

V_{us} FROM HADRONIC τ DECAY

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OUTLINE

- *The basics*
- *A few relevant technical issues*
- *Results, assessment and the future*

THE BASIC IDEA

- With $I_{ij}^w(s_0)$, $ij = ud, us$, the $s \leq s_0$, $w(s)$ -weighted integrals over flavor ud , us $V+A$ τ decay distributions, $[\delta J^w(s_0)]_{OPE}$ the OPE representation for

$$\delta J^w(s_0) \equiv \frac{I_{ud}^w(s_0)}{|V_{ud}|^2} - \frac{I_{us}^w(s_0)}{|V_{us}|^2}$$

$$|V_{us}| = \sqrt{I_{us}^w(s_0) / \left[\frac{I_{ud}^w(s_0)}{|V_{ud}|^2} - [\delta J^w(s_0)]_{OPE} \right]}$$

- Typically $[\delta J^w(s_0)]_{OPE} \sim$ few to several % of $I_{ud}^w(s_0)$
 \Rightarrow modest OPE errors to get accurate $|V_{us}|$ [Gamiz et al., JHEP 0301: 060]

- V, A $ij = ud, us$, $(J) = (0 + 1), (0)$ spectral functions from experimental differential decay distributions

$$dR_{V/A;ij}/ds = 12\pi^2 |V_{ij}|^2 S_{EW} \left[w_{(00)}(y_\tau) \rho_{V/A;ij}^{(0+1)}(s) + w_L(y_\tau) \rho_{V/A;ij}^{(0)}(s) \right] / m_\tau^2$$

with $R_{V/A;ij} \equiv \frac{\Gamma[\tau \rightarrow \nu_\tau \text{ hadrons}_{V/A;ij}(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]}$, $y_\tau = s/m_\tau^2$

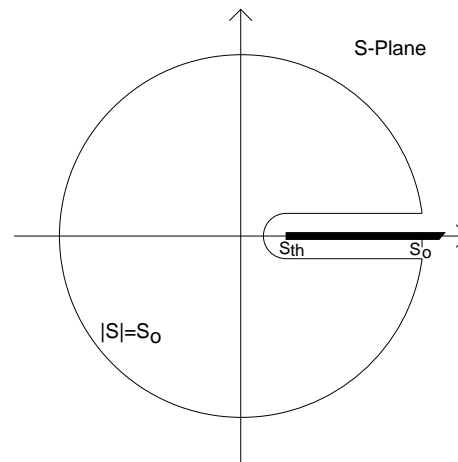
$$w_{(00)}(y) = (1 - y)^2(1 + 2y), \quad w_L(y) = -2y(1 - y)^2$$

- “longitudinal”: (0) part of $(0 + 1)/(0)$ decomposition

- $[\delta J^w(s_0)]_{OPE}$: OPE on RHS of FESR relation

$$\int_0^{s_0} w(s) \rho(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s) \Pi(s) ds$$

for correlators $\Pi(s) = \Pi_{ud,us;V/A}^{(0+1)}(s)$, $s\Pi_{ud,us;V/A}^{(0)}(s)$



(Data on LHS, OPE on RHS)

- Bad convergence of integrated $(J) = (0) D = 2$ OPE series \Rightarrow must treat phenomenologically. Fortunately
 - π, K contributions accurately known
 - strong continuum suppression ($\propto (m_i \mp m_j)^2$)
 - small us continuum contribution from us scalar, PS analyses (constrained by m_s)
 - impact on V_{us} small (~ 0.0002 or less)
- \Rightarrow essentially $\Delta\Pi \equiv \Pi_{ud;V+A}^{(0+1)} - \Pi_{us;V+A}^{(0+1)}$ FESRs

A PUZZLE: THE CURRENT KINEMATIC $w_{(00)}(y)$ WEIGHT CASE RESULTS

- $s_0 = m_\tau^2$, kinematic weight $w_{(00)}(s) \Rightarrow I_{ud,us}^w$ from $B_{ud;TOT}, B_{us;TOT}$
- \Rightarrow recent improved us branching fractions sufficient for improved $|V_{us}|$ determination (for $s_0 = m_\tau^2$ AND $w_{(00)}(s)$ choice only)
- Experimentally more difficult inclusive $dR_{us;V+A}/ds$ distribution required for other $w(s)$ and/or s_0 [expected from BaBar, Belle, but still some time in future]

