Vud from Nuclear Decays

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Advantages of Nuclear Decays

- select pure vector transitions: $0^+ \to 0^+$ decays
  - 10 cases, experimental precision $\leq 0.1\%$
  - 3 cases, experimental precision $\leq 0.4\%$
- conserved vector current (CVC) hypothesis
  - $G_V = G_F V_{ud}$ is a 'true' constant, nucleus-independent
  - $\mathcal{F}t$ values are constant, nucleus-independent
  - provides consistency checks

Disadvantages of Nuclear Decays

- $SU(2)$ -symmetry breaking correction needed
  - requires nuclear-structure calculation
  - typically $\sim 0.5\%$ – small and testable

Radiative correction in nuclear decays

In total: $RC \sim 4\%$

- Nucleus-independent component

$$\Delta_R = 2.361 \pm 0.038\%$$

error reduction, Marciano-Sirlin PRL 96, 032002 (2006)

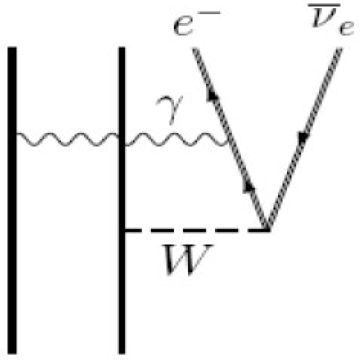
- Trivially nucleus-dependent component

$$\delta'_R \simeq 1.44 \pm 0.05\% \qquad \text{(typically)}$$

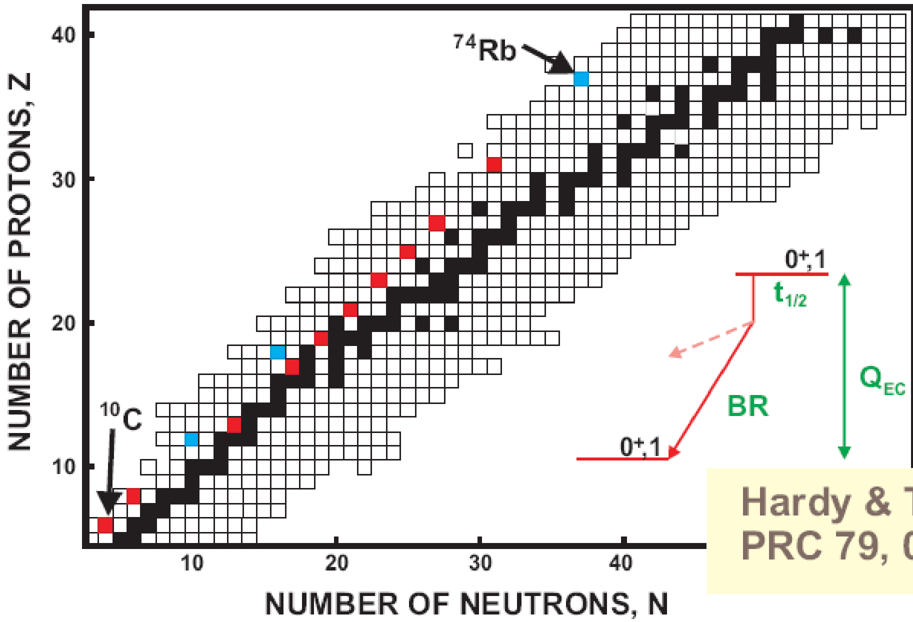
principally a QED calculation, depending on nuclear charge, Z, and electron energy, E_e.

