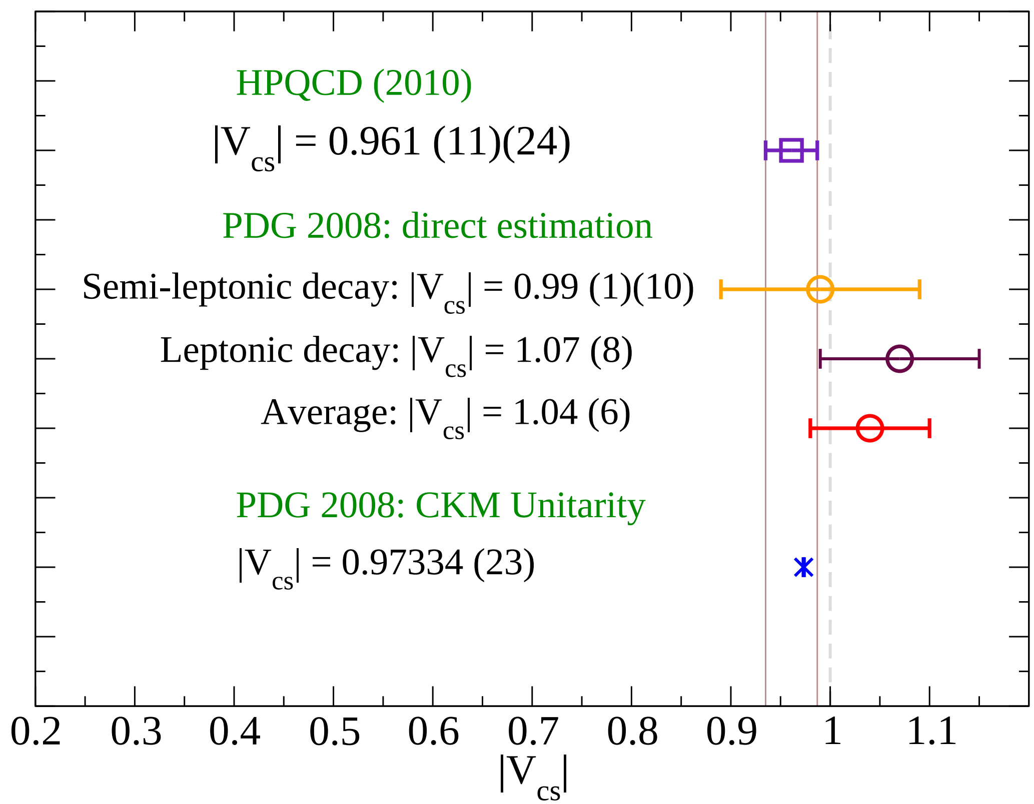
— **sophisticated data analysis tools**

— **HISQ is computationally cheap**

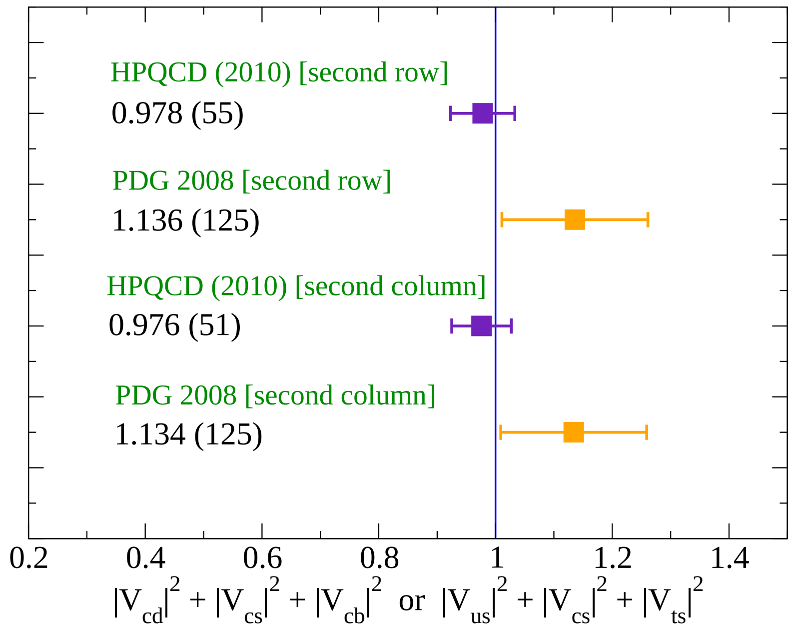
Direct Determination of $|V_{cs}|$

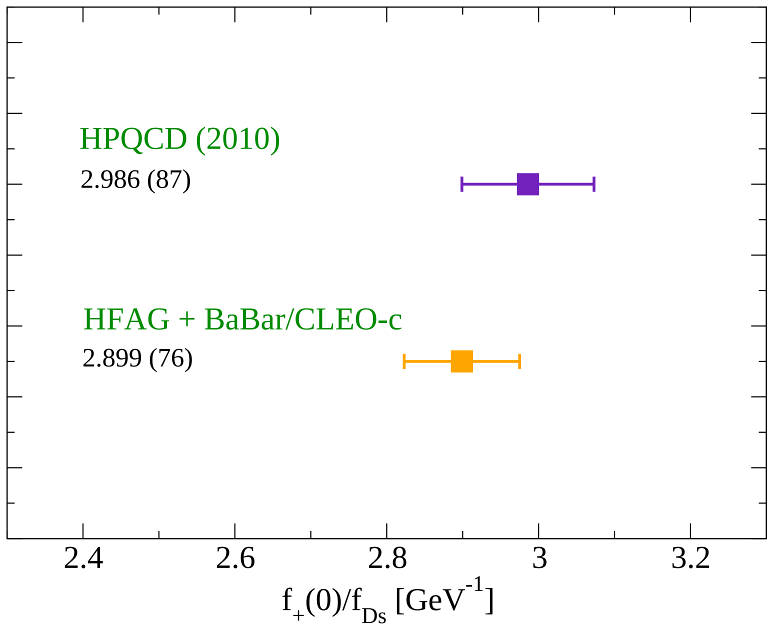
Use $f_+^{D\to K}(0) * |V_{cs}|$ from CLEO-c and BaBar

