Theory status of $|V_{cb}|$ inclusive

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The need to reexamine inclusive V_{cb}

- Discrepancy with exclusive determination, importance of $|V_{cb}|$ in UT determination: ε_K etc
- Results of fits to semileptonic & radiative moments are crucial input in inclusive $|V_{ub}|$ determination (mostly m_b and μ_π^2) and in normalizing $B\to X_s\gamma$ and $B\to X_s l^+l^-$
- b quark mass determinations from e^+e^- have recently improved significantly: how do they compare with fits? do we understand/trust theory errors? (see also Hoang talk)
- central m_b value from fits depends on radiative moments whose calculation is more problematic see G. Paz's talk

in collaboration with C. Schwanda, in progress

Inclusive semileptonic B decays: basic features

- ***Simple idea:*** inclusive decay do not depend on final state, factorize long distance dynamics of the meson. OPE allows to express it in terms of matrix elements of local operators

$$T\, J(x)\, J(0) \approx c_1 \bar{b} b + c_2 \bar{b}\, \vec{D}^2 b + c_3 \bar{b} \sigma \cdot G b + \ldots$$

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b} \left(i\vec{D} \right)^2 b \right| B \right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b} \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b \right| B \right\rangle_\mu$$

The total s.l. width in the OPE

$$\Gamma[\bar{B} \to X_c e \bar{\nu}] = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 g(r) \left[1 + \frac{\alpha_s}{\pi} p_c^{(1)}(r,\mu) + \frac{\alpha_s^2}{\pi^2} p_c^{(2)}(r,\mu) \right.$$

$$- \frac{\mu_\pi^2}{2m_b^2} + \left(\frac{1}{2} - \frac{2(1-r)^4}{g(r)} \right) \frac{\mu_G^2 - \frac{\rho_{LS}^3 + \rho_D^3}{m_b}}{m_b^2}$$

$$\left. + \left(8 \ln r - \frac{10r^4}{3} + \frac{32r^3}{3} - 8r^2 - \frac{32r}{3} + \frac{34}{3} \right) \frac{\rho_D^3}{g(r)\, m_b^3} \right]$$

$$+ O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)$$

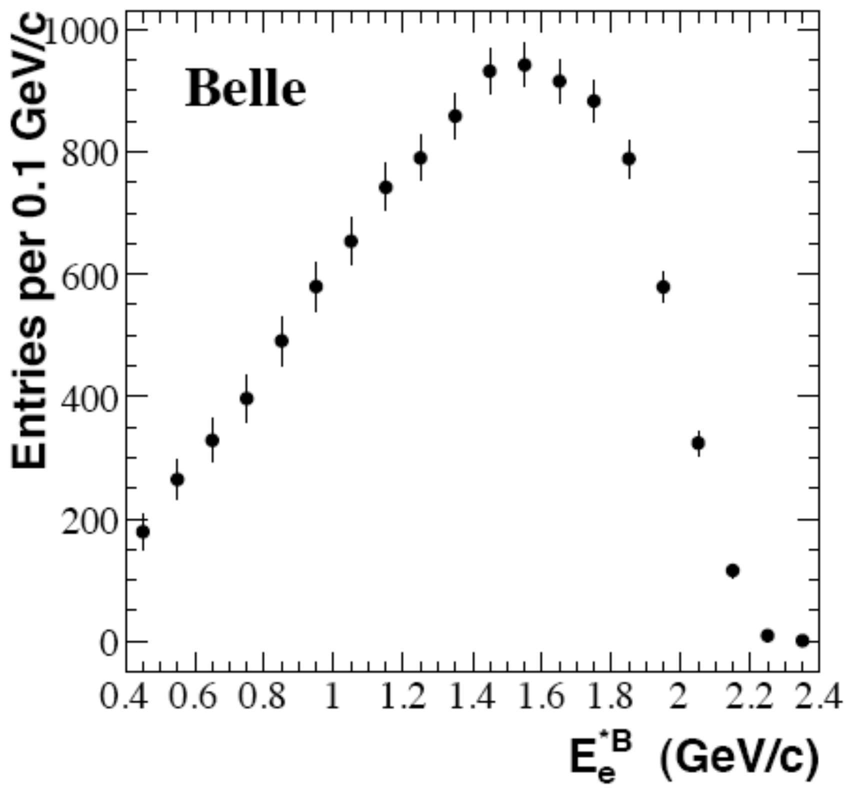
$$r = \frac{m_c^2}{m_b^2}$$

OPE valid for inclusive enough measurements, away from perturbative singularities ➠ moments

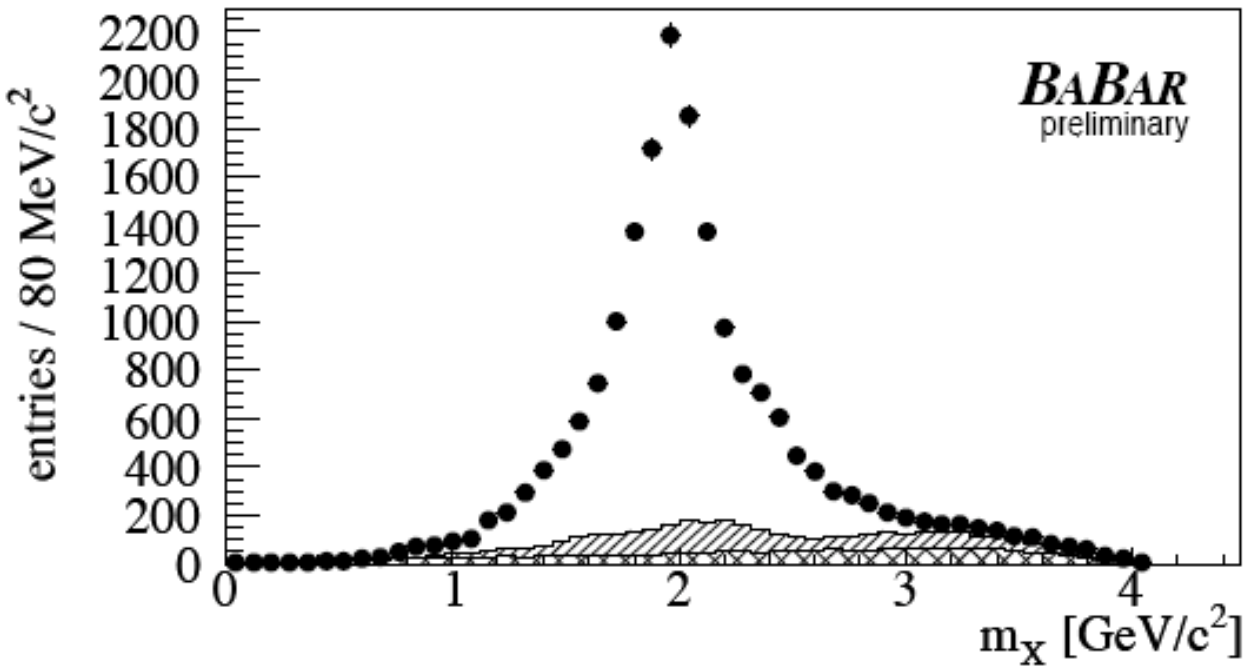
Present implementations include all terms through $O(\alpha_s^2 \beta_0, 1/m_b^3)$: $m_{b,c}$, $\mu^2_{\pi,G}$, $\rho^3_{D,LS}$ 6 parameters

