$|V_{cb}|$ from $B \to D^* l\nu$ and Lattice QCD

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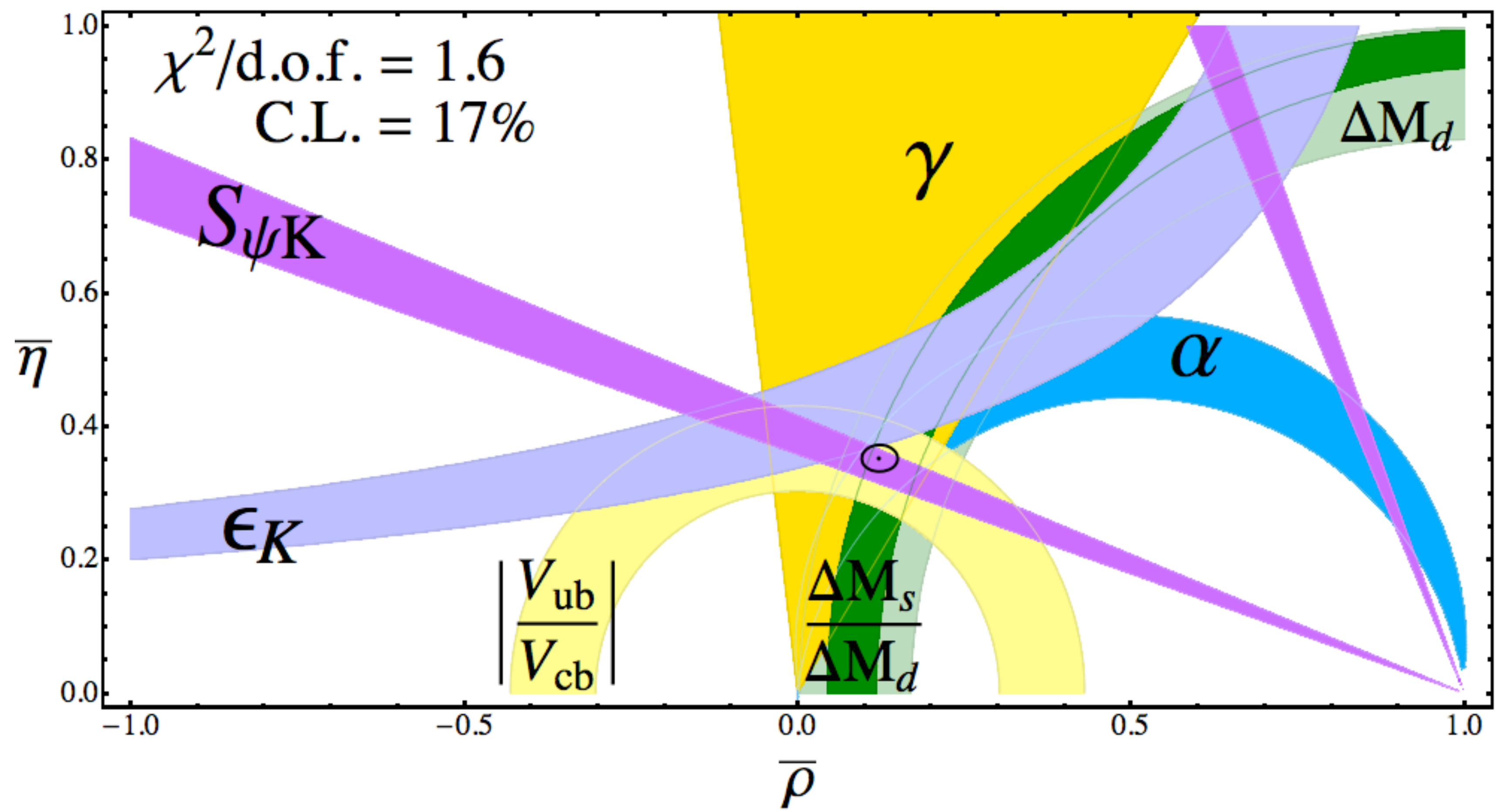
$|V_{cb}|$ Normalizes the whole Unitarity Triangle

c.f., Laiho, Lunghi, Van de Water, arXiv:0910.2928



So, for example, given that $A = \frac{|V_{cb}|}{\lambda^2}$
and that theoretical δA ~ 2% and that δB_K ~ 5%, the uncertainty in the lattice determination of Vcb contributes more uncertainty to the analysis of εK than does B_K.

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta} \{ -\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho}) \}$$

Double ratios

It is possible to obtain Vcb from B→D*lν particularly accurately because can be obtained from double ratios such as

$$\frac{\langle D^* | \bar{c}\gamma_j\gamma_5 b | \overline{B} \rangle \langle \overline{B} | \bar{b}\gamma_j\gamma_5 c | D^* \rangle}{\langle D^* | \bar{c}\gamma_4 c | D^* \rangle \langle \overline{B} | \bar{b}\gamma_4 b | \overline{B} \rangle}$$

in which most errors cancel in the symmetry limit.

History

2001, quenched calculation, Hashimoto et al.

PRD66:014503, 2002

$$\mathcal{F}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}{}^{+0.000}_{-0.016}{}^{+0.006}_{-0.014}$$

stats, match, a, χPT, quenching

Used complicated set of double ratios
that guaranteed cancellation of many errors in the HQS limit.

2008, unquenched 2+1 staggered sea, Laiho et al.

PRD79:014506, 2009

$$\mathcal{F}(1) = 0.921 \pm 0.013 \pm 0.008 \pm 0.008 \pm 0.014 \pm 0.006 \pm 0.003 \pm 0.004$$

stats, $g_{DD^*\pi}$, χPT, disc., $\varkappa_{b,c}$, match, u_0

$$|V_{cb}| = (38.9 \pm 0.7_{\text{expt}} \pm 1.0_{\text{LQCD}}) \times 10^{-3}$$

Used single double ratio at w=1.
Errors need not cancel as completely, but in practice many do.
Much faster than Hashimoto et al. method.