$f_+^{D\to K}(0) = 0.747(19)$ $\Longrightarrow$ $|V_{cs}| = 0.961(11)(24)$



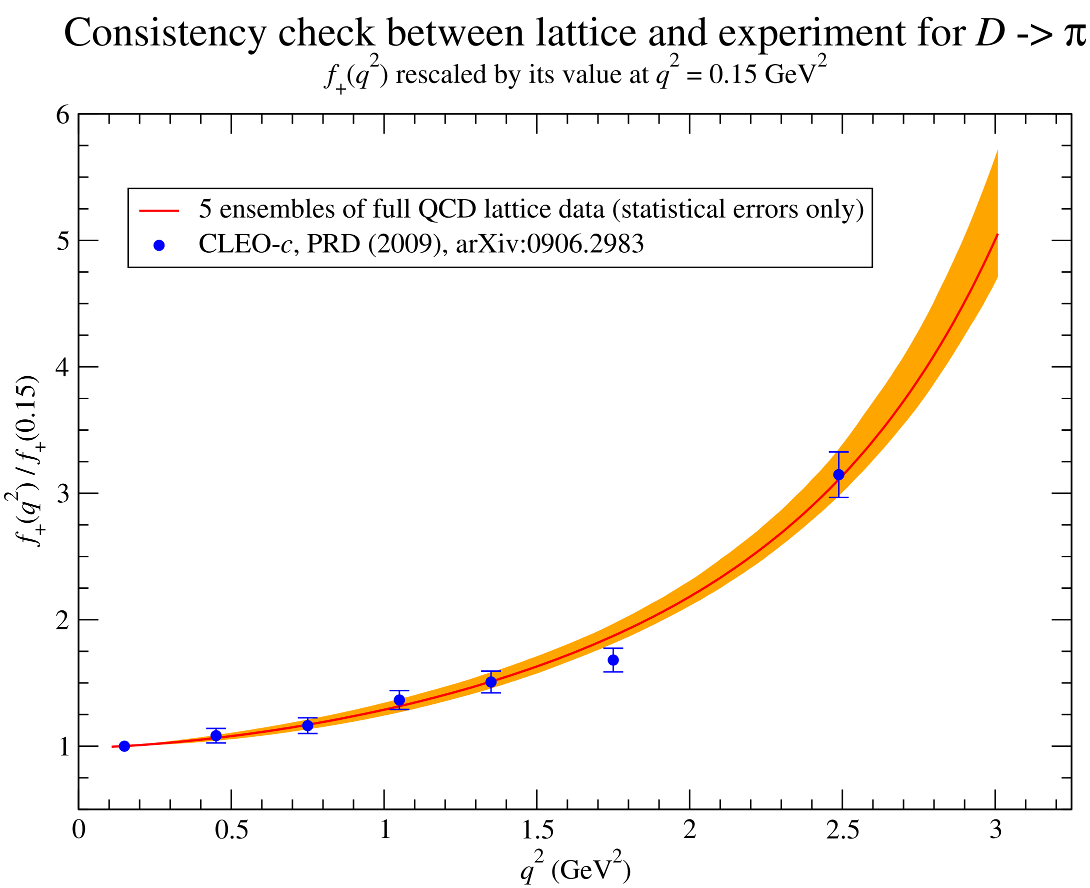
2nd Row & Column Unitarity and $f_+^{D\to K}(0)/f_{D_s}$





D Semileptonic Decays from Fermilab/MILC

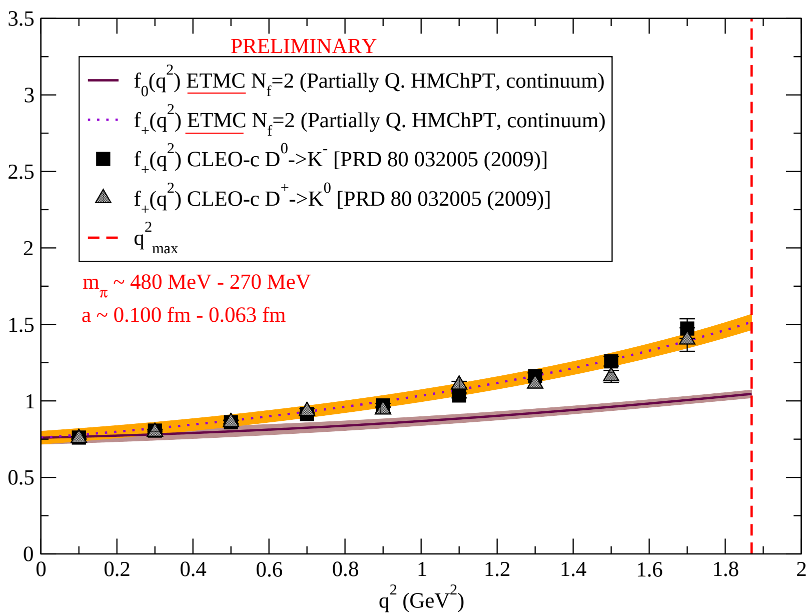
(Many improvements on their 2005 calculations)

