Calculations of QCD Instanton Processes in Deep Inelastic Scattering

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Topological Effects in the Standard Model
Virtual Workshop
16 Dec 2020

Introduction

New Physics within the Standard Model

- Standard Model of electroweak (QFD) and strong (QCD) interactions extremely successful

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4} W_a^{\mu\nu} W_{\mu\nu}^a - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 |\phi|^2 - \lambda |\phi|^4 \\
& + \sum_i \left(\bar{L}^i i \not{D} L^i + \bar{R}^i i \not{D} R^i + \bar{Q}_L^i i \not{D} Q_L^i + \bar{u}_R^i i \not{D} u_R^i + \bar{d}_R^i i \not{D} d_R^i \right) \\
& - \sqrt{2} \sum_{ij} \left(\lambda^{ij} \bar{L}^i \phi R^j + \lambda_d^{ij} \bar{Q}_L^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_L^i \phi^c u_R^j + \text{h.c.} \right) \\
& - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_f \bar{q}^f i \not{D}_{\text{QCD}} q^f
\end{aligned}
$$

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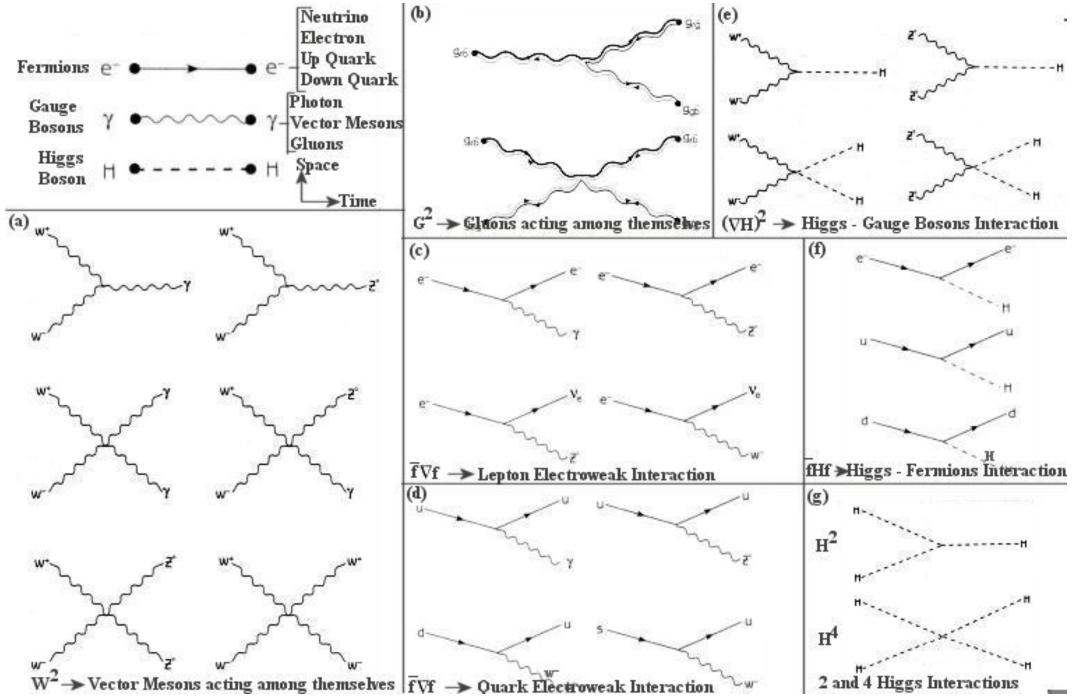

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{4}W_a^{\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2|\phi|^2 - \lambda|\phi|^4 \\
&+ \sum_i \left(\bar{L}^i i\not{D} L^i + \bar{R}^i i\not{D} R^i + \bar{Q}^i_L i\not{D} Q^i_L + \bar{u}^i_R i\not{D} u^i_R + \bar{d}^i_R i\not{D} d^i_R\right) \\
&- \sqrt{2}\sum_{ij}\left(\lambda^{ij}\bar{L}^i\phi R^j + \lambda_d^{ij}\bar{Q}^i_L\phi d^j_R + \lambda_u^{ij}\bar{Q}^i_L\phi^c u^j_R + \text{h.c.}\right) \\
&- \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} + \sum_f \bar{q}^f i\not{D}_{\text{QCD}} q^f
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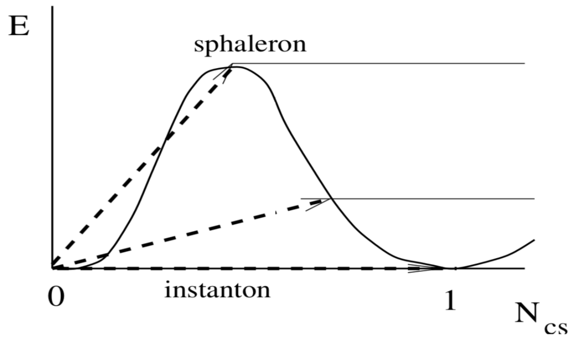
- There are processes inaccessible to ordinary perturbation theory

  B+L/Chirality-violating processes in QFD/QCD

- Induced by topological fluctuations of non-Abelian gauge fields, in particular instantons

[Belavin et al. \`75; \`t Hooft \`76]



- $B+L/Q_5$ are anomalous,

[Adler \`69; Bell,Jackiw \`69; Bardeen \`69]

$$\Delta(B+L) = -2\, n_g\, \Delta N_{\rm CS}[W]$$
$$\Delta Q_5 = 2\, n_f\, \Delta N_{\rm CS}[G]$$

- Topological fluctuations of the gauge fields W/G, i.e. fluctuations with integer $\triangle N_{\rm CS} \neq 0$, induce anomalous processes
- Instanton: lowest Euclidean action configuration with $\Delta N_{\rm CS} = 1 \Rightarrow$ tunneling
- Sphaleron: lowest static energy configuration with $N_{\rm CS} = 1/2 \Rightarrow$ barrier