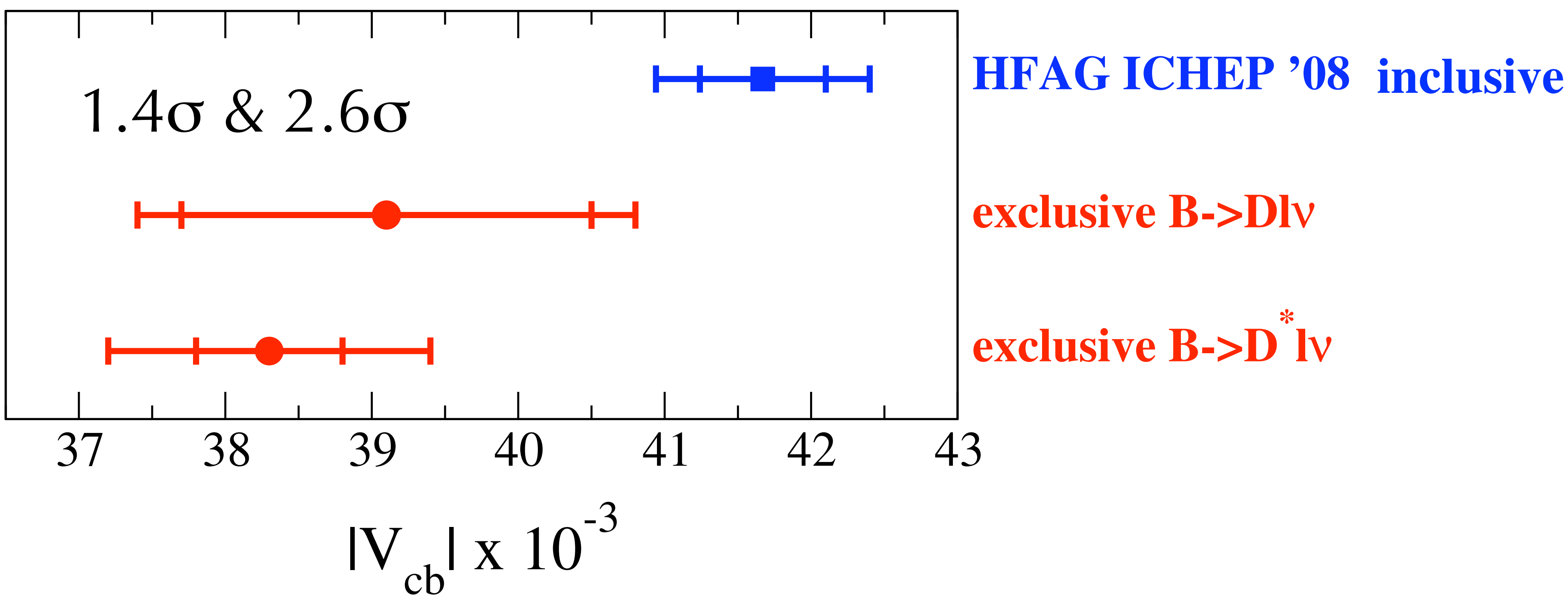
2010, Laiho et al., this talk

Quadruple statistics, smaller lattice spacings, generated completely new data set with retuned parameters and some inconsistencies removed.

Exclusive/Inclusive Tension in V_{cb}

2009

Van de Water, Lattice 2009

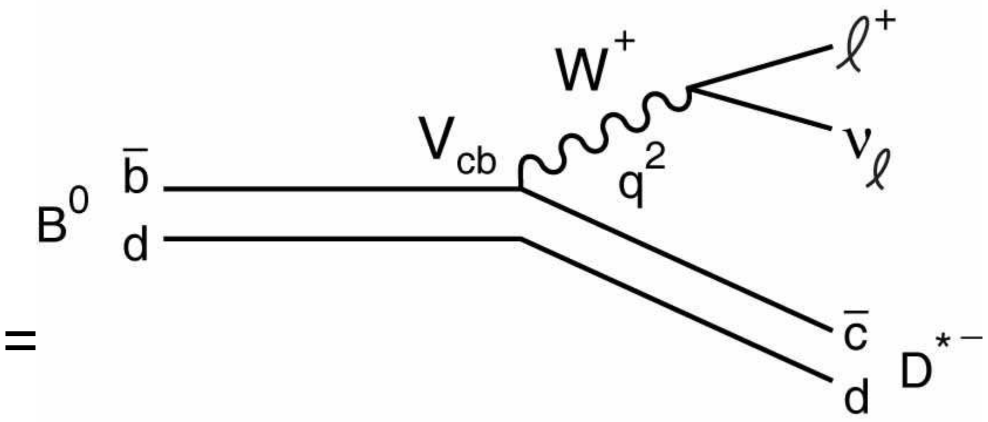


- Determinations of $|V_{cb}|$ via *exclusive* (+ LQCD) and *inclusive* (+ OPE & pQCD) decays haven't agreed perfectly (discrepancy >2σ).

To reduce tension

- Exclusive:
  - firm up existing lattice-QCD calculations (this talk); cross-check from other groups;
  - re-examine extrapolation $w \to 1$;
  - determine $|V_{cb}|$ at $w \neq 1$.
  - Improved experiment.
- Inclusive: higher-order corrections being computed.

Semileptonic Form Factors



- Kinematics: $q^2 = M_B^2 + M_{D^*}^2 - 2wM_BM_{D^*}$, $w =$

$$\frac{\langle D|\mathcal{V}^\mu|B\rangle}{\sqrt{m_Bm_D}} = (v_B+v_D)^\mu h_+(w) + (v_B - v_D)^\mu h_-(w),$$

$$\frac{\langle D^*_\alpha|\mathcal{V}^\mu|B\rangle}{\sqrt{m_Bm_{D^*}}} = \varepsilon^{\mu\nu\rho\sigma} v_B^\nu v_{D^*}^\rho \varepsilon_\alpha^{*\sigma} h_V(w),$$

$$\frac{\langle D^*_\alpha|\mathcal{A}^\mu|B\rangle}{\sqrt{m_Bm_{D^*}}} = i\varepsilon_\alpha^{*\nu}\{(1+w)g^{\nu\mu}h_{A_1}(w) - v_B^\nu[v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w)]\},$$

$$\frac{d\Gamma(B\to D\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B+m_D)^2 (w^2-1)^{3/2}|V_{cb}|^2|\mathcal{G}(w)|^2$$

$$\mathcal{G}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w) \propto f_+(q^2)$$

$B \to D^* l\nu$ at Zero Recoil, $w \to 1$:

$$\frac{1}{\sqrt{w^2-1}}\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 |V_{cb}|^2 \chi(w) |\mathcal{F}(w)|^2$$

$$\chi(w) = \frac{w+1}{12}\left(5w+1-\frac{8w(w-1)m_B m_{D^*}}{(m_B-m_{D^*})^2}\right) \to 1$$

$$\mathcal{F}(w) = h_{A_1}(w)\frac{1+w}{2}\sqrt{\frac{H_0^2(w)+H_+^2(w)+H_-^2(w)}{3\chi(w)}} \to h_{A_1}(1)$$