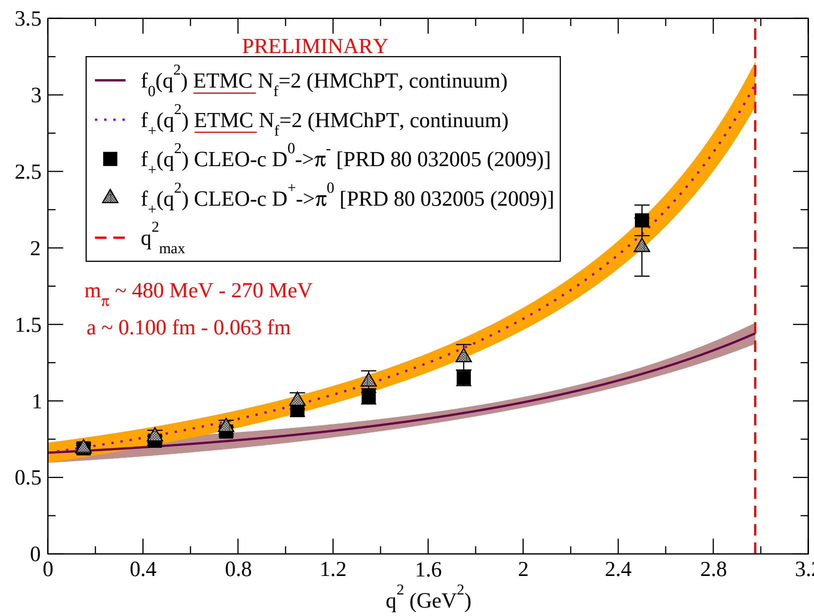


(preliminary)

D Semileptonic Decays with Twisted Mass Quarks

$N_f = 2$





ETMC (preliminary): $f^{D\to\pi}(0) = 0.66(6)_{stat}$

$$f^{D\to K}(0) = 0.76(4)_{stat}$$

$|V_{cs}|$ from D_s Leptonic Decays

Experimental determinations of f_{D_s} start from the branching fraction $\mathcal{B}(D_s \to l\nu)$ (corrected for e.& m.)

$$f_{D_s} = \frac{1}{G_F|V_{cs}|m_l(1 - m_l^2/m_{D_s}^2)}\sqrt{\frac{8\pi\mathcal{B}(D_s \to l\nu)}{m_{D_s}\tau_{D_s}}},$$

assuming values for $|V_{cs}|$ based on CKM unitarity. One can turn things around and,

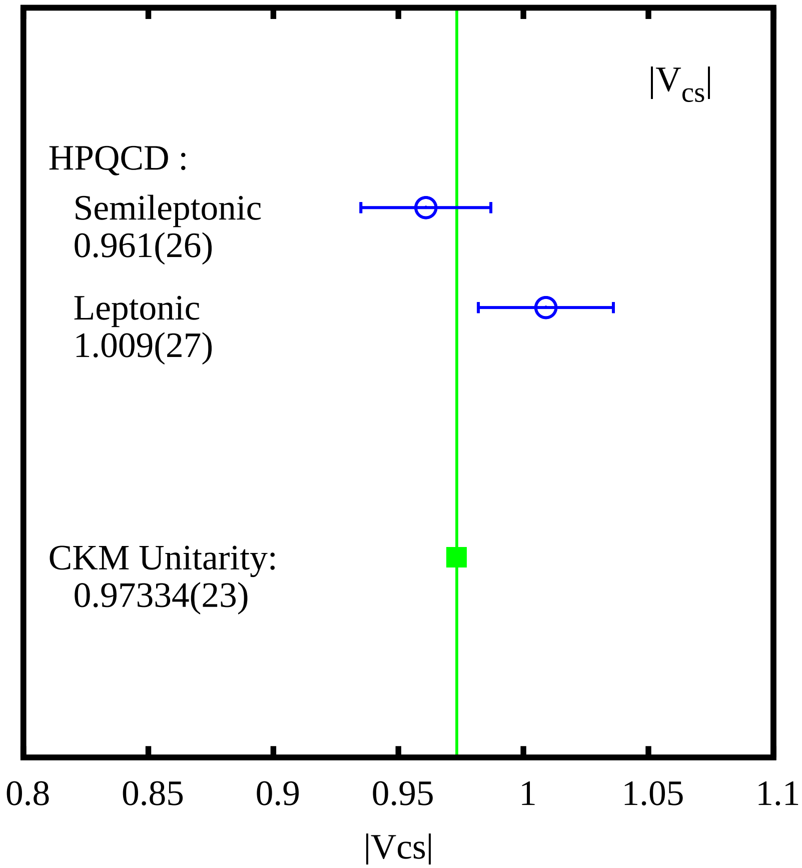
$$\boxed{f_{D_s}|_{lattice} + \mathcal{B}(D_s \to l\nu)|_{exp.} \Longrightarrow |V_{cs}|}$$

From HPQCD's f_{D_s} $\Longrightarrow$ $\boxed{|V_{cs}|_{leptonic} = 1.009(27)}$

This is an average over $D_s \to \mu\nu$ and $D_s \to \tau\nu$ channels. Note that PDG08 has $|V_{cs}|_{leptonic} = 1.07(8)$.