Fitting OPE parameters to the moments

E_l spectrum



m_X spectrum



Total ***rate*** *gives $|V_{cb}|$, global* ***shape*** *parameters (moments of the distributions) tell us about B structure, m_b and m_c*

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications

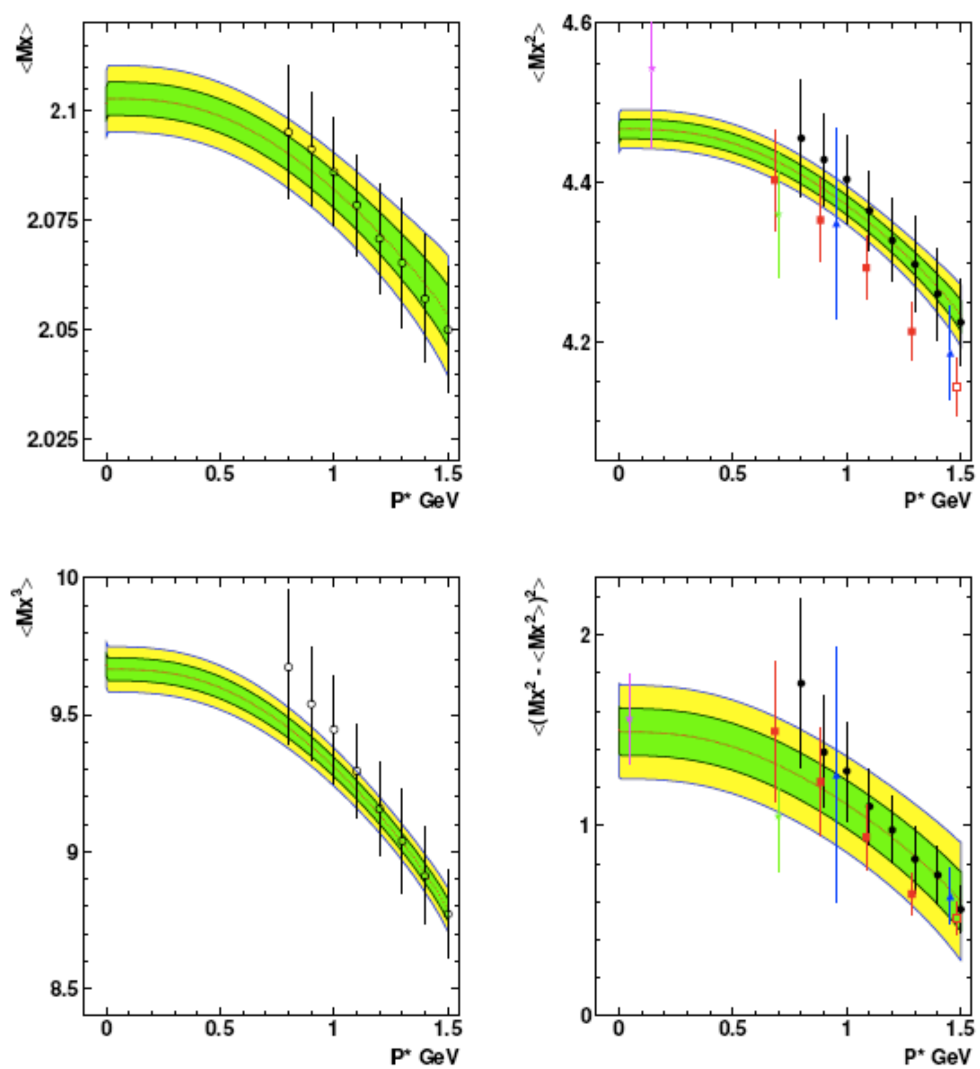
Global HFAG fit (kinetic scheme)

Inputs	$\lvert V_{cb}\rvert\ 10^3$	m_b^{kin}	χ^2/ndf
b→c & b→sγ	41.85(44)(58)	4.590(31)	29.7/59
b→c only	41.68(48)(58)	4.646(47)	24.2/48

Based on PG, Uraltsev, Benson et al

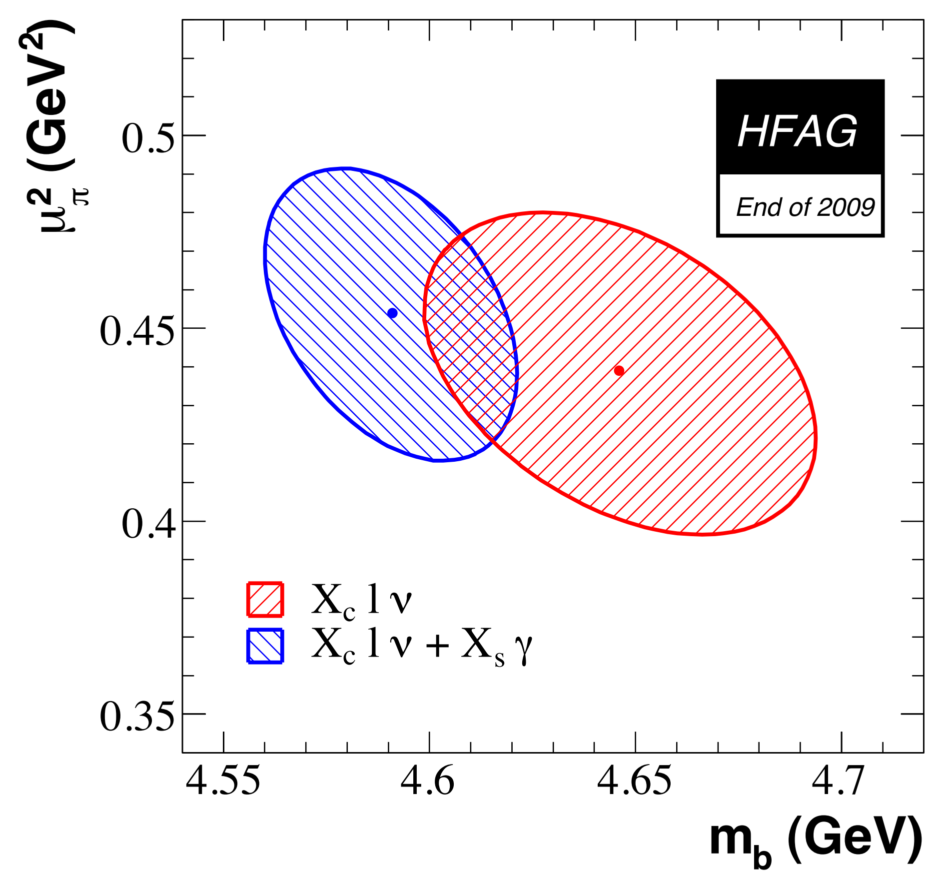
***In the kinetic scheme** the contributions of gluons with energy below $\mu\approx$ 1GeV are absorbed in the OPE parameters*