$$H_0(w) = \frac{m_B w - m_{D^*} - m_B(w-1)R_2(w)}{m_B - m_{D^*}} \to 1$$

$$H_\pm(w) = t(w)\left[1 \mp \sqrt{(w-1)/(w+1)}R_1(w)\right] \to 1$$

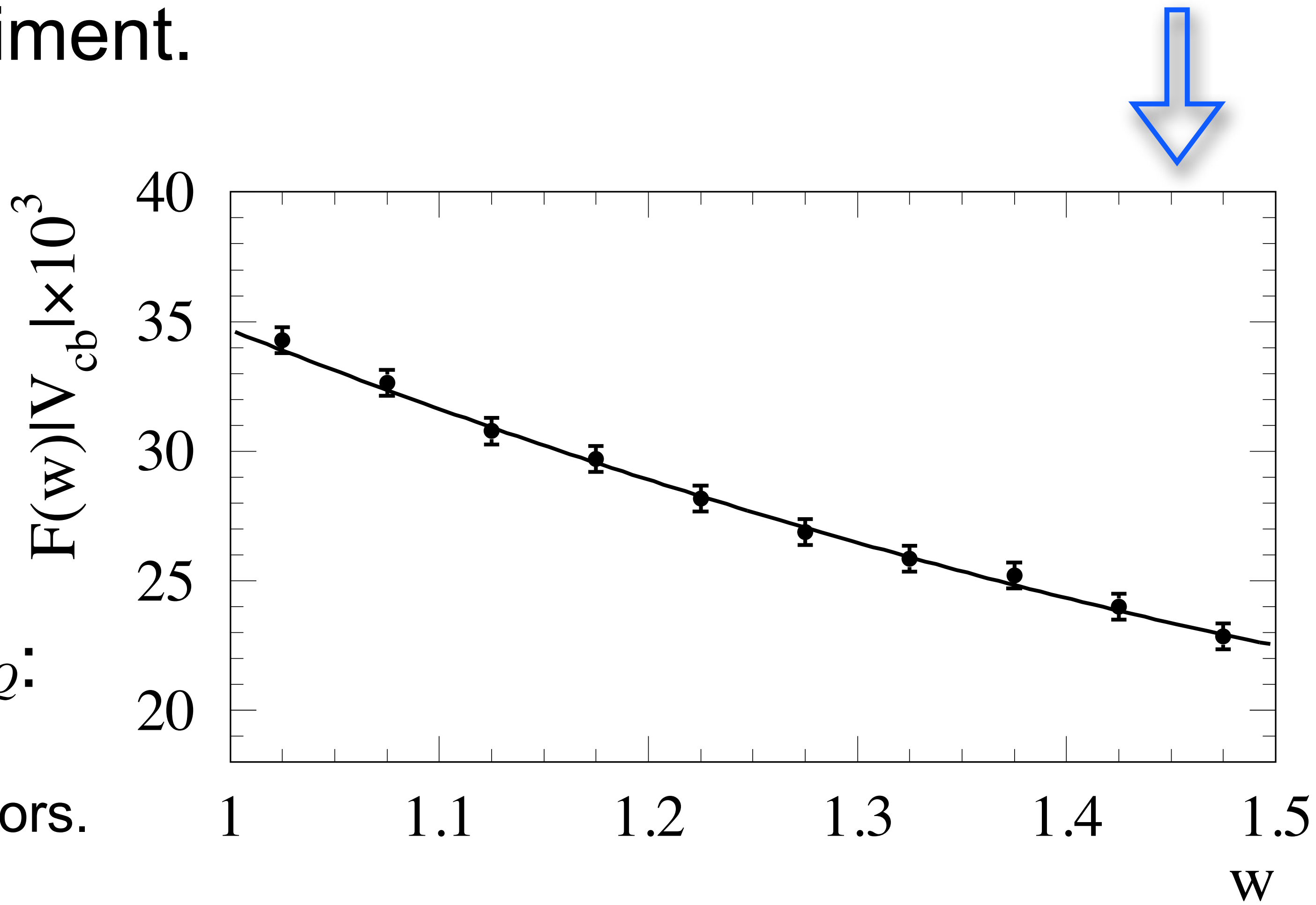
$$t^2(w) = [m_B^2 + m_{D^*}^2 - 2w m_B m_{D^*}]/(m_B - m_{D^*})^2 \to 1$$

$$R_1(w) = h_V(w)/h_{A_1}(w)$$

$$R_2(w) = [m_B h_{A_3}(w) + m_{D^*} h_{A_2}(w)]/m_B h_{A_1}(w)$$

Advantages of Zero Recoil

- Simpler: one number to compute, not four functions: take shape from experiment.
- More powerful HQS:
  - Luke's theorem, $1/m_Q^2$;
  - control errors.
- Nonzero recoil has $1/m_Q$:
  - *e.g.*, larger discretization errors.



BaBar

- In the end, of course, adopt strategy that minimizes error in $|V_{cb}|$.

Ingredients

- Gluon fields from MILC ensembles:
  - Lüscher-Weisz improved action with g^2N_c corrections but not g^2n_f—O($\alpha_s^{\text{“}2\text{”}}a^2$), O($a^4$);
  - 2+1 flavors of sea quarks: rooted asqtad determinant—O($\alpha_s a^2$), O(a^4) “small” ⇐ Fat7.
- Light spectator quark: asqtad action—O($\alpha_s a^2$), O(a^4) “small” ⇐ Fat7.
- Heavy quarks: Sheikholeslami-Wohlert (aka clover) action with Fermilab interpretation:
  - discretization effects O($\alpha_s a^2 b_{\Sigma\cdot B}^{[1]}(ma)$), O($\alpha_s a^2 d_1^{[1]}(ma)$), O($a^2 b_i^{[0]}(ma)$);
  - functions $b_i^{[0]}(ma)$ derived from HQET matching (see below).

MILC Gauge Field Ensembles

MILC asqtad ensembles

a (fm)	lattice	# confs	(am_l, am_s)	am_q	$\varkappa_b$	$\varkappa_c$	c_{SW}
≈ 0.15	$16^3\times48$	596	(0.0290, 0.0484)	{0.0484, 0.0068,			
medium	$16^3\times48$	640	(0.0194, 0.0484)	0.0453, 0.0421,			
coarse	$16^3\times48$	631	(0.0097, 0.0484)	0.0290, **0.0194**,	0.0781	0.1218	1.570
	$20^3\times48$	603	(0.0048, 0.0484)	*0.0097*, 0.0048}			
≈ 0.12	$20^3\times64$	2052	(0.02, 0.05)	{0.05, 0.03,	0.0918	0.1259	1.525
coarse	$20^3\times64$	2259	(0.01, 0.05)	0.0415, 0.0349,	0.0901	0.1254	1.531
	$20^3\times64$	2110	(0.007, 0.05)	***0.02***, *0.01*,	0.0901	0.1254	1.530
	$24^3\times64$	2099	(0.005, 0.05)	*0.007*, *0.005*}	0.0901	0.1254	1.530
≈ 0.09	$28^3\times96$	1996	(0.0124, 0.031)	{0.031, 0.0261,	0.0982	0.1277	1.473
fine	$28^3\times96$	1946	(0.0062, 0.031)	0.0093,	0.0979	0.1276	1.476
	$32^3\times96$	983	(0.00465, 0.031)	***0.0124***, *0.0062*	0.0977	0.1275	1.476
	$40^3\times96$	1015	(0.0031, 0.031)	0.0047, *0.0031*}	0.0976	0.1275	1.478
≈ 0.06	$48^3\times144$	668	(0.0072, 0.018)	{0.0188, 0.0160,	0.1052	0.1296	1.4287
superfine	$48^3\times144$	668	(0.0036, 0.018)	0.0054,	0.1052	0.1296	1.4287
	$56^3\times144$	800	(0.0025, 0.018)	***0.0072***, *0.0036*			
	$64^3\times144$	826	(0.0018, 0.018)	0.0025, 0.0018}			
≈ 0.045 ultrafine	$64^3\times192$	860	(0.0028, 0.014)	0.014, 0.0028			

Scope of analysis

- This update encompasses the ensembles highlighted in red:
  - mass and decay correlators for all $am_q = am_l$ aka "full QCD" or "unitary" (in *italics*);
  - also for all $am_q = 0.4 am_s$ (in **bold**).;
  - hence 2 + 7 + 5 + 3 (partially-quenched) correlators at $am_q = 0.15, 0.12, 0.09, 0.06$ fm.
- Bare quark mass (aka $\varkappa$) determined from spin-averaged kinetic meson mass:
  - improving strategies with twisted b.c. and, eventually, better sources.
- Tree-level tadpole improved $c_{\mathrm{SW}} = 1/u_0^2$, where $u_0^4 = \langle\mathrm{plaquette}\rangle$.