$|V_{cs}|$ from Semileptonic and Leptonic Decays

agreement at $\sim 1.3\sigma$ level



$\Longleftarrow$ theory errors dominate (Lattice QCD)

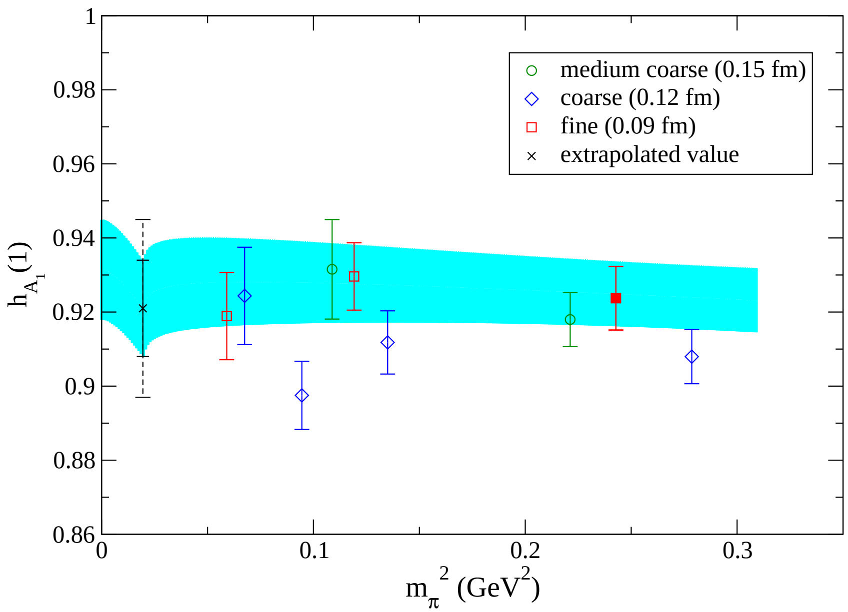
$\Longleftarrow$ exper. errors dominate (BESIII, ...)

It will be interesting to see how this plot develops as $3\% \Rightarrow 1 \sim 2\%$ errors

B PHYSICS

$B \to D^*$ at Zero Recoil from Fermilab/MILC

(see talk by P.Mackenzie WGII)



Chiral/Continuum Extrapolation

2008

$\mathcal{F}(1) = 0.921(13)(20)$

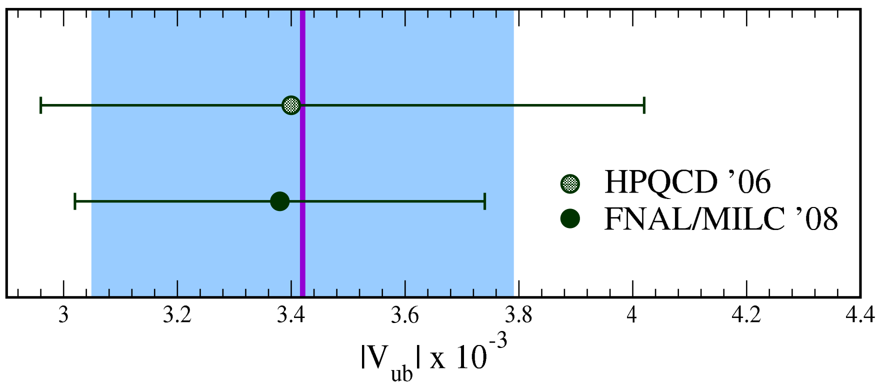
Lattice error : 2.6 %

2010

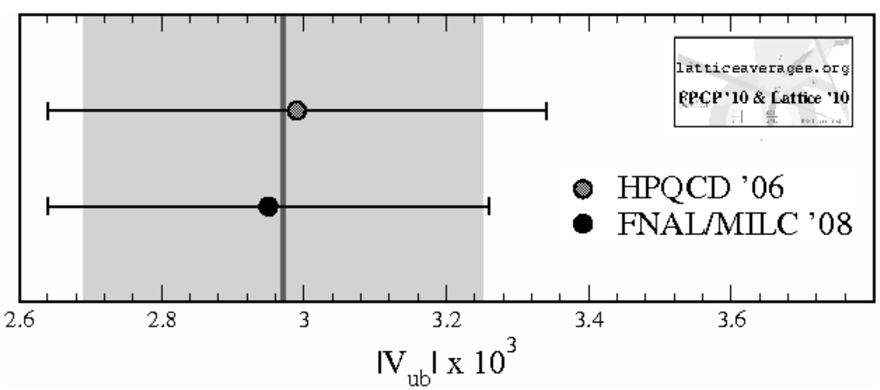
Update with $\leq$2.0 % errors.

