$B \to \pi e \nu$ from QCD Sum Rules on the Light-Cone

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What is $B \longrightarrow \pi e \nu$ good for?

- sensitive to $|V_{ub}|$
- compared to inclusive channel:
  - good experimental accuracy (largely reduced $b \to c$ background)
  - hadronic parameters: just one: form factor $f_+(q^2)$, q^2 = invariant lepton mass (no heavy quark expansion or other theo approximations)
- form factor to be calculated by non-perturbative methods, e.g. lattice (see previous talk) or QCD sum rules on the light-cone or quark models or else

Definition of form factor:

$$\langle \pi(p_\pi) | \bar{u}\gamma_\mu b | B(p_B) \rangle = f_+(q^2)(p_B + p_\pi)_\mu + f_-(q^2)(p_B - p_\pi)_\mu$$

with $q = p_B - p_\pi$ and $0 \leq q^2 \leq (m_B - m_\pi)^2 = 26.4\,\mathrm{GeV}^2$.
f_- enters decay rate as $m_e^2 f_-^2$ and is hence irrelevant.

QCD Sum Rules in a Nutshell I

Basic quantity: correlation function:

$$\Pi_\mu \equiv i \int d^4 y e^{iqy} \langle \pi(p) | T[\bar{u}\gamma_\mu b](y)[m_b \bar{b} i\gamma_5 d](0)|0\rangle \stackrel{\text{LCE}}{=} \sum_n T_H^{(n)} \otimes \phi_\pi^{(n)}$$

- $\phi_\pi^{(n)}$: π distribution amplitudes (DAs): non-perturbative
- $T_H^{(n)}$: perturbative amplitudes
- n: twist
- LCE: light-cone expansion
- B meson described by PS current + plus analytic continuation (in p_B^2):

$$\Pi_\mu = 2p_\mu \left(\boxed{f_+(q^2)} \frac{m_B^2 f_B}{m_B^2 - p_B^2} + \text{higher-mass poles and cuts} \right) + \ldots$$

QCD Sum Rules in a Nutshell II

Features of LCSRs:

- LCE effectively in $1/m_b \to$ need to include higher-twist terms
- $\sum T_H^{(n)} \otimes \phi_\pi^{(n)}$ implies factorization – valid at higher twist?
  - calculate $O(\alpha_s)$, known for
    T2 (π (Khodjamirian et al. 97, Ball et al. 97), ρ (Ball/Braun 98))
    T3 (π (Ball/Zwicky 2001))
    $\to$ factorization OK, i.e. no "end-point" singularities upon convolution
- info on non-pert. transition amplitudes from conformal expansion, pion transition form factor $\gamma + \gamma^* \to \pi$, lattice and QCD sum rules
  - could do with some improvement!
    (QCDSF/UKQCD 2006 quote 50% error on a_2^π [most important non-pert. parameter of ϕ_π])
- use standard SR techniques to suppress contribution of higher-mass states to correlation function: Borel-transformation, continuum model
  - introduce irreducible systematic uncertainty $\sim 10\%$

Milestone Publications

Khodjamirian & Bagan et al. 1997:	twist-2 to $O(\alpha_s)$
Ball/Zwicky 2004:	2-particle twist-3 to $O(\alpha_s)$, use of b pole mass
Khodjamirian et al. 2006:	alternative LCSR with B instead of π DA
Duplancic et al. 2008:	2-particle twist-3 to $O(\alpha_s)$, use of $\overline{\mathrm{MS}}$ b mass

$f_+(q^2)$ or $f_+(0)$?

Calculation of *full* q^2 dependence not feasible by any known method:

- lattice best for “large” q^2 (small $q^2 \leftrightarrow$ large pion energy, can’t be simulated directly on lattice $\rightarrow$ “moving NRQCD” may help)
- LCSR best for “small” q^2 (LCE breaks down for large $q^2 \leftrightarrow$ small pion energy)

Experiment can help:

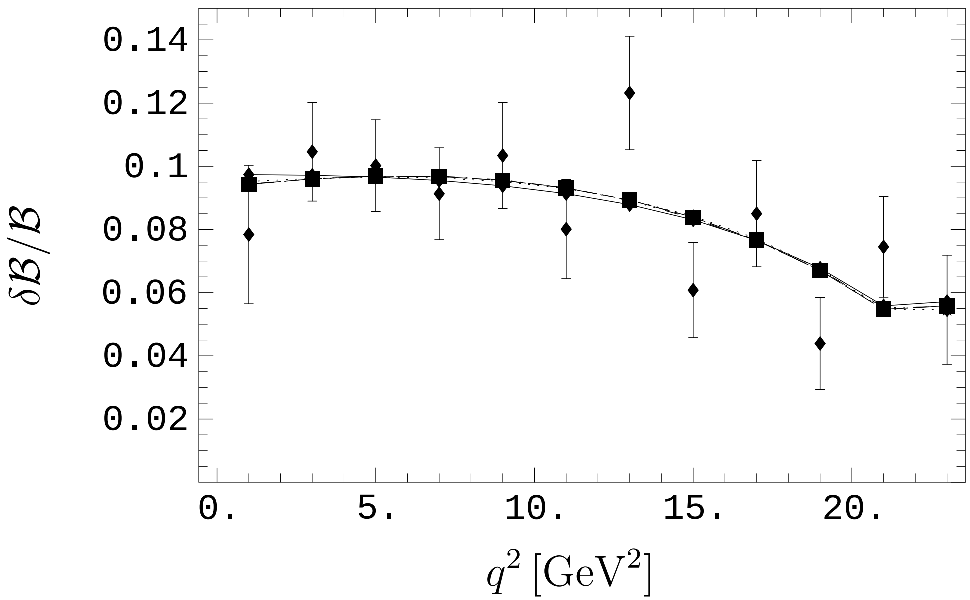
1. $d\Gamma/dq^2$ measured in several bins in q^2
2. parametrisation of q^2 dependence of form factor in terms of, for instance, z-expansion (Boyd/Grinstein/Lebed 1995)

$\rightsquigarrow$ model-independent experimental result for $|V_{ub}|f_+(0)|$

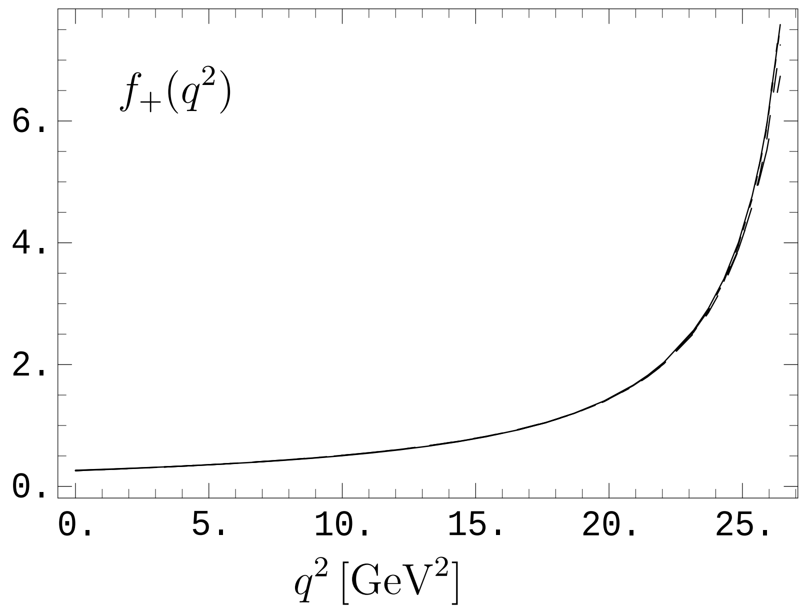
(normalisation point arbitrary; $q^2 = 0$ best for LCSR)

First done in Ball 2006 using BaBar 2006 data: $|V_{ub}|f_+(0)| = (9.1 \pm 0.7) \times 10^{-4}$.

A few Details (Ball 2006)



BaBar data 2006 in 12 bins in q^2 together with best-fit results based on 5 different parametrisations of $f_+(q^2)$



best-fit shape of form factor from data, using 5 different parametrisations

The Issue of f_B

The LCSR yields value for $f_B f_+(q^2)$. What value of f_B to use?

1. Lattice: difficult to average various results (and errors). Most recent result quoted at Lattice 10: $f_B = 212(6)(6)$ MeV. (FNAL/MILC)

2. QCD sum rule results known to $O(\alpha_s^2)$:
   - Jamin/Lange 2001: $210(19)$ MeV
   - Steinhauser 2001: $206(20)$ MeV

   Value very sensitive to m_b, large radiative corrections.

LCSR only known to $O(\alpha_s)$. Expect some cancellation of radiative corrections in ratio $(f_B f_+(q^2))/f_B$, so use f_B as determined from QCD sum rule to the same $O(\alpha_s)$ accuracy (and using the same QCD sum rule parameters):

$$f_B(\text{1 loop}) = 170\,\text{MeV} \quad \text{(for central input parameters)}$$

How realistic is this expectation?

A new Calculation: $f_+(0)$ to $O(\alpha_s^2\beta_0)$ (Ball/Bharucha 2010)

Complete $O(\alpha_s^2)$ pretty difficult (two scales, one dimensionless parameter).

Meaningful subset of diagrams: two-loop diagrams with internal fermion loop: $\propto N_f \to -\frac{3}{2}\,\beta_0$, aka BLM approximation.