Correlators and Ratios of Correlators

- Our objective is

$$\mathcal{R}_{A_1} = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\overline{B}\rangle\langle\overline{B}|\bar{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\overline{B}|\bar{b}\gamma_4 b|\overline{B}\rangle} = |h_{A_1}(1)|^2$$

- We define 3-point correlations functions:

$$C^{B\to D^*}(t_i,t_s,t_f) = \sum_{\mathbf{x},\mathbf{y}}\langle 0|O_{D^*}(\mathbf{x},t_f)\overline{\Psi}_c\gamma_j\gamma_5\Psi_b(\mathbf{y},t_s)O_B^\dagger(\mathbf{0},t_i)|0\rangle,$$

$$C^{B\to B}(t_i,t_s,t_f) = \sum_{\mathbf{x},\mathbf{y}}\langle 0|O_B(\mathbf{x},t_f)\overline{\Psi}_b\gamma_4\Psi_b(\mathbf{y},t_s)O_B^\dagger(\mathbf{0},t_i)|0\rangle,$$

$$C^{D^*\to D^*}(t_i,t_s,t_f) = \sum_{\mathbf{x},\mathbf{y}}\langle 0|O_{D^*}(\mathbf{x},t_f)\overline{\Psi}_c\gamma_4\Psi_c(\mathbf{y},t_s)O_{D^*}^\dagger(\mathbf{0},t_i)|0\rangle.$$

- So look for plateau in

matching ρ_A

$$R_{A_1}(t) = \frac{C^{B\to D^*}(0,t,T)C^{D^*\to B}(0,t,T)}{C^{D^*\to D^*}(0,t,T)C^{B\to B}(0,t,T)} = \rho_A^{-2}\mathcal{R}_{A_1}$$

Oscillating states:

- A staggered correlator couples to opposite-parity states with $(-1)^t$:

$$C^{X\to Y}(0,t,T) = \sum_{k=0}\sum_{\ell=0}(-1)^{kt}(-1)^{\ell(T-t)}A_{\ell k}e^{-m_X^{(k)}t}e^{-m_Y^{(\ell)}(T-t)}$$
$$= A_{00}^{X\to Y}e^{-m_X t-m_Y(T-t)} + (-1)^{T-t}A_{01}^{X\to Y}e^{-m_X t-m'_Y(T-t)}$$
$$+ (-1)^t A_{10}^{X\to Y}e^{-m'_X t-m_Y(T-t)} + (-1)^T A_{11}^{X\to Y}e^{-m'_X t-m'_Y(T-t)} + \dots$$

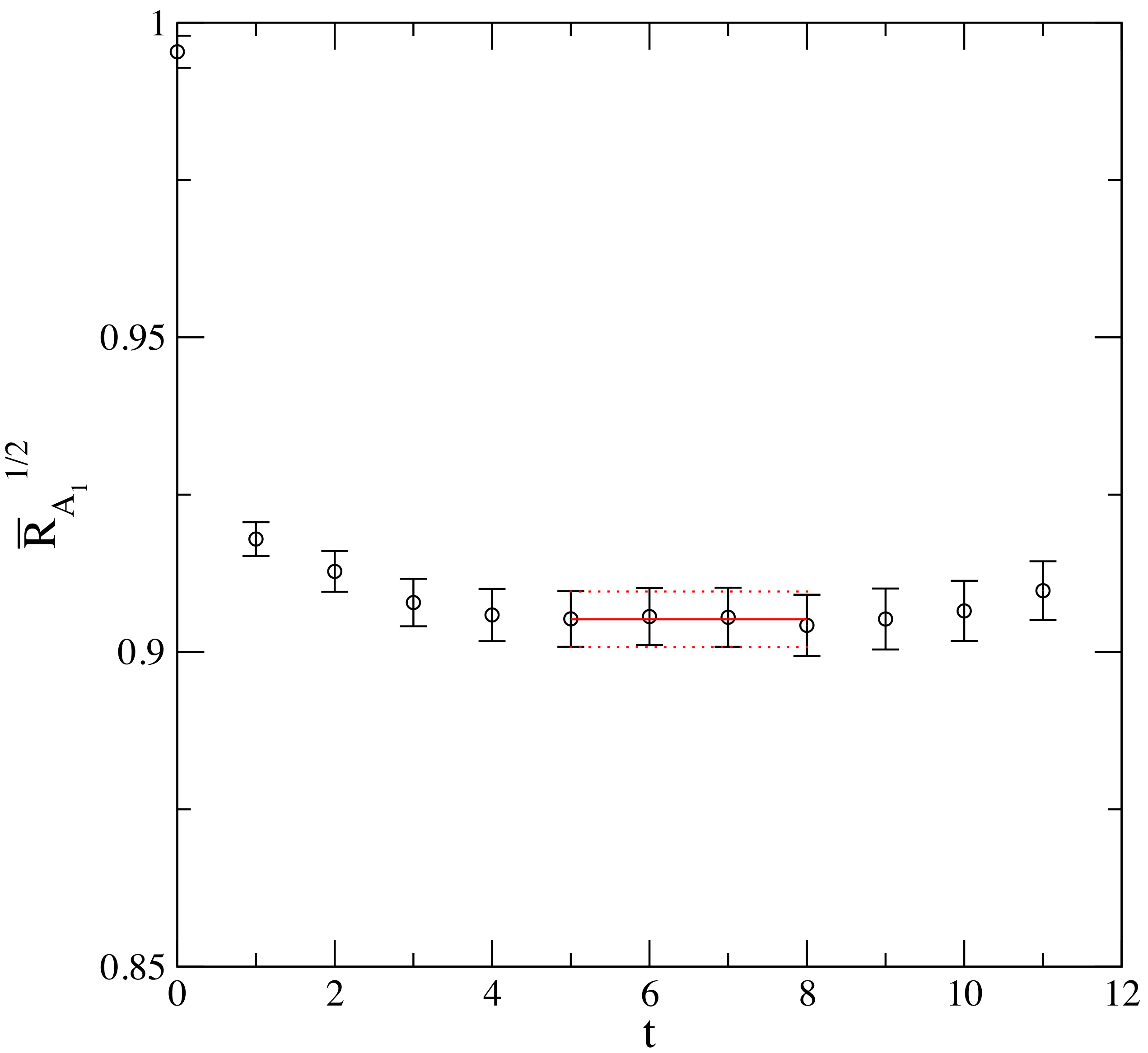
- Last term is wrong-parity–to–wrong-parity transition, and doesn't oscillate in t.

