QCD instantons at hadron colliders

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with Frank Krauss & Matthias Schott 1911.09726 : JHEP
and Dan Milne & Michael Spannowsky 2010.02287 : PRD
QCD Instantons

- Yang-Mills vacuum has a nontrivial structure
- Instantons are tunnelling solutions between the vacua.
- At the classical level there is no barrier in QCD. The sphaleron is a quantum effect
- Transitions between the vacua change chirality (result of the ABJ anomaly).
- All light quark-anti-quark pairs must participate in the reaction
- Not described by perturbation theory.

\[ g + g \rightarrow n_g \times g + \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf}) \]
QCD Instantons

Instanton-induced processes with 2 gluons in the initial state:

\[
g + g \rightarrow n_g \times g + \sum_{f=1}^{N_f} (q_R f + \bar{q}_L f) \]

All light flavours of quark-antiquark pairs must be present. Light =>

\[
m_f \leq 1/\rho. \]

( tends to be large \(\sim 1/\alpha_s\) )

Can also have quark-initiated processes e.g. :

\[
\begin{align*}
 u_L + \bar{u}_R &\rightarrow n_g \times g + \sum_{f=1}^{N_f-1} (q_R f + \bar{q}_L f), \\
u_L + d_L &\rightarrow n_g \times g + u_R + d_R + \sum_{f=1}^{N_f-2} (q_R f + \bar{q}_L f)
\end{align*}
\]
\[ g + g \rightarrow n_g \times g + \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf}) \]

The amplitude takes the form of an integral over instanton collective coordinates. The classical result (leading order in the instanton perturbation theory) is simply:

\[ S_I = \frac{8\pi^2}{g^2} = \frac{2\pi}{\alpha_s(\mu_r)} \]

\[ A_{2\rightarrow n_g+2N_f} \sim \int d^4x_0 \, d\rho \, D(\rho) \, e^{-S_I} \left[ \prod_{i=1}^{n_g+2} A_{LSZ}^{a_i \text{inst}}(p_i, \lambda_i) \right] \left[ \prod_{j=1}^{2N_f} \psi_{LSZ}^{(0)}(p_j, \lambda_j) \right] \]

- the integrand: a product of bosonic and fermionic components of the instanton field configurations
- the factorised structure implies that emission of individual particles in the final state is uncorrelated and mutually independent.

[This is correct at the LO in instanton pert. theory approximation]

LO Instanton vertex
The Optical Theorem approach: to include final state interactions

- Cross-section is obtained by squaring the instanton amplitude.
- Final states have been instrumental in combatting the exp. suppression.
- Now also the interactions between the final states (and the improvement on the point-like I-vertex) are taken into account.
- Use the Optical Theorem to compute Im part of the 2->2 amplitude in around the Instanton-Anti-instanton configuration.
- Varying the energy E changes the dominant value of I-Ibar separation R. At R=0 instanton and anti-instanton mutually annihilate.
- The suppression of the EW instanton cross-section is gradually reduced at lower R(E).

\[ \hat{\sigma}_{\text{tot}}^{\text{inst}} = \frac{1}{E^2} \text{Im} A^I_2(p_1, p_2, -p_1, -p_2) \]

VVK & Ringwald 1991
The Optical Theorem approach: to include final state interactions

- Instanton — anti-instanton valley configuration has $Q=0$; it interpolates between infinitely separated instanton—anti-instanton and the perturbative vacuum at $R=0$

\[
\sigma_{\text{tot}}^{(\text{cl})\text{inst}} \approx \frac{1}{s} \text{Im} \int_0^\infty d\rho \int_0^\infty d\bar{\rho} \int d^4R \int d\Omega \; D(\rho)D(\bar{\rho}) \; e^{-S_{II}} \; K_{\text{ferm}} \times A_{\text{LSZ}}(p_1) A_{\text{LSZ}}(p_2) A_{\text{LSZ}}(-p_1) A_{\text{LSZ}}(-p_2),
\]

\[
S_{II}(\rho, \bar{\rho}, R) = \frac{4\pi}{\alpha_s(\mu_r)} \hat{S}
\]

instanton-anti-instanton action (see next slide)

