# Search for the critical point

(in NA61/SHINE)

Tobiasz Czopowicz

Jan Kochanowski University
Warsaw University of Technology

SHINE Autumn School
October 26 – 30, 2020

# Outline

## 1 Critical point search strategies

- Critical point of QGP
- Exploring the phase diagram with heavy-ion collisions

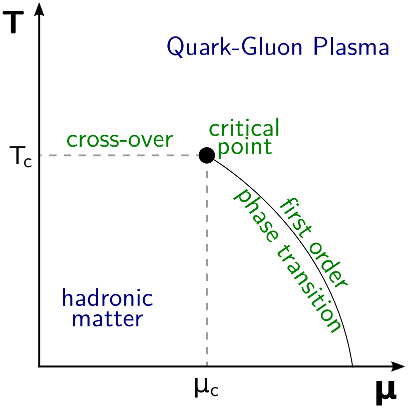
## 2 Experimental measures

## 3 Experimental results

## 4 Summary

# Critical point of QGP

## Critical point search strategies




Critical point (CP) – a hypothetical end point of first order phase transition line (QGP-HM) that has properties of second order phase transition.

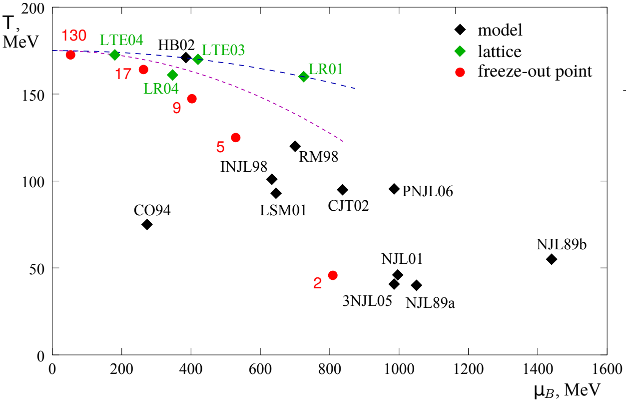
2nd order phase transition $\longrightarrow$ scale invariance $\longrightarrow$ power-law form of correlation function.

These expectations are for fluctuations and correlations in the configuration space which are expected to be projected to the momentum space via quantum statistics and/or collective flow.

Asakawa, Yazaki NPA 504 (1989) 668
Barducci, Casalbuoni, De Curtis, Gatto, Pettini, PLB 231 (1989) 463

# Critical point of QGP

## Critical point search strategies



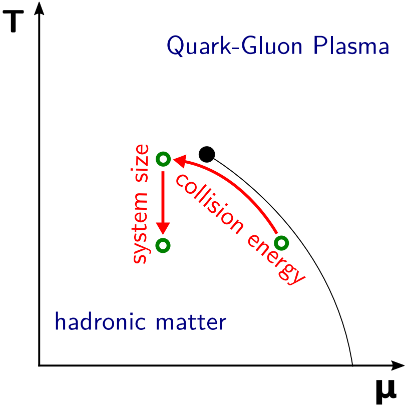

The main signal of the CP is anomaly in fluctuations in a narrow domain of the phase diagram.

However predictions on the CP existence, its location and what and how should fluctuate are model-dependent.

Stephanov, PoS LAT2006 (2006) 024

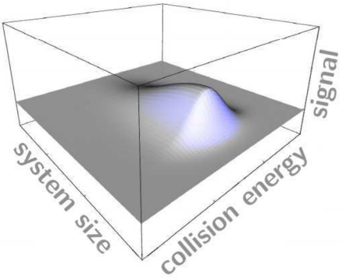
# Exploring the phase diagram with heavy-ion collisions

## Critical point search strategies




The experimental search for the critical point requires a two-dimensional scan in collision energy and size of the colliding nuclei (centrality).

Search for the critical end point in heavy-ion collisions is performed by a scan in the parameters controlled in laboratory (collision energy and nuclear mass number, centrality). By changing them, we change freeze-out conditions (T, $\mu_B$).