- *Does* oscillate in T, so control by computing $C^{X\to Y}(0,t,T)$ and $C^{X\to Y}(0,t,T+1)$:

$$\overline{R}_{A_1}(0,t,T) = \tfrac{1}{2}R_{A_1}(0,t,T) + \tfrac{1}{4}R_{A_1}(0,t,T+1) + \tfrac{1}{4}R_{A_1}(0,t+1,T+1)$$

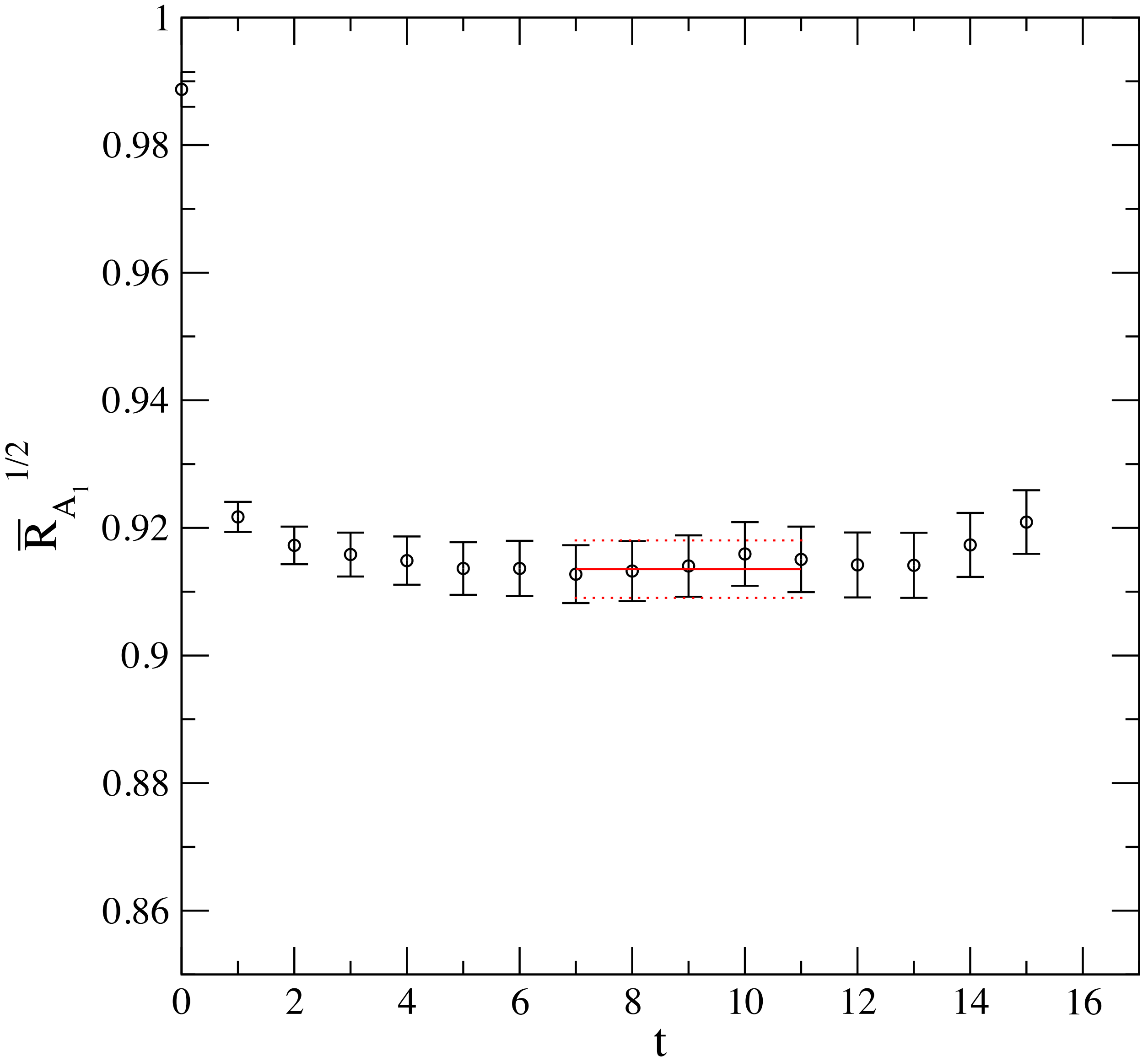
Plateau on a Coarse Ensemble

$(am_l, am_s) = (0.01, 0.05)$



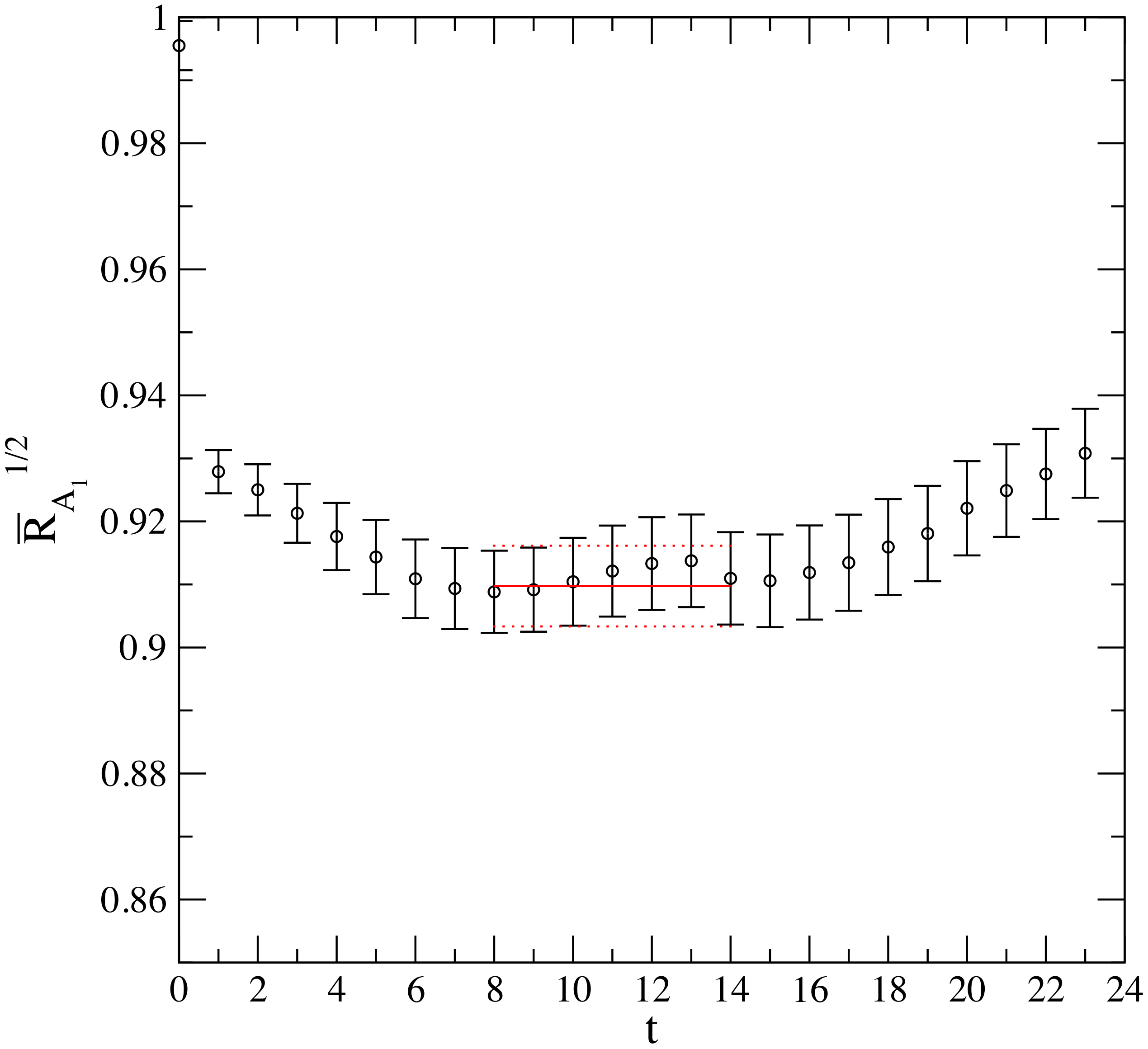
Plateau on a Fine Ensemble

$(am_l, am_s) = (0.0062, 0.031)$



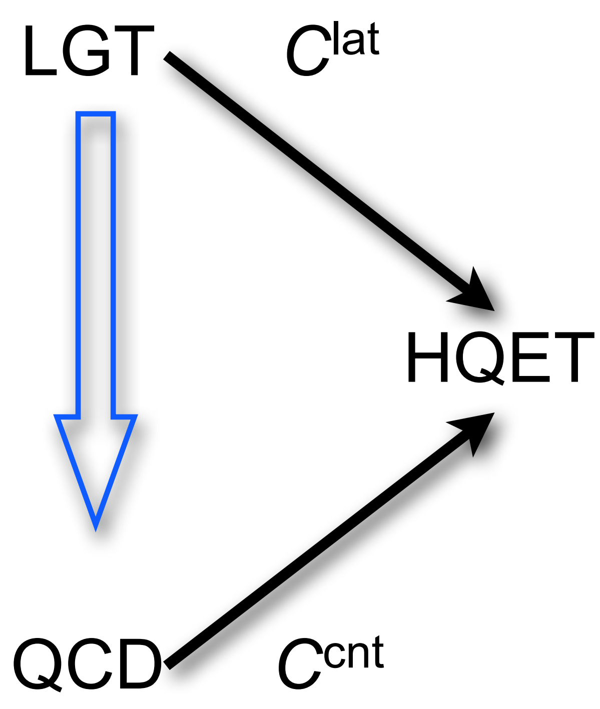
Plateau on a Superfine Ensemble

$(am_l, am_s) = (0.0036, 0.018)$



Matching and Discretization via HQET

- Heavy-light hadrons can be described by heavy-quark effective theory.
- Founded on basic dynamics and emerging symmetries.