- Exponential suppression is gradually reduced at lower $R$ (Energy-dependent)
- no radiative corrections from hard initial states included in this approximation
\( \sigma_{\text{tot}}^{(\text{cl}) \text{inst}} = \frac{1}{s} \text{Im} \ A_{4\bar{I}}(p_{1}, p_{2}, -p_{1}, -p_{2}) \)

\[ \simeq \frac{1}{s} \text{Im} \int_{0}^{\infty} d\rho \int_{0}^{\infty} d\bar{\rho} \int d^{4}R \int d\Omega \ D(\rho) D(\bar{\rho}) \ e^{-S_{I\bar{I}}} \mathcal{K}_{\text{ferm}} \times \]

\[ A_{\text{LSZ}}^{\text{inst}}(p_{1}) A_{\text{LSZ}}^{\text{inst}}(p_{2}) A_{\text{LSZ}}^{\text{inst}}(-p_{1}) A_{\text{LSZ}}^{\text{inst}}(-p_{2}), \]

\[ S(\chi) \simeq 1 - 6/\chi^{4} + 24/\chi^{6} + \ldots \quad \chi = \frac{R}{\rho} \]

\[ S_{I\bar{I}}(\rho, \bar{\rho}, R) = \frac{4\pi}{\alpha_{s}(\mu_{r})} S \]

- Exponential suppression is gradually reduced at lower and lower \( \chi = \frac{R}{\rho} \)
- no radiative corrections from hard initial states included in this approximation
\[
\sigma_{\text{tot}}^{(\text{cl}) \ \text{inst}} = \frac{1}{s} \text{Im} \ A_4^{I\bar{I}}(p_1, p_2, -p_1, -p_2) \\
\simeq \frac{1}{s} \text{Im} \int_0^\infty d\rho \int_0^\infty d\bar{\rho} \int d^4 R \int d\Omega \ D(\rho) D(\bar{\rho}) e^{-S_{I\bar{I}}} K_{\text{ferm}} \times \ \\
A_{\text{LSZ}}^{\text{inst}}(p_1) A_{\text{LSZ}}^{\text{inst}}(p_2) A_{\text{LSZ}}^{\text{inst}}(-p_1) A_{\text{LSZ}}^{\text{inst}}(-p_2),
\]

\[
A_{\text{LSZ}}^{\text{inst}}(p_1) A_{\text{LSZ}}^{\text{inst}}(p_2) A_{\text{LSZ}}^{\text{inst}}(-p_1) A_{\text{LSZ}}^{\text{inst}}(-p_2) = \frac{1}{36} \left( \frac{2\pi^2}{g} \rho^2 \sqrt{s'} \right)^4 e^{i\mathbf{R}(p_1+p_2)}
\]

\[
D(\rho, \mu_r) = \kappa \frac{1}{\rho^5} \left( \frac{2\pi}{\alpha_s(\mu_r)} \right)^6 (\rho \mu_r)^{b_0}
\]

But the instanton size has not been stabilised. In QCD - rho is a classically flat direction — need to include and re-sum quantum corrections!
in the EW theory:

\[ G_{4\text{Eucl}} \sim \int d^4 R \, d\rho_I d\rho_{\bar{I}} \ldots \exp \left[ i(p_1 + p_2) \cdot R - S_{I\bar{I}}(z) - \pi^2 v^2 (\rho_I^2 + \rho_{\bar{I}}^2) \right] \]

\[ \sigma_{B+L} \sim \text{Im} \int d^4 R \, d\rho_I d\rho_{\bar{I}} \ldots \exp \left[ ER - S_{I\bar{I}}(R) - \pi^2 v^2 (\rho_I^2 + \rho_{\bar{I}}^2) \right] \]

- Exponential suppression is gradually reduced with energy [in the EW theory]

Higgs vev: EW theory - not QCD!