$B \to \pi$ Semileptonic Decays

Plots by LLV



HPQCD + HFAG '08
FNAL/MILC + BABAR '06
$|V_{ub}| = 3.42(37) \times 10^{-3}$



HPQCD & FNAL/MILC + BABAR '10
$|V_{ub}| = 2.97(28) \times 10^{-3}$

Theory errors at the 9% (FNAL/MILC) or 14% (HPQCD) level

Neutral B Meson Mixing

This is a vast subject and Lattice QCD has so far addressed only parts of it. In order to cover both SM and BSM physics need (i, j are color indices),

$$Q1 = \left(\overline{\Psi}_b^i \gamma^\nu P_L \Psi_q^i\right)\left(\overline{\Psi}_b^j \gamma_\nu P_L \Psi_q^j\right) \quad (1)$$

$$Q2 = \left(\overline{\Psi}_b^i P_L \Psi_q^i\right)\left(\overline{\Psi}_b^j P_L \Psi_q^j\right) \quad (2)$$

$$Q3 = \left(\overline{\Psi}_b^i P_L \Psi_q^j\right)\left(\overline{\Psi}_b^j P_L \Psi_q^i\right) \quad (3)$$

$$Q4 = \left(\overline{\Psi}_b^i P_L \Psi_q^i\right)\left(\overline{\Psi}_b^j P_R \Psi_q^j\right) \quad (4)$$

$$Q5 = \left(\overline{\Psi}_b^i P_L \Psi_q^j\right)\left(\overline{\Psi}_b^j P_R \Psi_q^i\right) \quad (5)$$

+ 1/M corrections (R_i in Lenz/Nierste notation).
($N_f = 0$) all five Q_i's: (Becirevic et al. '02)
($N_f = 2 + 1$) Q_1, Q_2, Q_3: for B_s (HPQCD '07)
Q_1, Q_2: for B_s and B_d (HPQCD '09, RBC/UKQCD '10)
Several calculations underway (Fermilab/MILC, RBC/UKQCD)

Neutral B Meson Mixing: Results

$\langle Q_1 \rangle \Longrightarrow f_{B_q}\sqrt{B_{B_q}}$, $\qquad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$

$\langle Q_1 \rangle$ and $\langle Q_3 \rangle \Longrightarrow \Delta\Gamma_s$

HPQCD (2007,2009) $N_f = 2+1$:

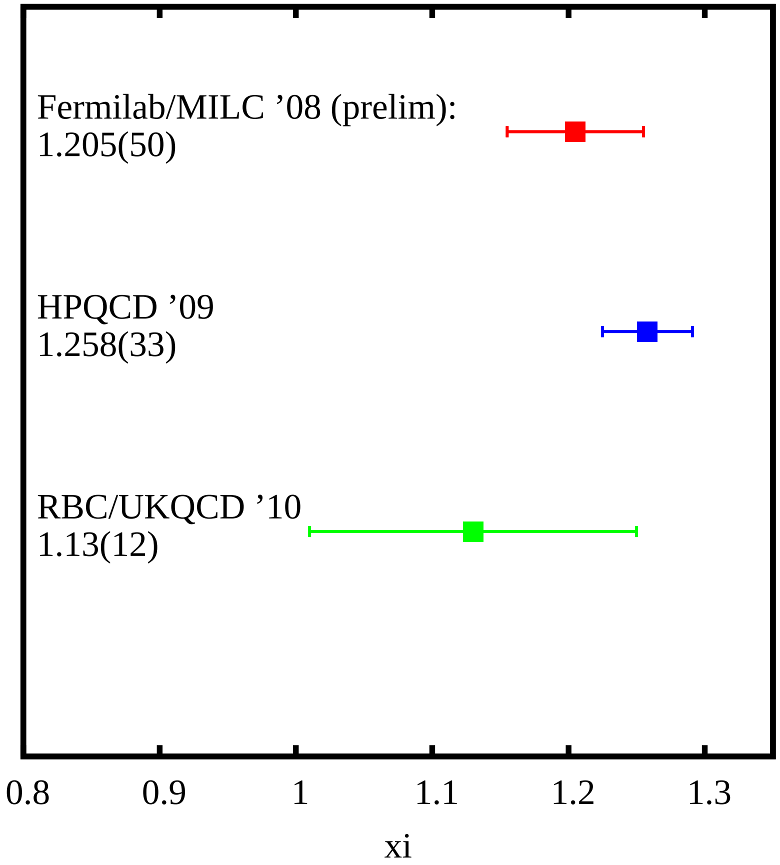
$f_{B_d}\sqrt{\hat{B}_{B_d}} = 216(15) MeV$, $\qquad f_{B_s}\sqrt{\hat{B}_{B_s}} = 266(18) MeV$,

