Form factors and long-distance effects in $B \to V(P)\ell^+\ell^-$ and $B \to V\gamma$

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$B \to V(P)\,\ell^+\ell^-$, $B \to V\gamma$ in Standard Model

- exclusive $b \to s$ FCNC decays:
  $B \to K^*\ell^+\ell^-$, $B \to K\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$, $B_s \to \eta^{(\prime)}\ell^+\ell^-$,
  $B \to K^*\gamma$, $B_s \to \phi\gamma$
- $b \to d$ channels:
  $B \to \rho(\omega)\ell^+\ell^-$, $B \to \pi(\eta)\ell^+\ell^-$, $B \to \rho\gamma$, $B_s \to \dots$,
  CKM suppressed
- exclusive decay amplitude:

$$A(B \to K^{(*)}\ell^+\ell^-) = \langle K^{(*)}\ell^+\ell^- \mid H_{eff} \mid B\rangle \,,$$

$$\boxed{H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\,,}$$

- dominant $b \to s$ effective operators: $O_{7,9,10}$
  $C_7(m_b) \simeq -0.3$, $C_9(m_b) \simeq 4.4$, $C_{10}(m_b) \simeq= -4.7$

$B \to P, V$ form factors

- hadronic matrix elements of $O_{7,9,10}$ factorize, e.g.,

  $O_9 = (\bar{s}_L\gamma^\mu b_L)(\bar{\ell}\gamma_\mu\ell) \;\Rightarrow\; \langle K^{(*)}(p)|\bar{s}\gamma_\mu b|B(p+q)\rangle$

  $\Rightarrow$ $B \to P$ and $B \to V$ form factors

- form factors have to be calculated in QCD, functions of q^2,
  $0 < q^2 < (m_B - m_{K^{(*)}})^2$ - inv. mass of lepton pair,
  $q^2 = 0$ -radiative decays

- the remaining operators $O_{1,2,...,6,8g}$, combined
  with e.m. interactions, also contribute to $B \to K^{(*)}\ell^+\ell^-$,
  $O_{1,2}^{(c)} \sim (\bar{c}b)(c\bar{s})$
  $C_1(m_b) \simeq 1.1$, $C_2 \simeq -0.25$, $\quad C_{3,4,5,6} < 0.03$
  $\Rightarrow$ new matrix elements, not reducible to form factors !

$B \to P, V$ form factors, flavour symmetries

- use exp. data on $B \to \pi, (\rho)\ell\nu_\ell$ and
  $SU(3)_{fl}$ symmetry to obtain $B \to K^{(*)}$ form factors

  but! $SU(3)_{fl}$ is violated up to 20%, e.g.,
  $f_K/f_\pi \simeq 155\,MeV/130\,MeV, \quad f_{DK}(0)/f_{D\pi}(0) \sim 1.2 \div 1.1$

- heavy-quark symmetry $m_b, m_c \to \infty$,
  nontrivial relations between B and D form factors,
  use measured $D \to K, K^*$ to obtain $B \to K, K^{(*)}$

  but! $1/m_{c,b}$ corrections are not small: e.g.,
  $f_B \sim f_D \sim 200$ MeV, although HQET predicts $f_H \sim 1/\sqrt{m_Q}$

- to reach $< 20\%$ accuracy of the form factors we need
  QCD calculation !

$B \to P$, V form factors from lattice QCD

- $B \to \pi$ form factors, accessible at large $q^2 > 15\ \mathrm{GeV}^2$: unquenched, $n_f = 2 + 1$, $\sim 10\%$ accuracy achieved
  [HPQCD, FNAL-MILC, see the talk by J.Shigemitsu]
- $B \to K$ some recent results available [QCDSF, (2009)],(quenched)
- $B \to K^*, \rho$, older results (≤ 2005)
  calculated in quenched approximation
- will lattice QCD in future be able to calculate the $B \to V$ form factors as precise as $B \to P$?
  need a lattice treatment of the total width (e.g., $K^* \to K\pi$)