- Small nuclear-structure dependent component

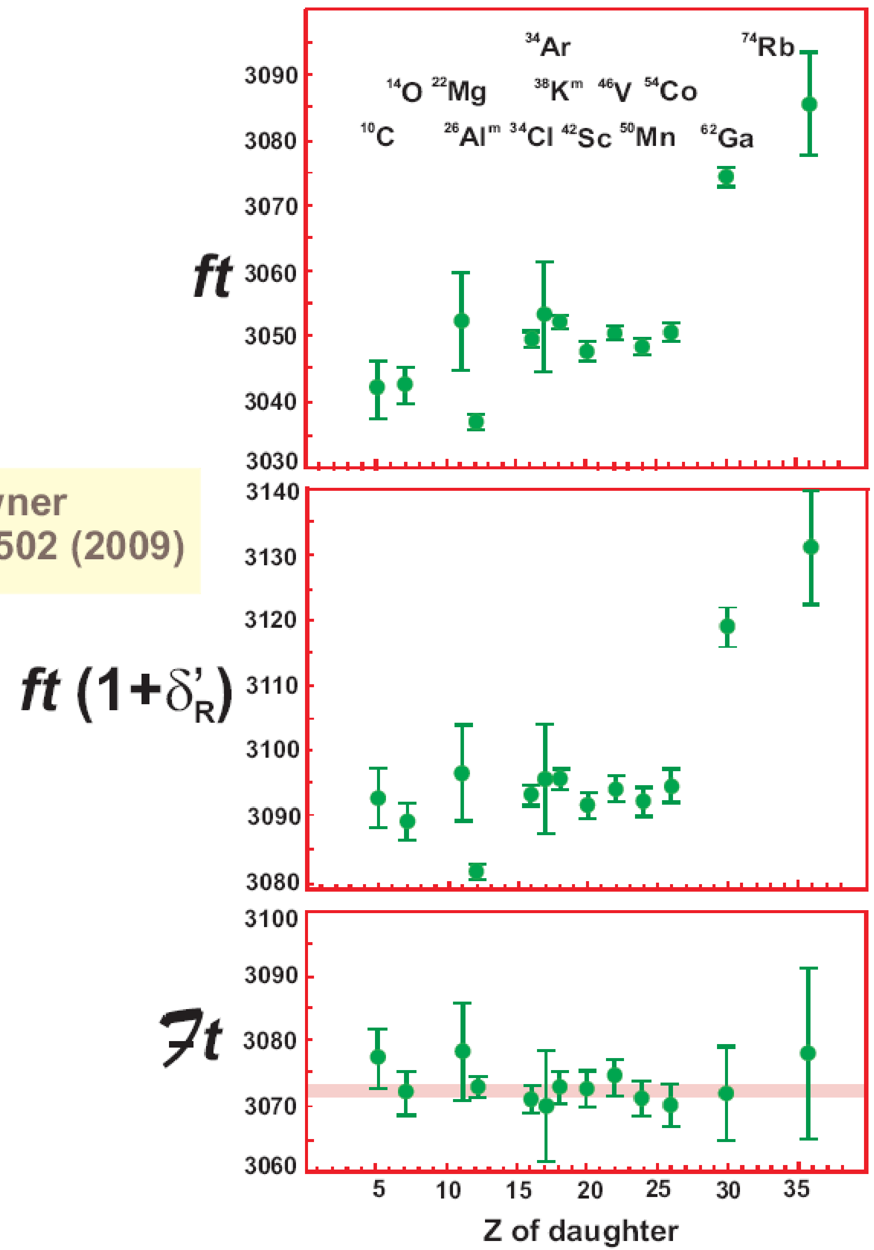
$$\begin{aligned} \delta_{NS} &= -0.20 \pm 0.02\% \quad T_z = -1 \text{ nuclei} \\ &= -0.05 \pm 0.02\% \quad T_z = 0 \text{ nuclei} \end{aligned}$$



WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2009



Hardy & Towner
PRC 79, 055502 (2009)



- **10 cases with *ft*-values measured to ~0.1% precision; 3 more cases with <0.3% precision.**
- **~150 individual measurements with compatible precision**

$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY IN 2009

1) G_V constant $\quad \mathcal{F}t = \dfrac{K}{2G_V^2\,(1+\Delta_R)}$ ✔ verified to ± 0.013%

2) Correction terms validated ✔

3) Scalar current zero ✔ limit, C_S/C_V = 0.0011 (14)

4) Precise value determined for $|V_{ud}|$ = 0.97425 ± 0.00022

$$V_{ud} = G_V / G_\mu$$

5) CKM unitarity established ✔ $V_{ud}^2 + V_{us}^2 + V_{ub}^2$ = 0.99990 ± 0.00060

$|V_{ud}|$ nuclear decays, muon decay: 0.9743 ± 0.0002 (± 0.0001 exp't)

$|V_{us}|$ kaon decays: 0.2246 ± 0.0012

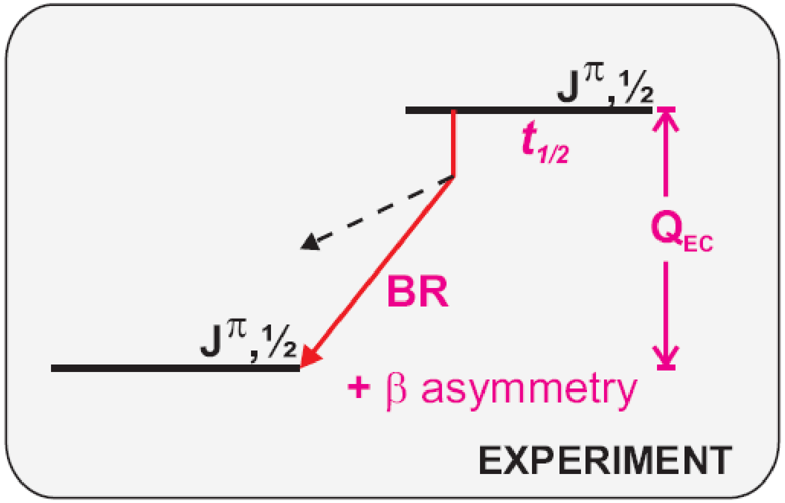
$|V_{ub}|$ B decays: 0.0039 ± 0.0004

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_V^2\langle\tau\rangle^2 + G_A^2\langle\sigma\tau\rangle^2}$$

f = statistical rate function: $f(Z, Q_{EC})$
t = partial half-life: $f(t_{1/2}, BR)$
$G_{V,A}$ = coupling constants
< > = Fermi, Gamow-Teller matrix elements



INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{G_V^2\,(1 + \Delta_R)(1 + \lambda^2\langle\sigma\tau\rangle^2)}$$