Higgs vev cuts-off large instantons
**In QCD:**

\[
G_{4 \text{Eucl}} \sim \int d^4 R \, d\rho_I d\rho_{\bar{I}} \cdots \exp [i(p_1 + p_2) \cdot R - S_{I\bar{I}}(z)]
\]

\[
\sigma_{B+L} \sim \text{Im} \int d^4 R \, d\rho_I d\rho_{\bar{I}} \cdots \exp [ER - S_{I\bar{I}}(R)]
\]

- new in QCD

Quantum effects to cut-off

Instanton size integrations

new in QCD
**Initial state interactions in the instanton approach**

LO instanton process

\[
\begin{align*}
 & p_1 \quad A(p_1) \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & p_2 \quad A(p_2)
\end{align*}
\]

NLO instanton process

\[
\begin{align*}
 & p_1 \quad A(p_1) \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & k_1 \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & k_2 \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & \ldots \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & k_n \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & p_2 \quad G(p_1, p_2) \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & k_1 \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & k_2 \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & \ldots \\
 & \quad \bigcirc \bigcirc \bigcirc \\
 & k_n
\end{align*}
\]

Propagator in the instanton background

\[
G^{ab}_{\mu\nu}(p_1, p_2) \rightarrow -\frac{g^2 \rho^2 s}{64\pi^2} \log(s) A^a_\mu(p_1) A^b_\nu(p_2)
\]

\[
p_1^2 = 0 = p_2^2, \quad 2p_1p_2 = s \gg 1/\rho^2
\]

Include now higher order corrections in the high-energy limit:

\[
\sum_{r=1}^{N} \frac{1}{r!} \left( -\frac{g^2 \rho^2 s}{64\pi^2} \log(s) \right)^r A^a_\mu(p_1) A^b_\nu(p_2)
\]

Mueller 1991
\[ e^{-\frac{\alpha_s(\mu_r)}{16\pi} \rho^2 E^2 \log E^2/\mu_r^2} \]
Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}_{\text{tot}}^{\text{inst}} \sim \frac{1}{s'} \text{Im} \frac{\kappa^2 \pi^4}{36 \cdot 4} \int \frac{d\rho}{\rho^5} \int \frac{d\bar{\rho}}{\bar{\rho}^5} \int d^4 R \int d\Omega \left( \frac{2\pi}{\alpha_s(\mu_r)} \right)^{14} (\rho^2 \sqrt{s'})^2 (\bar{\rho}^2 \sqrt{s'})^2 K_{\text{ferm}}$$

$$(\rho \mu_r)^{b_0} (\bar{\rho} \mu_r)^{b_0} \exp \left( R_0 \sqrt{s'} - \frac{4\pi}{\alpha_s(\mu_r)} \hat{S}(z) - \frac{\alpha_s(\mu_r)}{16\pi} (\rho^2 + \bar{\rho}^2) s' \log \left( \frac{s'}{\mu_r^2} \right) \right)$$

Instanton size is cut-off by $\sim \sqrt{s}$

this is what sets the effective QCD sphaleron scale

Mueller’s result for quantum corrections due to in-in states interactions

Basically, in QCD one can never reach the effective sphaleron barrier — it’s height grows with the energy.

$\Rightarrow$ Among other things, no problems with unitarity.