$B \to P, V$ form factors from QCD light-cone sum rules (LCSR)

see also the talk by P. Ball at WGII

Correlator of quark currents = *hadronic sum (disp.relation)*

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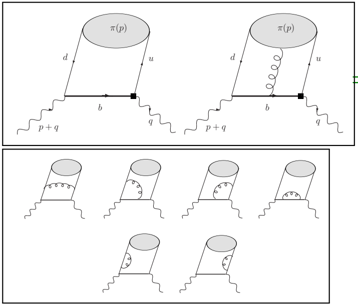
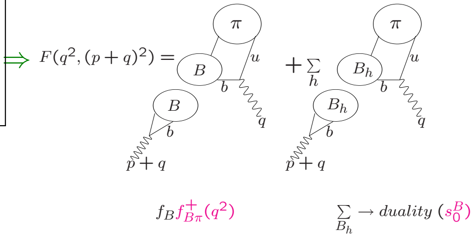




{light-cone OPE, pion DA's} {quark-hadron duality}

Status and accuracy of LCSR calculations

- $q^2 \leq 12 - 15$ GeV^2 accessible, complementing the lattice
- $B \to \pi, K$ recent updates ($\overline{MS}$ b-quark mass):
  [G.Duplancic, AK, Mannel, B.Melic, N.Offen (2008)] ,
  $B \to K$ [G.Duplancic, B.Melic (2008)] ,
- the method/input recently checked for $D \to \pi, K$
  A.K.,Ch.Klein, Th.Mannel, N.Offen 0907.2842[hep-ph]
- within uncertainties ($\pm 12 - 15\%$)
  $B \to \pi, K$ form factors agree with
  [Ball-Zwicky (2005)] (pole b-mass)
- $B_{(s)} \to \rho, \omega, K^*, \phi$, [Ball-Zwicky (2005)]
  LCSR in "quenched" approxim.: the width of ρ, K^* is neglected
- new perspective: LCSR with B-meson distribution amplitudes [A.K., N. Offen, Th. Mannel '06]
  need radiative corrections, better understanding of B DA's
- accuracy $< 10\%$ will hardly be accessible with LCSR

Form factors from LCSR with B-meson DA's

form factor	this work	LCSR with light-meson DA's
		[P.Ball and R.Zwicky] ([Duplancic et al])
$f^{+}_{B\pi}(0)$	0.25$\pm$0.05	0.258$\pm$0.031, $(0.26^{+0.04}_{-0.03})$
$f^{+}_{BK}(0)$	0.31$\pm$0.04	0.301$\pm$0.041$\pm$0.008
$f^{T}_{B\pi}(0)$	0.21$\pm$0.04	0.253$\pm$0.028
$f^{T}_{BK}(0)$	0.27$\pm$0.04	0.321$\pm$0.037$\pm$0.009
$V^{B\rho}(0)$	0.32$\pm$0.10	0.323$\pm$0.029
$V^{BK^*}(0)$	0.39$\pm$0.11	0.411$\pm$0.033$\pm$0.031
$A_1^{B\rho}(0)$	0.24$\pm$0.08	0.242$\pm$0.024
$A_1^{BK^*}(0)$	0.30$\pm$0.08	0.292$\pm$0.028$\pm$0.023
$A_2^{B\rho}(0)$	0.21$\pm$0.09	0.221$\pm$0.023
$A_2^{BK^*}(0)$	0.26$\pm$0.08	0.259$\pm$0.027$\pm$0.022
$T_1^{B\rho}(0)$	0.28$\pm$0.09	0.267$\pm$0.021
$T_1^{BK^*}(0)$	0.33$\pm$0.10	0.333$\pm$0.028$\pm$0.024

Other non-lattice tools

- effective theories (HQET, QCD factorization, SCET),
  $B \to K^* \ell^+ \ell^-$ [Beneke, Feldmann, Seidel(2001)], ...
- access factorizable contributions/mechanisms in the form-factors/ decay amplitudes
  non-trivial relations betw. form factors
- need soft form factors and meson DA's as an input
- the issue of $1/m_b$ terms
- LCSR in SCET [F. De Fazio, Th. Feldmann T.Hurth (2006)