- The experimental situation:
 - $I_{ud}^w(s_0)$: $\sim 0.5\%$ errors for range of w and s_0 [ALEPH 2005 data and covariances]
 - $I_{us}^w(s_0)$: pre-2007 us errors $\sim 3 - 4\%$ [ALEPH99 distribution, rescaled mode-by-mode for exclusive us B changes] ($\Rightarrow \sim 1.5 - 2\%$ on $|V_{us}|$)
 - Recent improved B values for several us exclusive modes [BaBar, Belle] (but not dR_{us}/ds)
 - Current $B_{us;TOT}$ error 2.0% [Lusiani, ICHEP10] ($\Rightarrow 1\%$ on $|V_{us}|$)

- Results of conventional $w_{(00)}(s)$ analysis:
 - CIPT+Adler function/CIPT+correlator $D = 2$ OPE evaluations used previously in literature, $K_{\mu 2}$ for K contribution, yield updated $|V_{us}|$ results
 - $0.2166(22)_{exp}(5??)_{th}$ (CIPT + Adler function)
 - $0.2162(22)_{exp}(5??)_{th}$ (CIPT + correlator)
 - $|V_{us}|$ nominally 3.6σ low c.f. 3-family unitarity expectations, $K_{\ell 3}$ and $\Gamma[K_{mu2}]/\Gamma[\pi_{\mu 2}]$
 - (More on ?? in nominal theory error later)

WHAT'S GOING ON?

- Problem(s) with the ud $V+A$ data?
- Problem(s) with the us $V+A$ data? (Especially possible missing higher multiplicity modes at higher s)
- Underestimate of theory uncertainties/unreliable central OPE values?
- None of the above, i.e., new physics?

INVESTIGATING THE POSSIBILITIES

- Definite problem(s) if $|V_{us}|$ not independent of *both* s_0 and $w(s)$
- Use of polynomial $w(y) = \sum_m b_m y^m$, $y = s/s_0$ to test/check higher D OPE contributions ($D = 2k \propto 1/s_0^{k-1}$, “absent” if $b_{k-1} = 0$)
- Alternate $D = 2$ OPE prescriptions differing only at higher order in α_s than truncation order (CIPT+correlator, CIPT+Adler function, FOPT) should give results compatible within $D = 2$ truncation error estimate

- “Supplementary” us V+A FESRs
 - Remove ud V+A data as potential problem source
 - $O(m_s^2 \alpha_s^m)$ $D = 2$ coefficients almost identical to those of $ud - us$ V+A series
 - IF OPE OK, problem due to missing higher s us spectral strength $\Rightarrow |V_{us}|$ *must* be larger at lower s_0
 - $|V_{us}|$ lower at lower $s_0 \Rightarrow$ definite OPE problem (additional us data problem not precluded)
 - CAUTION: presence of $D = 0$, $\langle \alpha_s G^2 \rangle$ $D = 4$ OPE contributions \Rightarrow some increase in OPE error

Problems with the ud data?

- τ vs CVC+IB electroproduction expectation discrepancy for $\pi\pi$ [minor for BaBar EM, non-trivial for KLOE, Novosibirsk]
- Similar τ vs EM discrepancy for 4π [still non-trivial, even for preliminary BaBar LP07 4π EM]
- HOWEVER, correlations in PDG global τ branching fractions fit dominantly to “nearby multiplicity” non-strange modes \Rightarrow impact on $|V_{us}|$ likely small

Problems with the us data?

- B for some moderately large exclusive modes not yet remeasured by B factories [Table]
- Missing modes above $s \sim 2 \text{ GeV}^2$ ($\delta V_{us} \sim 0.0004$ for each $\delta B \sim 10^{-4}$), e.g., for $w_{(00)}(s)$, $s_0 = m_\tau^2$,
 - $B[K^- \pi^0 \pi^0 \nu_\tau]$ up $3\sigma \Rightarrow \delta|V_{us}| = +0.0025$
 - $B[(K 3\pi)^- \nu_\tau]$ up $3\sigma \Rightarrow \delta|V_{us}| = +0.0030$
 - ALEPH99 $K 4\pi$ rough estimate $\Rightarrow \delta|V_{us}| = +0.0013$
- (See later, however, re s_0 stability issues etc.)