This is the main idea of the approach:

[1] VVK, Krauss, Schott
[2] VVK, Milne, Spannowsky
\[
\hat{\sigma}_{\text{tot}}^{\text{inst}} \approx \frac{1}{s'} \operatorname{Im} \frac{\kappa^2 \pi^4}{36 \cdot 4} \int \frac{d\rho}{\rho^5} \int \frac{d\tilde{\rho}}{\tilde{\rho}^5} \int d^4 R \int d\Omega \left( \frac{2\pi}{\alpha_s(\mu_r)} \right)^{14} (\rho^2 \sqrt{s'})^2 (\tilde{\rho}^2 \sqrt{s'})^2 K_{\text{ferm}} \\
(\rho \mu_r)^{b_0} (\tilde{\rho} \mu_r)^{b_0} \exp \left( R_0 \sqrt{s'} - \frac{4\pi}{\alpha_s(\mu_r)} \hat{S}(z) - \frac{\alpha_s(\mu_r)}{16\pi} (\rho^2 + \tilde{\rho}^2) s' \log \left( \frac{s'}{\mu_r^2} \right) \right)
\]

1. Externalise the holy-grail function in the exponent by finding a saddle-point in variables:

\[
\tilde{\rho} = \frac{\alpha_s(\rho)}{4\pi} \sqrt{s\rho}, \quad \chi = \frac{R}{\rho}
\]

\[
\mathcal{F} = \rho \chi \sqrt{s} - \frac{4\pi}{\alpha_s(\rho)} S(\chi) - \frac{\alpha_s(\rho)}{4\pi} \rho^2 s \log(\sqrt{s\rho})
\]
Combined effect of initial and final states interactions in QCD

\[
\hat{\sigma}_{\text{tot}}^{\text{inst}} \approx \frac{1}{s'} \text{Im} \frac{\kappa^2 \pi^4}{36 \cdot 4} \int \frac{d\rho}{\rho^5} \int \frac{d\bar{\rho}}{\bar{\rho}^5} \int d^4R \int d\Omega \left( \frac{2\pi}{\alpha_s(\mu_r)} \right)^{14} (\rho^2 \sqrt{s'})^2 (\bar{\rho}^2 \sqrt{s'})^2 K_{\text{ferm}}  \\
(\rho \mu_r)^{b_0} (\bar{\rho} \mu_r)^{\bar{b}_0} \exp \left( R_0 \sqrt{s'} - \frac{4\pi}{\alpha_s(\mu_r)} \hat{S}(z) - \frac{\alpha_s(\mu_r)}{16\pi} (\rho^2 + \bar{\rho}^2) s' \log \left( \frac{s'}{\mu_r^2} \right) \right)
\]

1st Approach: VVK, Krauss, Schott

1. Exteriorise the holy-grail function in the exponent by finding a saddle-point in variables:

\[
\tilde{\rho} = \frac{\alpha_s(\rho)}{4\pi} \sqrt{s' \rho}, \quad \chi = \frac{R}{\rho}
\]

- II action

\[
\mathcal{F} = \rho \chi \sqrt{s} - \frac{4\pi}{\alpha_s(\rho)} S(\chi) - \frac{\alpha_s(\rho)}{4\pi} \rho^2 s \log(\sqrt{s' \rho})
\]

The holy-grail in units of \( \frac{4\pi}{\alpha_s} \)

\[
u = \sqrt{s}/\mu = \sqrt{s' \rho}
\]
Combined effect of initial and final states interactions in QCD

\[ \hat{\sigma}_{\text{inst}}^{\text{tot}} \approx \frac{1}{s'} \text{Im} \frac{\kappa^2 \pi^4}{36 \cdot 4} \int \frac{d\rho}{\rho^5} \int \frac{d\rho}{\rho^5} \int d^4R \int d\Omega \left( \frac{2\pi}{\alpha_s(\mu_r)} \right)^{14} (\rho^2 \sqrt{s'})^2 (\rho^2 \sqrt{s'})^2 K_{\text{ferm}} \]

\[ (\rho \mu_r)^b_0 (\bar{\rho} \mu_r)^b_0 \exp \left( R_0 \sqrt{s'} - \frac{4\pi}{\alpha_s(\mu_r)} \hat{S}(z) - \frac{\alpha_s(\mu_r)}{16\pi} (\rho^2 + \bar{\rho}^2) s' \log \left( \frac{s'}{\mu_r^2} \right) \right) \]

