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On the consistency between CP violation in the K^0 vs. B_d^0 systems within the Standard Model

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In the K^0 and B_d^0 systems, indirect CP violation is quantified by the parameters ϵ_K and $\sin 2\beta$ respectively. Within the Standard Model, the uniqueness of the CP violating phase implies that the measurement of either between ϵ_K and $\sin 2\beta$ permits to predict the other. Since both these parameters are very well measured, this turns into a powerful test of consistency. I discuss the status of this test, especially in the light of recent advances on the ϵ_K formula.

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1 Short statement of the problem

Within the Standard Model (SM), all of the wide phenomenology of flavor and CP violating (CPV) transitions is described in terms of one unitary matrix. Taking into account unphysical phase redefinitions in the quark fields, the CKM matrix, V, can be parameterized in terms of 3 Euler angles and one phase δ. Given the simplicity of this picture, the known high sensitivity of flavor and CPV processes to physics beyond the SM, and the amount of data at our disposal, the verification of the CKM mechanism is one of the most important SM tests.

In this contribution we focus on CP violation, in particular on its realization in meson-antimeson mixings, so-called 'indirect'. The latter is very accurately known in the $K^0 - \overline{K}^0$ system, from the parameter ϵ_K, of $O(10^{-3})$, and in the $B_d^0 - \overline{B}_d^0$ system, from the parameter $\sin 2\beta$, of O(1) instead. Data are now available also for the $B_s^0 - \overline{B}_s^0$ system, but, while very interesting, they are not yet comparable with those on ϵ_K and $\sin 2\beta$, and I will confine my discussion to the latter two quantities. Because of the unique CPV phase δ, the parameters ϵ_K and $\sin 2\beta$ are necessarily correlated within the SM, and this offers one of the most stringent tests of the CKM picture of CPV.

The main conclusion of this contribution is that the performance of the SM in the $\epsilon_K - \sin 2\beta$ correlation is less than perfect. In short, the problem can be stated as follows. Since the dominant contribution to ϵ_K is proportional to $\mathrm{Im}(V_{ts}V_{td}^*)^2$, with $V_{ts} \simeq -A\lambda^2$ and $V_{td} \simeq A\lambda^3(1 - \overline{\rho} - i\overline{\eta})$ [1], one has $\epsilon_K \propto (1 - \overline{\rho})\overline{\eta}$. Then, recalling that $1 - \overline{\rho} = R_t \cos\beta$ and $\overline{\eta} = R_t \sin\beta$ (with R_t and β one of the sides and one of the angles of the 'unitarity triangle' implied by the CKM matrix [2]), one sees that $\epsilon_K \propto \sin 2\beta$. Because the proportionality factor is calculable, knowledge of either between ϵ_K and $\sin 2\beta$ allows to predict the other. Concerning this proportionality factor, it has been pointed out in recent work [3, 4, 5] that subleading contributions to the ϵ_K formula – the main topic of this contribution – can be bundled in a non-negligible, negative, multiplicative correction, κ_ϵ, to ϵ_K. Therefore, for fixed $\sin 2\beta$, this correction would bring the $|\epsilon_K|$ central value down. Numerically, if one plugs in the golden-mode determination $\sin 2\beta_{\psi K_S} \simeq 0.67$, one finds a central value for $|\epsilon_K|$ of about 1.85×10^{-3}, to be compared with $|\epsilon_K|^{\rm exp} \simeq 2.2 \times 10^{-3}$.* Without (yet) any reference to associated errors, this difference seems to indicate that the level of consistency of the CKM picture for CPV is at no better than the 20% level (!). These statements will be made more quantitative in section 4.

*Equivalently, enforcing the experimental $|\epsilon_K|$ constraint instead, one finds a prediction for $\sin 2\beta$ higher than the experimental determination [6].

2 The ϵ_K formula and the κ_ϵ correction

A general theoretical formula for ϵ_K is the following

$$\epsilon_K \;=\; e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im} M_{12}}{\Delta M_K} + \xi \right) \;, \tag{1}$$