PRE-2007 vs Lusiani ICHEP10 *us* \mathcal{B} VALUES

Mode	\mathcal{B}_{2006} (%)	$\mathcal{B}_{ICHEP10}$ (%)
K^- [τ decay]	0.685(23)	0.696(10)
(Alt: [$K_{\mu 2}$])	(0.715(3))	(0.715(3))
$K^- \pi^0$	0.454(30)	0.431(15)
$\bar{K}^0 \pi^-$	0.878(38)	0.827(18)
$K^- \pi^0 \pi^0$	0.058(24)	0.060(22) [**]
$\bar{K}^0 \pi^0 \pi^-$	0.360(40)	0.349(15)
$K^- \pi^- \pi^+$	0.330(50)	0.294(7)
$K^- \eta$	0.027(6)	0.016(2)
$(\bar{K} \eta \pi)^-$	0.029(9)	0.0141(19)
$(\bar{K} 3\pi)^-$	0.141(37)	0.165(39) [**]
$K \phi$		0.0037(1)
$(\bar{K} 4\pi)^-$ (est'd)	0.011(7)	[**]
$(\bar{K} 5\pi)^-$ (est'd)	0.006	[**]
TOTAL	2.973(86)	2.857(58)
	(3.003(83))	(2.876(58))

OPE Problems?

- Key OPE problem: slow $D = 2$ ($0 + 1$) series convergence at the correlator level
- $\Delta\Pi(Q^2) \equiv \Pi_{ud;V+A}^{(0+1)} - \Pi_{us;V+A}^{(0+1)}$, $\Delta\rho(s)$: correlator and corresponding spectral function for $ud-us$ $V+A$ FESRs
- $D = 2$ OPE series, $\bar{m}_s = m_s(Q^2)$, $\bar{a} = \alpha_s(Q^2)/\pi$, \overline{MS} scheme [Baikov, Chetyrkin, Kuhn PRL95:012003]

$$\begin{aligned} [\Delta\Pi(Q^2)]_{D=2} = & \frac{3}{2\pi^2} \frac{\bar{m}_s}{Q^2} \left[1 + 2.333\bar{a} + 19.933\bar{a}^2 \right. \\ & \left. + 208.746\bar{a}^3 + (2378 \pm 200)\bar{a}^4 + \dots \right] \end{aligned}$$

- $a(m_\tau^2) \sim 0.1 \Rightarrow$ very slow convergence at spacelike point on $|s| = s_0$, even for maximum $s_0 = m_\tau^2$
- (Not surprisingly) integrated $D = 2$ ($0 + 1$) series convergence typically also slow, e.g., for $s_0 = m_\tau^2$, $w_{(00)}(s)$ with CIPT+Adler function (1^{st} line), CIPT+correlator (2^{nd} line), FOPT (3^{rd} line) $D = 2$ prescriptions, , to $O(\bar{a}^4)$:

$$\sim [1 + 0.286 + 0.103 - 0.039 - (0.197) + \dots]$$

$$\sim [1 + 0.151 + 0.017 - 0.120 - (0.293) + \dots]$$

$$\sim [1 + 0.405 + 0.257 + 0.154 + (0.081) + \dots]$$

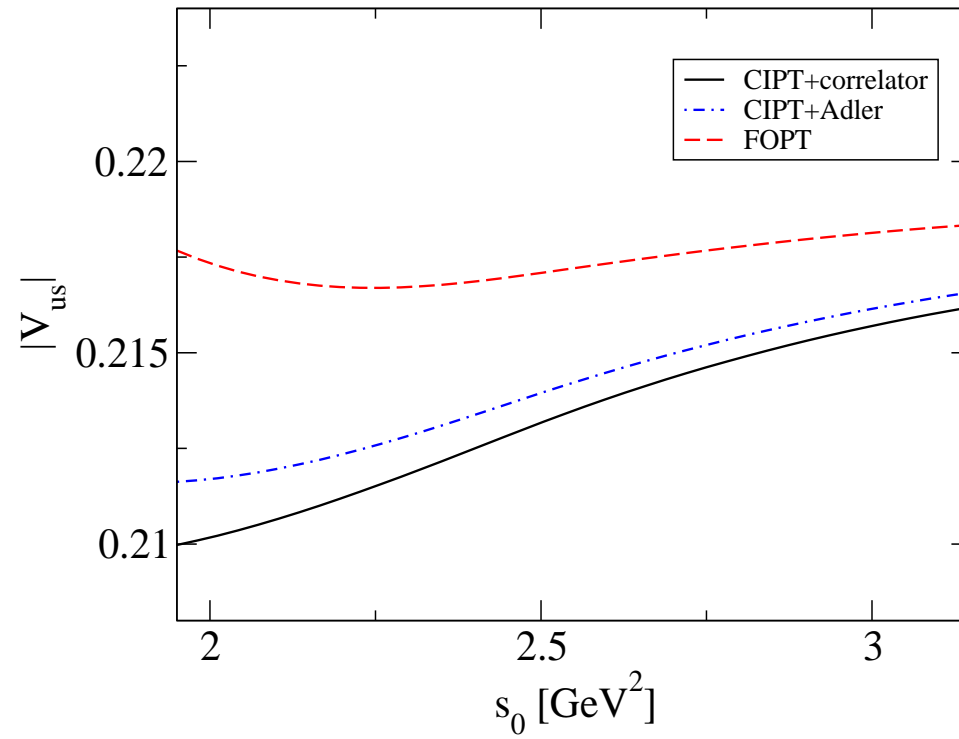
- Options for dealing with the slow $D = 2$ convergence:
 - take advantage of improved convergence in CIPT away from spacelike point via choice of weight [Here: w_{20} , \hat{w}_{10} , w_{10} of PRD62 (2000) 093020]
 - FESRs for alternate flavor-breaking correlator combinations with suppressed $D = 2$ OPE at correlator level [involves combination of EM, τ decay data]
 - s_0 -stability checks as crucial test