1st Approach: VVK, Krauss, Schott

1. Extermise the holy-grail function in the exponent by finding a saddle-point in variables:

\[ \mathcal{F} = \rho \chi \sqrt{s} - \frac{4\pi}{\alpha_s(\rho)} S(\chi) - \frac{\alpha_s(\rho)}{4\pi} \rho^2 s \log(\sqrt{s} \rho) \]

\[ \tilde{\rho} = \frac{\alpha_s(\rho)}{4\pi} \sqrt{s} \rho, \quad \chi = \frac{R}{\rho} \]

2. Carry out all integrations using the steepest descent method evaluating the determinants of quadratic fluctuations around the saddle-point solution

3. Pre-factors are very large — they compete with the semiclassical exponent which is very small!
Results

1st Approach: VVK, Krauss, Schott

\[ \hat{\sigma}_{\text{tot}}(\sqrt{s'}) = \frac{1}{\rho} \]

\[ n_g(\sqrt{s'}) \]

\[ \frac{1}{\rho} \quad (\text{GeV}) \]

\[ \rho^{\text{Inverse}}(\sqrt{s'}) \]

\[ \alpha_s[\rho](\sqrt{s'}) \]
Results and partonic cross-section

<table>
<thead>
<tr>
<th>(\sqrt{s'}) [GeV]</th>
<th>(1/\rho) [GeV]</th>
<th>(\alpha_s(1/\rho))</th>
<th>(\langle n_g \rangle)</th>
<th>(\hat{\sigma}) [pb]</th>
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<tbody>
<tr>
<td>10.7</td>
<td>0.99</td>
<td>0.416</td>
<td>4.59</td>
<td>4.922 \cdot 10^9</td>
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<tr>
<td>11.4</td>
<td>1.04</td>
<td>0.405</td>
<td>4.68</td>
<td>3.652 \cdot 10^9</td>
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<td>13.4</td>
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<td>0.382</td>
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<td>15.7</td>
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<td>0.360</td>
<td>5.13</td>
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<tr>
<td>22.9</td>
<td>1.76</td>
<td>0.315</td>
<td>5.44</td>
<td>85.94 \cdot 10^6</td>
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<tr>
<td>29.7</td>
<td>2.12</td>
<td>0.293</td>
<td>6.02</td>
<td>17.25 \cdot 10^6</td>
</tr>
<tr>
<td>40.8</td>
<td>2.72</td>
<td>0.267</td>
<td>6.47</td>
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<tr>
<td>61.8</td>
<td>3.64</td>
<td>0.223</td>
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<tr>
<td>89.6</td>
<td>4.98</td>
<td>0.206</td>
<td>7.67</td>
<td>2.733 \cdot 10^3</td>
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<tr>
<td>118.0</td>
<td>6.21</td>
<td>0.195</td>
<td>8.25</td>
<td>235.4</td>
</tr>
<tr>
<td>174.4</td>
<td>8.72</td>
<td>0.180</td>
<td>8.60</td>
<td>6.720</td>
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<td>0.169</td>
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<tr>
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<td>15.90</td>
<td>0.159</td>
<td>9.49</td>
<td>0.012</td>
</tr>
<tr>
<td>496.3</td>
<td>21.58</td>
<td>0.150</td>
<td>9.93</td>
<td>5.112 \cdot 10^{-4}</td>
</tr>
<tr>
<td>704.8</td>
<td>29.37</td>
<td>0.142</td>
<td>10.37</td>
<td>21.65 \cdot 10^{-6}</td>
</tr>
<tr>
<td>1001.8</td>
<td>40.07</td>
<td>0.135</td>
<td>10.81</td>
<td>0.9017 \cdot 10^{-6}</td>
</tr>
<tr>
<td>1425.6</td>
<td>54.83</td>
<td>0.128</td>
<td>11.26</td>
<td>36.45 \cdot 10^{-9}</td>
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<tr>
<td>2030.6</td>
<td>75.21</td>
<td>0.122</td>
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</tr>
<tr>
<td>2895.5</td>
<td>103.4</td>
<td>0.117</td>
<td>12.14</td>
<td>52.07 \cdot 10^{-12}</td>
</tr>
</tbody>
</table>

