Kaon physics lattice calculations

CKM 2010

Warwick, 06-10.09.2010

Andreas Jüttner
CERN Theory Division

Lattice QCD results relevant for flavour physics

- assessment of the quality of current lattice results
- status: f_K/f_π, $f_+^{K\to\pi}(0)$
- phenomenological implications
- will there be improvements in the near future?
- outlook

Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulnes of lattice results relevant to flavor physics

- G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig

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  - continuum extrapolation
  - finite volume effects
  - renormalization
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- FLAG averages/best values soon on arXiv
- also: Lattice results relevant for CKM triangle analysis
  Laiho, Lunghi, van de Water PRD, 81, 034503 (2008)

in practice:

- measure decay rates $\Gamma(i \to j)$ experimentally
- compute process in SM (FF, RC)
- $\Gamma(i \to j) = const. \times G_F^2 |V_{ij}|^2 \times FF \times RC$

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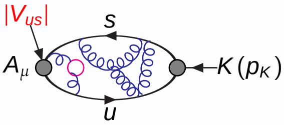

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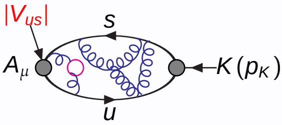

$$\langle 0|A_\mu(0)|K(p_K)\rangle|_{\rm QCD}$$

In 2004 Marciano first used the lattice determination of f_K/f_π to determine $|V_{us}|$:
(Marciano, Phys.Rev.Lett. 2004)

$$\frac{\Gamma(K\to\mu\bar\nu_\mu(\gamma))}{\Gamma(\pi\to\mu\bar\nu_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1-m_\mu^2/m_K^2)}{m_\pi(1-m_\mu^2/m_\pi^2)} \times 0.9930(35)$$

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	f_K/f_π
BMW 10	2+1	A	★	★	★	1.192(7)(6)
JLQCD/TWQCD 09B	2+1	C	●	■	■	$1.210(12)_{\rm stat}$
MILC 09A	2+1	C	●	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	A	●	★	★	$1.197(3)(^{+6}_{-13})$
ALVdW 08	2+1	C	●	●	●	1.191(16)(17)
PACS-CS 08, 08B	2+1	A	★	■	■	1.189(20)
HPQCD/UKQCD 08	2+1	A	●	●	★	1.189(2)(7)
RBC/UKQCD 08	2+1	A	●	★	■	1.205(18)(62)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 09	2	A	●	●	★	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	★	●	1.21(3)

$\rightarrow$ precision of $\approx 0.6 - 0.8\%$ possible

BMW Phys.Rev.D81:054507,2010

Source uncertainty/error	uncertainty/error on f_K/f_π
statistics	0.6%
chiral extrapolation	
- functional form	0.3%
- pion mass range	0.3%
continuum extrapolation	0.3%
exited states	0.2%
scale setting	0.1%
finite volume	0.1%
total syst	0.5%
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dominant syst. uncertainties: chiral and continuum extrapolation

- **!** often huge number of fit parameters
- **!** incomplete ChPT expressions including terms of NNNNLO ChPT are being used

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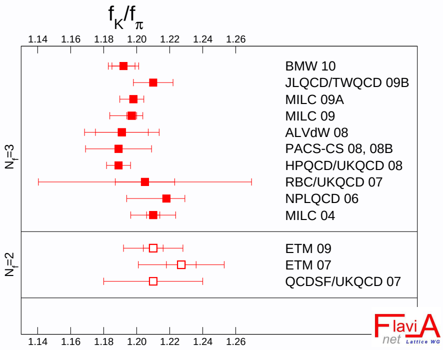
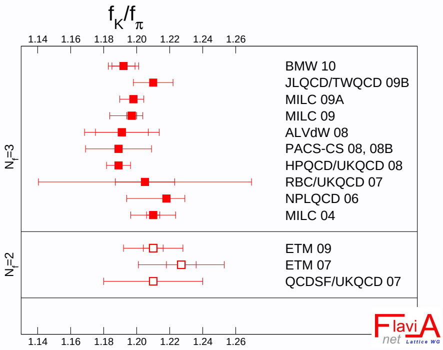
- **!** incomplete ChPT expressions including terms of NNNNLO ChPT are being used
- results for physical pion masses are now becoming available making the chiral extrapolation obsolete
- **!** there are issues with taking the continuum limit (cf. later)