Here *scheme* means also a number of different assumptions, inclusion of different data, and a recipe for theory errors



Very close result for $\lvert V_{cb}\rvert$ in 1S scheme

Bauer Ligeti Luke Manohar Trott see *Christoph's talk*



Perturbative corrections

Complete 2loop corrections to width and moments with cuts known, either in expansion m_c/m_b or numerically Melnikov, Pak, Czarnecki, Biswas

In kinetic scheme with μ=1GeV

$$\Gamma[\bar{B} \to X_c e \bar{\nu}] \propto 1 - 0.96\,\frac{\alpha_s}{\pi} - 0.48\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + 0.82 \left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3) \approx 0.916$$

Good convergence, higher BLM studied by Uraltsev et al, small. Residual th error O(1%).

Perturbative corrections (II)

In normalized *leptonic moments* pert corrections cancel to large extent, in any scheme, for any cut: hard gluon emission is comparatively suppressed. In the kin scheme

$$\langle E_l \rangle_{E_l > 1\text{GeV}} = 0.681 \frac{m_b}{2} \left[1 + (3.179 - 3.199) \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 ((4.30 - 4.35)\beta_0 + 3.49(7) - 3.36(8) + 5.91 - 5.91) + O(1/m_b^2, \alpha_s^3) \right] \quad (1)$$

- same pattern of cancellations at $O(\alpha_s)$ $O(\beta_0\alpha_s^2)$ $O(\alpha_s^2)$ confirms our estimate of th error, no appreciable change in fit
- *Additional* cancellations in higher central moments due to endpoint enhancement: existing results confirm cancellation pattern but numerical precision is not always sufficient.

Implementation in hadronic moments under way, but we don't expect important effects

$O(\alpha_s/m_b^2)$ effects in $B\to X_s\gamma$

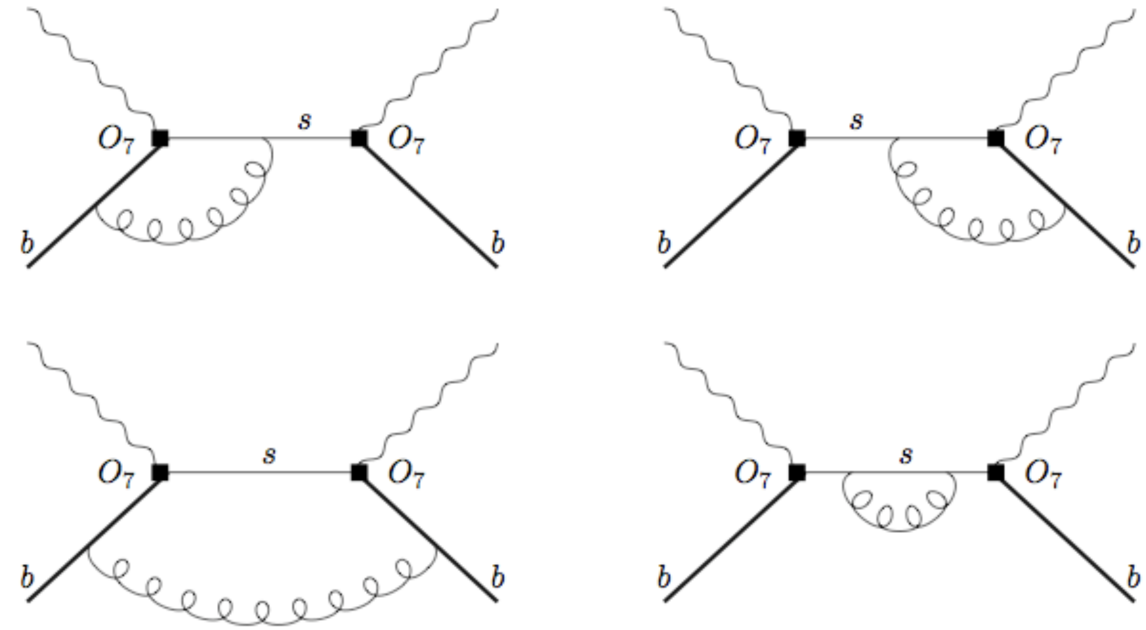
Ewerth,Nandi,PG arXiv:0911.2175

$$T\{\bar b(x)\sigma_{\mu\nu}P_L s(x)\bar s(0)\sigma_{\alpha\beta}P_R b(0)\} =$$

$$c_{\dim 3}O_{\dim 3} + \frac{1}{m_b}c_{\dim 4}O_{\dim 4} + \frac{1}{m_b^2}c_{\dim 5}O_{\dim 5} + \dots$$

$$O_b^\mu = \bar b\gamma^\mu b, \qquad O_2^{\mu\nu} = \bar b_v \frac{1}{2}\{iD^\mu, iD^\nu\}b_v\,,$$

$$O_1^\mu = \bar b_v iD^\mu b_v, \qquad O_3^{\mu\nu} = \bar b_v \frac{g_s}{2}G^{\alpha\mu}{}_\alpha\sigma^{\alpha\nu}T^a b_v\,,$$



One-loop matching onto local operators with HQET fields in dim reg

$$\frac{d\Gamma_{77}}{dz} = \Gamma_{77}^{(0)}\left[c_0^{(0)} + c_{\lambda_1}^{(0)}\frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(0)}\frac{\lambda_2(\mu)}{2m_b^2} + \frac{\alpha_s(\mu)}{4\pi}\left(c_0^{(1)} + c_{\lambda_1}^{(1)}\frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(1)}\frac{\lambda_2(\mu)}{2m_b^2}\right)\right]$$

$\lambda_{1,2}$ are HQET analogues of $\mu^2_{\pi,G}$

The coefficients are highly singular at the endpoint z=1:
$\delta(1\text{-}z)$, $\delta'(1\text{-}z)$, $\delta''(1\text{-}z)$, $[1/(1\text{-}z)^n]_+$ with $n\leq 3$

The NLO effect 10-20% in coefficients of first few moments, leading to $\delta m_b\sim 10$MeV, $\delta\mu_\pi^2 \sim 0.04$GeV^2 Extension to semileptonic case in progress

More on Higher Orders

- $O(\alpha_s \mu^2_\pi/m_b^2)$ are known numerically   Becher,Boos,Lunghi 2007
  they are not implemented yet, waiting for complete $O(\alpha_s/m_b^2)$

- $O(1/m_b^3)$ corrections ~3% in width, to have 1% accuracy we will need to compute $O(\alpha_s/m_b^3)$

- $O(1/m_b^4)$ corrections first computed by Dassinger et al. in 2006, new refined analysis by Mannel, Turczyk, Uraltsev to appear soon with $1/m^5$ as well.