Series parametrizations: conformal mapping

- Form factors are analytic functions of q^2
- e.g., $B \to \pi$ form factor $f^+_{B\pi}(q^2)$ has no singularities at $q^2 < (m_B + m_\pi)^2$, except the pole at $q^2 = m^2_{B^*}$.
- map the complex q^2-plane onto $|z| < 1$ in the z-plane:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_B + m_\pi)^2, t_0 < t_+$$

[N. Meiman (1963)]; [S.Okubo (1971)], [C.G.Boyd, B.Grinstein, R.Lebed (1995)], [L.Lellouch (1996)],..., [T.Becher, R.Hill (2006)],..

- $|z| \ll 1$ in semileptonic region $0 < q^2 < (m_B - m_\pi)^2$
  a Taylor expansion around $z = 0$ describes the form factor

Series parametrization: predicting the q^2 shape

- The last (and simplest) version of z-parametrization:
  [C. Bourrely, I. Caprini, L. Lellouch (2008)]

$$f^+_{B\pi}(q^2) = \frac{1}{1 - q^2/m^2_{B*}} \sum_{k=0}^{k_{max}} a_k \Big(z(q^2, t_0) \Big)^k$$

- this shape was fitted to QCD lattice ($q^2 > 15$ GeV^2) and LCSR ($q^2 = 0$) points to predict the $B \to \pi$ form factor
- additional perturbative QCD bounds
  (from the unitarity for the 2-point correlation function)
- generalization to all $B \to P, V$ form factors possible, different sub-threshold $B(J^P)$ resonance poles

Combined analysis of $B \to K$ and $B \to K^*$ form factors

[A.Bharucha, Th.Feldmann, M.Wick, 1004.3249[hep-ph].

- use LCSR (Ball-Zwicky (2005)) and some lattice results $\oplus$ series parameterization
- typical uncertainties: $\pm(12-15)\%$ for $B \to P$, $\sim \pm 20\%$ for $B \to V$ form factors
- an example: $B \to K$ form factor $f^+_{BK}(q^2) \equiv A_{V,0}(q^2)$

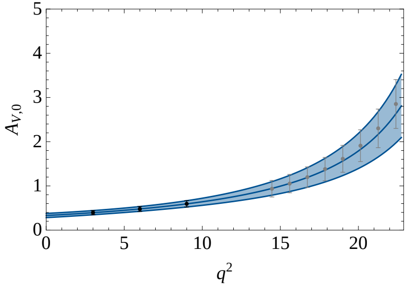
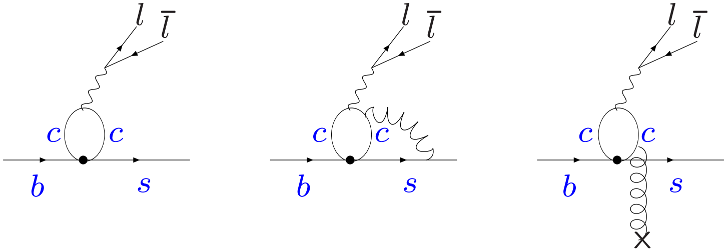


fig. from the above paper

Charm-loops in $B \to K^{(*)}\ell^+\ell^-$

- Charm-loop effect: a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\bar{\ell}\ell)$

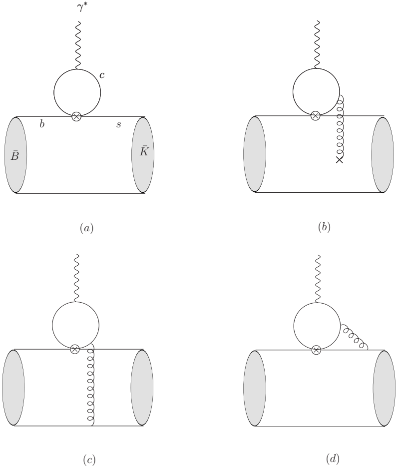


- charm-loops mimic FCNC, "contaminate" decay ampl.
- similar effects:
  u, d, s, c, b-quark loops (quark-penguin operators O_{3-6}),
  u-loops from $O^u_{1,2}$ (CKM suppressed in $b \to s$),
- new hadronic matrix elements, not simply form factors
- at $q^2 \to m^2_{J/\psi}, ...$ charm-loop goes on-shell:
  $B \to J/\psi K \otimes J/\psi \to \ell^+\ell^-$, cuts by exp.