$\hat{B}_{B_d} = 1.26(11)$, $\qquad \hat{B}_{B_s} = 1.33(6)$,

$\Delta\Gamma_s = 0.10(3)\ ps^{-1}$ **(using Lenz/Nierste)**

To date several $N_f = 2+1$ calculations of ξ

Unquenched Results for $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$



Heavy Clover b

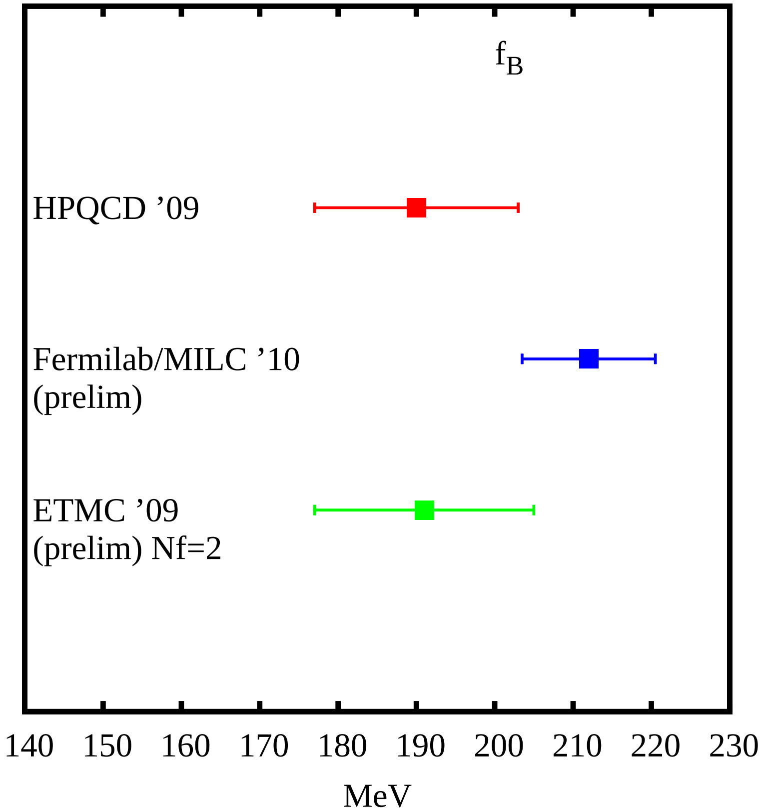
NRQCD b

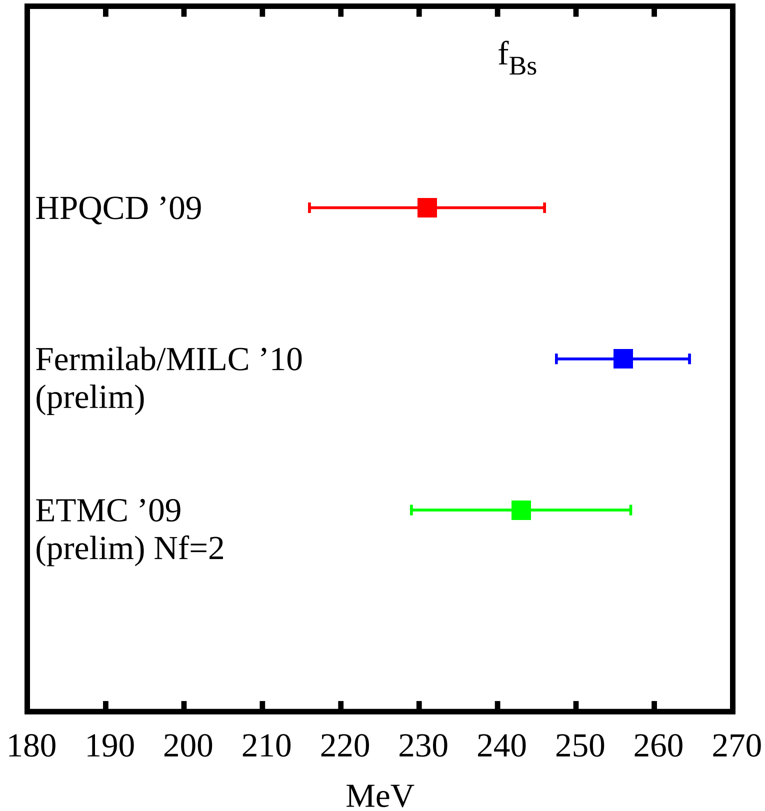
Static b

Non static b calculations underway

HPQCD $\xi \Longrightarrow \boxed{\frac{|V_{td}|}{|V_{ts}|} = 0.214(1)(5)}$

B Meson Decay Constants





HPQCD: new scale not implemented yet, better tuning of b-quark mass, start using HISQ light quarks.

Rare B Decays

Starting point : $\mathcal{H}_{eff}^{b\to s} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$

Short distance contributions dominated by Q_7, Q_9, Q_{10}.

$$Q_7 = \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu}(1+\gamma_5) b_i F_{\mu\nu} \quad (6)$$

$$Q_9 = \frac{e}{8\pi^2} (\bar{s}b)_{V-A} (\bar{l}l)_V \quad (7)$$

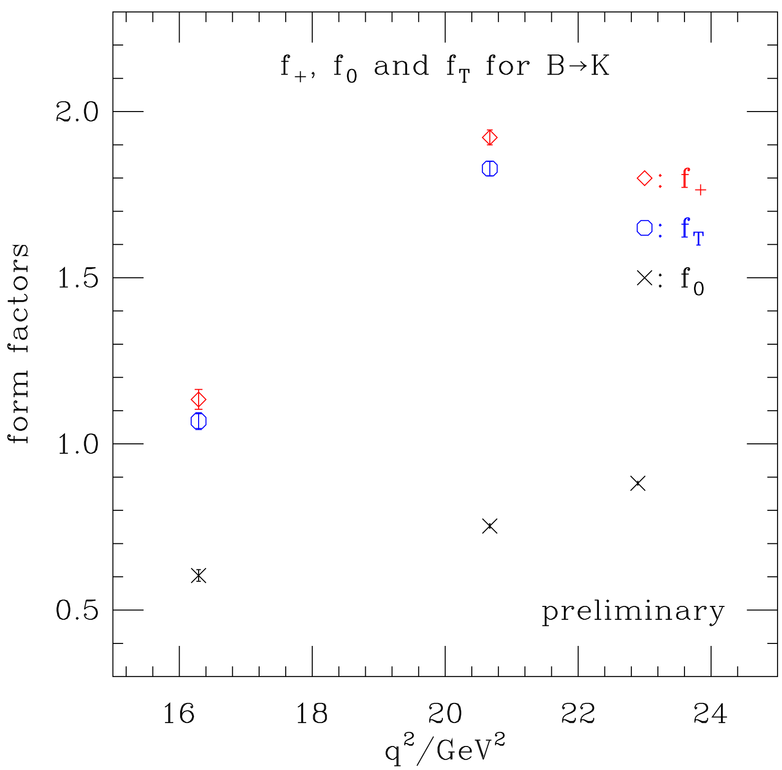
$$Q_{10} = \frac{e}{8\pi^2} (\bar{s}b)_{V-A} (\bar{l}l)_A \quad (8)$$