where $\Delta M_K \equiv m_{K_L} - m_{K_S} \simeq 3.5 \times 10^{-15}$ GeV and $\Delta\Gamma_K \equiv \Gamma_{K_L} - \Gamma_{K_S} \simeq -7.4 \times 10^{-15}$ GeV. Accidentally, $\Delta\Gamma_K \simeq -2\Delta M_K$, hence one defines the phase $\phi_\epsilon \equiv \arctan(-\Delta M_K / \frac{1}{2}\Delta\Gamma_K) \simeq 43.5°$. The other quantities in eq. (1) are the amplitude for $\overline{K}^0 - K^0$ mixing, $M_{12} = \langle K^0 | \mathcal{H}_{\Delta S = 2} | \overline{K}^0 \rangle$, that is the part sensitive to non-SM contributions, and ξ, quantity on which I will return in more detail below and in section 3. Typical approximations in the literature consist in setting $\phi_\epsilon = 45°$ and $\xi = 0$. Refs. [3, 4] pointed out that both these corrections are negative and sum up to a total -8% correction. In fact, the combined effect of $\phi_\epsilon \neq 45°$ and $\xi \neq 0$ can be described by a multiplicative factor κ_ϵ such that

$$\epsilon_K = \kappa_\epsilon \, \epsilon_K(\phi_\epsilon = 45°, \xi = 0) \;, \tag{2}$$

and one gets the estimate $\kappa_\epsilon = 0.92 \pm 0.02$ [3]. This estimate can be understood as follows. By the κ_ϵ definition one easily obtains

$$\kappa_\epsilon \;=\; \frac{\sin\phi_\epsilon}{1/\sqrt{2}} \times \left(1 + \frac{\xi}{\sqrt{2}|\epsilon_K|} \right) \;, \tag{3}$$

where, in view of the smallness of ξ, we identify $|\epsilon_K|$ on the r.h.s. with $|\epsilon_K(\phi_\epsilon = 45°, \xi = 0)|$ The hard part is a quantitative estimate of ξ. One can however note that the combination $\xi/\sqrt{2}|\epsilon_K|$ also appears in the theoretical formula for the parameter ϵ'/ϵ_K, namely $|\epsilon'/\epsilon_K| = -\omega \frac{\xi}{\sqrt{2}|\epsilon_K|}(1 - \Omega)$ [2], where $\omega = 0.045$ is known very precisely, and quantifies the '$\Delta I = 1/2$ rule'. Enforcing equality of this formula with $|\epsilon'/\epsilon_K|_{\rm exp} = (1.65 \pm 0.26) \cdot 10^{-3}$, the problem of estimating ξ is restated into that of estimating Ω. This quantity is the ratio between so-called EW-penguin and QCD-penguin contributions to ϵ'/ϵ_K: both of them are calculable – with caveats on the knowledge of the hadronic matrix elements – using the work of refs. [7]. Proceeding as explained in sec. 4 of ref. [4], one finds $\Omega = 0.33(1 \pm 20\%)$, assuming the SM. Note that the still quite poor theoretical control on, especially, the QCD penguins is reflected in the 20% error on Ω, and the Ω estimate, in turn, results in $\xi/\sqrt{2}|\epsilon_K| = -0.054(1 \pm 25\%)$, where the error takes into account also the 15% experimental error on ϵ'/ϵ_K. It is to be observed that the large final error, 25%, on κ_ϵ, affects only marginally the total error on ϵ_K, since it is an error on a correction.

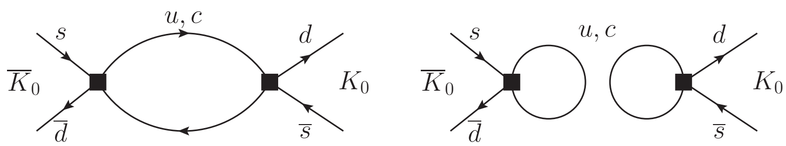

Figure 1: Contractions of the leading $|\Delta S| = 1$ four-quark effective operators contributing to the mixing amplitude at $O(G_F^2)$.

3 A closer look at the OPE for ϵ_K

Let us now look more closely at the meaning of the parameter ξ. This parameter is defined as $\xi = \mathrm{Im}A_0/\mathrm{Re}A_0$, with A_0 the amplitude for $K^0 \to (\pi\pi)_{I=0}$. A more inspiring interpretation arises by observing that the $|(\pi\pi)_{I=0}\rangle$ state largely saturates the neutral kaon decay widths. Denoting the absorptive part of the $\overline{K}^0 - K^0$ amplitude as Γ_{12}, one has therefore

$$\Gamma_{21} = \Gamma_{12}^* = \sum_f \mathcal{A}(K^0 \to f)\mathcal{A}(\overline{K}^0 \to f)^* \simeq (A_0)^2 \ , \tag{4}$$

where, as mentioned, in the last step we have approximated the final states to consist only of $|(\pi\pi)_{I=0}\rangle$. Eq. (4) implies

$$\frac{\mathrm{Im}\Gamma_{12}}{\mathrm{Re}\Gamma_{12}} \approx -2\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = -2\xi \ . \tag{5}$$