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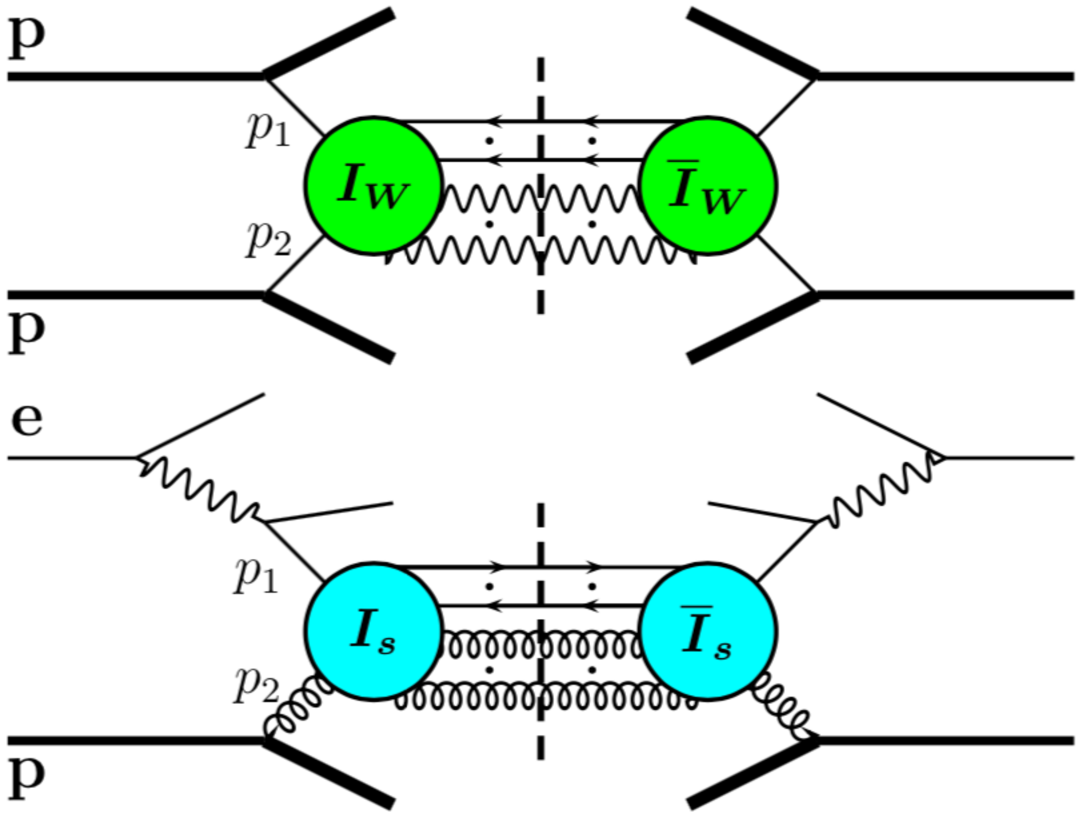
  B+L/Chirality-violating processes in QFD/QCD

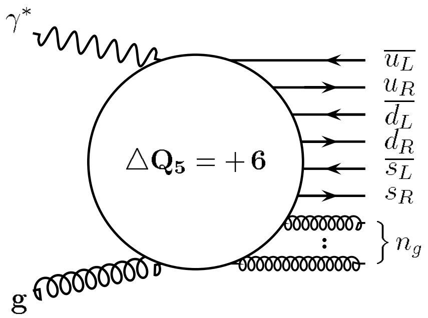
- Induced by topological fluctuations of non-Abelian gauge fields, in particular instantons [Belavin et al. `75; `t Hooft `76]
- Are anomalous instanton-induced events directly observable at high-energy colliders?
  - Electroweak B+L violation at SSC? [AR `90; Espinosa `90;...]
  - QCD-instanton induced processes in Deep Inelastic Scattering (DIS) at HERA? [AR,F. Schrempp `94;...]



QCD-Instanton Induced Processes in DIS

Generic Processes

- Generic QCD-instanton-induced chirality-violating processes in DIS:



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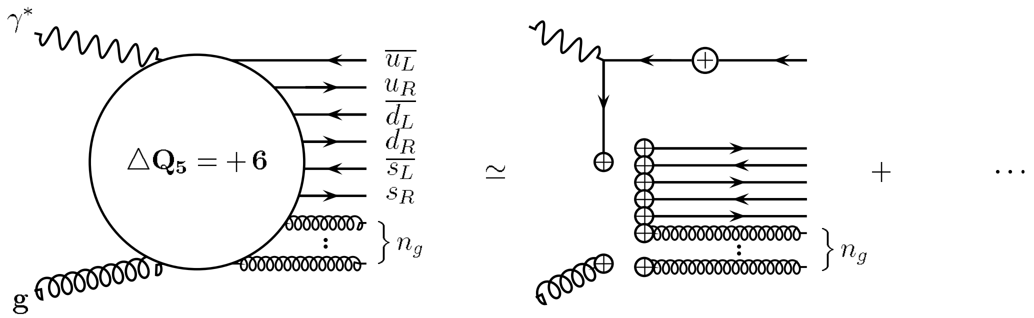

- In instanton-perturbation theory, amplitude given in terms of integral over instanton collective coordinates:

$$\mathcal{T}^{a\,a_1\ldots a_{n_g}}_{\mu\,\mu'}\left(\gamma^* + g \to \sum_{\text{flavours}}^{n_f} [\overline{q_L} + q_R] + n_g\, g\right) = \int_0^\infty \frac{d\rho}{\rho^5}\, d(\rho,\mu_r) \int dU\, \mathcal{A}^{a\,a_1\ldots a_{n_g}}_{\mu\,\mu'}(\rho, U)$$

  - Size ρ and color orientation U

$$A^{(I)}_\mu(x;\rho,U) = -\frac{i}{g}\,\frac{\rho^2}{x^2}\, U\, \frac{\sigma_\mu\,\overline{x} - x_\mu}{x^2+\rho^2}\, U^\dagger \qquad \mathcal{L}\left(A^{(I)}_\mu(x;\rho,U)\right) = \frac{12}{\pi\alpha_s}\cdot\frac{\rho^4}{(x^2+\rho^2)^4} \qquad S\left[A^{(I)}_\mu\right] = \frac{2\,\pi}{\alpha_s}$$

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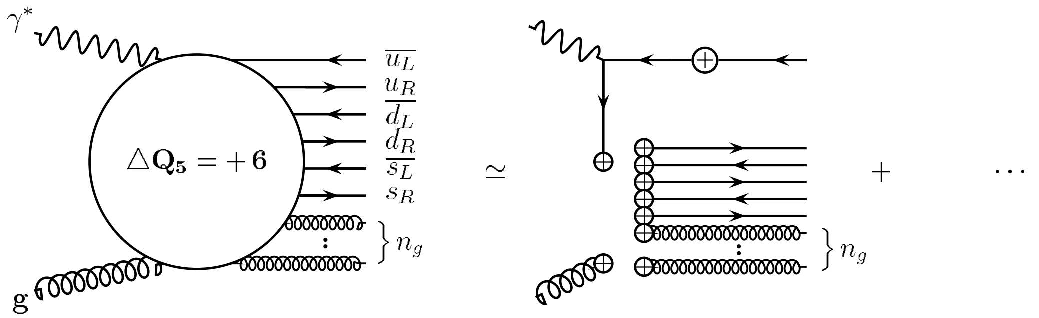