- LGT has the same basic dynamics and symmetries, so an HQET description exists here too.
- Relating HQET for two underlying theories (LGT & QCD) yields
  - theory of cutoff effects;
  - definition of matching factors;
  - relationships between observables.



- As $v' \to v$, $1/m$ corrections vanish.
- From (tree-level) HQET matching, zero recoil [ASK, hep-lat/00020085]:

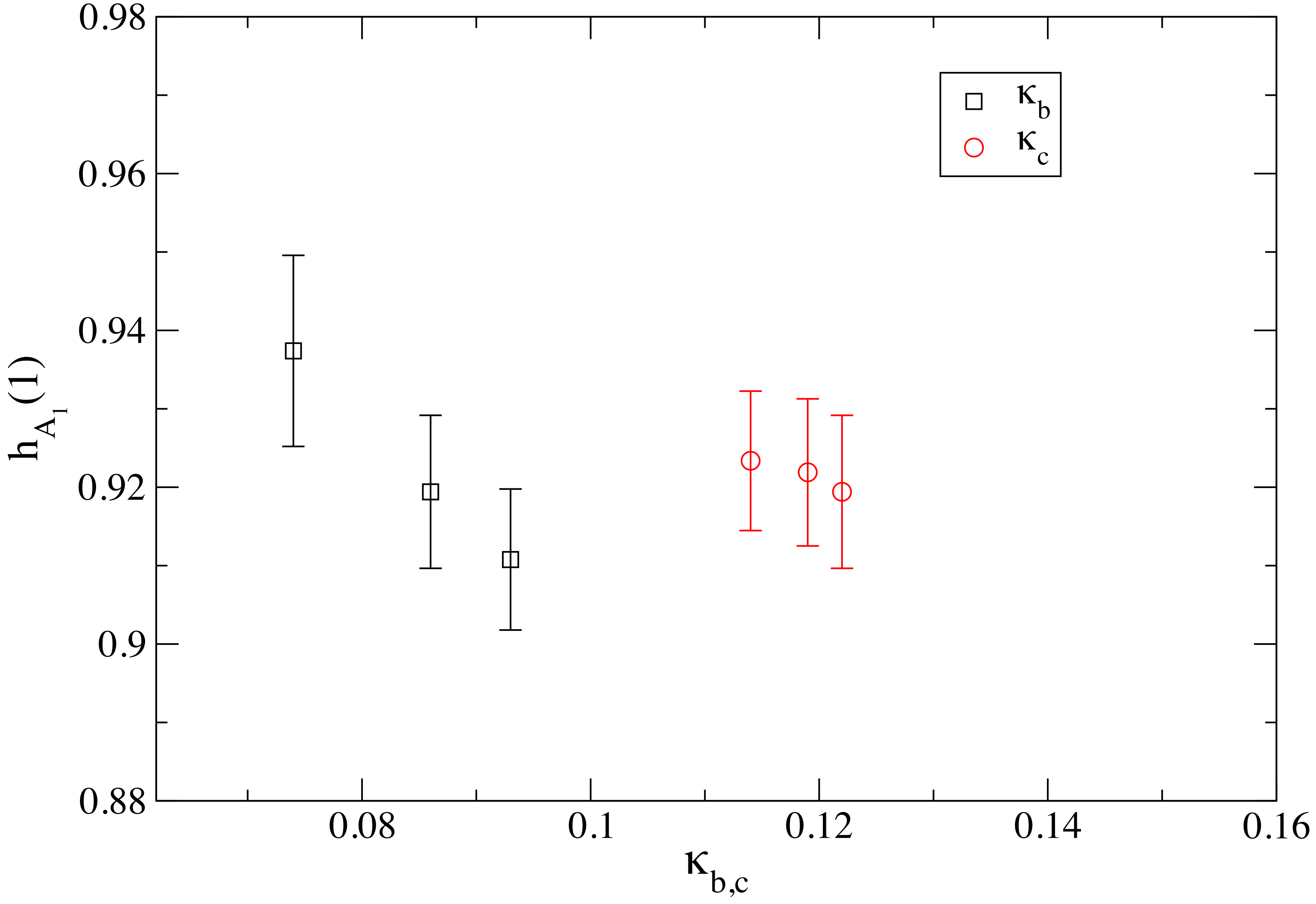
$$\delta h_{A_1}(1) = \left(\frac{1}{8m_{3c}^2} - \frac{1}{8m_{D_\perp^2 c}^2} + \frac{1}{8m_{3b}^2} - \frac{1}{8m_{D_\perp^2 b}^2} \right) \mu_\pi^2 + \left(\frac{1}{8m_{3c}^2} - \frac{1}{8m_{sBc}^2} - \frac{3}{8m_{3b}^2} + \frac{3}{8m_{sBb}^2} \right) \frac{\mu_G^2}{3}$$