Schäfer et al. arXiv:0910.1465, Lüscher Commun.Math.Phys.293:899-919,2010



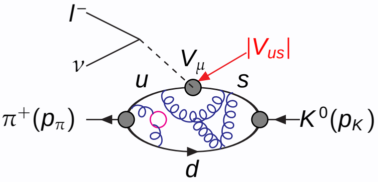
- very good agreement
- no (sea-)strange-quark effects visible at the current precision of data



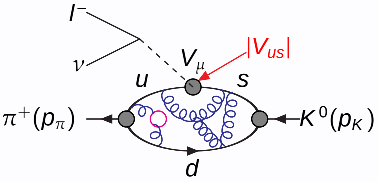
- very good agreement
- no (sea-)strange-quark effects visible at the current precision of data
- FLAG averages:

$N_f = 2+1$	$f_K/f_\pi = 1.193(6)$	BMW 10, MILC 09A, HPQCD/UKQCD 08
$N_f = 2$	$f_K/f_\pi = 1.210(18)$	ETM 09

Results for $f_+^{K\pi}(0)$



$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K + p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K - p_\pi)_\mu$$



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$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\rm EW}[1+\Delta_{SU(2)}+\Delta_{\rm EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$

- I phase space integral (via FF shape from experiment)
- $S_{\rm EW}$ short distance EW corrections
- $\Delta_{SU(2)}$ Iso-spin breaking corrections
- $\Delta_{\rm EM}$ long distance EM corrections

Antonelli et al., arXiv:1005.2323 (KLOE, KTeV, ISTRA+, NA48): $|V_{us} f_+^{K\pi}(0)| = 0.2163(5)$

$\rightarrow$ sub-1%-precision for $f_+^{K\pi}(0)$ required

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	$f_+(0)$	FlaviA net Lattice WG
RBC/UKQCD 10	2+1	A	●	★	■	$0.9599(34)(^{+31}_{-47})(14)$	
RBC/UKQCD 07	2+1	A	●	★	■	0.9644(33)(34)(14)	
ETM 09A	2	A	●	●	●	0.9560(57)(62)	
QCDSF 07	2	C	■	★	■	$0.9647(15)_{stat}$	
RBC 06	2	A	■	★	■	0.968(9)(6)	
JLQCD 05	2	C	■	★	■	0.967(6) and 0.952(6)	

$\rightarrow$ precision $\approx 0.5\%$ possible

RBC+UKQCD Phys.Rev.Lett.100:141601,2008, arXiv:1004.0886

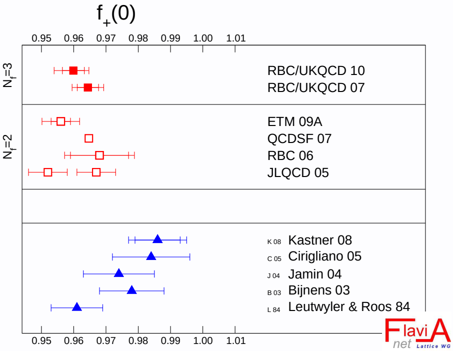
Source uncertainty/error	uncertainty/error on $f_+^{K\pi}(0)$
statistical	0.3%
chiral extrapolation	0.4%
continuum extrapolation	0.1%
total systematic	0.4%
total	0.5%

RBC+UKQCD Phys.Rev.Lett.100:141601,2008, arXiv:1004.0886

Source uncertainty/error	uncertainty/error on $f_+^{K\pi}(0)$	
statistical	0.3%	
chiral extrapolation	0.4%	$\gtrsim$ 330MeV
continuum extrapolation	0.1%	
total systematic	0.4%	
total	0.5%	

- dominant uncertainty from chiral extrapolation

Results for $f_+^{K\pi}(0)$



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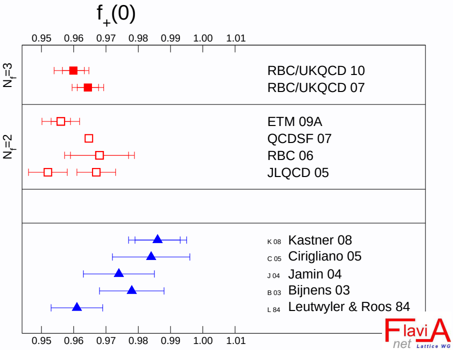

Comments:

- only two state-of-the art calculations:

$$
\begin{array}{lll}
N_f = 2+1 & f_+^{K\pi}(0) = 0.960(5) & \text{RBC+UKQCD 10} \\
N_f = 2 & f_+^{K\pi}(0) = 0.956(8) & \text{ETM 09A}
\end{array}
$$