SOME ILLUSTRATIVE RESULTS

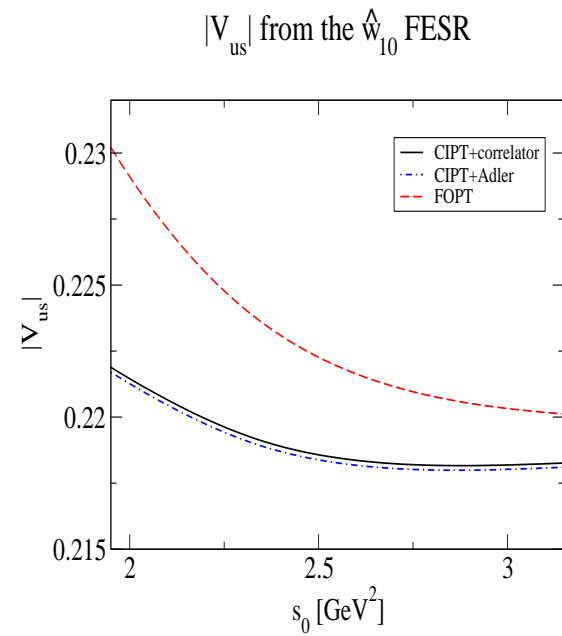
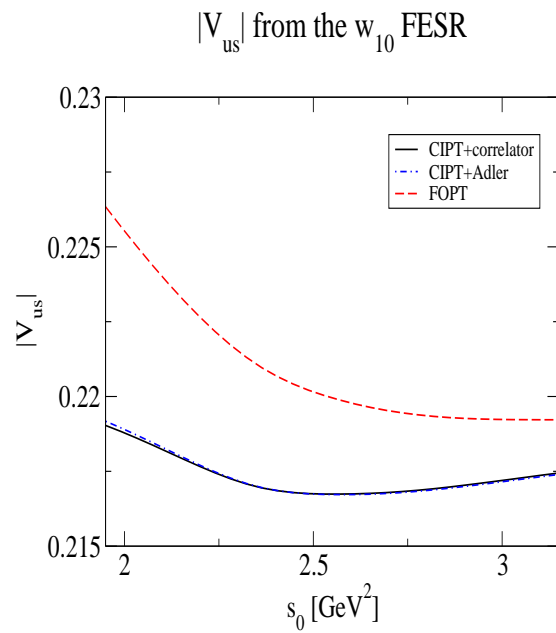
- ud V+A spectral integrals, errors from ALEPH 2005 data, covariances
- us V+A spectral integrals results using $K_{\mu 2}$ input, mode-by-mode rescaled ALEPH 1999 us distribution to handle $w \neq w_{(00)}$, $s_0 \neq m_{\tau}^2$ cases
- Rescaling necessary as updated distributions currently available only for $K^{-}\pi^{+}\pi^{-}$, $K^{-}K^{+}K^{-}$ [BaBar]
- However, test of rescaling for weighted $K^{-}\pi^{+}\pi^{-}$ integrals (BaBar vs rescaled ALEPH99) shows rescaling very reliable for central values, despite large rescaling