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  - Size distribution, for $\alpha_s(\mu_r)\log(\rho\mu_r) \ll 1$: [`t Hooft `76; Bernard `79; Morris et al. `85]

$$d(\rho,\mu_r) = d\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right] (\rho\,\mu_r)^{\beta_0 + \frac{\alpha_s(\mu_r)}{4\,\pi}(\beta_1 - 12\,\beta_0)}$$

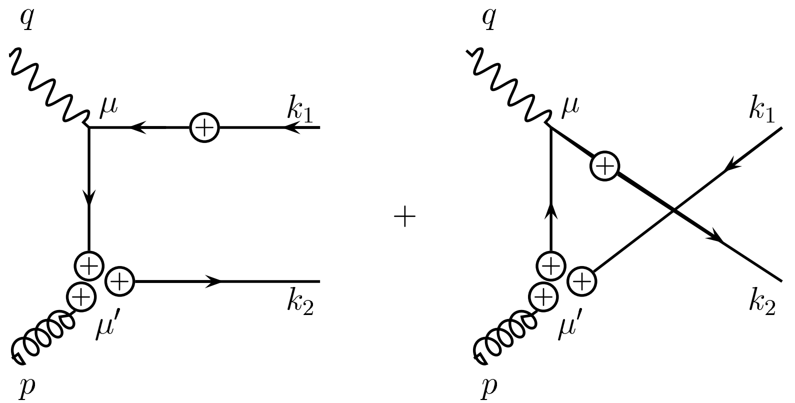
$$d = \frac{C_1}{2}\,\mathrm{e}^{-3\,C_2 + n_f\,C_3} \qquad\qquad \beta_0 = 11 - \frac{2}{3}\,n_f\,; \qquad \beta_1 = 102 - \frac{38}{3}\,n_f$$

QCD-Instanton Induced Processes in DIS

Simplest Process

- Simplest process, in leading-order:

[Moch,AR,F Schrempp `97]



$$\mathcal{A}^{a}_{\mu\,\mu'} = -i\,e_q \lim_{p^2\to 0} p^2\,\mathrm{tr}\left(\lambda^a\,A^{(I)}_{\mu'}(p)\right)\times$$

$$\chi^\dagger_R(k_2)\left[\lim_{k_2^2\to 0}(ik\!\!\!/_2)\,\kappa(-k_2)\,\mathcal{V}^{(t)}_\mu(q,-k_1) + \mathcal{V}^{(u)}_\mu(q,-k_2)\lim_{k_1^2\to 0}\overline{\phi}(-k_1)\,(-i\,\overline{k\!\!\!/}_1)\right]\chi_L(k_1)$$

- LSZ amputated Fourier transforms of instanton gauge field $A^{(I)}_{\mu'}$, quark zero modes, κ and $\overline{\phi}$, of the Dirac operator in instanton background, and of quark currents involving zero modes and quark propagators in instanton background:

$$\mathcal{V}^{(t)}_\mu(q,-k_1) \equiv \int d^4x\,\mathrm{e}^{-i\,q\cdot x}\left[\overline{\phi}(x)\,\overline{\sigma}_\mu \lim_{k_1^2\to 0} S^{(I)}(x,-k_1)\,(-i\,\overline{k\!\!\!/}_1)\right];$$

$$\mathcal{V}^{(u)}_\mu(q,-k_2) \equiv \int d^4x\,\mathrm{e}^{-i\,q\cdot x}\left[\lim_{k_2^2\to 0}(ik\!\!\!/_2)\,\overline{S}^{(I)}(-k_2,x)\,\sigma_\mu\,\kappa(x)\right]$$

QCD-Instanton Induced Processes in DIS

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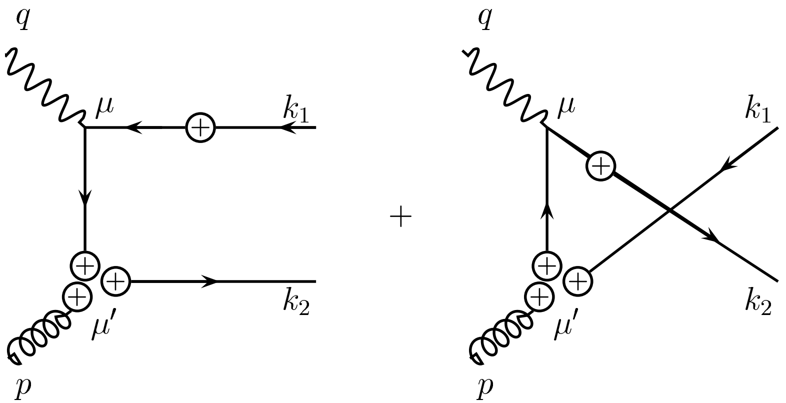
[Moch,AR,F Schrempp `97]

$$Q^2 \equiv -q^2$$

$$x \equiv \frac{Q^2}{2\,p\cdot q}$$



$$t = (q-k_1)^2 = (p-k_2)^2$$

$$u = -t - Q^2/x$$

$$\mathcal{T}^a_{\mu\,\mu'}\,(\gamma^* + g \to \overline{q_L} + q_R) = -i\,\frac{\pi^4}{2}\,\frac{e_q}{g_s}\,\lambda^a \int\limits_0^\infty d\rho\, d(\rho,\mu_r)\,\times$$

$$\times\,\chi_R^\dagger(k_2)\left[(\sigma_{\mu'}\overline{p} - p\overline{\sigma}_{\mu'})\,V(q,k_1;\rho)\overline{\sigma}_\mu - \sigma_\mu\overline{V}(q,k_2;\rho)\,(\sigma_{\mu'}\overline{p} - p\overline{\sigma}_{\mu'})\right]\chi_L(k_1)$$

- Quark current factor:

$$V_\lambda(q,k;\rho) \equiv \left[\frac{(q-k)_\lambda}{-(q-k)^2} + \frac{k_\lambda}{2q\cdot k}\right]\rho\sqrt{-(q-k)^2}\,K_1\left(\rho\sqrt{-(q-k)^2}\right) - \frac{k_\lambda}{2q\cdot k}\rho\sqrt{-q^2}\,K_1\left(\rho\sqrt{-q^2}\right)$$

- Contribution of large instantons exponentially suppressed, as long as q^2, $(q-k_1)^2$, and $(q-k_2)^2$ space-like
- In DIS ($Q^2 = -q^2 > 0$), amplitudes well defined away from collinear singularities occuring at $t, u \to 0_-$