- the $N_f = 2$ result is technically more advanced: lighter pion masses, 3 lattice spacings
- no (sea-)strange-quark effects visible at the current precision of data

input from experiment:

$$|V_{us}|f_+^{K\pi}(0) = 0.2163(5) \text{ and } \left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| = 0.2758(5)$$

Antonelli et al., arXiv:1005.2323

there are now two types of analysis that one can do:

1) assume Standard Model unitarity $|V_{ud}|^2 + |V_{us}|^2 = 1$ - using experimental input the four quantities $|V_{ud}|$, $|V_{us}|$, $f_+(0)$ and f_K/f_π reduce to a single unknown

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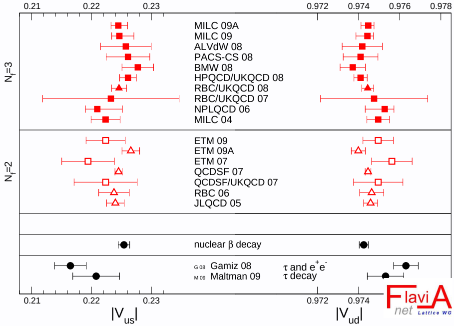

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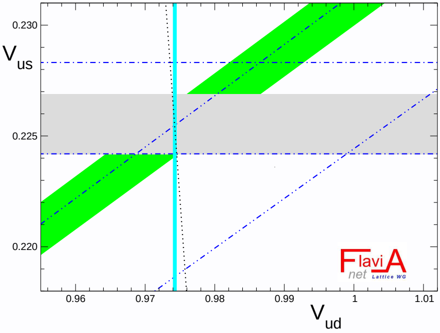

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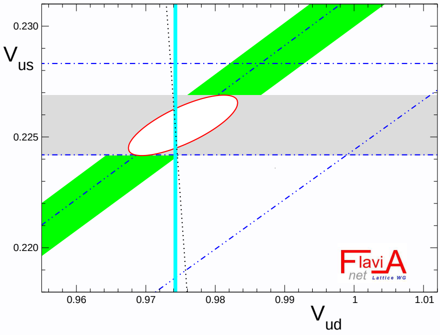

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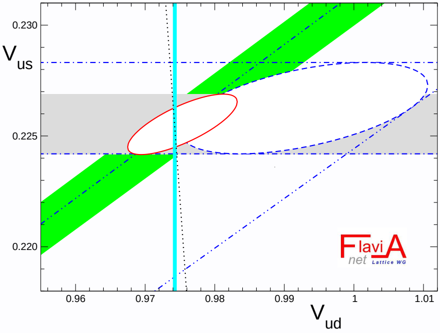

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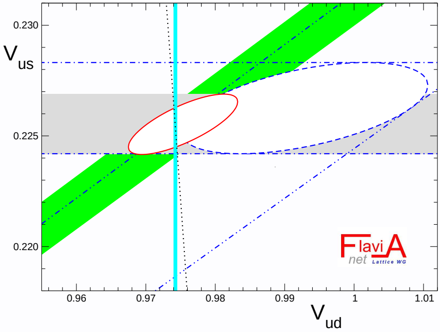

	exp. input	w/o $\|V_{ud}\|$	w/ $\|V_{ud}\|$
$\|V_u\|^2$:	$N_f = 2+1$	1.002(16)	$1.0000(7)_{f_+^{K\pi}(0)}$, $0.9999(7)_{f_K/f_\pi}$
	$N_f = 2$	1.037(36)	$1.0004(10)_{f_+^{K\pi}(0)}$, $0.9985(16)_{f_K/f_\pi}$

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$$|V_{us}|f_+^{K\pi}(0) = 0.2163(5) \text{ and } \left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| = 0.2758(5)$$

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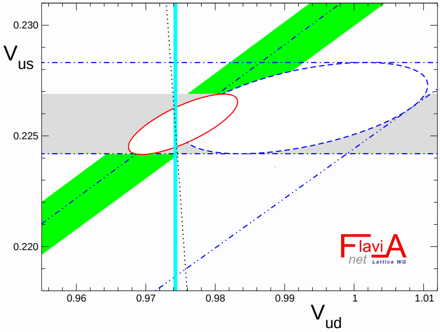

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- reducing *a* beyond ≈ 0.06fm turns out to be problematic
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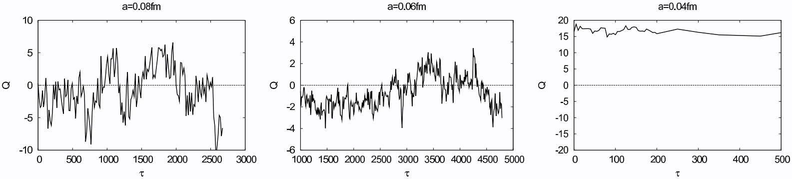
  - the problem gets worse as the lattice spacing is reduced as seen for example by CLS and MILC for the topological charge:

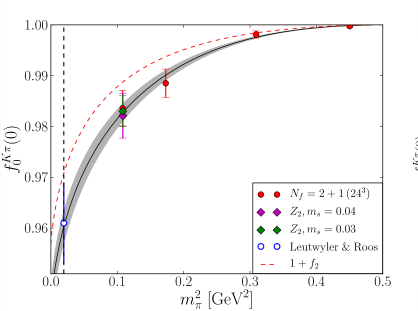


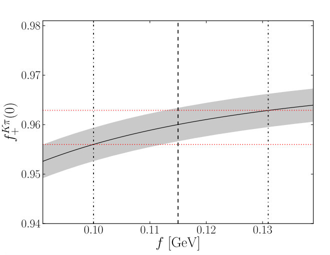
  - there is currently no cure and it seems that all simulations with *a* much smaller than 0.1fm are affected, i.e., have modes with very long correlation times
  - there is no theoretical understanding of which observables couple to slow modes and which ones don't *Schäfer's talk at Confinement 2010*
  - estimation of statistical errors at fine lattice spacing is therefore a delicate issue; risk of biased data

- reducing m_π: improved algorithms and/or more FLOP/s

example: $$f_0^{K\pi}(0) = 1 + \underbrace{f_2(f_\pi, m_\pi, m_K)}_{} + \underbrace{\Delta f}_{\text{lattice}}$$

Gasser & Leutwyler
Nucl. Phys. B250 (1985) 517538





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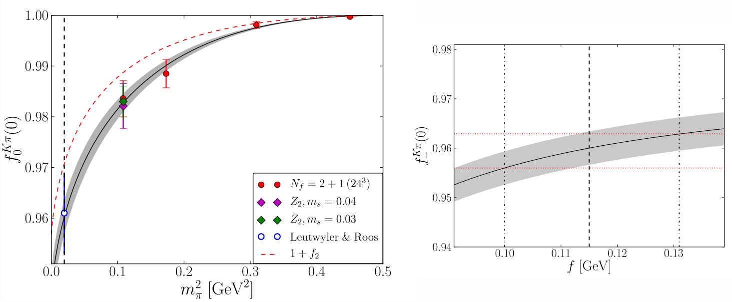

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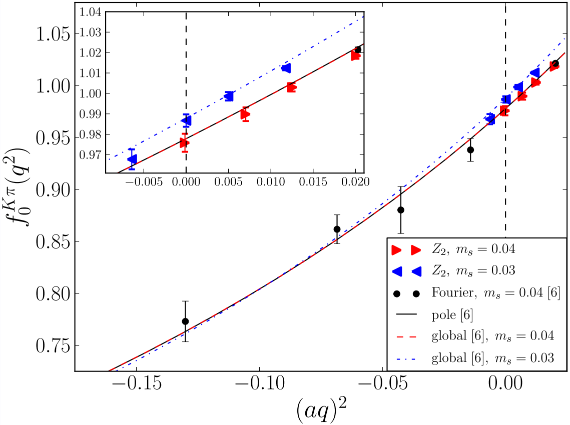
- systematics dominated by limited control of chiral extrapolation
- parametrization of NLO-term?; $f_0^{K\pi}(0)$ in NNLO χPT???
- some collaborations could in principle do simulation at the physical point - need to convince them to carry out a $f_+^{K\pi}(0)$-project
  what about BMW???

interpolation in q^2 - systematic due to interpolation in q^2 entirely removed through using partially twisted boundary conditions



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f_K/f_π

- reduction of m_π will soon reduce/remove systematic in chiral extrapolation
- lattice spacing a is already in critical range in advanced computations, is estimate of stat. error correct or is there even a bias?
- once algorithmic problems solved, increase statistics $\sigma \propto 1/\sqrt{N}$

$f_+^{K\pi}(0)$

- most important: reduce pion mass in simulations and motivate other collaborations to compute it

  if stat. error does not increase as m_π is reduced the overall error on $f_+^{K\pi}(0)$ has the potential to be reduced significantly!!! (currently chiral extrapolation error is dominant and about 0.4%)
- luckily systematic due to lattice spacing a is sub-dominant here
- increase statistics $\sigma \propto 1/\sqrt{N}$ (2nd largest contribution to overall error)