Charm-loop in $B \to K^{(*)}\ell^+\ell^-$

[A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, 1006.4945 [hep-ph]]

$(q^2 \ll 4m_c^2)$



- factorizable c-quark loop

$C_9 \to C_9 + (C_1 + 3C_2)g(m_c^2, q^2)$

- perturbative gluons $\to$ (nonfactorizable) corrections being factorized in $O(\alpha_s)$ and added to C_9

[M. Beneke,T. Feldmann,D. Seidel (2001)]

- how important are the soft gluons (low-virtuality, nonvanishing momenta) emitted from the c-quark loop ?

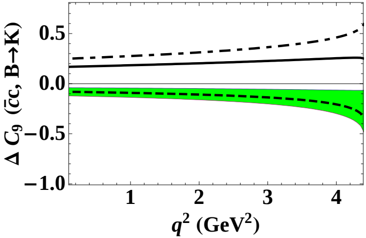
Estimate of the soft-gluon effect

- soft-gluon emission at $q^2 \ll 4m_c^2$ using light-cone OPE:
  ▶ nonlocal operator, $\sim 1/(4m_c^2 - q^2)$-suppression
  effective resummation of local operators,

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q, m_c, \omega)\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L \ ,$$

- LCSR with B meson DAs used to calculate
  $\langle K^{(*)}|\bar{s}\tilde{G}b|B\rangle$, not a simple form factor

  ▶ correction to the effective coefficient of O_9 operator,
  $B \to K\ell^+\ell^-$:



The local OPE limit

- $\omega \to 0$ in the nonlocal operator, no derivatives of $G_{\mu\nu}$

$$\widetilde{\mathcal{O}}_\mu^{(0)}(q) = I_{\mu\rho\alpha\beta}(q)\bar{s}_L\gamma^\rho \widetilde{G}_{\alpha\beta} b_L \ ,$$

$$I_{\mu\rho\alpha\beta}(q, m_c) = (q_\mu q_\alpha g_{\rho\beta} + q_\rho q_\alpha g_{\mu\beta} - q^2 g_{\mu\alpha} g_{\rho\beta}) \times \frac{1}{16\pi^2} \int_0^1 dt \ \frac{t(1-t)}{m_c^2 - q^2 t(1-t)}$$

At $q^2 = 0$, the quark-gluon operator obtained
in $B \to X_s\gamma$ in *[M.Voloshin (1997)]*
in $B \to K^*\gamma$ *[A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]*

- the neccesity of resummation was discussed before
*[Z. Ligeti, L. Randall and M.B. Wise,(1997);
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei (1997);
J. W. Chen, G. Rupak and M. J. Savage,(1997);
G. Buchalla, G. Isidori and S.J. Rey (1997)]*

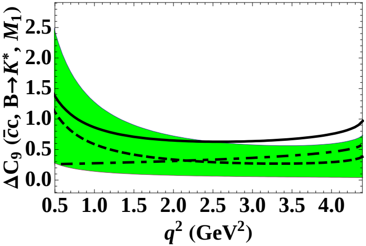
Charm-loop effect for $B \to K^* \ell^+ \ell^-$

- factorizable part determined by the three $B \to K^*$ form factors $V^{BK^*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- three kinematical structures for the nonfactorizable part:

$$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) - 2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*})\widetilde{A}_V(q^2)}{q^2\, V^{BK^*}(q^2)} ,$$