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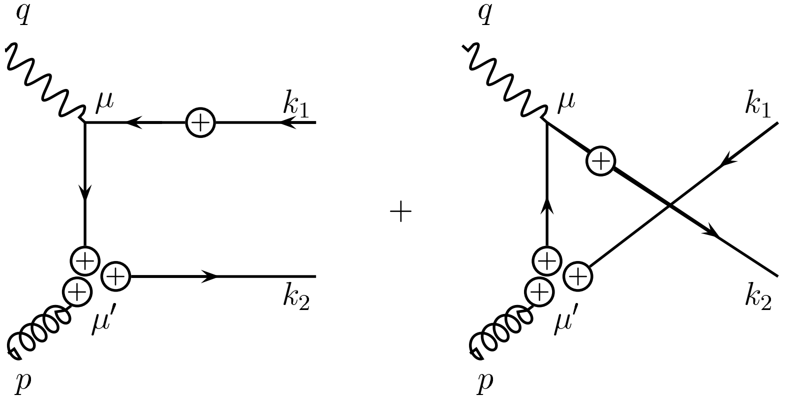


$$t = (q-k_1)^2 = (p-k_2)^2 \qquad u = -t - Q^2/x$$

$$\mathcal{T}^{a}_{\mu\,\mu'}\,(\gamma^* + g \to \overline{q_L} + q_R) = -i\frac{\sqrt{2}}{3}d\pi^3 e_q \left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^{13/2} \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]\, 2^{\,b}\Gamma\left(\frac{b+1}{2}\right)\Gamma\left(\frac{b+3}{2}\right)\lambda^a \times$$

$$\times\,\chi^\dagger_R(k_2)\left[(\sigma_{\mu'}\overline{p} - p\overline{\sigma}_{\mu'})\,v(q,k_1;\mu_r)\overline{\sigma}_\mu - \sigma_\mu\overline{v}(q,k_2;\mu_r)\,(\sigma_{\mu'}\overline{p} - p\overline{\sigma}_{\mu'})\right]\chi_L(k_1)$$

- Quark current factor:

$$v_\lambda(q,k;\mu_r) \equiv \frac{1}{\mu_r}\left\{\left[\frac{(q-k)_\lambda}{-(q-k)^2} + \frac{k_\lambda}{2q\cdot k}\right]\left(\frac{\mu_r}{\sqrt{-\,(q-k)^2}}\right)^{b+1} - \frac{k_\lambda}{2q\cdot k}\left(\frac{\mu_r}{\sqrt{-q^2}}\right)^{b+1}\right\}$$

- Fractional power suppression in virtualities:

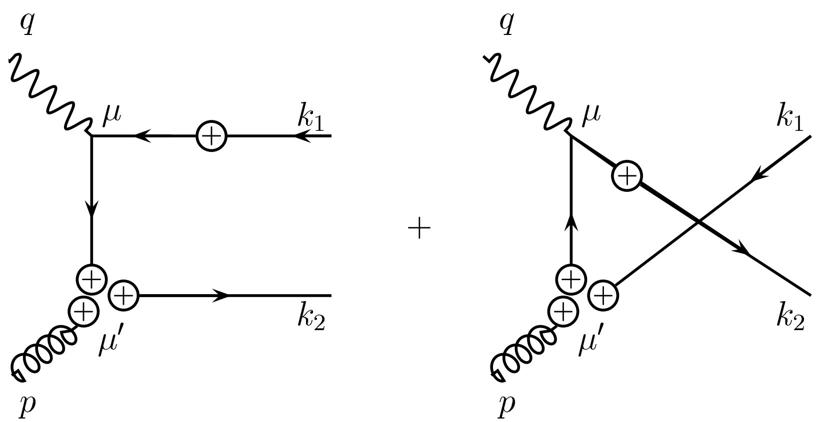
$$b \equiv \beta_0 + \frac{\alpha_s(\mu_r)}{4\,\pi}\,(\beta_1 - 12\,\beta_0)$$

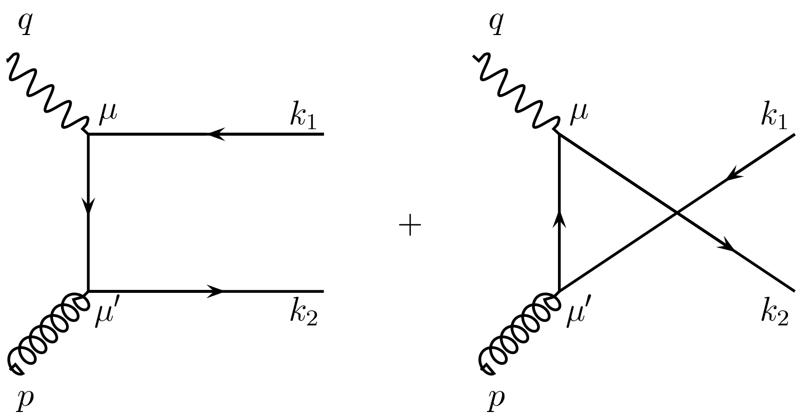
QCD-Instanton Induced Processes in DIS

Simplest Process

- Fixed angle cross-section of simplest QCD-instanton induced DIS process compared with analogue ordinary QCD process:

[Moch,AR,F Schrempp `97]