- Structure of the expansion:
  Two large scales m_b and m_c

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 + \frac{1}{m_b^4}\Gamma_4 + \frac{1}{m_b^3}\log(m_c)\Gamma_{3,0} + \frac{1}{m_b^3}\frac{\alpha_s(m_b)}{m_c}\Gamma_{3,1} + \frac{1}{m_b^3}\frac{1}{m_c^2}\Gamma_{3,2} + \cdots$$

- The Γ_i and $\Gamma_{i,j}$ are regular as $m_c \to 0$
- The Γ_i and $\Gamma_{i,j}$ have perturbative expansions

see Bigi,Mannel,Turczyk,Uraltsev
Bigi,Uraltsev,Zwicki

Higher power corrections

Proliferation of non-pert parameters: for ex at $1/m_b^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by Ground State Saturation

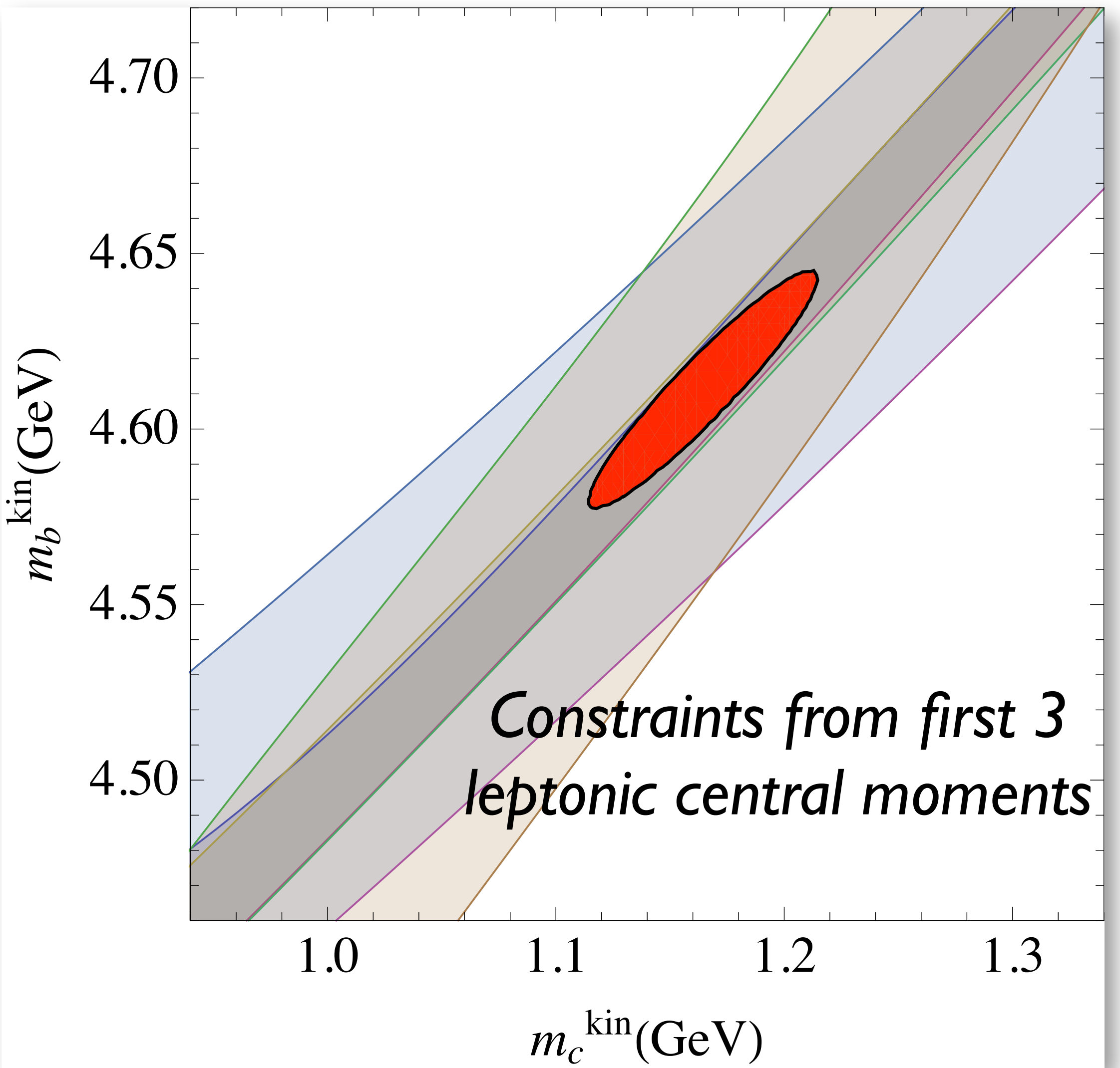
$$\frac{\delta\Gamma_{1/m^4} + \delta\Gamma_{1/m^5}}{\Gamma} \approx 0.013 \qquad \frac{\delta V_{cb}}{V_{cb}} \approx +0.4\%$$