Therefore ξ is generated by the imaginary part of the *absorptive* contribution to the mixing amplitude, namely, by the absorptive part of the diagrams in Fig. 1 Comparing ξ as in eq. (5) with the ϵ_K formula (1), one realizes [5] that, if one is to include ξ, one should also compute and include for consistency the *dispersive* part of the diagrams in Fig. 1, because $\mathrm{Re}M_{12}$ and $\mathrm{Re}\Gamma_{12}$, appearing in the denominators (recall that $\Delta M_K \simeq 2\mathrm{Re}M_{12}$) are of the same order.

A consistent, beyond-leading-order evaluation of ϵ_K requires therefore to take into account: *(a)* non-local contributions to both $\mathrm{Im}M_{12}$ and $\mathrm{Im}\Gamma_{12}$ generated by the $O(G_F)$ dimension-six $\Delta S = 1$ operators; *(b)* local contributions to $\mathrm{Im}M_{12}$ generated by dimension-eight $\Delta S = 2$ operators of $O(G_F^2)$. The non-local contributions to $\mathrm{Im}\Gamma_{12}$, giving rise to ξ, have been estimated in Ref. [3] with the strategy outlined beneath eq. (3).

Concerning $\mathrm{Im}M_{12}$, it assumes the form $\mathrm{Im}M_{12} = \mathrm{Im}M_{12}^{SD} + \mathrm{Im}M_{12}^{LD}$, with $\mathrm{Im}M_{12}^{LD} = \mathrm{Im}M_{12}^{\rm non-local} + \mathrm{Im}M_{12}^{(8)}$, where $\mathrm{Im}M_{12}^{\rm non-local}$ and $\mathrm{Im}M_{12}^{(8)}$ are not separately scale independent. The structure of the dimension-eight operators obtained integrating out the charm, and an estimate of their impact on ϵ_K, has been presented in Ref. [8]. According to this estimate, $\mathrm{Im}M_{12}^{(8)}$ is less than 1% of the leading term.

The only potentially large long-distance contribution to $\mathrm{Im}M_{12}$ is therefore the contribution of the non-local terms enhanced by the $\Delta I = 1/2$ rule. Given their long-distance nature, these terms can be described within Chiral Perturbation Theory (ChPT), the limit of QCD valid at scales below the threshold for production of vector-meson states. Within ChPT, the $\Delta I = 1/2$ part of $\mathcal{H}_{\Delta S=1}$ consists of one operator only, with effective coupling G_8. Then by definition one must have $A_0 \propto G_8$. Thus, decomposing $\mathrm{Im}M_{12}^{LD}$ as a leading term proportional to G_8^2, plus a subleading term with different effective coupling, namely $\mathrm{Im}M_{12}^{LD} = \mathrm{Im}M_{12}^{LD}|_{G_8^2} + \mathrm{Im}M_{12}^{LD}|_{\mathrm{non}-G_8^2}$, one can write

$$\mathrm{Im}M_{12}^{LD}\big|_{G_8^2} = \mathrm{Re}M_{12}^{LD}\big|_{G_8^2} \times \frac{\mathrm{Im}[(G_8^*)^2]}{\mathrm{Re}[(G_8^*)^2]} \;, \tag{6}$$

whence, using $A_0 \propto G_8$ as mentioned, and eq. (5) one arrives at

$$\mathrm{Im}M_{12}^{LD}\big|_{G_8^2} \approx \mathrm{Re}M_{12}^{LD}\big|_{G_8^2} \times (-2\xi) \approx -\xi \times \left(\Delta m_K^{LD}|_{G_8^2}\right) \;. \tag{7}$$

This allow us to rewrite eq. (1) as follows [5]

$$|\epsilon_K| = \sin\phi_\epsilon \Big[\frac{\mathrm{Im}M_{12}^{(6)}}{\Delta m_K} + \xi \underbrace{\left(1 - \frac{\Delta m_K^{LD}|_{G_8^2}}{\Delta m_K}\right)}_{\rho} + \delta_{\mathrm{Im}M_{12}} \Big] \;, \tag{8}$$

where $\delta_{\mathrm{Im}M_{12}}$ encodes any contribution to $\mathrm{Im}M_{12}^{LD}$ *not* generated by the double insertion of the G_8 operator, and also $\mathrm{Im}M_{12}^{(8)}$. Eq. (8) defines the parameter ρ, whose deviation from 1 quantifies long-distance effects to $\mathrm{Im}M_{12}$. The estimate of the parameter ρ within ChPT is summarized in the section to follow.