QCD-Instanton Induced Processes in DIS

Simplest Process

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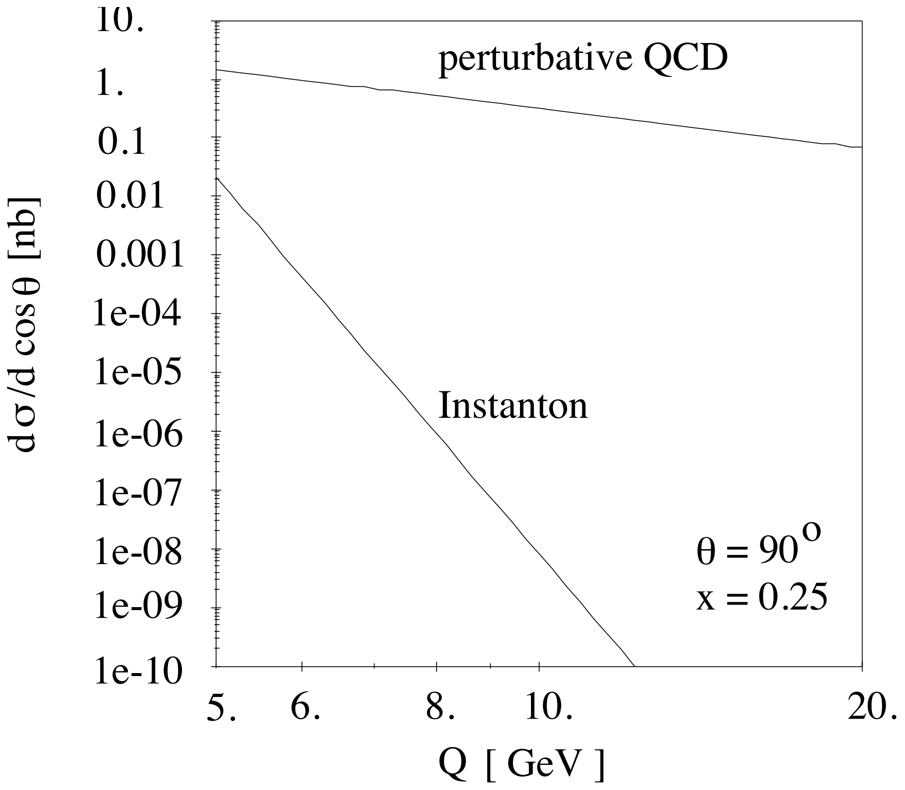
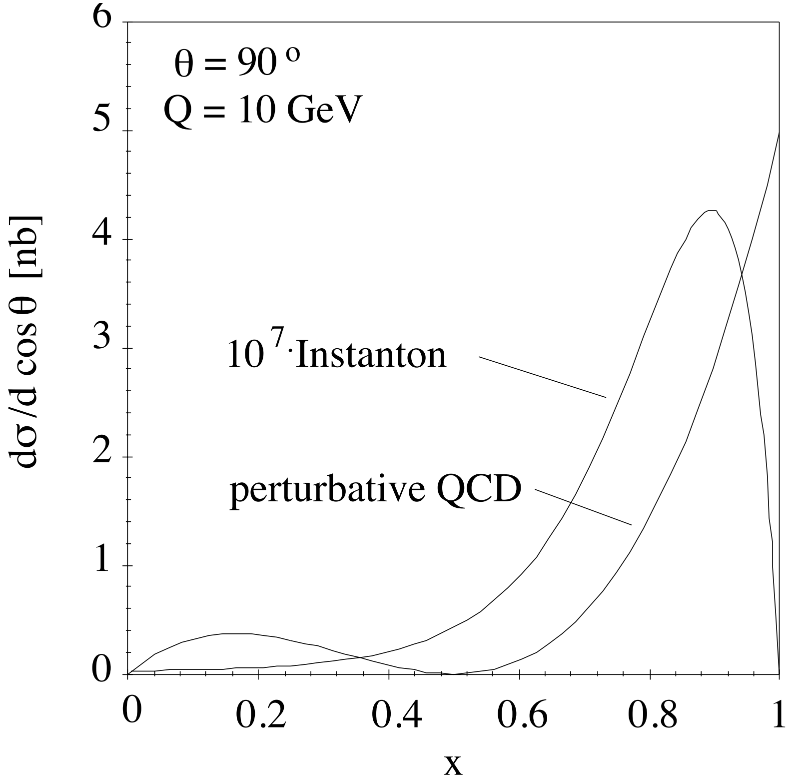
[Moch,AR,F Schrempp `97]

$$t = -\frac{Q^2}{2\,x}\;(1-\cos\theta)$$





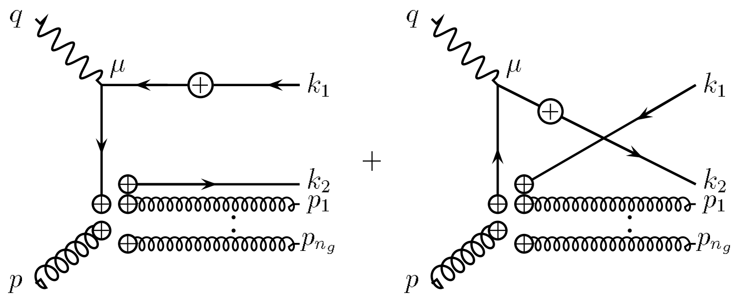
QCD-Instanton Induced Processes in DIS

Generic Processes

- Generic leading-order QCD-instanton induced process in DIS also well behaved away from collinear singularities

[Moch,AR,F Schrempp `97]

- E.g., for $n_f = 1$:



$$\mathcal{T}_\mu^{a\,a_1\ldots a_{n_g}}\,(\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_R + n_g\,\mathrm{g}) = i\,e_q\,4\,\pi^2\left(\frac{\pi^3}{\alpha_s}\right)^{\frac{n_g+1}{2}}\int dU\int_0^\infty d\rho\, d(\rho,\mu_r)\,\rho^{2\,n_g}$$

$$\times\,\mathrm{tr}\left[\lambda^a\,U\;[\epsilon_g(p)\cdot p - \epsilon_g(p)\,\overline{p}]\;U^\dagger\right]\prod_{i=1}^{n_g}\mathrm{tr}\left[\lambda^{a_i}\,U\;[\epsilon_g(p_i)\,\overline{p_i} - \epsilon_g(p_i)\cdot p_i]\;U^\dagger\right]$$

$$\times\left\{\left[U\chi_R^\dagger(k_2)\epsilon\right]\left[\epsilon V(q,k_1;\rho)\overline{\sigma}_\mu\chi_L(k_1)\,U^\dagger\right] - \left[U\chi_R^\dagger(k_2)\sigma_\mu\overline{V}(q,k_2;\rho)\epsilon\right]\left[\epsilon\,\chi_L(k_1)\,U^\dagger\right]\right\}$$

- Quark current factor exponentially suppresses contribution from large instantons in non-collinear DIS kinematics:

$$V_\lambda(q,k;\rho) \equiv \left[\frac{(q-k)_\lambda}{-(q-k)^2} + \frac{k_\lambda}{2q\cdot k}\right]\rho\sqrt{-(q-k)^2}\,K_1\left(\rho\sqrt{-(q-k)^2}\right) - \frac{k_\lambda}{2q\cdot k}\rho\sqrt{-q^2}\,K_1\left(\rho\sqrt{-q^2}\right)$$

QCD-Instanton Induced Processes in DIS

Total Cross-Section

- Total instanton-induced cross-section in DIS:

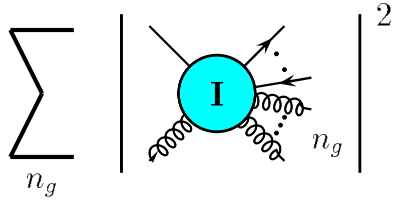
[AR,F Schrempp `98]

$$\hat{\sigma}^{(I)}_{q'p} \sim \int \mathrm{d}^4 R \int_0^\infty \mathrm{d}\rho \int_0^\infty \mathrm{d}\bar{\rho}\, D(\rho) D(\bar{\rho}) \int \mathrm{d}U \,\ldots\, \mathrm{e}^{-\frac{4\pi}{\alpha_s}\Omega\left(U, \frac{R^2}{\rho\bar{\rho}}, \frac{\bar{\rho}}{\rho}\right)}\; \mathrm{e}^{\mathrm{i}(q'+p)\cdot R - Q'\,(\rho+\bar{\rho})}$$