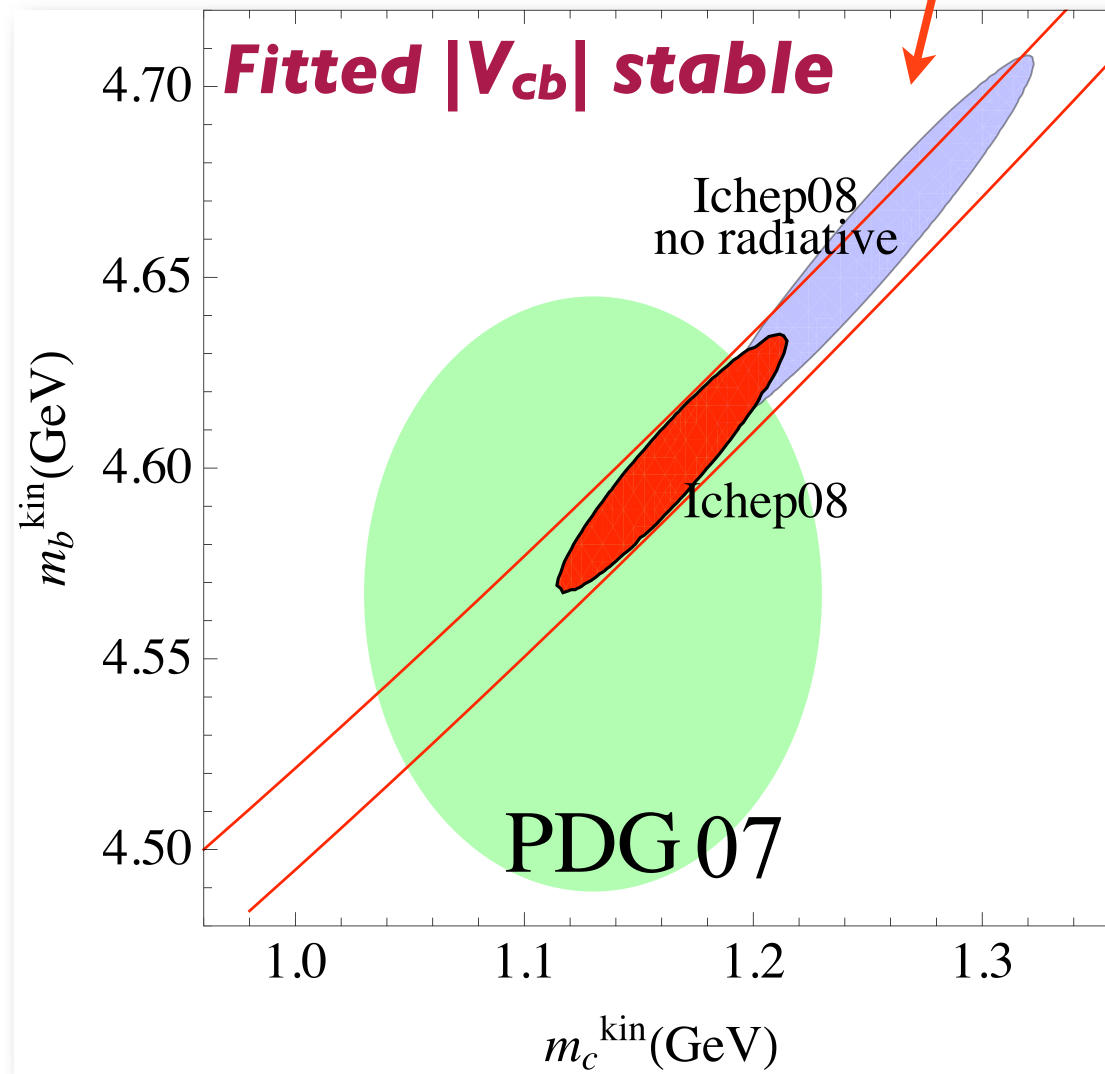
PRELIMINARY

after inclusion of the corrections in the moments. While this might set the scale of effect, *how much does it depend on assumptions on expectation values?*

A strip in the m_b-m_c plane

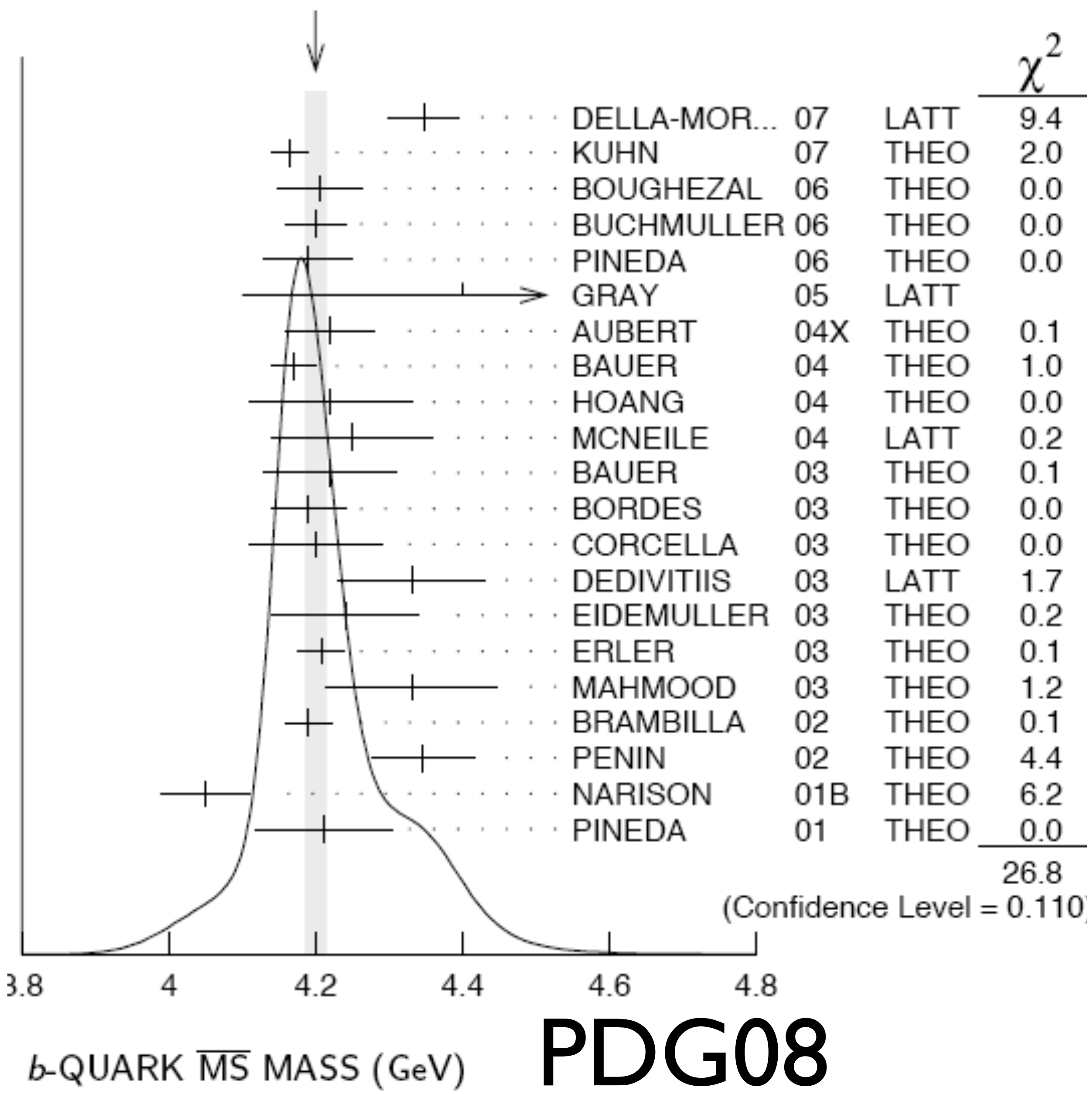
Constant values of s.l. width at fixed V_{cb}