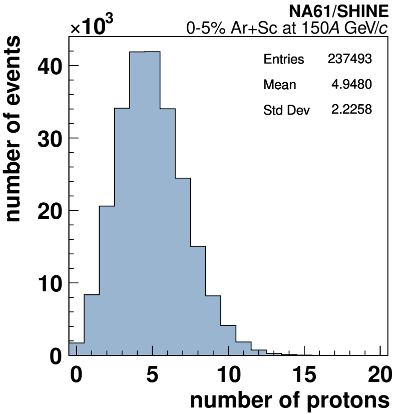
# Introduction

## Experimental measures

We measure a fraction of particles produced in the collision.

A single collision is too complex to be studied. Therefore we perform many collisions and try to conclude on the probabilities.

Most commonly used graphical way of showing the data are histograms.



# Extensive quantities

## Experimental measures

A quantity proportional to W (WNM) or V in (IB-GCE) is called an extensive quantity. The most popular are particle number (multiplicity) distribution P(N) cumulants:

- $\kappa_1 = \langle \mathrm{N} \rangle$
- $\kappa_2 = \left\langle (\delta \mathrm{N})^2 \right\rangle = \sigma^2$
- $\kappa_3 = \left\langle (\delta \mathrm{N})^3 \right\rangle = \mathrm{S}\sigma^3$
- $\kappa_4 = \left\langle (\delta \mathrm{N})^4 \right\rangle - 3 \left\langle (\delta \mathrm{N})^2 \right\rangle^2 = \kappa\sigma^4$

These multiplicity cumulants characterize the shape of multiplicity distribution and quantify fluctuations.

WNM – Wounded Nucleon Model ($\left\langle \mathrm{N}_{\mathrm{A+B}} \right\rangle = \left\langle \mathrm{W}_{\mathrm{A+B}} \right\rangle /2 \cdot \left\langle \mathrm{N}_{\mathrm{N+N}} \right\rangle$)

IB-GCE – Ideal Boltzmann Grand Canonical Ensamble

Bialas, Bleszynski Czyz, NPB 111 461 (1976)

# Intensive quantities

## Experimental measures

Ratio of any two extensive quantities is independent of W (WNM) or V (IB-GCE). It is an intensive quantity.
For example:

$$\langle A \rangle / \langle B \rangle = W \cdot \langle a \rangle / W \cdot \langle b \rangle = \langle a \rangle / \langle b \rangle$$

where A and B are any extensive event quantities, i.e. $\langle \mathrm{A} \rangle \sim \mathrm{W}$, $\langle \mathrm{B} \rangle \sim \mathrm{W}$ and $\langle \mathrm{a} \rangle = \langle \mathrm{A} \rangle$ and $\langle \mathrm{b} \rangle = \langle \mathrm{B} \rangle$ for $\mathrm{W} = 1$.

Popular examples:

- $\frac{\kappa_2}{\kappa_1} = \omega[\mathrm{N}] = \frac{\sigma^2[\mathrm{N}]}{\langle \mathrm{N} \rangle} = \frac{\mathrm{W} \cdot \sigma^2[\mathrm{n}]}{\mathrm{W} \cdot \langle \mathrm{n} \rangle} = \omega[\mathrm{n}]$ (scaled variance)
- $\frac{\kappa_3}{\kappa_2} = \mathrm{S}\sigma$
- $\frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$

# Strongly intensive quantities

## Experimental measures

For an event sample with varying W, cumulants are not extensive quantities any more. For example:

$$\kappa_2 = \sigma^2[N] = \sigma^2[n]\langle W\rangle + \langle n\rangle^2 \sigma^2[W]$$

But having two extensive event quantities, one can construct quantities that are independent of P(W)!