3.1 Estimate of long-distance effects to $\mathrm{Im}M_{12}$ within ChPT

The lowest-order ChPT Lagrangian describing non-leptonic $\Delta S = 1$ decays has only two operators, transforming as $(8_L, 1_R)$ and $(27_L, 1_R)$ under the $SU(3)_L \times SU(3)_R$ chiral group [9]. Of these operators, only the $(8_L, 1_R)$ one has a phenomenologically large coefficient, describing the observed enhancement of $\Delta I = 1/2$ amplitudes. Hence, the only term in the effective Lagrangian relevant to our calculation is

$$\mathcal{L}^{(2)}_{|\Delta S|=1} = F^4 G_8 \left(\partial^\mu U^\dagger \partial_\mu U\right)_{23} + \mathrm{h.c.} \;, \tag{9}$$

where, as usual, the set of pion fields Φ is defined in $U = \exp(i\sqrt{2}\Phi/F)$ and F can be identified with the pion decay constant ($F \approx 92\,\mathrm{MeV}$). The fact that the $(8_L, 1_R)$ operator must describe $\Delta I = 1/2$ amplitudes implies that, at tree level, the phase of G_8 coincides with ξ: $\xi \equiv \mathrm{Im}A_0/\mathrm{Re}A_0 = \mathrm{Im}(G_8)/\mathrm{Re}(G_8)$. Furthermore, from the full ChPT formula for A_0 one can also estimate the G_8 magnitude to be $\mathrm{O}(G_F)$,

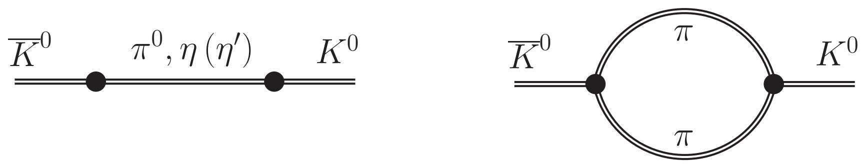

Figure 2: Tree-level and one-loop diagrams contributing to $\overline{K}^0$–K^0 mixing within ChPT.

$|G_8| \approx 9 \times 10^{-6}$ (GeV)$^{-2}$ [9]. Hence the full G_8 coupling can be determined from data on $K \to 2\pi$ amplitudes.

The ChPT diagrams contributing to Δm_K up to O(p^4) are depicted in Fig. 2. The tree-level, O(p^2), diagram in the left panel vanishes when using the so-called Gell-Mann–Okubo formula, namely the O(p^2) relation among π^0, η and kaon masses [10]. As a result, the first non-vanishing contribution to M_{12} generated by $\mathcal{L}^{(2)}_{|\Delta S|=1}$ arises only at O(p^4).

At O(p^4) the amplitude to be calculated consists of loop contributions with two insertions of $\mathcal{L}^{(2)}_{|\Delta S|=1}$, plus tree-level contributions with the insertion of appropriate O(p^4) counterterms, cancelling the renormalization scale dependence. Among all these O(p^4) contributions, the only model-independent, and presumably dominant, contribution to M_{12} is the non-analytic one generated by the pion-loop amplitude, $A^{(\pi\pi)}$, in Fig. 2 (right). Our calculation has been confined to this contribution, for the reasons to follow: (1) $A^{(\pi\pi)}$ is the only contribution which has an absorptive part. Hence it's the only contribution whose weak phase (the information relevant to our calculation) can be extracted from data; (2) It is the only contribution that survives in the limit of $SU(2)_L \times SU(2)_R$ ChPT, which is known to be successful phenomenologically (see e.g. Ref. [9]); (3) In the words of ref. [11], it is almost a theorem that kaon loops go into the redefinition of the local terms; and besides, there are doubts about the meaning of kaon loops, their effective threshold lying at $2m_K > m_\rho$ (!) [11].