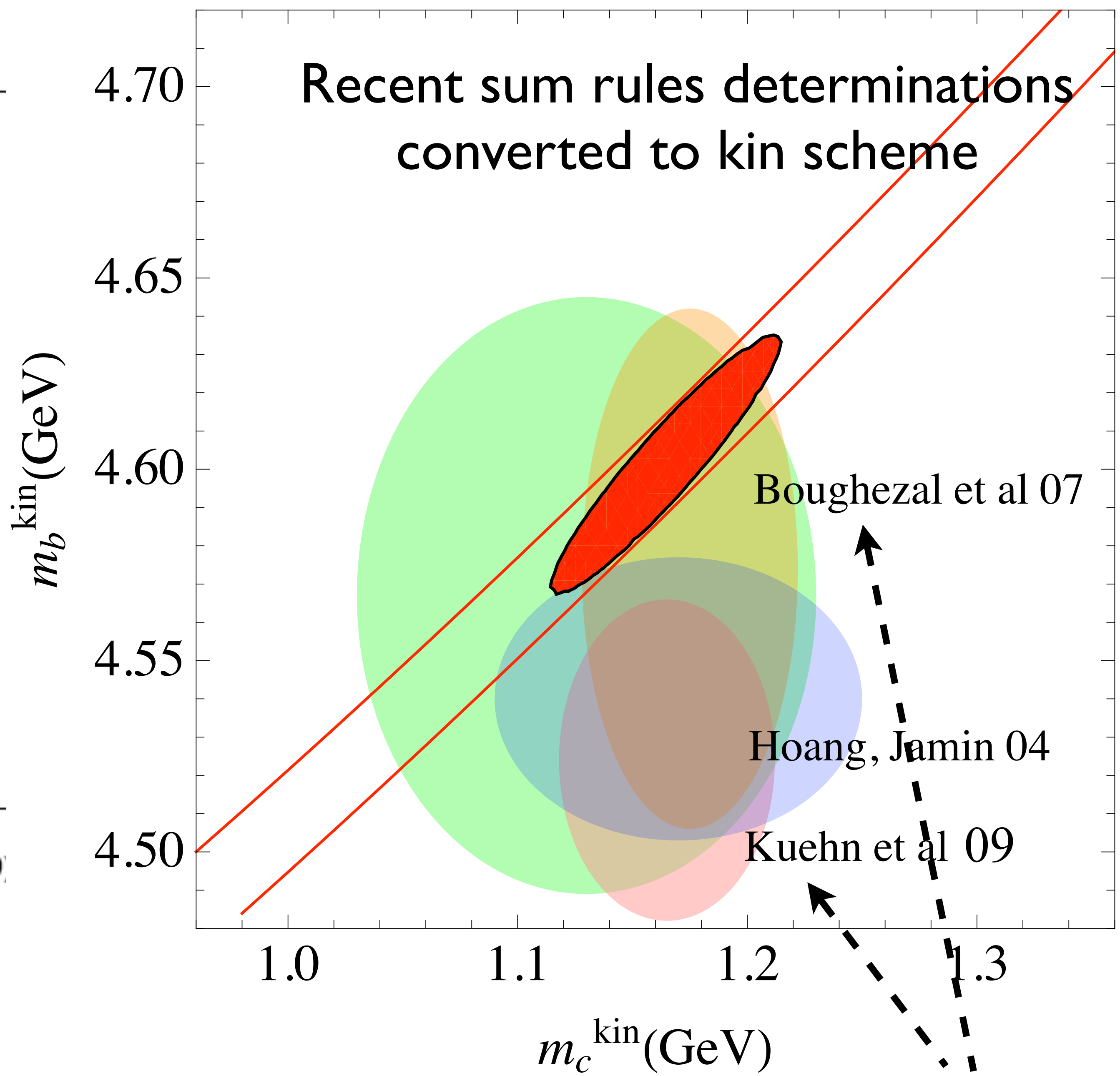


*Semileptonic moments do not measure m_b well. They rather identify a strip in (m_b,m_c) plane along which the minimum is **shallow**.*