1st Approach: VVK, Krauss, Schott
**HOW QCD instantons address criticism of EW sphaleron production in high-E collisions:**

The sphaleron is a semiclassical configuration with

\[ \text{Size}_{\text{sph}} \sim m_W^{-1}, \quad E_{\text{sph}} = \text{few} \times m_W / \alpha_W \approx 10 \text{ TeV}. \]

It is ‘made out’ of \(~1/\alpha_W\) particles (i.e. it decays into \(~1/\alpha_W\) W’s, Z’s, H’s).

\[ 2_{\text{initial hard partons}} \rightarrow \text{Sphaleron} \rightarrow (\sim 1/\alpha_W)_{\text{soft final quanta}} \]

The sphaleron production out of 2 hard partons is unlikely.

![Diagram of a fish](image)

**Fig. 3.** “You can’t make a fish in a pp collider.”

from Mattis PRpts 1991

**But in QCD instantons are small**

[A ‘small fish’ compared to the EW case]

**This criticism does not apply to our QCD calculation**
compute all (but one) collective coordinate integrals numerically using numerical techniques

introduce dimensionless integration variables,

\[ r_0 = R_0 E, \quad r = |\vec{R}| E, \]
\[ y = \rho \overline{\rho} E^2, \quad x = \rho / \overline{\rho}, \]

numerically evaluate (no saddle point approximation needed):

\[
G(r_0, E) = \frac{k^2 \pi^4}{2^{17}} \sqrt{\frac{\pi}{3}} \int_0^\infty r^2 \, dr \int_0^\infty \frac{dx}{x} \int_0^\infty \frac{dy}{y} \left( \frac{4\pi}{\alpha_s} \right)^{21/2} \left( \frac{1}{1 - S(z)} \right)^{7/2} \]
\[
\kappa_{\text{ferm}}(z) \exp \left( - \frac{4\pi}{\alpha_s} S(z) - \frac{\alpha_s}{4\pi} x + \frac{1/x}{4} y \log y \right). \]

and use this to compute the final integral (in the saddle-point approximation to get Im):

\[
\hat{\sigma}_{\text{tot}}^{\text{inst}} (E) = \frac{1}{E^2} \operatorname{Im} \int_{-\infty}^{+\infty} dr_0 \, e^{r_0} G(r_0, E),
\]
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$\sqrt{s}$ [GeV] & 50 & 100 & 150 & 200 & 300 & 400 & 500 \\
\hline
$\langle n_g \rangle$ & 9.43 & 11.2 & 12.22 & 12.94 & 13.96 & 14.68 & 15.23 \\
\hline
$\hat{\sigma}_{\text{tot}}$ [pb] & 207.33×$10^3$ & 1.29×$10^3$ & 53.1 & 5.21 & 165.73×$10^{-3}$ & 13.65×$10^{-3}$ & 1.89×$10^{-3}$ \\
\hline
\end{tabular}

\begin{align*}
G(r_0, E) &= \frac{\kappa^2 \pi^4}{217} \sqrt{\frac{\pi}{3}} \int_0^\infty r^2 dr \int_0^\infty dx \int_0^\infty dy \left( \frac{4\pi}{\alpha_s} \right)^{21/2} \left( \frac{1}{1 - S(z)} \right)^{7/2} K_{\text{ferm}} \\
\sum_{n_g=0}^{\infty} \frac{1}{n_g!} (U_{\text{int}})^{n_g} \exp \left( -\frac{4\pi}{\alpha_s} - \frac{\alpha_s}{4\pi} \frac{x + 1/x}{4} y \log y \right) .
\end{align*}