- Interpretation of integration variable R_μ and function Ω in terms of summation of exclusive cross-sections:

[Arnold,Mattis `91;A Mueller `91]

  - Energy-momentum conservation written in terms of integration over R_μ:

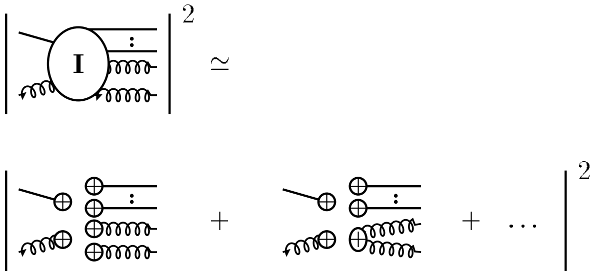


$$(2\,\pi)^4\,\delta^{(4)}(p + q' - \sum_i k_i) = \int d^4 R \;\exp\left[i\,(p + q' - \sum_i k_i)\cdot R\right]$$

Phase space integration over final state gluons and quarks performed via:

$$\int \frac{d^4 k_i}{(2\,\pi)^3}\,\delta^{(+)}(k_i^2)\;\exp\left[-i\,k_i \cdot R\right] = \frac{1}{(2\pi)^2}\,\frac{1}{-R^2 + i\epsilon R_0}$$

  - Function Ω takes into account exponentiation of final state tree-graph corrections:



$$\Omega\left(\frac{R^2}{\rho\bar{\rho}}, \frac{\bar{\rho}}{\rho}\right) = -6\left(\frac{\rho\bar{\rho}}{-R^2 + i\epsilon R_0}\right)^2 + 12\left(\frac{\rho\bar{\rho}}{-R^2 + i\epsilon R_0}\right)^3\left(\frac{\rho}{\bar{\rho}} + \frac{\bar{\rho}}{\rho}\right) + \ldots$$

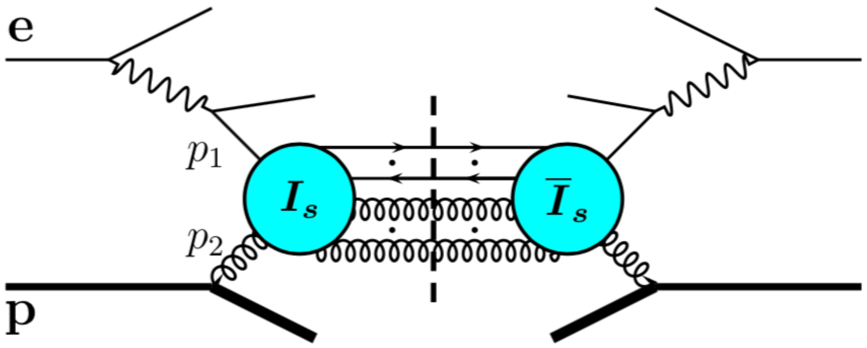
QCD-Instanton Induced Processes in DIS

Total Cross-Section

- Total instanton-induced cross-section in DIS: [AR,F Schrempp `98]

$$\hat{\sigma}^{(I)}_{q'p} \sim \int \mathrm{d}^4 R \int_0^\infty \mathrm{d}\rho \int_0^\infty \mathrm{d}\overline{\rho}\, D(\rho) D(\overline{\rho}) \int \mathrm{d}U \; \ldots \; \mathrm{e}^{-\frac{4\pi}{\alpha_s}\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \frac{\overline{\rho}}{\rho}\right)} \; \mathrm{e}^{\mathrm{i}(q'+p)\cdot R - Q'\,(\rho+\overline{\rho})}$$

- Interpretation of integration variable R_μ and function Ω in terms of the instanton-anti-instanton interaction: Exploit optical theorem and instanton-anti-instanton valley method: [Zakharov `90;VV Khoze,AR `91]



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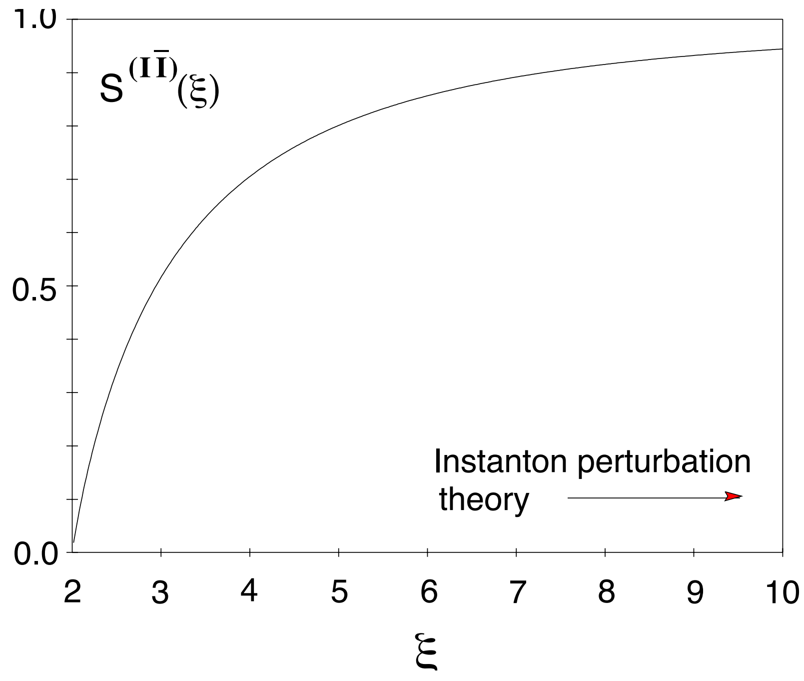
- Interpretation of integration variable R_μ and function Ω in terms of the instanton-anti-instanton interaction: Exploit optical theorem and instanton-anti-instanton valley method: [Zakharov `90;VV Khoze,AR `91]
  - R_μ: distance between instanton and anti-instanton
  - Ω : interaction between instanton and anti-instanton: $\Omega \equiv S^{(I\bar{I})} - 1$
    - Classical conformal invariance:
      - Depends only on conformal distance: $\xi \equiv \frac{R^2}{\rho\overline{\rho}} + \frac{\rho}{\overline{\rho}} + \frac{\overline{\rho}}{\rho}$

$$\Omega(1,\xi) = -\frac{12}{f(\xi)} - \frac{96}{f(\xi)^2} + \frac{48}{f(\xi)^3}\left[3f(\xi)+8\right]\ln\left[\frac{1}{2\xi}\left(f(\xi)+4\right)\right]$$