Mass determinations



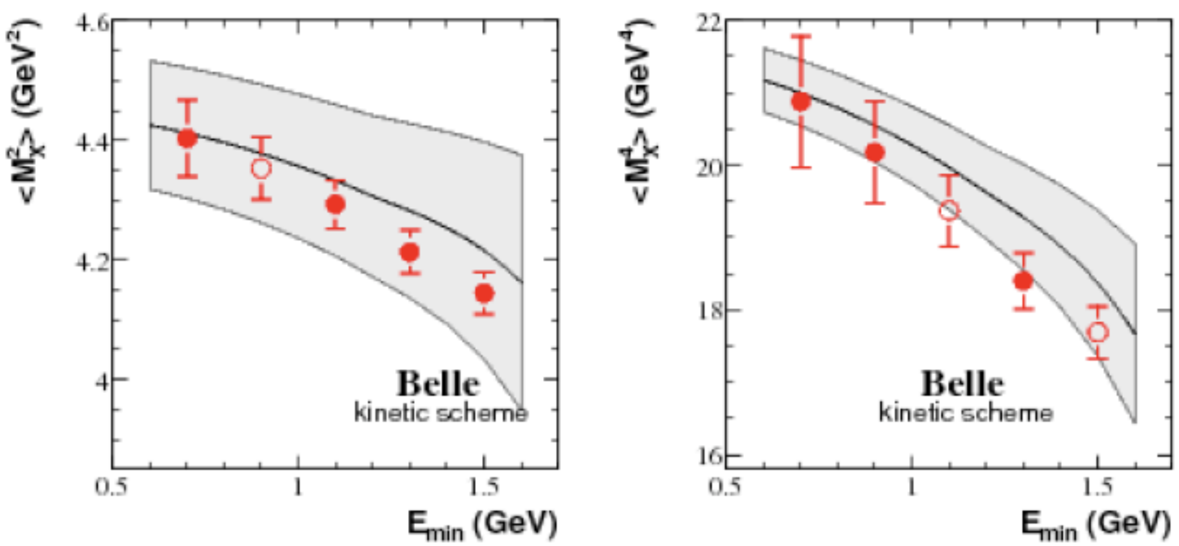
PDG08



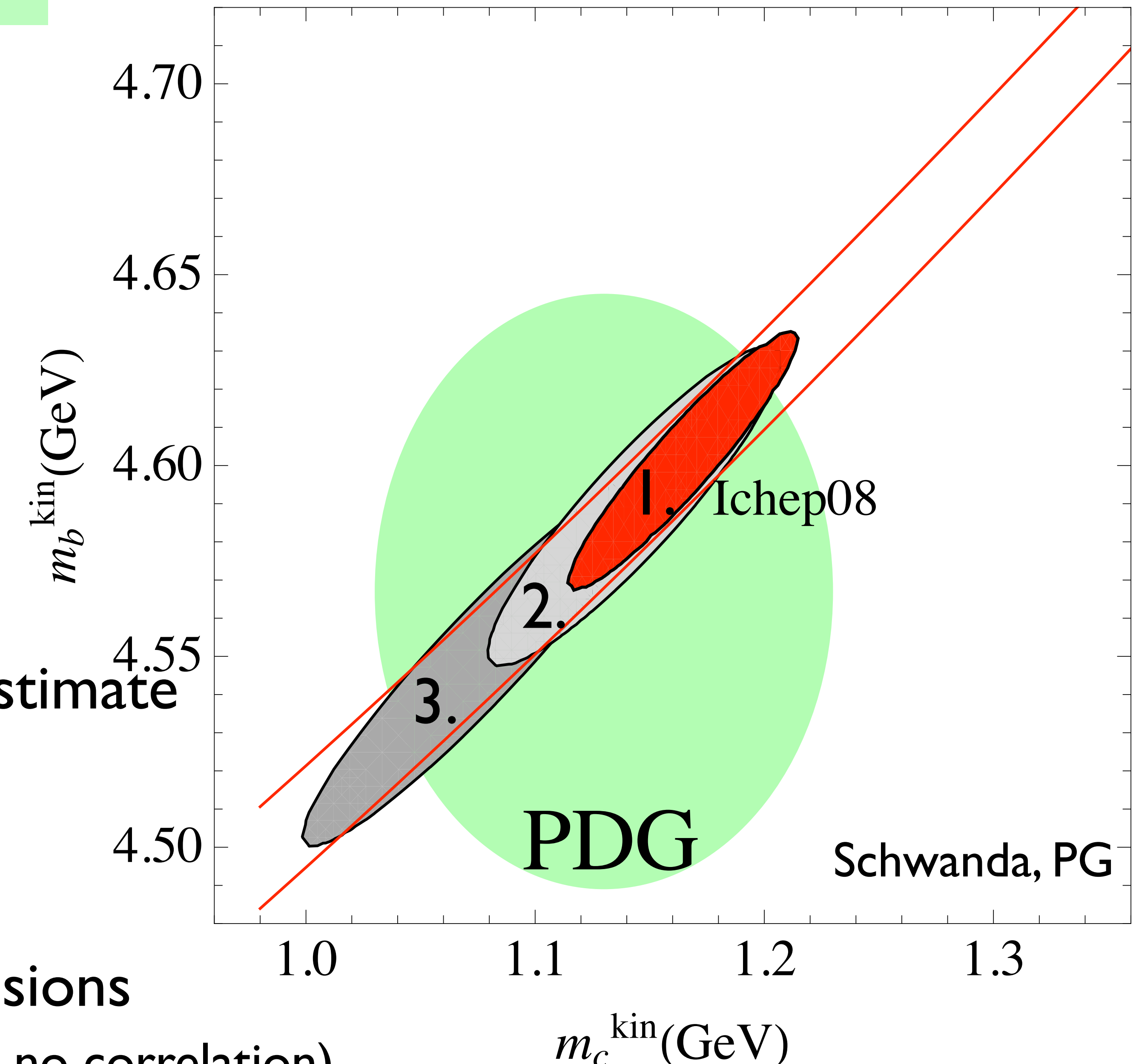
neglecting correlations between $m_{c,b}$ (α_s)

How reliable are mass determinations?

1. Theoretical correlations



Correlations between theory errors of moments with different cuts difficult to estimate



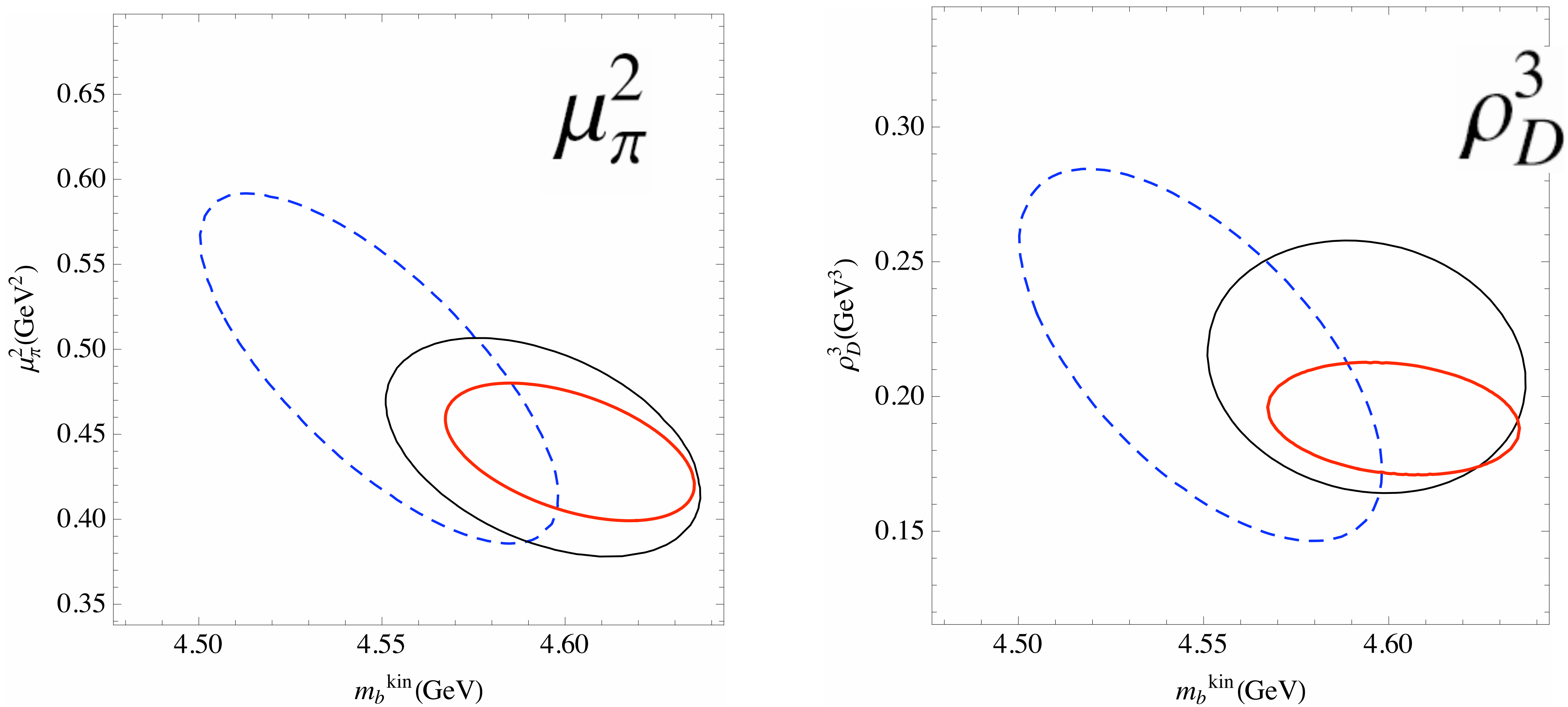
Examples:

1. 100% correlations
2. corr. computed from low-order expressions
3. experimental correlations (very similar to no correlation)

always assume different central moments uncorrelated

Theoretical correlations (II)

Th correlations are also important for other OPE parameters



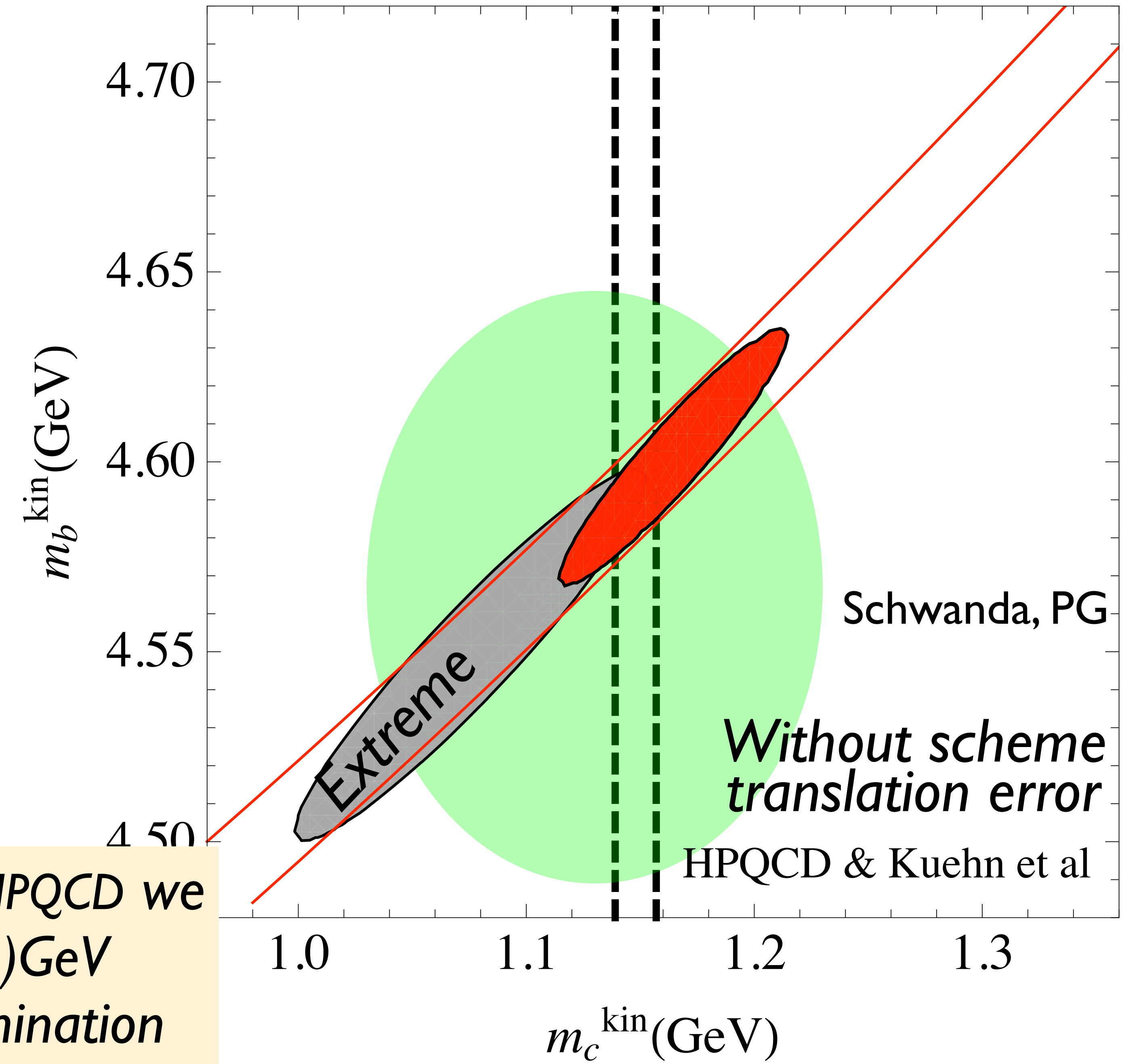
Not all assumptions are reasonable, as high correlations are inevitable.
Black: correlations between different cuts computed using th error recipe, encodes existing correlations in computation: probably a good default!

2. How important are radiative moments?
3. Can we include other constraints?

OPE fails for bsγ, but only at $O(\alpha_s)$ with operators $\neq O_7$. Unlikely to be relevant for normalized moments, but it must be studied

At the moment the role of radiative moments in the fits is almost ***identical*** *to using PDG07 bound $m_b(m_b)$=4.20(7)GeV*

the inclusion of additional constraints can be very useful:



Using m_c(3GeV)=0.986(13) by Karlsruhe/HPQCD we get m_b^{kin}=4.535(21) ➨ $m_b(m_b)$=4.165(45)GeV in perfect agreement with their m_b determination

PRELIMINARY

Which scale for $\overline{\text{MS}}$ m_c?

$\mu_c = m_c$

$$\Gamma[\bar{B} \to X_c e \bar{\nu}] \propto 1 - 0.45\,\frac{\alpha_s}{\pi} + 0.23\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + 1.3\left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3) \approx 0.985$$

μ_c=2GeV

$$\Gamma[\bar{B} \to X_c e \bar{\nu}] \propto 1 - 1.24\,\frac{\alpha_s}{\pi} - 0.29\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 - 0.4\left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3) \approx 0.899$$

μ_c=3GeV

$$\Gamma[\bar{B} \to X_c e \bar{\nu}] \propto 1 - 1.66\,\frac{\alpha_s}{\pi} - 0.46\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 - 2.2\left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3) \approx 0.854$$

The best scale seems to be close to m_c, as a result of accidental cancellations. Width expressed in terms of m_c(3GeV) and $m_c(m_c)$ differs by almost 3%. In the moments?

Towards a new standard fit

Radiative moments are not crucial ingredients in the fits. Their role is almost identical to using PDG07 bound $m_b(m_b)$=4.20(7)GeV → m_b^{kin} =4.57(8)GeV.

But we need additional external constraints. Precise determinations of m_c can be used to fix m_b. First preliminary results are consistent with Kuhn et al./HPQCD.

New important calculation of higher order power corrections by Mannel et al. needs further study of parameter dependence.
Complete $O(\alpha_s/m_b^2)$ coming soon.

Theoretical error on V_{cb} can reach 1% but still some work to be done.