s_0 -STABILITY, $w_{(00)}$ FESR

$|V_{us}|$ from the $w_{(00)}$ FESR



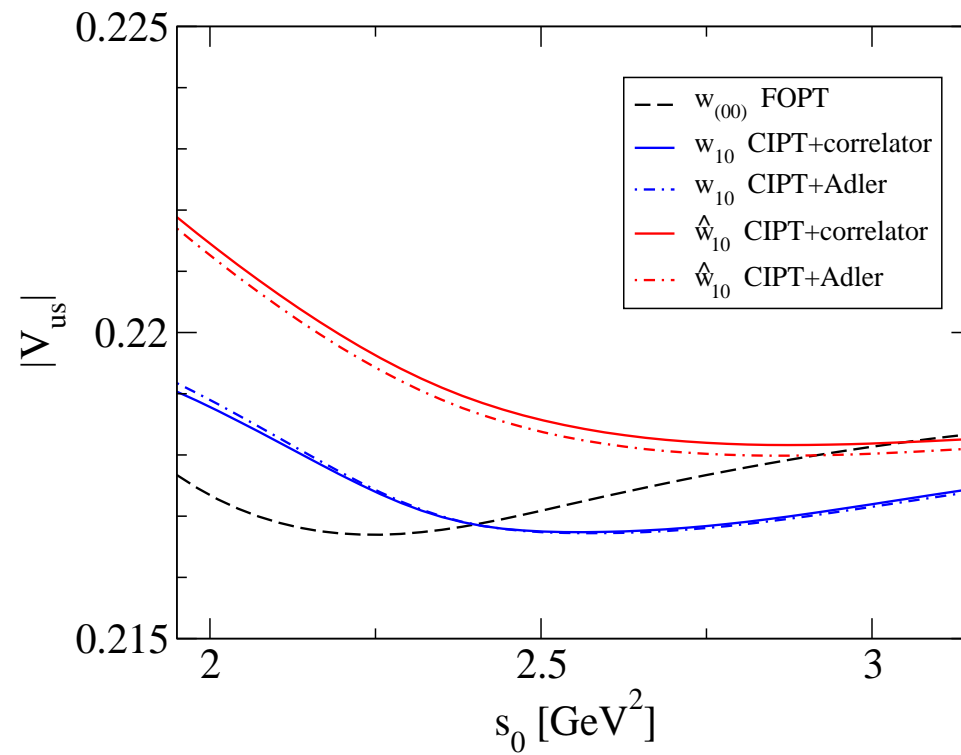
s_0 -STABILITY, w_{10} , \hat{w}_{10} FESRs