$$f(\xi) = \xi^2 + \sqrt{\xi^2 - 4}\xi - 4$$



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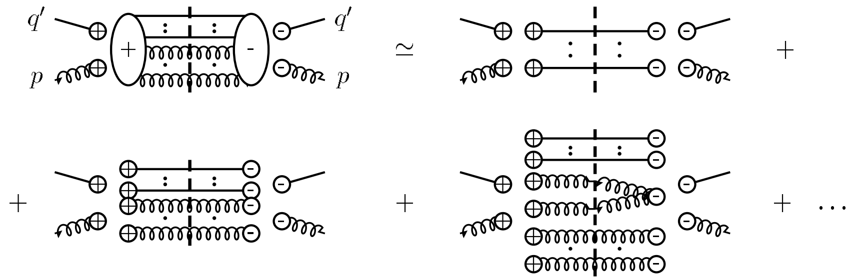
  - R_μ: distance between instanton and anti-instanton
  - Ω : interaction between instanton and anti-instanton: $\Omega \equiv S^{(I\bar{I})} - 1$
    - Classical conformal invariance:
      - Depends only on conformal distance: $\xi \equiv \frac{R^2}{\rho\overline{\rho}} + \frac{\rho}{\overline{\rho}} + \frac{\overline{\rho}}{\rho}$
    - Leading term in asymptotic expansion for large conformal distance,

$$\Omega(\xi) = -\frac{6}{\xi^2} + \mathcal{O}(\ln(\xi)/\xi^4))$$

exactly reproduces the leading and next-to-leading final state corrections



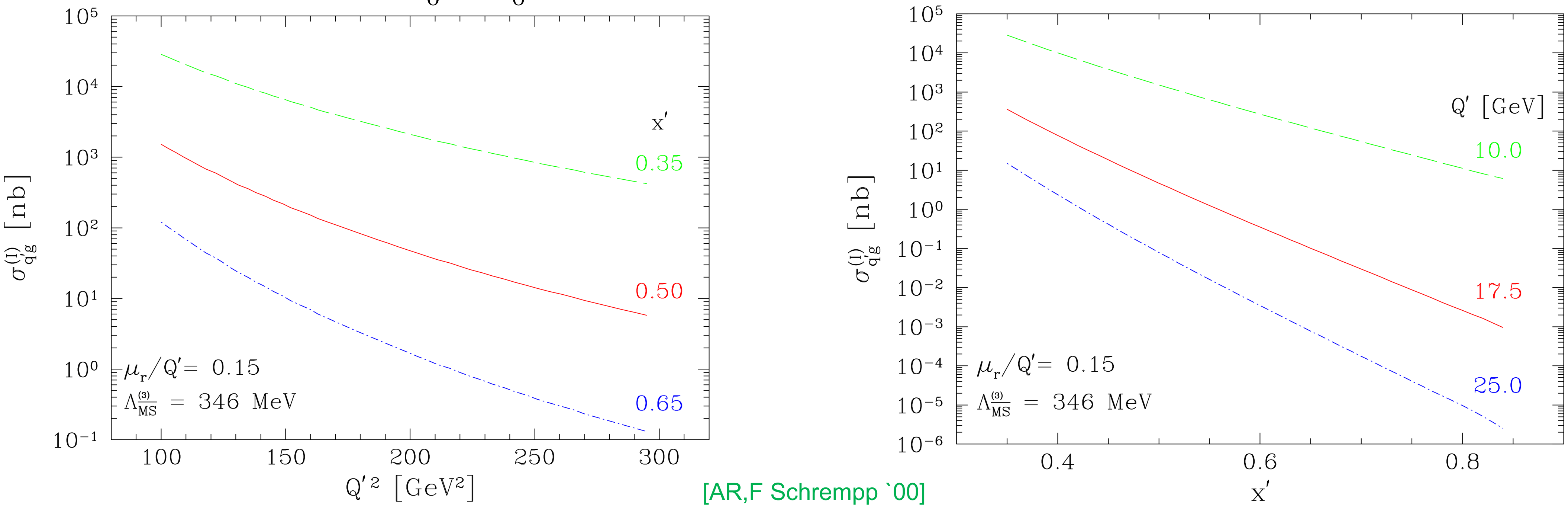
- Method effectively sums up gluonic final state tree graph corrections to leading semiclassical result

QCD-Instanton Induced Processes in DIS

Total Cross-Section

- Saddle point evaluation of total instanton-induced cross-section in DIS: $U_* = 1,\ \rho_* = \bar\rho_* \sim 1/Q',\ R_*^2 \sim 1/(p+q')^2$

$$\hat\sigma^{(I)}_{q'p} \sim \int \mathrm{d}^4 R \int_0^\infty \mathrm{d}\rho \int_0^\infty \mathrm{d}\bar\rho\, D(\rho) D(\bar\rho) \int \mathrm{d}U \ldots \mathrm{e}^{-\frac{4\pi}{\alpha_s}\Omega\left(U, \frac{R^2}{\rho\bar\rho}, \frac{\bar\rho}{\rho}\right)}\ \mathrm{e}^{\mathrm{i}(q'+p)\cdot R - Q'(\rho+\bar\rho)}$$



[AR,F Schrempp `00]

- Educated guess for region of validity from lattice simulations of instanton size distribution and instanton-anti-instanton interaction: $\left(\rho_* \Lambda^{(0)}_{\overline{\mathrm{MS}}} \lesssim 0.4;\, R_*/\rho_* \gtrsim 1\right) \Rightarrow \left(Q' \gtrsim 30.8\, \Lambda^{(n_f)}_{\overline{\mathrm{MS}}};\, x' \gtrsim 0.35\right)$

QCD-Instanton Induced Processes in DIS

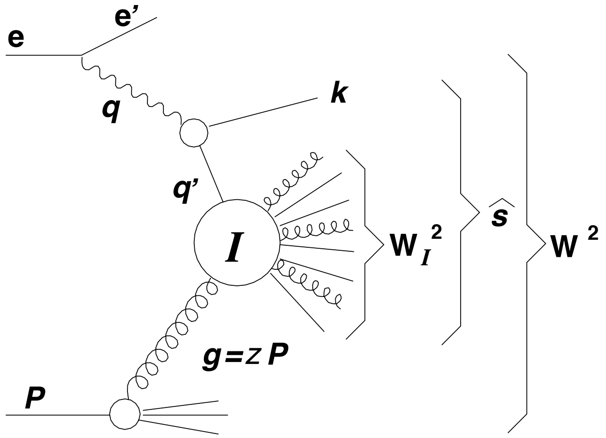
Final States in DIS

- Event generator QCINS based on instanton-anti-instanton valley approach

  [Gibbs,AR,F Schrempp `95; AR,F Schrempp `00]