Popular examples:

- $\langle \mathrm{K}\rangle / \langle \pi\rangle$
- $\Delta[\mathrm{N}, \mathrm{P_T}] = \frac{1}{\mathrm{C}}(\omega[\mathrm{N}]\langle \mathrm{P_T}\rangle - \omega[\mathrm{P_T}]\langle \mathrm{N}\rangle)$
- $\Sigma[\mathrm{N}, \mathrm{P_T}] = \frac{1}{\mathrm{C}}(\omega[\mathrm{N}]\langle \mathrm{P_T}\rangle + \omega[\mathrm{B}]\langle \mathrm{N}\rangle - 2(\langle \mathrm{NP_T}\rangle - \langle \mathrm{P_T}\rangle\langle \mathrm{N}\rangle)$

where $\mathrm{P_T} = \sum_{i=1}^{N} \mathrm{p_{T,i}}$ and C is any extensive quantity (e.g.$\langle \mathrm{N}\rangle$).

Gazdzicki, Gorenstein, PRC 84 (2011) 014904
Gazdzicki, Gorenstein, Mackowiak-Pawlowska, PRC 88 (2013) 024907

# Short-range correlations

## Experimental measures

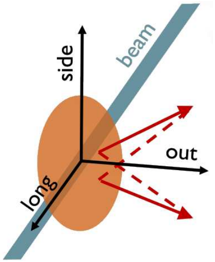
Quantum statistics leads to short-range correlations in momentum space, which are sensitive to particle correlations in configuration space (e.g. of CP origin).

Popular measure:
Momentum difference in LCMS, $\mathbf{q}$, is decomposed into three components:
$q_{\text{long}}$ – momentum difference along the beam
$q_{\text{out}}$ – parallel to the pair transverse momentum vector $\mathbf{k}_{\text{t}} = (\mathbf{p}_{\text{T},1} + \mathbf{p}_{\text{T},2})/2$
$q_{\text{side}}$ – perpendicular to $q_{\text{out}}$ and $q_{\text{long}}$

The two-particle correlation function C is often approximated by a three-dimensional Gauss function:

$$C(\mathbf{q}) \cong 1 + \lambda \cdot exp\left(-R^2_{\text{long}} q^2_{\text{long}} - R^2_{\text{out}} q^2_{\text{out}} - R^2_{\text{side}} q^2_{\text{side}}\right)$$

where $\lambda$ describes the correlation strength and $R_{\text{out}}, R_{\text{side}}, R_{\text{long}}$ denote Gaussian HBT radii.



# Short-range correlations

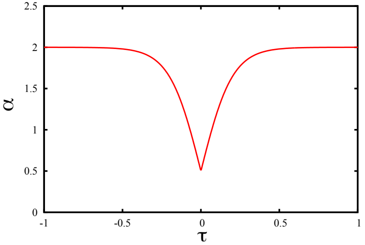
## Experimental measures

A more general idea – Lévy-shaped source (1-D):

$$C(q) \cong 1 + \lambda \cdot e^{(-qR)^{\alpha}}$$

where $q = |p_1 - p_2|_{LCMS}$, $\lambda$ describes correlation length, R determines the length of homogenity and Lévy exponent $\alpha$ determines source shape:

- $\alpha = 2$
  Gaussian, predicted from simple hydro
- $\alpha < 2$
  anomalous diffusion, generalized central limit theorem
- $\alpha = 0.5$
  conjectured value at the critical point




Csorgo, Hegyi, Novak, Zajc, AIP Conf.Proc. 828 (2006) 525

# Fluctuations as a function of momentum bin size

## Experimental measures

Momentum space is partitioned into M bins.
Second factorial moment is calculated as a function of cell size (number of cells, M):

$$F_2(M) \equiv \frac{\left\langle \frac{1}{M} \sum_{i=1}^{M} n_i(n_i - 1) \right\rangle}{\left\langle \frac{1}{M} \sum_{i=1}^{M} n_i \right\rangle}$$

At the second order phase transition the system is a simple fractal and the factorial moment exhibits a power law dependence on M:

$$F_2(M) \sim (M)^{\varphi_2}$$

Prediction for critical point: $\varphi_2 = 5/6$.