$\langle n_g \rangle = \langle U_{\text{int}} \rangle$

2nd Approach: VVK, Milne, Spannowsky
hadronic total cross-section

\[ \sigma_{pp \to I}(\hat{s} > \hat{s}_{\text{min}}) = \int_{\hat{s}_{\text{min}}}^{s_{pp}} dx_1 dx_2 \ f(x_1, Q^2) \ f(x_2, Q^2) \ \hat{\sigma}(\hat{s} = x_1 x_2 s_{pp}) \]

<table>
<thead>
<tr>
<th>(E_{\text{min}}) [GeV]</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{pp \to I}) (\sqrt{s_{pp}}=1.96) TeV</td>
<td>2.62 µb</td>
<td>2.61 nb</td>
<td>29.6 pb</td>
<td>1.59 pb</td>
<td>6.94 fb</td>
<td>105 ab</td>
<td>3.06 ab</td>
</tr>
<tr>
<td>(\sigma_{pp \to I}) (\sqrt{s_{pp}}=14) TeV</td>
<td>58.19 µb</td>
<td>129.70 nb</td>
<td>2.769 nb</td>
<td>270.61 pb</td>
<td>3.04 pb</td>
<td>114.04 fb</td>
<td>8.293 fb</td>
</tr>
<tr>
<td>(\sigma_{pp \to I}) (\sqrt{s_{pp}}=30) TeV</td>
<td>211.0 µb</td>
<td>400.9 nb</td>
<td>9.51 nb</td>
<td>1.02 nb</td>
<td>13.3 pb</td>
<td>559.3 fb</td>
<td>46.3 fb</td>
</tr>
<tr>
<td>(\sigma_{pp \to I}) (\sqrt{s_{pp}}=100) TeV</td>
<td>771.0 µb</td>
<td>2.12 µb</td>
<td>48.3 nb</td>
<td>5.65 nb</td>
<td>88.3 pb</td>
<td>4.42 pb</td>
<td>395.0 fb</td>
</tr>
</tbody>
</table>
If the instanton is recoiled by a high pT jet emitted from one of the initial state gluons \( \Rightarrow \) hadronic cross-section is tiny

\[
\exp(-Q(\rho + \bar{\rho})) = \exp\left(-\frac{Q}{E}\sqrt{y(x + 1/x + 2)}\right)
\]

<table>
<thead>
<tr>
<th>( \sqrt{s} ) [GeV]</th>
<th>310</th>
<th>350</th>
<th>375</th>
<th>400</th>
<th>450</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\sigma}^\text{inst}_{\text{tot}} ) [pb]</td>
<td>(3.42\times10^{-23})</td>
<td>(1.35\times10^{-18})</td>
<td>(1.06\times10^{-17})</td>
<td>(1.13\times10^{-16})</td>
<td>(9.23\times10^{-16})</td>
<td>(3.10\times10^{-15})</td>
</tr>
</tbody>
</table>

**Table 3.** The instanton partonic cross-section recoiled against a hard jet with \(p_T = 150\) GeV emitted from an initial state and calculated using Eq. (3.7). Results for the cross-section are shown for a range of partonic C.o.M. energies \(\sqrt{s}\).

<table>
<thead>
<tr>
<th>( \sqrt{s} ) [GeV]</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\sigma}^\text{inst}_{\text{tot}} ) [pb]</td>
<td>(1.68\times10^{-7})</td>
<td>(1.20\times10^{-9})</td>
<td>(3.24\times10^{-11})</td>
<td>(1.84\times10^{-13})</td>
<td>(4.38\times10^{-15})</td>
<td>(2.38\times10^{-16})</td>
</tr>
</tbody>
</table>

**Table 4.** The cross-section presented for a range of partonic C.o.M. energies \(\sqrt{s} = E\) where the recoiled \(p_T\) is scaled with the energy, \(p_T = \sqrt{s}/3\).
QCD instanton cross-sections can be very large at hadron colliders.