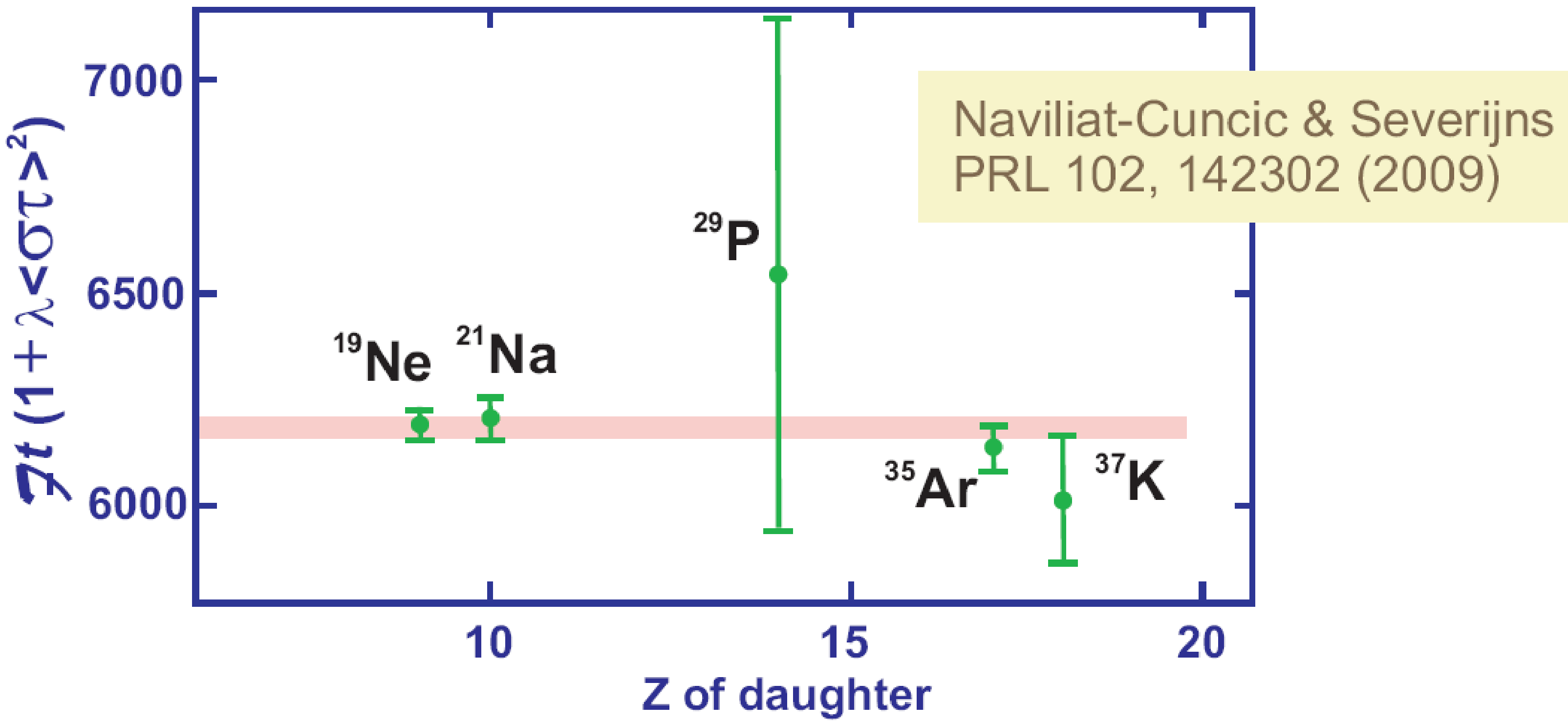
$\lambda = G_A/G_V$

NEUTRON DECAY

Requires additional experiment: for example, β asymmetry

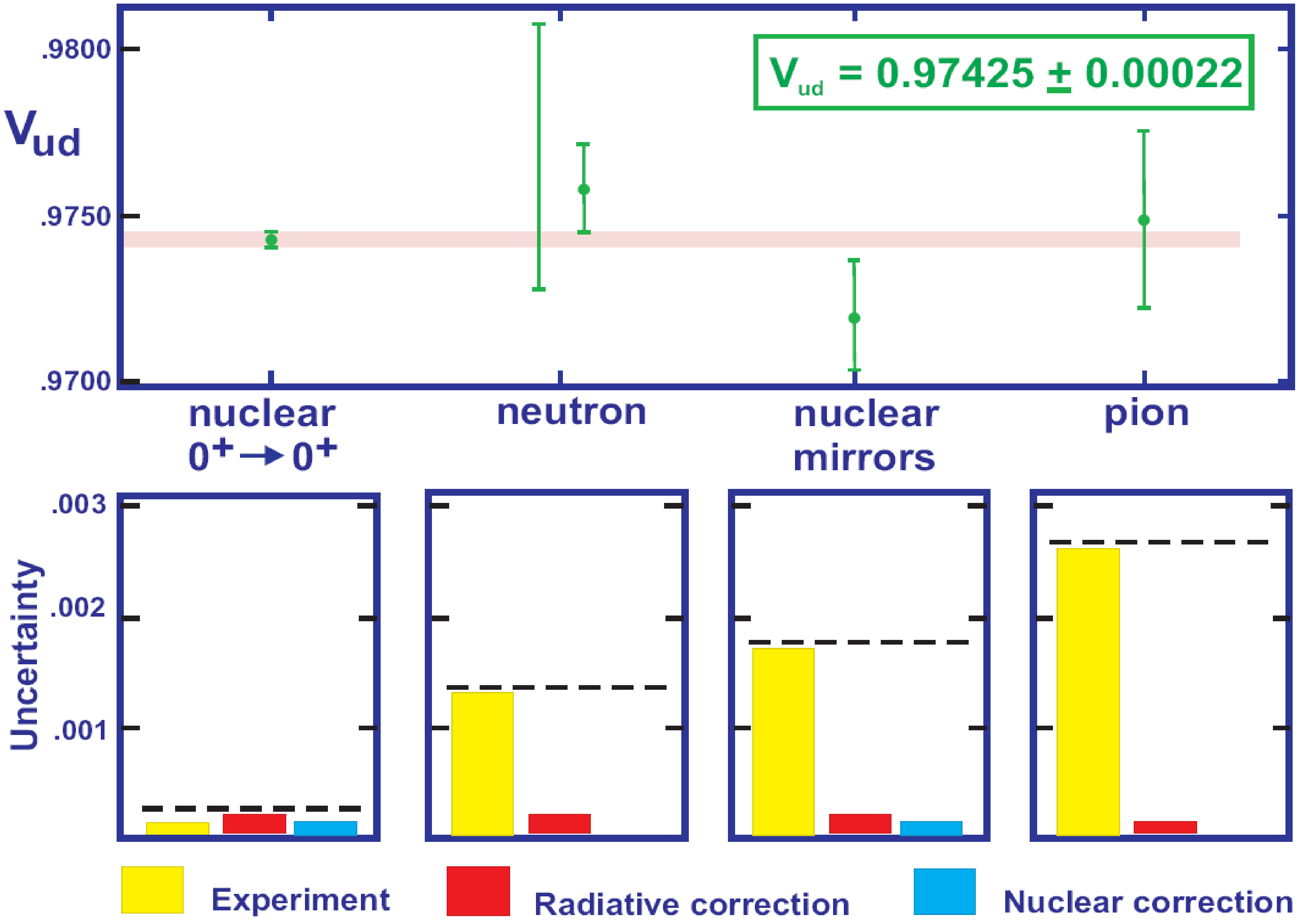
NUCLEAR T=1/2 MIRROR DECAY DATA 2009

$$\mathcal{F}t = ft\,(1+\delta'_R)\left[1-(\delta_C-\delta_{NS})\right] = \frac{K}{G_V^{\,2}\,(1+\Delta_R)(1+\lambda^2\langle\sigma\tau\rangle^2)}$$



$V_{ud} = 0.9719 \pm 0.0017$

CURRENT STATUS OF V_{ud} – 2009



SU(2)-symmetry breaking correction

In determining V_{ud} from nuclear decays, a nuclear-structure dependent correction, δ_C, required.

Uncertainty in $\delta_C \Rightarrow$ second-largest uncertainty in V_{ud}.

Is this correction under control?

Recent work:

- Nuclear shell model with radial functions from:
  - Saxon-Woods potential — PR C77, 025501