Wosiek, APPB 19 (1988) 863
Bialas, Hwa, PLB 253 (1991) 436
Bialas, Peschanski, NPB 273 (1986) 703
Antoniou, Diakonos, Kapoyannis, Kousouris, PRL 97 (2006) 032002

# Fluctuations as a function of momentum bin size

## Experimental measures

However, to cancel the $F_2(M)$ dependence on the single particle inclusive momentum distribution, one needs a uniform distribution of particles in bins.
One can either subtract $F_2(M)$ for mixed events:

$$\Delta F_2(M) = F_2^{\text{data}}(M) - F_2^{\text{mixed}}(M)$$

or use cumulative quantities, e.g.

$$Q_x = \int_{x_{min}}^{x} \rho(x)\mathrm{d}x \Bigg/ \int_{x_{min}}^{x_{max}} \rho(x)\mathrm{d}x$$

Bialas, Gazdzicki, PLB 252 (1990) 483

# Light nuclei production

## Experimental measures

Based on coalescence model, particle ratios of light nuclei are sensitive to the nucleon density fluctuations at kinetic freeze-out and thus to CP.
In the vicinity of the critical point or the first order phase transition, density fluctuation becomes larger.

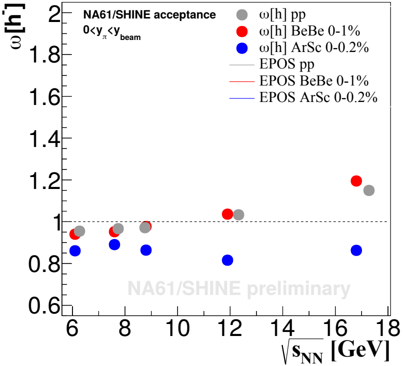
Neutron density fluctuation can be expressed by proton, triton and deuteron yields:

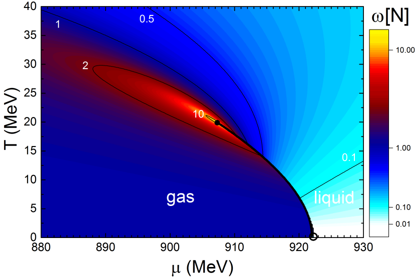
$$\Delta n = \frac{\left\langle (\delta n)^2 \right\rangle}{\langle n \rangle} \approx \frac{1}{2\sqrt{3}} \frac{N_p \cdot N_t}{N_d^2} - 1$$

Sun, Chen, Ko, Xu, PLB 774 (2017) 103
Shuryak, Torres-Rincon, NPA 982 (2019) 831

# Multiplicity fluctuations

## Experimental results: Fluctuations in large momentum bins



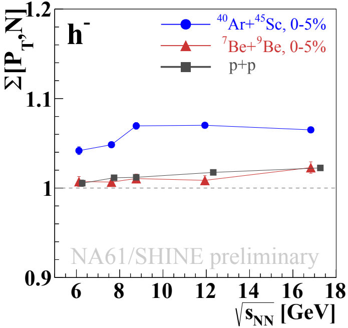



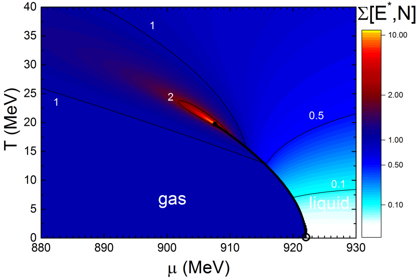

No prominent structures that could be related to the critical point are observed so far...

NA61/SHINE: PoS CPOD2017 (2018) 012
Vovchenko, Anchishkin, Gorenstein, Poberezhnyuk, Stoecker, Acta Phys.Polon.Supp. 10 (2017) 753

# Multiplicity-transverse momentum fluctuations

## Experimental results: Fluctuations in large momentum bins



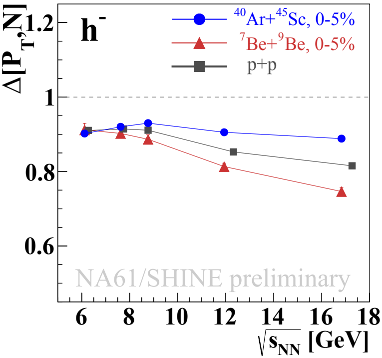




No prominent structures that could be related to the critical point are observed so far...