- $1/m^2$ corrections cancel well for c ($m_c a < 1$) and to some extent for b.
- Previous work: conservative power counting with $\Lambda = 500\text{–}700$ MeV.
- Future work:
  - use explicit formulae and experimental results or lattice data for μ_π^2 and μ_G^2.
  - Incorporate correction operators in Bayesian continuum extrapolations.
- Remaining matching error is overall normalization, computed in one-loop PT w/ BLM α_s:

$$\rho_A^2 = \frac{Z_A^2}{Z_{V^{bb}} Z_{V^{cc}}}$$

Heavy-quark mass (aka $\varkappa$) tuning

coarse $(am_q, am_l, am_s) = (0.02, 0.02, 0.05)$



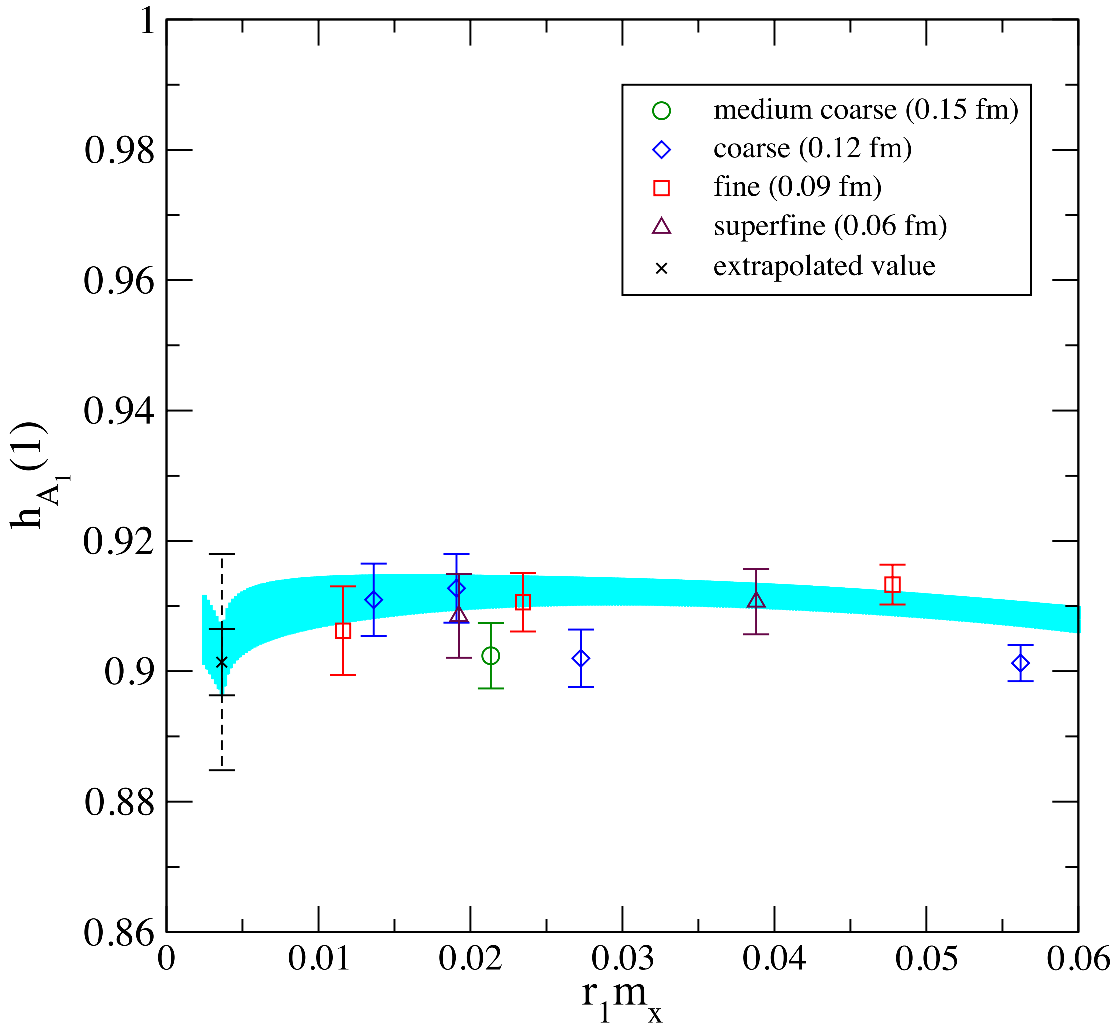
Chiral Extrapolation

- In arXiv:0808.2519 we introduce two intermediate quadruple ratios (ratios of double ratios) to disentangle chiral extrapolation from heavy-quark discretization errors.
- Now we carry out the chiral extrapolation without the quadruple ratios, but with equivalent information in the fit.
- Partially-quenched staggered PT available from Laiho & Van de Water [hep-lat/0512007].
- Incorporates a cusp when pion is light enough for $D^* \to D\pi$ to be physical.
- Show only “full QCD” points on plot:

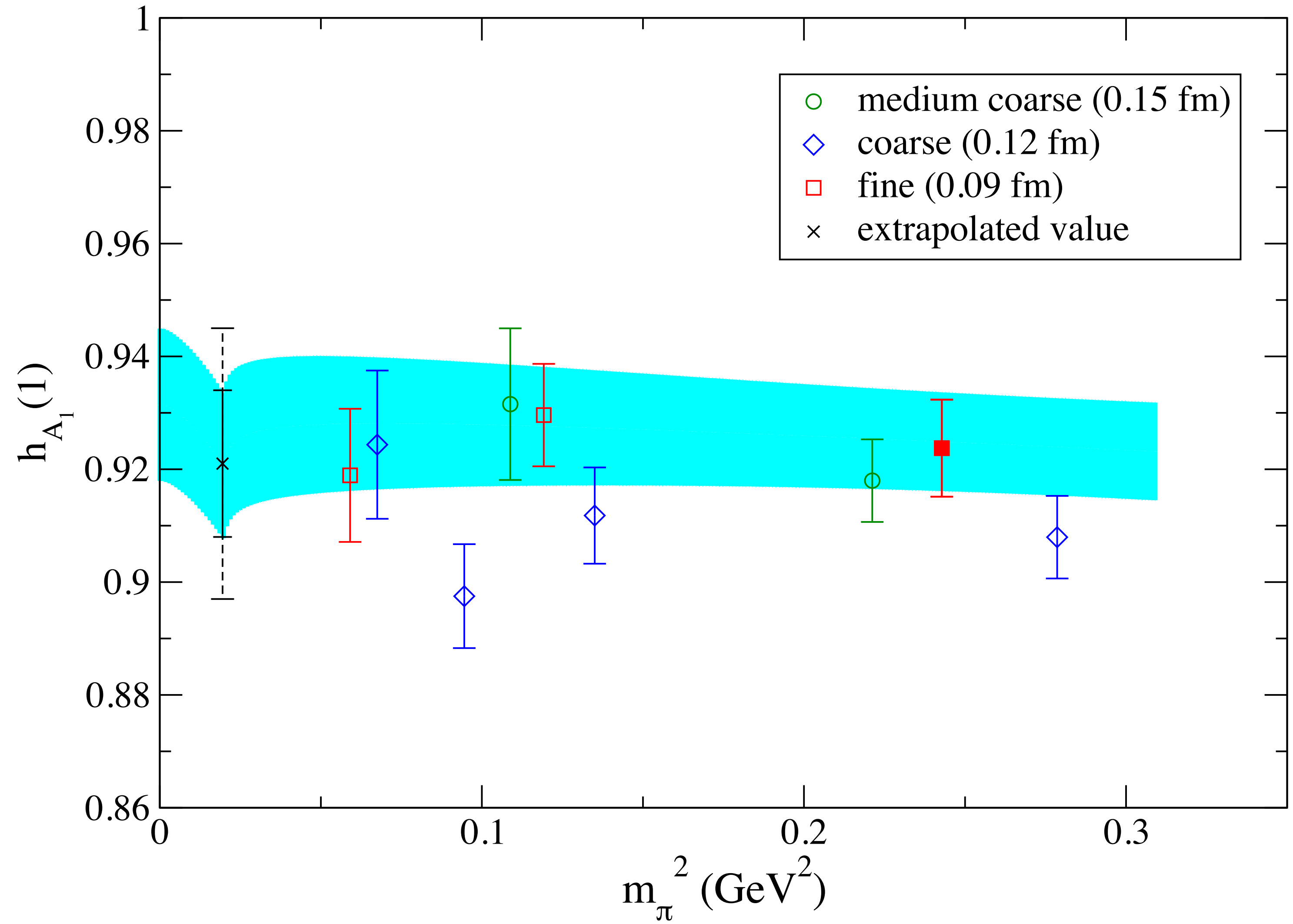
Chiral Extrapolation

2010

$\chi^2/dof = 8.9/12$, CL=0.72



Compare 2008



2010 Result

$h_{A1}(1)$=0.9077(51)(88)(84)(90)(33)(30)

stat, $g_{\pi DD^*}$, χ extrap, HQ disc., κ tune, PT

=0.9077(51)(159)

stat, sys

=0.9077(167)

$$
\begin{aligned}
h_{A1}(1) &= \\
F(1) &= 0.927(13)(8)(8)(14)(6)(3)(4) \\
F(1) &= 0.908(05)(9)(8)(09)(3)(3)
\end{aligned}
$$

2008, PRD79:014506, 2009

2010, this work

$|Vcb|\ F(1) \times 10^3 = 36.04$ +/- 0.52

HFAG, 09 End of Year.

(35.41(52)→ ('08))

$\Rightarrow |V_{cb}|=39.7(7)(7) \times 10^{-3}$, (theory, experiment)

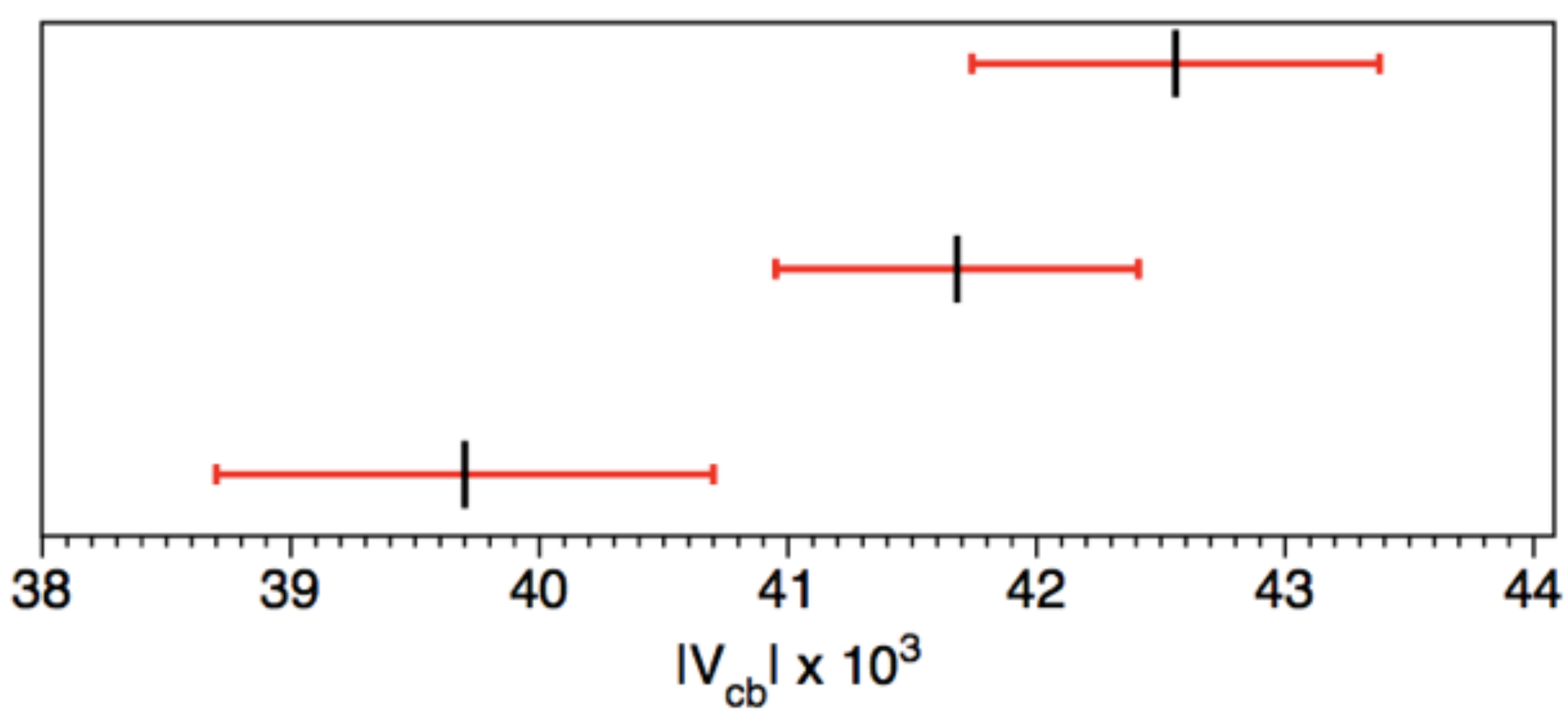
Discrepancies reduced

Result from global fit excluding direct determination of V_{cb}:
$V_{cb} = (42.56 \pm 0.82) \times 10^{-3}$. (Discrepancy: 2.2σ.)

Laiho, Lunghi, and Van de Water, latticeaverages.org, PRD81:034503, 2010.

Result from inclusive B decay:
$V_{cb} = (41.68 \text{ +/- } 0.44 \text{ +/- } 0.09 \text{ +/- } 0.58) \times 10^{-3}$
$= (41.68 \text{ +/- } 0.73) \times 10^{-3}$.
(Discrepancy: 1.6σ.)

HFAG, 09 End of Year.



Global fit

Inclusive

Exclusive $B \to D^* l\nu$

Outlook

- Should be possible to further reduce discretization error (largest current error) with smaller lattice and incorporation of known HQET behavior into Baysian priors for discretization into fitting program.
- $g_{\pi DD^*}$ and χ extrapolation uncertainties almost as large, and will take more thought.
- B_K analysis needs <1% uncertainty in this theory.
  - A lot still to accomplish!