  - Hartree-Fock mean field — PR C79, 055502
- Hartree-Fock + RPA. — PR C79, 064316
- Coupling to isovector monopole resonance (IVMR). — PR C79, 035502
- Criticisms of Miller-Schwenk. — PR C80, 064319

Experimental-based test

Proposal: Use the CVC hypothesis to test the efficacy of δ_C calculations.

Recall:

$$\begin{aligned} [\mathcal{F}\mathrm{t}]_\mathrm{i} &= [\mathrm{ft}(1+\delta_\mathrm{R})(1-\delta_\mathrm{C})]_\mathrm{i} = \mathrm{K} \\ \mathrm{K} &= \frac{2\pi^3 \ln 2/\mathrm{m}_\mathrm{e}^5}{2\mathrm{G}_\mathrm{V}^2(1+\Delta_\mathrm{R})} \\ \Delta_\mathrm{R} &= \text{ nucleus independent radiative correction} \end{aligned}$$

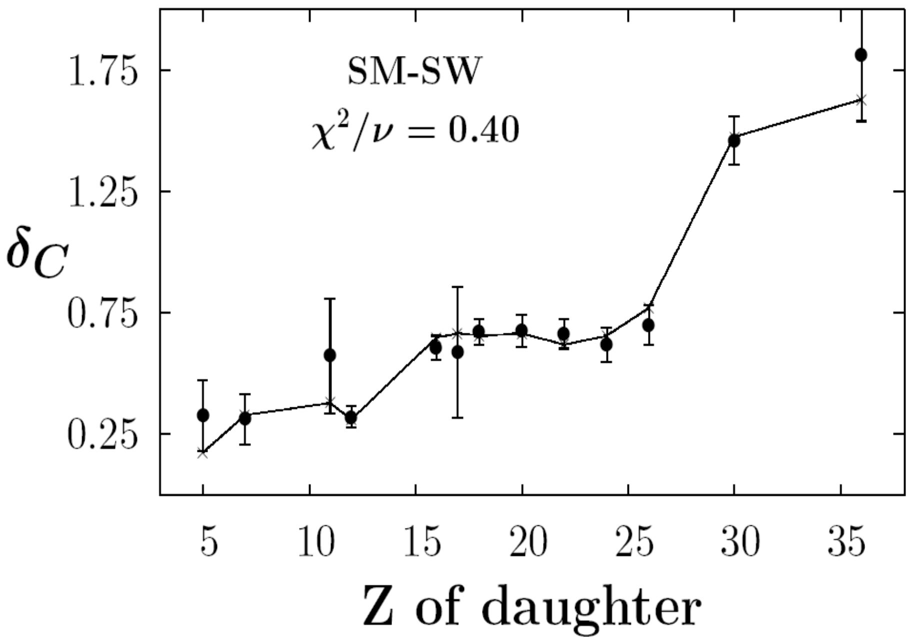
for $i = 1 \ldots 13$ different nuclei. CVC asserts G_V is not renormalized in nuclear medium, hence K is a constant.