NA61/SHINE: Acta Phys.Polon.Supp. 10 (2017) 449
Vovchenko, Anchishkin, Gorenstein, Poberezhnyuk, Stoecker, Acta Phys.Polon.Supp. 10 (2017) 753

# Multiplicity-transverse momentum fluctuations

## Experimental results: Fluctuations in large momentum bins

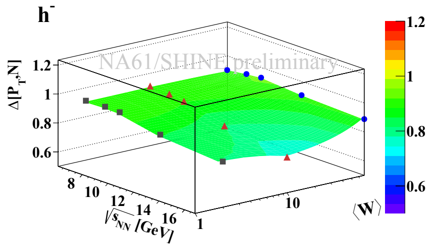


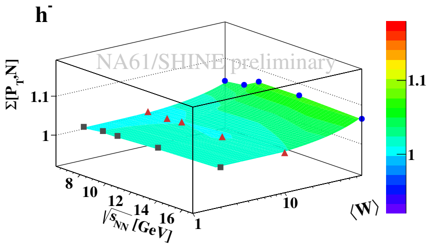

No prominent structures that could be related to the critical point are observed so far...

NA61/SHINE: Acta Phys.Polon.Supp. 10 (2017) 449

# Multiplicity-transverse momentum fluctuations

## Experimental results: Fluctuations in large momentum bins



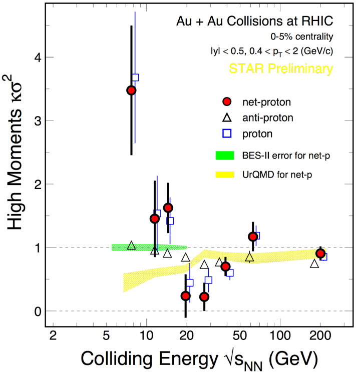



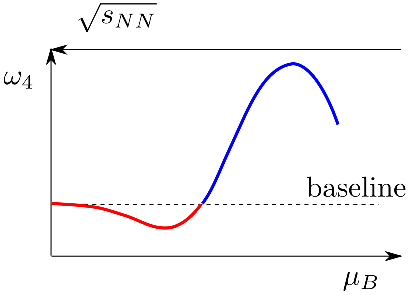

No prominent structures that could be related to the critical point are observed so far...

NA61/SHINE: Acta Phys.Polon.Supp. 10 (2017) 449

# Net-proton fluctuations

## Experimental results: Fluctuations in large momentum bins



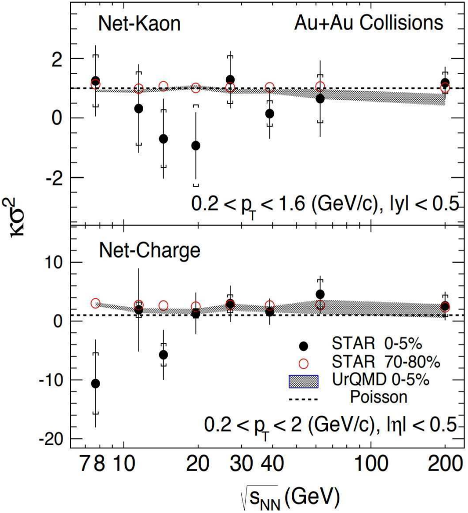




Observation of non-monotonic energy dependence of fourth order net-proton fluctuation.

STAR: PRL 112 (2014) 032302
Stephanov, PRL 107, 052301

# Net-kaon and net-charge fluctuations

## Experimental results: Fluctuations in large momentum bins



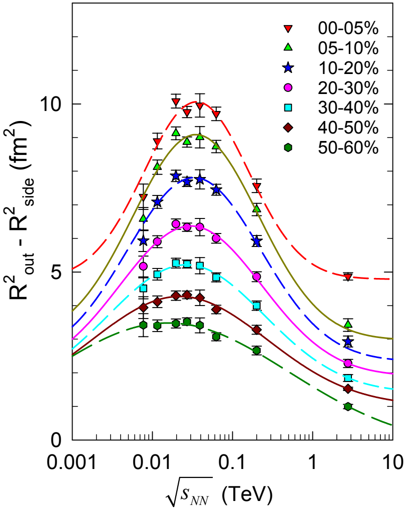

Within errors, the results on net-kaon and net-charge show flat energy dependence.