CAUTION: VERY slow FOPT convergence

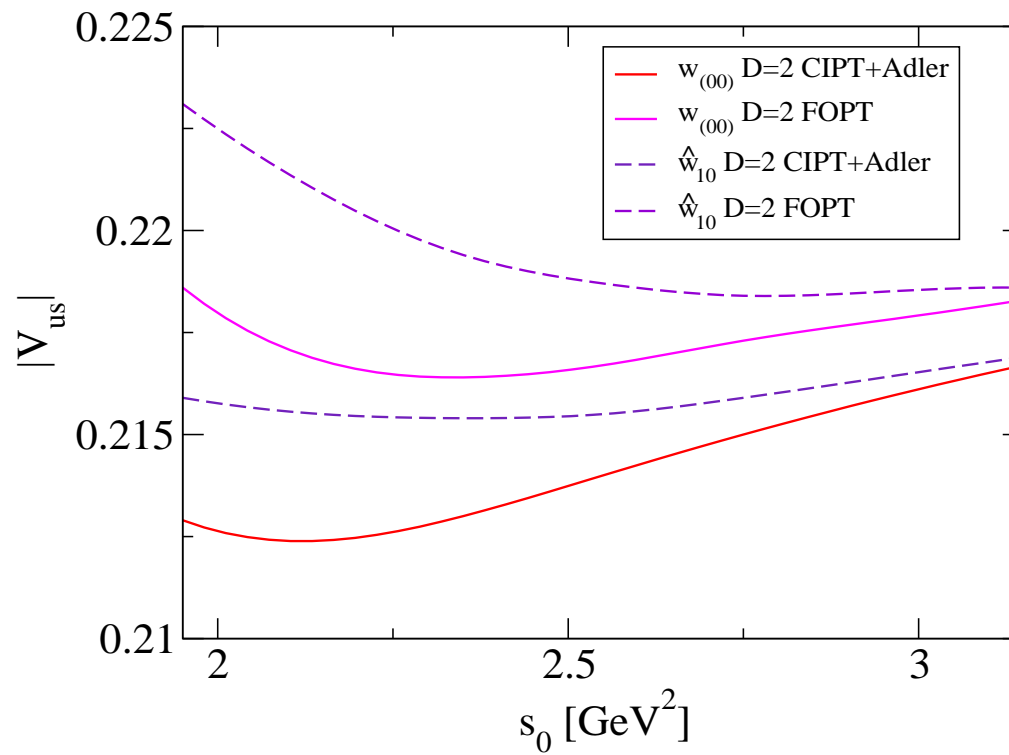
OK STABILITY, D=2 CONVERGENCE CASES

$|V_{us}|$ from reasonable stability/convergence ud-us FESRs



THE us V+A FESRs

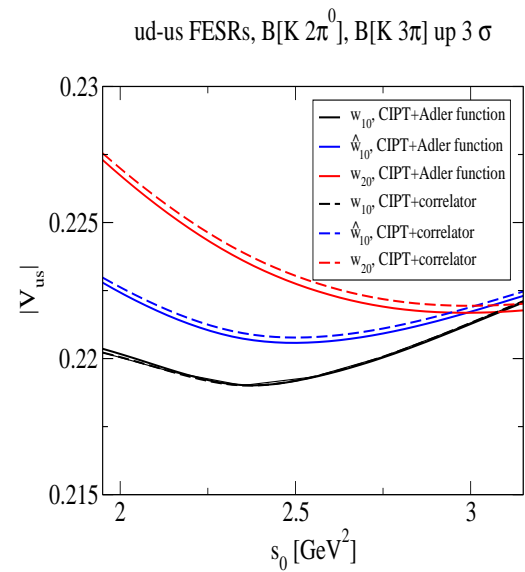
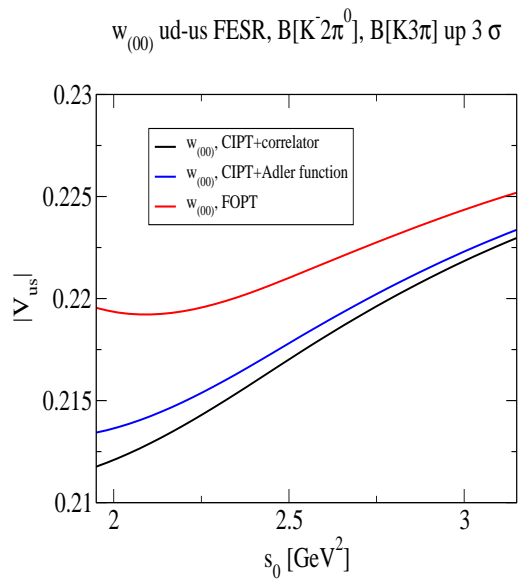
$|V_{us}|$ from the us V+A FESRs



Impact of 3σ B increases for largest us modes not yet remeasured by BaBar or Belle: $|V_{us}|$ vs s_0

$w_{(00)}$ FESR

$w_{10}, \hat{w}_{10}, w_{20}$ FESRs



ALTERNATE EM- τ FESRs

- Slow convergence of the integrated $D = 2$ OPE series for $\Delta\Pi$ due to slow convergence at the correlator level (for scales kinematically accessible in τ decay)
- Suggests trying alternate flavor-breaking combinations with suppressed $D = 2$ OPE contributions, e.g.,

$$\Delta\Pi^{EM,\tau} \equiv 9\Pi_{EM} - \left[5\Pi_{ud;V} - \Pi_{ud;A} + \Pi_{us;V+A} \right]$$

(same normalization for us V+A as in $\Delta\Pi$)

- $D = 2$ suppression choice also suppresses $D = 4$

– $D = 2$

$$\left[\Delta\Pi(Q^2) \right]_{D=2} = \frac{3}{2\pi^2} \frac{\bar{m}_s}{Q^2} \left[1 + \frac{7}{3}\bar{a} + 19.933\bar{a}^2 + 208.75\bar{a}^3 + \dots \right]$$

$$\left[\Delta\Pi^{EM,\tau}(Q^2) \right]_{D=2} = \frac{3}{2\pi^2} \frac{\bar{m}_s}{Q^2} \left[0 + \frac{1}{3}\bar{a} + 4.3839\bar{a}^2 + 44.943\bar{a}^3 + \dots \right]$$