Rearrange:

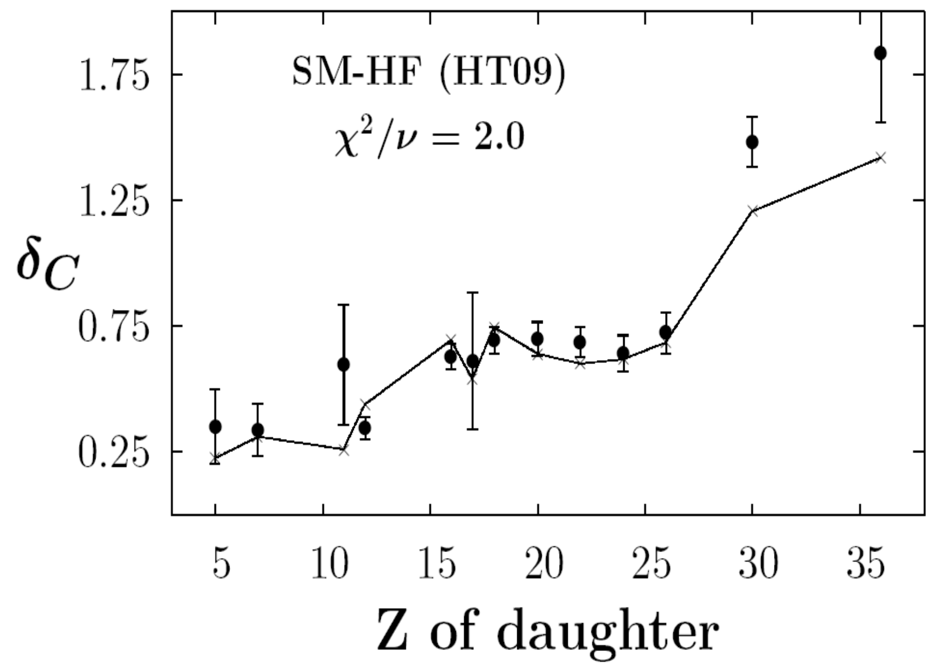
$$[\delta_\mathrm{C}]_\mathrm{i} = 1 + \frac{\mathrm{K}}{[\mathrm{ft}(1+\delta_\mathrm{R})]_\mathrm{i}}$$

Test: Compare calculated δ_C with RHS and find K that minimizes χ^2.

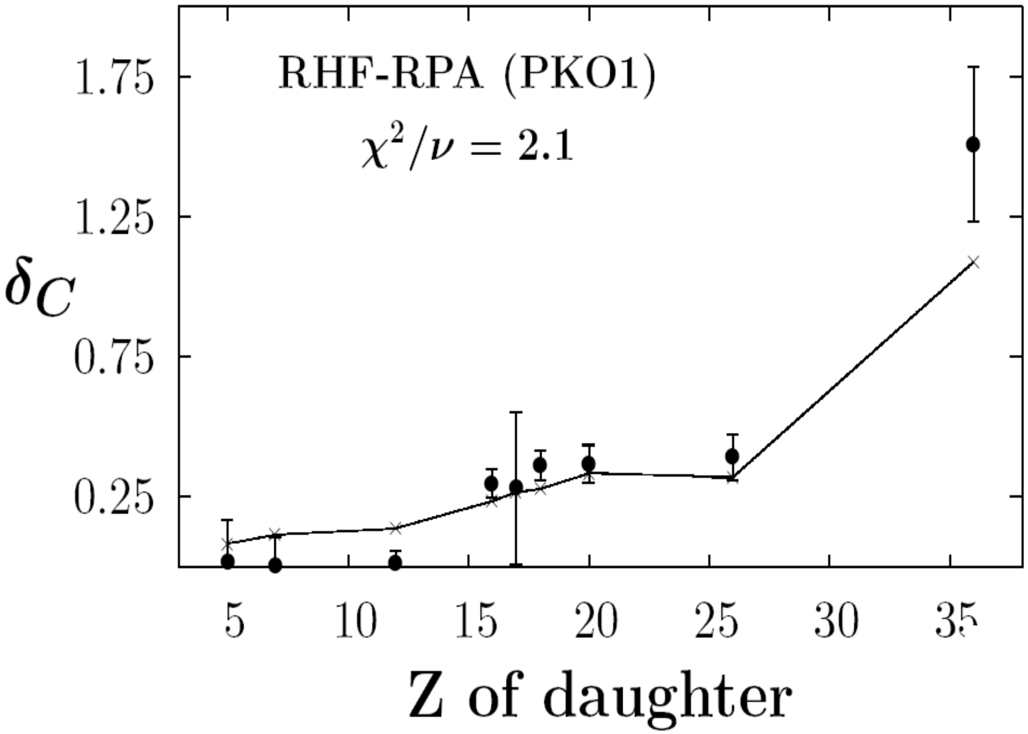
Figure of merit: The minimum χ^2.