No prominent structures that could be related to the critical point are observed so far...


STAR: PLB 785 (2018) 551
PRL 113 (2014) 092301

# Short-range correlations

## Experimental results




Data taken from:
STAR: Phys. Rev. C 92, 014904 (2015)
ALICE: Phys.Lett. B696 (2011) 328

Clear non-monotonic energy dependence in Au+Au/Pb+Pb

Lacey, PRL 114 (2015) 142301

# Short-range correlations

## Experimental results

**Finite size scaling analysis**

$L = \overline{R}$

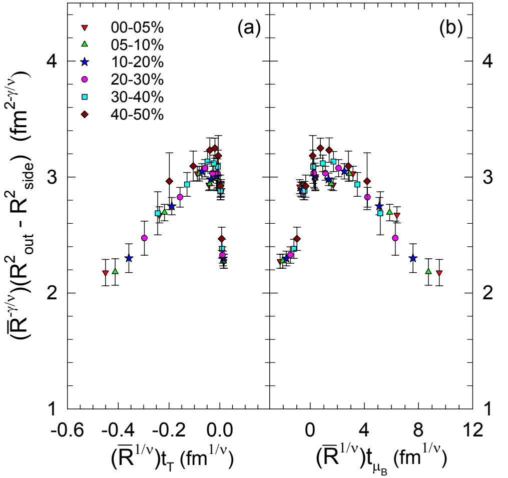
$1/\overline{R} = \sqrt{(1/\sigma_x)^2 + (1/\sigma_y)^2}$

$\sigma_x$, $\sigma_y$ - widths of source

$t_T = \dfrac{T - T_C}{T_C}$

$t_{\mu_B} = \dfrac{\mu - \mu_C}{\mu_C}$

CP in Au+Au at $\sqrt{s_{NN}} = 47.5$ GeV
$T_C = 165$ MeV
$\mu_C = 95$ MeV

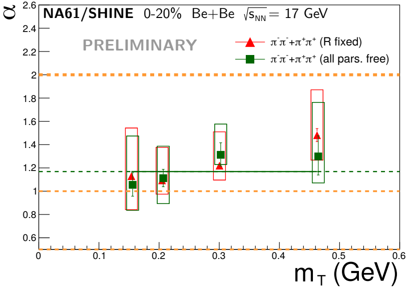


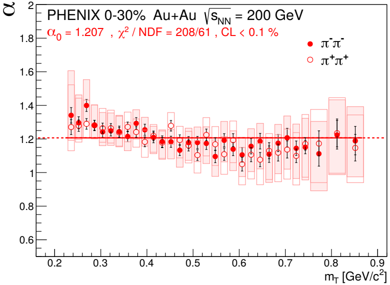

Lacey, PRL 114 (2015) 142301

# Short-range correlations

## Experimental results

Transverse mass dependence of Lévy exponent $\alpha$




$\alpha$(Be+Be at 17 GeV) $\approx$ $\alpha$(Au+Au at 200 GeV) $\approx$ 1.2

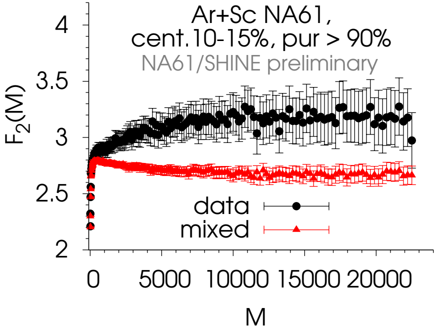
No indication of the critical point so far...