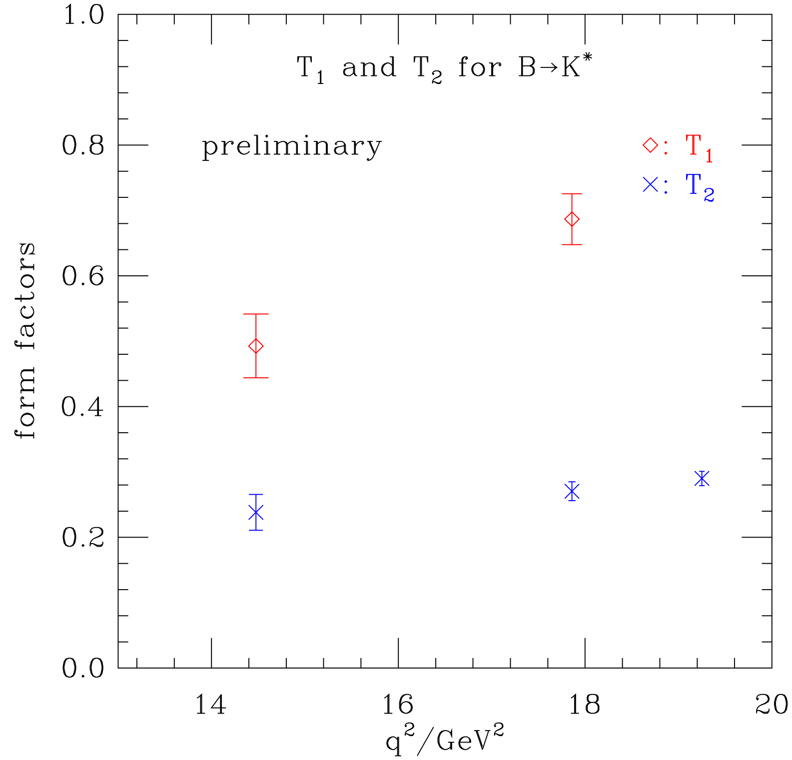
several form factors: f_+, f_0, f_T, V, A_0, A_1, A_2, T_1, T_2, T_3

Project initiated by Cambridge Group
Moving NRQCD b-quarks + AsqTad light, $N_f = 2{+}1$ **MILC lattices.**

$B \to K(K^*)$ Form Factors

(see talk by Z.Liu WGIII)





QUARK MASSES

Quark Masses from Lattice Simulations

Quark masses are free parameters of the Standard Model. They cannot be predicted from theory but must be determined from a combination of experimental and theory inputs.

One popular approach :

— Tune the bare mass m_0 in the lattice QCD action such that hadron masses agree with experiment.

— Convert the bare lattice mass to the $\overline{\mathrm{MS}}$ scheme

$$m^{\overline{\mathrm{MS}}}(\mu) = Z_m(\mu) m_0$$

— $Z_m(\mu)$ is evaluated either perturbatively (2-loop lattice + high order continuum) or using nonperturbative methods.

New Approach: Charm Mass from J-J Correlators

— No lattice perturbation theory or non-perturbative matching involved

— Compute t^n moments for correlators

$$G(t) = \sum_{\vec{x}} m_{0c}^2 \langle 0 | J_5(\vec{x}, t) J_5(0,0) | \rangle$$

$J_5 = \overline{\Psi}_c \gamma_5 \Psi_c$

— Exploit very high order (3 or 4 loop) continuum perturbation theory results.
(Karlsruhe Group: Chetyrkin, Kuehn, Steinhauser, Sturm)

— Extract $\boxed{\frac{m_{\eta_c}}{2\overline{m}_c(\mu)}}$ $\boxed{\alpha_{\overline{MS}}(\mu)}$

Results for Quark Masses

(HPQCD Collaboration)

One finds :

$\boxed{m_c(3GeV) = 0.986(6)GeV}$ $\boxed{m_c(m_c) = 1.273(6)GeV}$

Previously Kuehn et al, using e^+e^- data, found $m_c(3GeV) = 0.986(13)GeV$.

Using the same HISQ action for both charm and strange quarks allows for accurate determination of the mass ratio,

$$m_c/m_s = 11.85(16) \Longrightarrow \boxed{m_s(2GeV) = 92.2(1.3)MeV}$$

Use same method with *relativistic b-quarks*.

An extrapolation $m_H \to m_b$ from $m_H < m_b$ was required.

$$\Longrightarrow \boxed{m_b(m_b) = 4.164(23)GeV}$$

Compare with Kuehn et al. $m_b(m_b) = 4.163(16)GeV$.

Implications for Continuum Results of Kuehn et al.