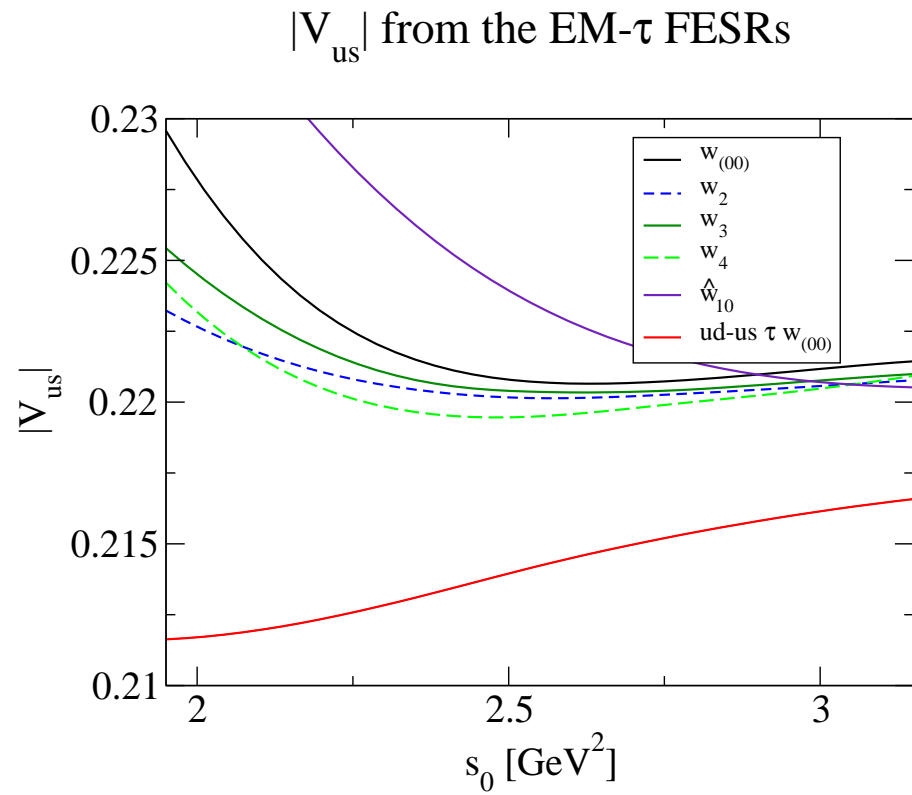
– $D = 4$

$$\left[\Delta\Pi(Q^2) \right]_{D=4} = \frac{\langle m_s \bar{s}s \rangle - \langle m_\ell \bar{\ell}\ell \rangle}{Q^4} \left[-2 - 2\bar{a} - \frac{26}{3}\bar{a}^2 \right]$$

$$\left[\Delta\Pi^{EM,\tau}(Q^2) \right]_{D=4} = \frac{\langle m_s \bar{s}s \rangle - \langle m_\ell \bar{\ell}\ell \rangle}{Q^4} \left[0 + \frac{8}{3}\bar{a} + \frac{59}{3}\bar{a}^2 \right]$$

- Strong suppression of $D = 2, 4$ contributions $\Rightarrow w(y)$ usable even without improved $D = 2$ convergence, hence e.g. $w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N$
 - single integrated $D > 4$ contribution ($D = 2N + 2$) (up to $O(\alpha_s^2)$ corrections)
 - $D = 2N + 2$ suppressed by relevant w_N coefficient, $1/(N - 1)$
 - $1/s_0^N$ dependence provides handle on integrated $D = 2N + 2$ contributions
- **NOTE: $D > 4$ typically NOT suppressed at correlator level:** E.g. in VSA, $D = 6$ a factor of $9/2$ larger for $\Delta\Pi^{EM,\tau}$ than for $\Delta\Pi \Rightarrow$ small relevant coefficient values useful

MIXED τ -EM vs. the $w_{(00)}$ $ud - us$ FESR



CURRENT RESULTS/OBSERVATIONS

- The $ud - us$ V+A FESRs:
 - Clear s_0 -stability problem for $w_{(00)}$, CIPT $D = 2$; us V+A results \Rightarrow significant OPE component
 - Better convergence, stability with FOPT for $w_{(00)}$,
 $|V_{us}| = 0.2183(5)_{ud}(22)_{us}(??)_{th}$
(~ 0.0020 higher c.f. CIPT)
 - Best of improved CIPT convergence weights, \hat{w}_{10} ,
yields $|V_{us}| = 0.2182(5)_{ud}(22)_{us}(??)_{th}$
 - OPE uncertainties (s_0 -instability, $w(y)$ -dependence)
clearly much larger than $\delta V_{us} \sim 0.0005$ at present

- Upward B shifts for as-yet-unremeasured us modes could still shift $|V_{us}|$ significantly, but N.B. re stability issues

- EM- τ FESR results:

- Good s_0 -stability, $w(y)$ independence
- For $w_{(00)}$, $s_0 = m_\tau^2$, including variation with weight-choice in theory error (totally dominant)

$$|V_{us}| = 0.2214 (22)_{us;V+A} (5)_{ud;V,A} (28)_{EM} (6)_{th}$$

- Theory errors much better BUT experimental errors much worse c.f. $ud-us$ $V+A$ (EM- τ spectral integral differences, with independent errors)

FUTURE PROSPECTS/DIRECTIONS

- The $ud - us$ V+A FESRs:
 - Many us B errors already reduced, others still needed
 - Ingredients for full remeasurement of actual us spectral distribution in place and work in progress
 - Some obvious targets for near term BaBar, Belle attention ($K^- \pi^0 \pi^0$, $K3\pi$, $K4\pi$, ...)
 - Updates on ud 2π , 4π τ decay modes desirable
 - Better understanding of $D = 2$ OPE truncation error needed to significantly reduce theory error

- The flavor-breaking EM- τ FESR:
 - us $V+A$ error reductions as per the $ud - us$ $V+A$ FESRs
 - Much improved s_0 -stability, $w(y)$ -independence compatible with OPE as significant error source for $ud - us$ $V+A$ FESRs
 - Need resolution of EM vs τ $\pi\pi$ and 4π issues
 - Significantly reduced $I = 0$ EM cross-section errors needed to make competitive with other methods

SUPPLEMENTARY PAGES

- Details on the handling of potential $D > 6$ OPE contributions
- Rough scale of longitudinal subtraction, $(0 + 1)$ OPE relative to ud spectral integrals
- Details on the integrated $D = 2$ for improved-CIPT-convergence Kambor-Maltman weights
- Impact of 3σ increases of $B[K^{-2}\pi^0]$, $B[K3\pi]$ on $|V_{us}|$ from the us V+A FESR

HIGHER D OPE CONTRIBUTIONS

- rough estimates for $D = 6$ condensates, $D > 6$ combinations unknown, usually assumed negligible
- $w(y) = \sum_m c_m y^m$, $y = s/s_0 \Rightarrow$ integrated $D = 2k + 2$
OPE $\propto c_k/s_0^k$ (up to logs) \Rightarrow avoid large c_k , $k \geq 2$
- neglect of non-negligible higher D terms $\Rightarrow s_0$ -instability of output \Rightarrow *need to study output as function of s_0*

RELATIVE SCALES IN THE $ud - us$ $V+A$ FESR

E.g., $ud - us$ $V+A$, $s_0 = m_\tau^2$ contributions:

- $R_{ud;V+A} = 3.478(16)$
- Longitudinal subtraction $\left[\delta R_\tau^{(0)} \right]_L = 0.1544(37)$
(0.1204 from K, π poles, 0.0340 from continuum)
- $\left[\delta R_\tau^{(0+1)} \right]_{OPE} = 0.0612(15)$ (Gamiz et al. 2008)
[90% of uncertainty from m_s^2 $D = 2$ scale]

CONVERGENCE OF w_{10} , \hat{w}_{10} and w_{20} -WEIGHTED
 $D = 2$ OPE SERIES FOR VARIOUS $D = 2$
PRESCRIPTIONS, $s_0 = m_\tau^2$

- First lines: CIPT + Adler function; second lines: CIPT + correlator; third lines: FOPT

- \hat{w}_{10} :

$$\sim [1 + 0.391 + 0.278 + 0.215 + (0.167) + \dots]$$

$$\sim [1 + 0.241 + 0.185 + 0.150 + (0.109) + \dots]$$

$$\sim [1 + 0.514 + 0.432 + 0.400 + (0.411) + \dots]$$

- w_{10} :

$$\sim [1 + 0.371 + 0.246 + 0.173 + (0.115) + \dots]$$

$$\sim [1 + 0.226 + 0.160 + 0.114 + (0.062) + \dots]$$

$$\sim [1 + 0.487 + 0.387 + 0.332 + (0.325) + \dots]$$

- w_{20} :

$$\sim [1 + 0.412 + 0.307 + 0.246 + (0.198) + \dots]$$

$$\sim [1 + 0.255 + 0.205 + 0.172 + (0.126) + \dots]$$

$$\sim [1 + 0.558 + 0.502 + 0.490 + (0.535) + \dots]$$

Impact of 3σ increases of $B[K^-2\pi^0]$, $B[K3\pi]$ on $|V_{us}|$
from the us V+A FESR