Instanton events are isotropic multi-particle final states. Their event topology is very distinct (see Event shape observables next slide).

Particles with large pT are rare. Especially hard to produce them at low partonic energies (for obvious kinematic reasons). Do not pass triggers.

At higher (partonic) energies instanton events can pass triggers but have suppressed cross-sections.

Propose to examine data collected with minimum bias trigger.

[Note: large theoretical uncertainty at low partonic energies (strong coupling in the IR) where the semiclassical approximation becomes suspect/invalid. Best would be to constrain theory prediction / normalisation with data.]
environment. One could speculate about the inclusion of event-shape observables directly would have to be developed for instantons to pass trigger requirements in such a jet-rich colliders, e.g. the FCC-hh, trigger thresholds for jets will have to be increased, which event reconstruction step is a highly challenging task.

processes with less than fairly high partonic centre-of-mass energies of \( 300 \text{ GeV} \) or six jets of \( 85 \text{ GeV} \). Generating 100000 signal events \( \tau \bar{\tau} \) and reconstructing them with the anti-kT jet algorithm, we find that the LHC has very little sensitivity to cross-section that passes such trigger cuts of \( \pm 1.7 \)–1.8\%.

Thus, one would have to resort to multijet triggers, either with four jets of \( \pm 7 \text{ GeV} \) or six jets of \( \pm 85 \text{ GeV} \). According to Table 4.1, the LHC has very little sensitivity to the \( \tau \bar{\tau} \) cross-section that passes such trigger cuts of \( \pm 1.7 \)–1.8\%.

As a result of the trigger cuts imposed, we find that the LHC has very little sensitivity to QCD instantons in current and future high-luminosity runs. QCD instanton events produce multi-particle events consisting of soft jets. No isolated leptons or a large amount of missing transverse energy, and so appear only as VVK, Milne, Spannowsky.
Event shape observables are good. Triggering based on high pT pose problems:

Example: High-luminosity LHC

Missing transverse energy higher-level triggers require at least $E_{T\text{mis}} \geq 70$ GeV while single jet triggers are as high as $p_{T,j} \geq 360$ GeV

If one of the instanton-induced partons has a transverse momentum to pass the single-jet trigger requirements, the centre-of-mass energy of the instanton $\sqrt{s'}$ has to be at least of $O(700)$ GeV.

$=>$ Cross-section is too small.

Thus, one would have to resort to multijet triggers, either with four jets of $p_{T,j} \geq 85$ GeV or six jets of $p_{T,j} \geq 45$ GeV. Both such trigger requirements result in for instantons fairly high partonic centre-of-mass energies of $O(300)$ GeV. Generating 100000 signal events

$=>$ none of the events passed trigger.

Need to be creative: low-luminosity LHC with minimal bias triggers does much better.

Can one incorporate event-shape observables into trigger strategies?

VVK, Milne, Spannowsky
Main sources of theoretical uncertainties (for discussion)

- QCD Instanton rates are interesting in the regime where they become large — lower end of partonic energies 10-80 GeV. The weak coupling approximation used in the semiclassical calculation can be problematic. How to address: vary s’ minimal partonic energy cutoff.

- What is the role of higher-order corrections to the Mueller’s term in the exponent?

- Possible corrections to the instanton-anti-instanton interaction at medium instanton separations in the optical theorem approach.

- Non-factorisation of the determinants in the instanton-anti-instanton background in the optical theorem. (Instanton densities D(rho) do not factorise at finite R/rho ~1.5 - 2.)

- Choice of the RG scale = 1/rho. (can vary by a factor of 2 or use other prescriptions to test. In Ref. [1] we checked that)

- A practical point for future progress is to test theory normalisation of predicted QCD instanton rates with data. [The unbiased and un-tuned theory prediction is promising.]