Questions have been raised about the error analysis of the original Kuehn et al. m_c and m_b determinations using e^+e^- data and high order continuum perturbation theory. Consistency with lattice results provides additional checks of their analysis. In the Lattice analysis one

- used both pseudo-scalar and vector correlators and several moments. Many cross checks possible

- avoided complications due to resonances etc.

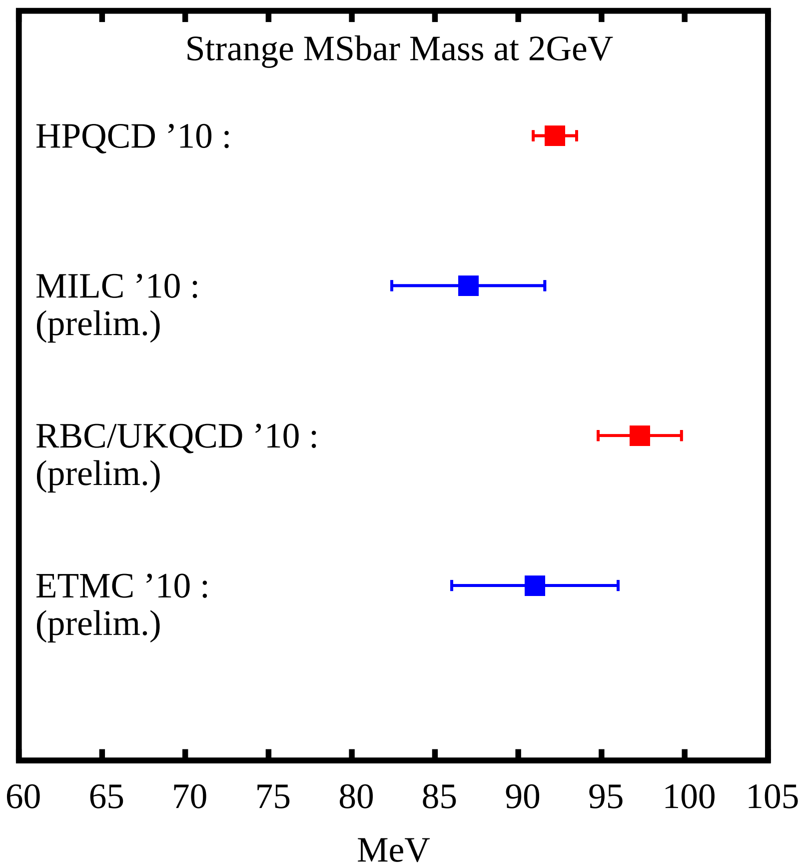
- obtained ratio $[m_b(\mu, n_f)/m_c(\mu, n_f)]$ consistent with fully nonperturbatively determined $\left(\frac{m_{0b}}{m_{0c}}\right)_{a\to 0}$

- **extracted a value for $\alpha_{\overline{MS}}(\mu)$ in complete agreement with other lattice and non-lattice determinations**

- **included terms up to $\mathcal{O}(\alpha^6_{\overline{MS}})$ with the unknown coefficients left as fit parameters**

Difficult to imagine that so many accurate results could emerge that are consistent with each other, if the perturbation theory of Kuehn et al. were not working as advertized.

Recent Determinations of the Strange Quark Mass



HISQ, m_s/m_c

Staggered, 2 - loop matching

Domain Wall, nonperturb. matching

Twisted Mass, nonperturb. matching

Compared to a couple of years ago results from different groups and approaches are converging.

TAKING STOCK : WHAT NEXT?

Kaons

$f_+^{K\to\pi}(0)$: more $N_f = 2+1$ or $N_f = 2+1+1$ results
different fermion formulations
more lattice spacings, closer to physical pions,
......
want < 0.5% errors

f_K/f_π : same as above

B_K : reduce matching error (e.g. $MOM \to \overline{MS}$)
better control over chiral extrapolation
shoot for $4 \Rightarrow 1-2$ % errors

$K \to \pi\pi$: $\Delta I = 3/2$ at $\sim 15-20$ % feasible
$\Delta I = 1/2$?

Charm

$f_{D_s}(f_D)$: more lattice calculations with 1 - 2% errors
look forward to reduced experimental errors
2.3 (4.3)% $\Rightarrow$ 1-2%

$f_+^{D\to K}(0)$: reduce statistical and continuum extrap. errors
2.5 $\Rightarrow\sim$1% errors
calculations with other lattice actions needed
experiment already at $\sim$1%

$f_+^{D\to \pi}(0)$: work underway to repeat $D \to K$ calculation
for $D \to \pi$
experiment currently at $\sim$3%

m_c : need J-J correlator results from several lattice groups

B Physics

$B \to D^*(D)$: How much better than $\sim$2% ?
Want $|V_{cb}|^4$ error $\leq$4% (B_K error)

$B\overline{B}$-Mixing : better statistics, matching, $a \to 0$
all 5 operators (even some R_i ?)

$B \to \pi$: better statistics (random wall sources)
better light quark action (HISQ)
better fitting and chiral/cont. extrapol. methods
10% $\Rightarrow$ <5%

f_B, f_{B_s} : HISQ light quarks
maybe even HISQ b-quarks ($a \leq 0.03$)

$B \to K(K^*)$: first $N_f = 2+1$ calculations coming

Summary

- Lattice QCD is working hard to do its share in Precision Flavor Physics.

- More and more results with few % or better errors becoming available, making accurate tests of the Standard Model possible

- Much work remains, however, especially in B Physics

- We look forward to exciting times ahead as we work together with our experimental and theory colleagues.