Shell-model: Saxon-Woods radial functions



Shell-model: Hartree-Fock radial functions

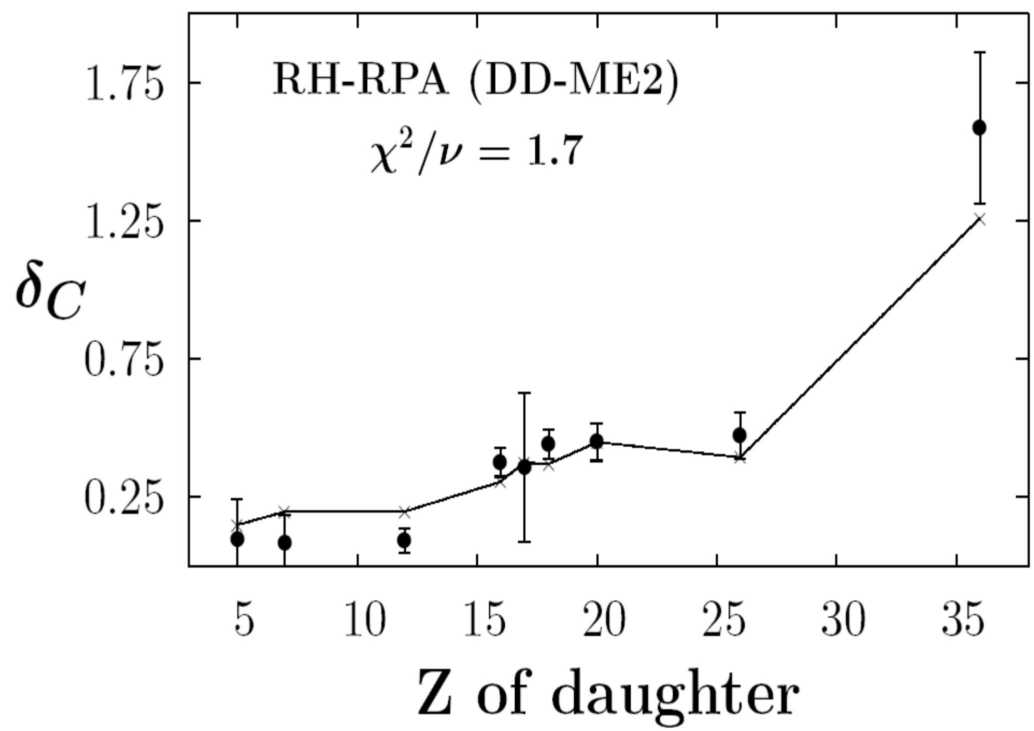


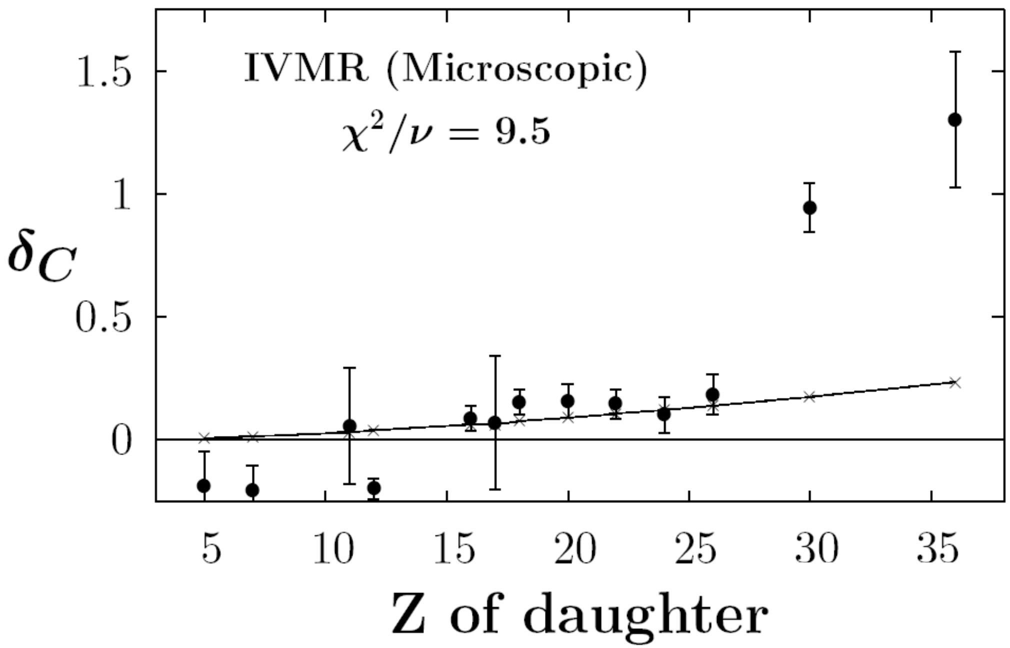
Relativistic Hartree-Fock + RPA

Interaction: PKO1

Relativistic Hartree + RPA

Density-dependent interaction: DD-ME2





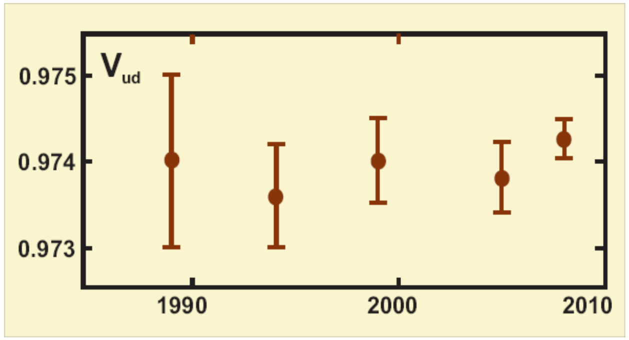
Isovector monopole resonance model

Summary

Model	χ^2	Comment
Shell-model: SW radial functions	0.4	Excellent agreement with CVC requirements
Shell-model: HF radial functions	2.0	Adequate agreement with CVC requirements; fails on high-Z cases
RHF-RPA: PKO1	2.1	
RH-RPA: DD-ME2	1.7	
IVMR	9.5	Fails to satisfy CVC requirements

- Superallowed β decay currently yields most precise value of V_{ud}, limited by theory uncertainties.

$$|V_{ud}| = 0.97425 \pm 0.00022$$



- Value of V_{ud} proving to be very robust.