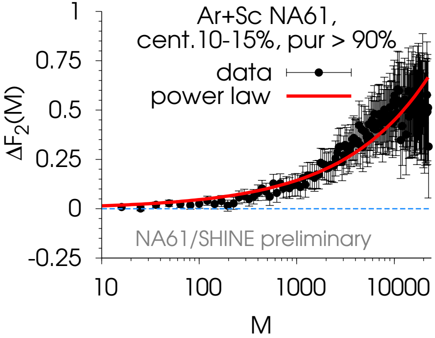
NA61/SHINE: Porfy, WPCF2019
PHENIX: PRC 97, 064911 (2018)

# Fluctuations as a function of momentum bin size

## Experimental results

Mid-rapidity protons at 17 GeV



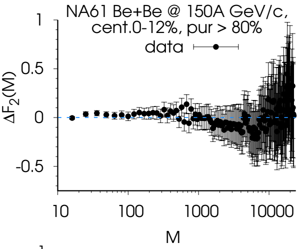

Note that points are strongly correlated.

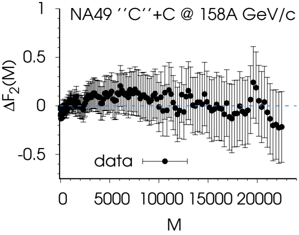
NA61/SHINE: PoS(CPOD2017) 054

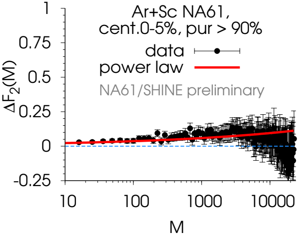
# Fluctuations as a function of momentum bin size

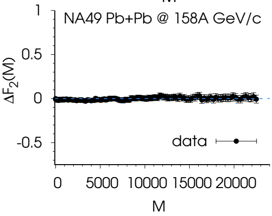
## Experimental results

$\Delta F_2$ for mid-rapidity protons at 17 GeV



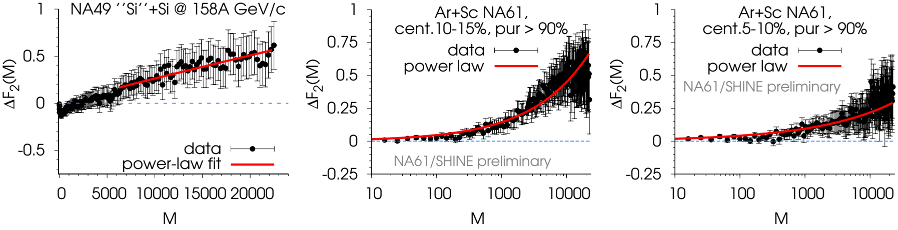




No signal visible in central Be+Be, C+C, Ar+Sc and Pb+Pb

NA49: EPJC 75 (2015) 587
NA61/SHINE: PoS(CPOD2017) 054

$\Delta F_2$ for mid-rapidity protons at 17 GeV

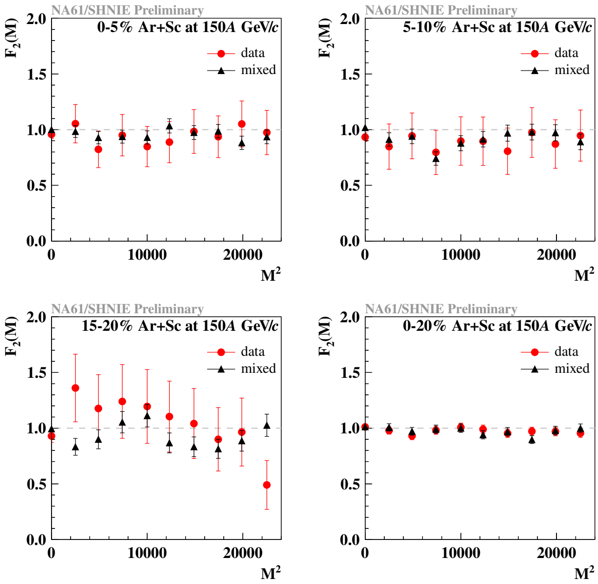



A deviation of $\Delta F_2$ from zero seems apparent in
central Si+Si and mid-central Ar+Sc