3.2 Final phenomenological formula for ϵ_K

Using the calculation of $A^{(\pi\pi)}$ described in the previous section (for more details, see ref. [5]), we can estimate the contribution to ImM_{12} proportional to G_8, namely the ρ parameter entering eq. (8). We find [5]

$$\frac{\Delta m_K^{LD}|_{G_8^2}}{\Delta m_K^{\rm exp}} = \frac{2{\rm Re}M_{12}^{(\pi\pi)}}{\Delta m_K^{\rm exp}} = 0.4 \pm 0.2 \ . \qquad (10)$$

where the central value and error are determined by setting the renormalization scale $\mu = 800$ MeV and respectively varying it in the interval $0.6 \div 1$ GeV.

As a cross-check, our calculation shows good agrement with what one would expect from the rest of the contributions to Δm_K known as dominant. One has roughly $\rho \simeq (\Delta m_K^{SD} + \Delta m_K^{LD}|_{\eta'})/\Delta m_K^{\rm exp}$, with $\Delta m_K^{(6)} = (0.7 \pm 0.1)\Delta m_K^{\rm exp}$ [12] and $\Delta m_K^{LD}|_{\eta'} \approx -0.3\Delta m_K^{\rm exp}$ according to a recent analysis [13].

To recapitulate, our final phenomenological formula for ϵ_K is eq. (8), with $\rho = 0.6 \pm 0.3$ [5] (we conservatively increase the error by 50% to account for the subleading non-G_8^2 contributions). In terms of κ_ϵ defined in eq. (2), this implies $\kappa_\epsilon = 0.94 \pm 0.02$.

4 Error budget in ϵ_K, and status of the problem

In sec. 1 we argued that the $|\epsilon_K^{\rm SM}|$ central value is about 20% beneath the experimental figure. Of course the significance of this difference is in the hands of the associated errors. The main components to the $|\epsilon_K^{\rm SM}|$ error can be understood intuitively by focusing on the top-top contribution to $\epsilon_K^{\rm SM}$, which constitutes about 75% of the total result. They are as follows. First, the calculation of ${\rm Im} M_{12}^{(6)}$ involves estimating the non-perturbative matrix element between the single dim-6 SM operator and the external kaon states. This affects linearly $|\epsilon_K|$, via the parameter $\hat{B}_K$, which is known with an uncertainty of O(5%). For recent unquenched LQCD estimates, see [14].

Second, there is the CKM error. One way to parameterize the CKM matrix useful in this discussion is through the parameters λ, $\sin 2\beta$, $|V_{cb}|$ and R_t. The first two parameters are very well known, so let's focus on the other two. Recalling the CKM combination relevant to $|\epsilon_K^{\rm SM}|$, namely $(V_{ts}V_{td}^*)^2$, and that $|V_{ts}| \approx |V_{cb}|$ and $|V_{td}| \approx \lambda |V_{cb}|$, one sees that $|\epsilon_K^{\rm SM}| \propto |V_{cb}|^4$. A $|V_{cb}|$ error presently at the 2.5% level translates into a 10% error on $|\epsilon_K^{\rm SM}|$. This is the dominant source of error. Finally, recalling from sec. 1 that $\epsilon_K \propto (1 - \overline{\rho})\overline{\eta}$, one finds $\epsilon_K \propto R_t^2$, contributing another O(8%) component to the ϵ_K relative error. Fortunately, this component is going to become irrelevant once a very accurate determination of the CKM angle γ will be available from LHCb data. The above mentioned three components build up a total error of order 15% in $|\epsilon_K^{\rm SM}|$.

I open a parenthesis on two further directions relevant to a more precise assessment of the $\epsilon_K - \sin 2\beta$ SM correlation. On the one side, the effort towards a NNLO calculation of the coefficient η_{ct} [15], appearing in the short-distance calculation. Note in fact that there are cancellations between the *t*-*t* (73%), *c*-*t* (41%) and *c*-*c* (−14%) contributions, so each of them should be known with the best possible accuracy. On the other side, the possible role of doubly-Cabibbo-suppressed corrections to the CP asymmetries leading to the $\sin 2\beta$ estimate [16].

To conclude, the most rigorous way to quantify a possible problem in the $\epsilon_K -$

$\sin 2\beta$ SM correlation remains that of a fit [17]. The UTfit group confirms that the new contributions in ϵ_K generate some tension in particular between ϵ_K and $\sin 2\beta$, tension manifested by an indirect determination of $\sin 2\beta$ larger than the experimental value by about 2.6σ (see [19]). Furthermore, the overall picture is very usefully summarized in so-called compatibility plots. The CKMfitter group, with the 'Rfit' treatment of theory errors, finds no discrepancy [18]. This treatment may however be too conservative, and in fact they conclude that the matter needs further investigation.

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