- $T = 1/2$ mirrors, neutron and pion decays yield V_{ud} consistent with nuclear result, but with larger experimental errors.

- SU(2)-symmetry breaking correction computed with nuclear shell model meets the requirements of CVC consistency. Other models less successful.

- CKM Unitarity now verified to 0.06% – dominant errors are from theory.

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99990 \pm 0.00060$$

Supplementary Slides

MASTER EQUATIONS

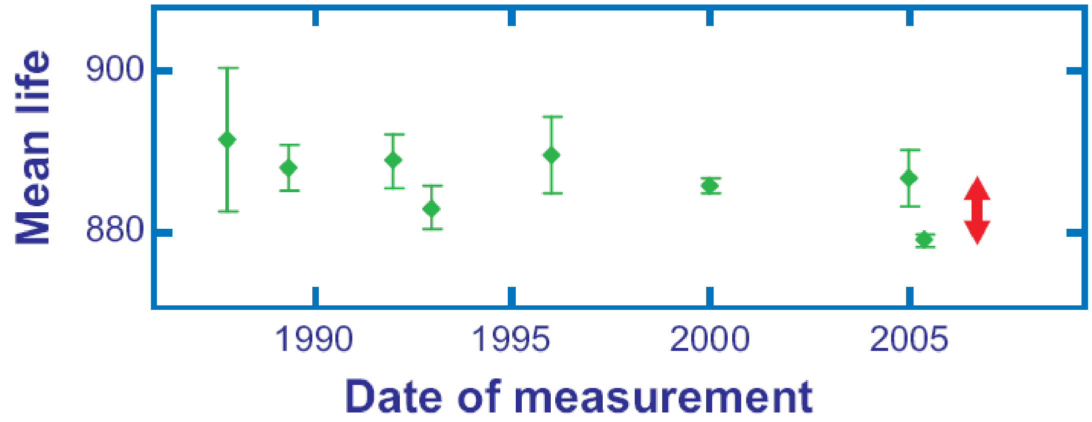
$$\mathrm{CVC}: \ \mathcal{F}t \ = \ ft(1+\delta'_R)(1-(\delta_C-\delta_{NS})) \ = \ \mathrm{constant}$$

$$V_{ud}^2 \ = \ \frac{K}{2G_F^2\overline{\mathcal{F}t}(1+\Delta_R)} \qquad\qquad \frac{K}{(\hbar c)^6} = \frac{2\pi^3\hbar\ln 2}{(m_e c^2)^5}$$

NEUTRON DECAY DATA 2009

Mean life:

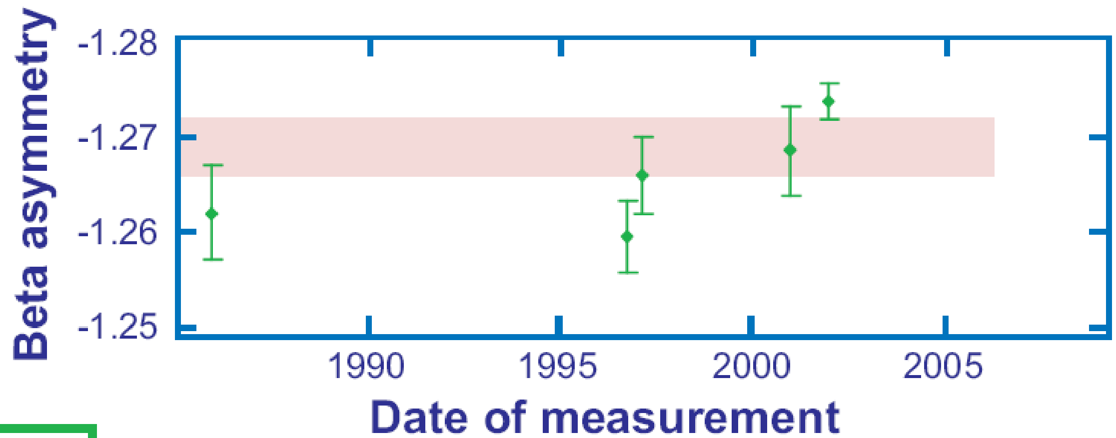
$878 < \tau < 886$ s



β asymmetry:

$\lambda = -1.269 \pm 0.003$

$\chi^2/N = 3.9$



$0.9727 < V_{ud} < 0.9807$

Most recent values only: $V_{ud} = 0.9758 \pm 0.0013$

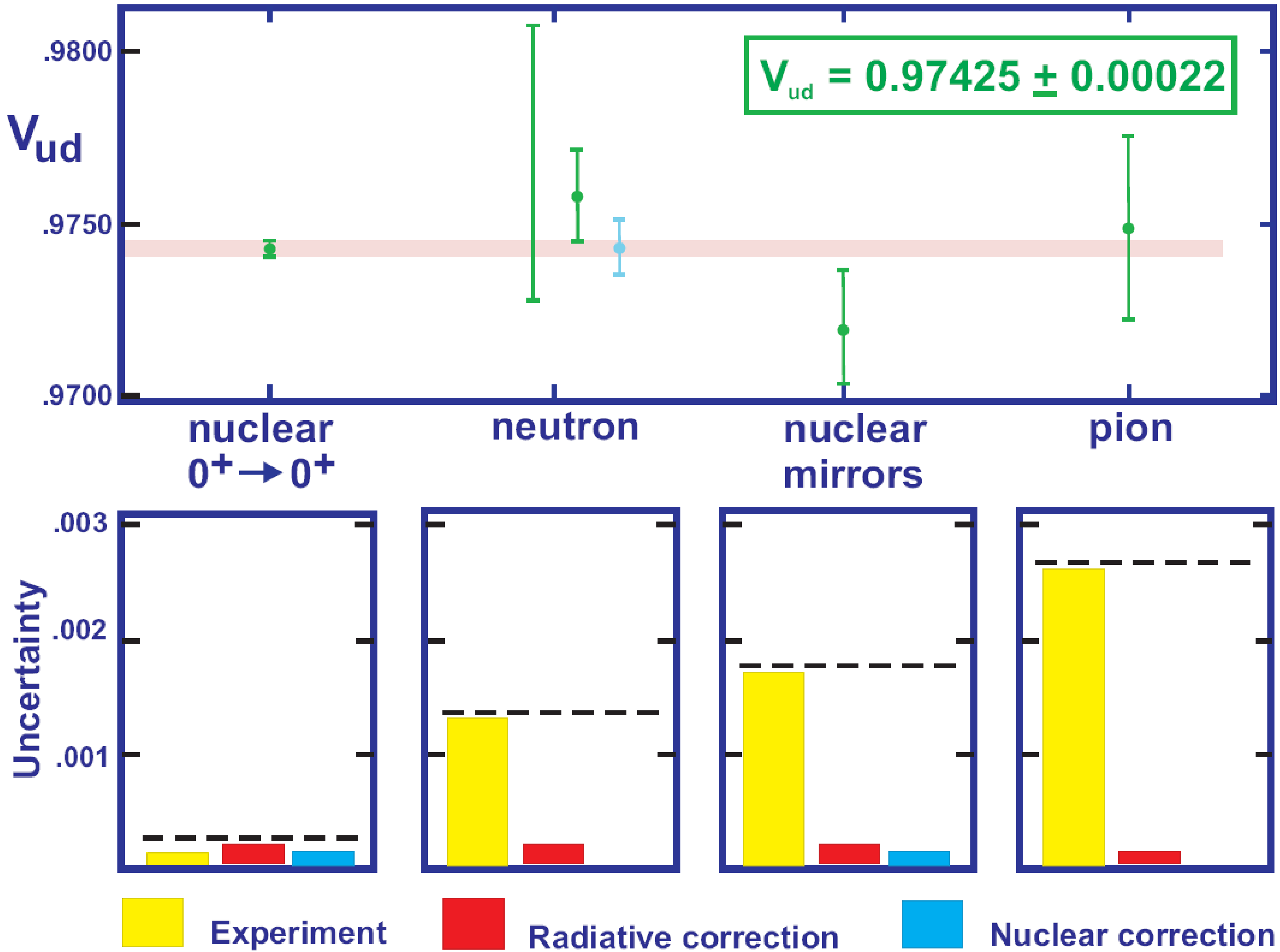
Serebrov/Fomin re-evaluation (2010): $V_{ud} = 0.9743 \pm 0.0008$

NEUTRON LIFETIME

Ref.	PDG10	SF10
Mo09		881.5(25)
Ez07		878.2(19)
Se05		878.5(8)
Ni05	886.3(34)	886.3(34)
Ar00	885.4(10)	879.9(26)
Pi00		881.0(30)
Ma93	882.6(27)	882.6(27)
Ne92	888.4(33)	
Sp88	891.0(90)	891.0(90)
		⋮
Avg.	885.7(8)	879.9(9)

SF10: A.P. Serebrov and A.K. Fomin, arXiv: 1005.4312

CURRENT STATUS OF V_{ud} – 2009



PION BETA DECAY

Decay process:

$$\pi^+ \longrightarrow \pi^0 e^+ \nu_e$$

$$0^-,1 \longrightarrow 0^-,1$$

Experimental data:

$$\tau = 2.6033 \pm 0.0005 \times 10^{-8}\ \text{s}$$ (PDG 2009)

$$\text{BR} = 1.036 \pm 0.007 \times 10^{-8}$$

Pocanic *et al,*
PRL 93, 181803 (2004)

Result:

$$V_{ud} = 0.9749 \pm 0.0026$$