NA49: EPJC 75 (2015) 587
NA61/SHINE: PoS(CPOD2017) 054

# Fluctuations as a function of momentum bin size

## Experimental results

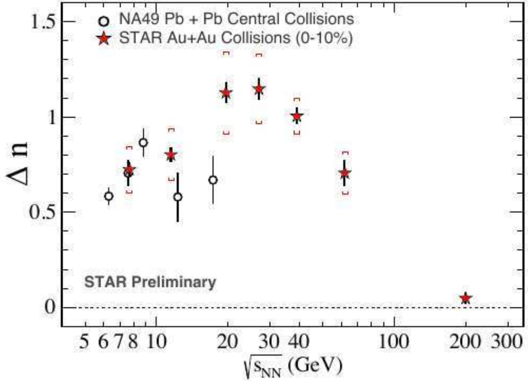



cumulative variables
uncorrelated points

**No signal observed in Ar+Sc at 150*A* GeV/*c***

# Light nuclei production

## Experimental results




$$\Delta n \approx \frac{1}{2\sqrt{3}} \frac{N_p \cdot N_t}{N_d^2} - 1$$

Data taken from:
NA49: PRC 94, 044906 (2016)
STAR: PRL 121, 032301 (2018)
NPA 967, 788

$\Delta$n shows a non-monotonic behavior on collision energy with a peak $\sqrt{s_{NN}} \approx 20$ GeV

STAR: Zhang, QM2018

1. Critical point search strategies
2. Experimental measures
3. Experimental results
4. **Summary**

# Summary

$4^{th}$ moment of
net-proton dist.:
$\approx$ 7 GeV
(Au+Au)

Proton intermittency:
$\approx$ 17 GeV
(Si+Si and Ar+Sc)

Light ion production:
$\approx$ 20 GeV
(Au+Au)

Pion interferometry:
$\approx$ 47 GeV
(Au+Au)

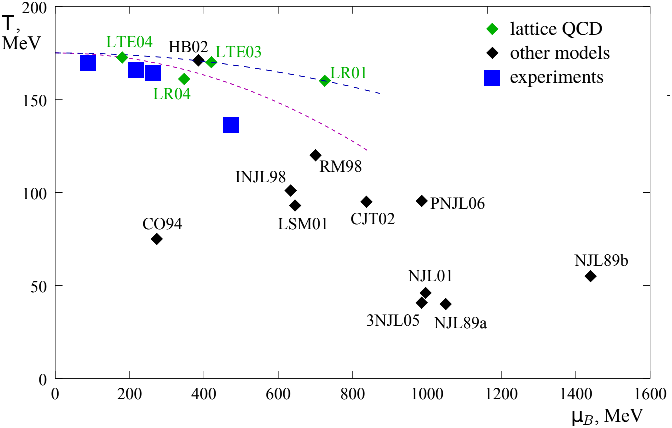
## Summary

4th moment of net-proton dist.: ≈ 7 GeV (Au+Au)

Proton intermittency: ≈ 17 GeV (Si+Si and Ar+Sc)

Light ion production: ≈ 20 GeV (Au+Au)

Pion interferometry: ≈ 47 GeV (Au+Au)



# Summary

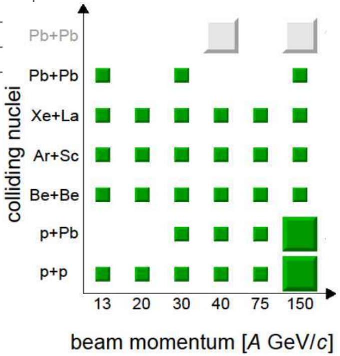
4th moment of net-proton dist.: $\approx$ 7 GeV (Au+Au)

Proton intermittency: $\approx$ 17 GeV (Si+Si and Ar+Sc)

Light ion production: $\approx$ 20 GeV (Au+Au)

Pion interferometry: $\approx$ 47 GeV (Au+Au)

| $\sqrt{s}$ (GeV) | Statistics(Millions) – BES-I | Statistics(Millions) – BES-II |
|---|---|---|
| 7.7 | ~4 | **~100** |
| 9.1 | - | **~160** |
| 11.5 | ~12 | **~230** |
| 14.5 | ~ 20 | **~300** |
| 19.6 | ~36 | **~400** |
| 27 | ~70 | **~500** |




**New, high-quality data is coming soon...**

**Thank You!**