- nonfactorizable part enhances the effect, $1/q^2$ factor

$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(1.0\text{GeV}^2) = 0.7^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_1)}(1.0\text{GeV}^2) = 0.8^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_2)}(1.0\text{GeV}^2) = 1.1^{+1.1}_{-0.7}$



Charm-loop effect in $B \to K^*\gamma$

- By-product of our calculation for $B \to K^*\ell^+\ell^-$ at $q^2 = 0$
- factorizable part vanishes,
  nonfactorizable part yields a correction to $C_7^{eff}(m_b) \simeq -0.3$
  in the two inv. amplitudes:

$$C_7^{eff} \to C_7^{eff} + [\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_{1,2} ,$$

$$\left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_1 \simeq \left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_2 = (-1.2^{+0.9}_{-1.6}) \times 10^{-2} ,$$

- the previous results in the local OPE limit , LCSR with K^* DA:

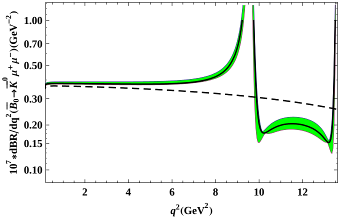
$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_1^{BZ} = (-0.39 \pm 0.3) \times 10^{-2} ,$$
$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_2^{BZ} = (-0.65 \pm 0.57) \times 10^{-2} . \qquad (1)$$

[P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]

- dispersion relation and data on $B \to \psi K$
  to access $q^2 \leq 4m_D^2$, nontrivial interference between ψ levels
- influence on observables: diff. width $B \to K\ell^+\ell^-$

differential distribution in q^2
with (solid) and without (dashed)
charm-loop effect



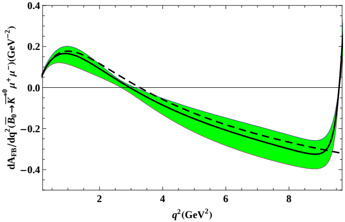
- forward-backward asymmetry in $B \to K^{(*)}\ell^+\ell^-$

$q_0^2 = 2.9^{+0.2}_{-0.3}\mathrm{GeV}^2$

$\sim$ 10% larger without

nonfactorizable correction,

no α_s correction inlcuded



Concluding remarks

- $B \to P$ form factors needed for $B \to P\ell^+\ell^-$ are accessible both on the lattice and with LCSR, future accuracy:
  $\sim$ 5% (lattice, [App.A SuperB report '07]
  $\sim$ 10% (LCSR)
- $B \to V$ form factors:
  difficulties of "unquenching" on the lattice
- LCSR techniques combined with series parameterization may play a decisive role in providing $B \to V$ form factors in future, very optimistically, with 10-15% accuracy
- there is also an experimental uncertainty related to the extraction of K^* (or ρ) from the data on $B \to K^*(\rho)\ell^+\ell^-$:
  $K\pi$ (or $\pi\pi$) nonresonant background, J^P analysis needed

Concluding remarks

- charm-loop: soft-gluon nonfact. effects are accessible using LC expansion and LCSR, the accuracy can further be improved, similar effects to be analysed
- are there "benefits of $B \to K^* \ell^+ \ell^-$ at low recoil" ?
  [C.Bobeth, G.Hiller, D.van Dyk, 1006.5013[hep-ph]]
  the region $q^2 > m_\psi^2$,
  based on the HQET at $q^2 \gg m_c^2$ limit [B.Grinstein, D. Pirjol (2005)]
- for physical masses $q^2 < (m_B - m_{K^*})^2 \sim 4m_D^2$,
  backgr. from resonant charm-loops,
  $1/m_b$ corrections, uncertainties in form factors
- an update is desirable:
  of the previous analysis of $B \to K^{(*)} \ell^+ \ell^-$
  A.Ali,P.Ball, L.T. Handoko, G.Hiller (2000) including:
  hard-gluon [M.Beneke,Th.Feldmann, Seidel (2001)]
  and soft-gluon [this work] nonfactorizable effects