Complication: both UV and IR divergencies (to be absorbed into pion DA).

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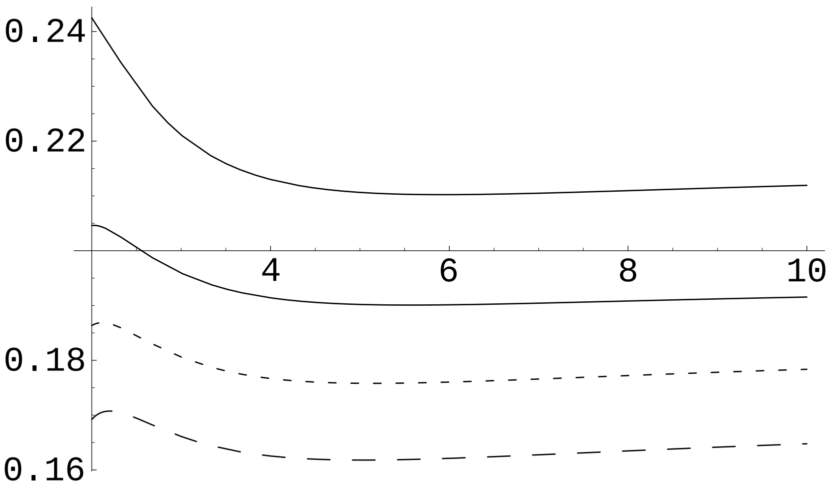
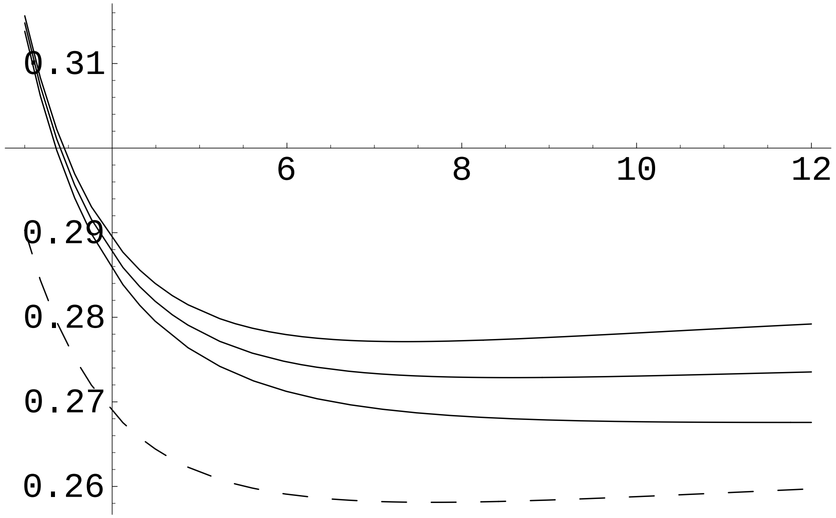
- Additional check: μ dependence of Π vanishes (as required by zero anomalous dimension)
- One of the most involved calcs I have ever done…

Results (preliminary)



f_B for $m_b = 4.8$ GeV, in 1L, BLM and 2L approximation. Also 2L for $m_b = 4.73$ GeV.
Recall: central lattice value is ~ 210 MeV.



dashes: $f_+(0)$ calculated with the same hadronic parameters as in BZ 04 ($m_b = 4.8$ GeV, $a_2(2.2\,\mathrm{GeV}) = 0.08$, $a_4(2.2\,\mathrm{GeV}) = -0.01$). Solid lines: ditto with new contributions added.

Central values: $f_+(0) = 0.258 \to 0.272(+5\%)$

New Experimental Results: BaBar & Belle 2010

BaBar 1005.3288: 349 fb^{-1}

- $B(B \to \pi \ell \nu) = (1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$
- $|V_{ub} f_+(0)| = (10.52 \pm 0.42) \times 10^{-4}$, using BK parametrisation (one parameter for shape); P = 14.8%
- fit of spectrum and MILC lattice data: $|V_{ub}| = (2.95 \pm 0.31) \times 10^{-3}$
- $\rightsquigarrow f_+(0) = 0.36 \pm 0.04$?????

Belle ICHEP 2010 (talk by Ha): 605 fb^{-1}

- $B(B \to \pi \ell \nu) = (1.49 \pm 0.04 \pm 0.07) \times 10^{-4}$
- $|V_{ub} f_+(0)| = (9.24 \pm 0.28) \times 10^{-4}$, using BK parametrisation; P = 62%
- fit of spectrum and MILC lattice data: $|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3}$
- fit of spectrum and Ball/Zwicky LCSR: $(3.64 \pm 0.11(\text{exp})^{+0.60}_{-0.40}(\text{th})) \times 10^{-3}$

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- looking forward to more analyses from BaBar & Belle!