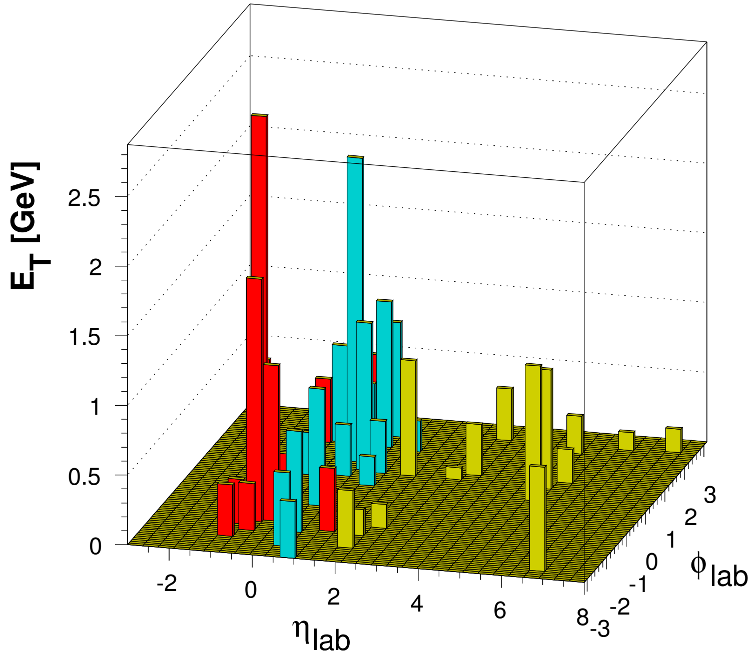
  - Hard subprocess
    - Isotropic in qprime p CM system
    - Flavour democratic
    - Large parton multiplicity

$$\langle n_q + n_g \rangle = 2\, n_f - 1 + \mathcal{O}(1)/\alpha_s \gtrsim 8$$



DIS variables:
$S = (e+P)^2$
$Q^2 = -q^2 = -(e-e')^2$
$x_{\rm Bj} = Q^2/\,(2P\cdot q)$
$W^2 = (q+P)^2 = Q^2(1/x_{\rm Bj}-1)$
$\hat{s} = (q+g)^2$
$z = x_{\rm Bj}\,(1+\hat{s}/Q^2)$

Variables of instanton-subprocess:
$Q'^2 = -q'^2 = -(q-k)^2$
$x' = Q'^2/\,(2\,g\cdot q')$
$W_I^2 = (q'+g)^2 = Q'^2(1/x'-1)$



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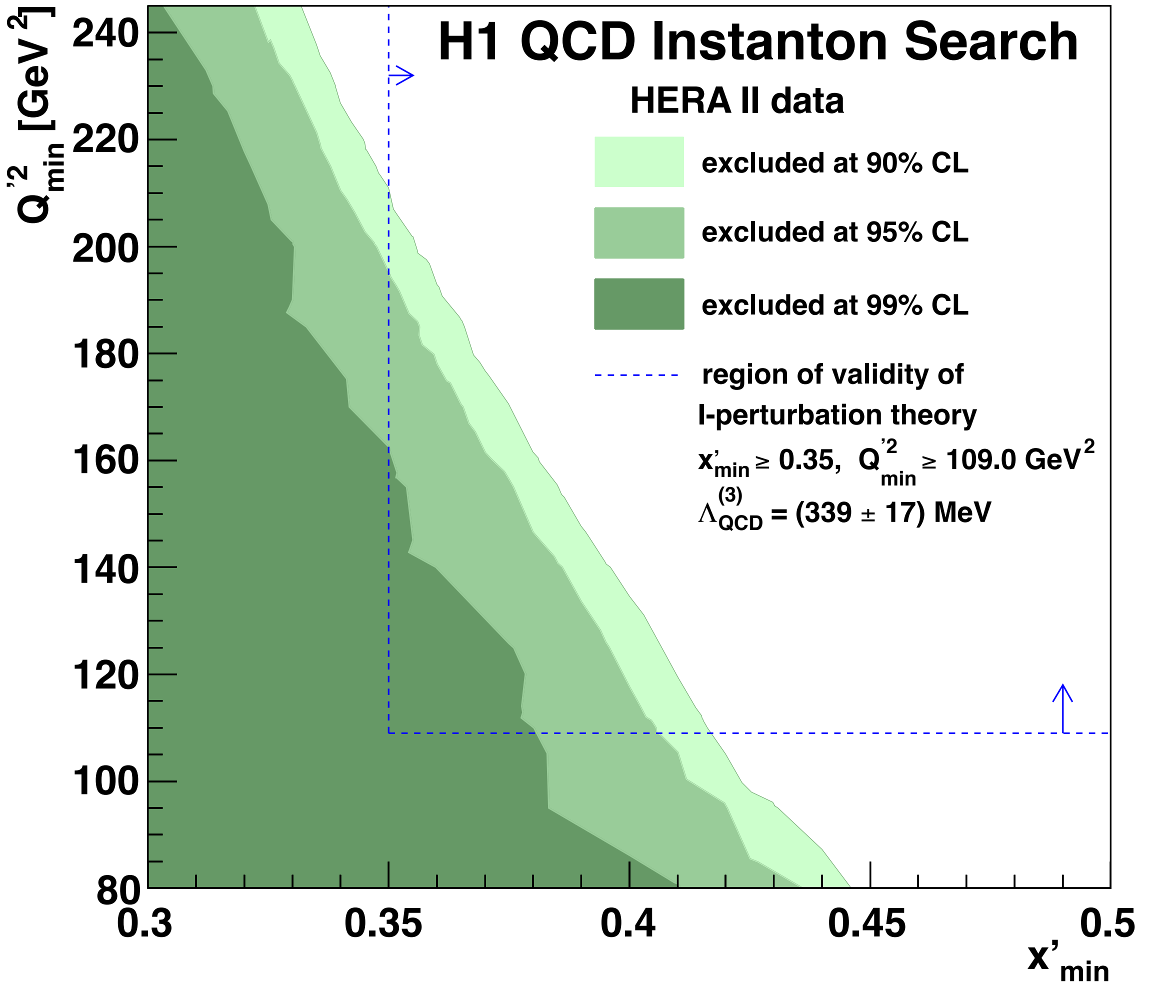
- Dedicated QCD-instanton searches by the H1 and ZEUS collaborations at HERA constrain the kinematic region of validity of the prediction from the instanton-anti-instanton valley approach



[H1 Collaboration, `16]

Summary

- QCD-instanton induced amplitudes in DIS calculable from first principles in instanton perturbation theory
- Total cross-section in pure instanton-anti-instanton valley approach tends to overshoot the HERA data in kinematic region where the probed instanton sizes and instanton-anti-instanton distances are still under perturbative control ...
- May be this problem is solved if one extends the instanton-anti-instanton valley approach by taking into account also initial state corrections to the total cross section ... see talk by Valya Khoze ...