QCD instantons at hadron colliders

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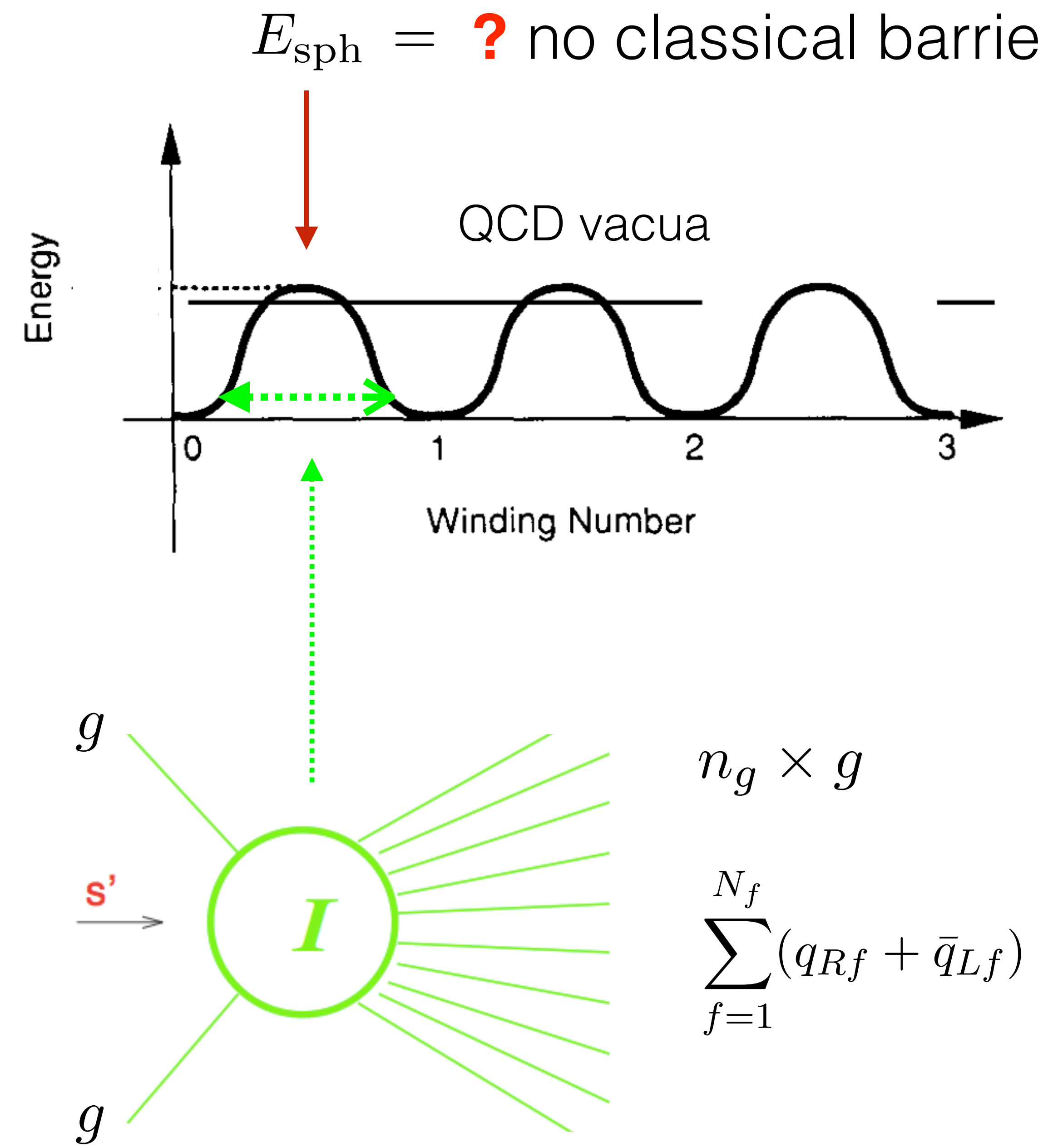
IPPP Durham

with Frank Krauss & Matthias Schott 1911.09726 : JHEP

and Dan Milne & Michael Spannowsky 2010.02287 : PRD

QCD Instantons

- Yang-Mills vacuum has a nontrivial structure
- *Instantons* are tunnelling solutions between the vacua.
- At the classical level there is no barrier in QCD. The *sphaleron is a quantum effect*
- Transitions between the vacua change chirality (result of the ABJ anomaly).
- All light quark-anti-quark pairs must participate in the reaction
- Not described by perturbation theory.



$$\boxed{g + g \to n_g \times g + \sum_{f=1}^{N_f}(q_{Rf} + \bar{q}_{Lf})}$$

QCD Instantons

Instanton-induced processes with 2 gluons in the initial state:

All light flavours of quark-antiquark pairs must be present. Light =>

$$m_f \leq 1/\rho.$$

instanton size

$$g + g \to n_g \times g + \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf})$$

arbitrary
(tends to be large ~1/alpha_s)

Can also have quark-initiated processes e.g. :

$$u_L + \bar{u}_R \to n_g \times g + \sum_{f=1}^{N_f-1} (q_{Rf} + \bar{q}_{Lf}) \,,$$

$$u_L + d_L \to n_g \times g + u_R + d_R + \sum_{f=1}^{N_f-2} (q_{Rf} + \bar{q}_{Lf})$$

$$g + g \;\to\; n_g \times g \;+\; \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf})$$

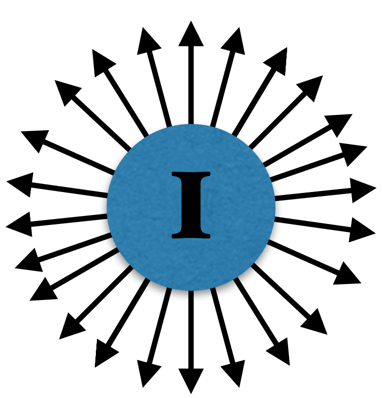
The amplitude takes the form of an integral over instanton collective coordinates.
The classical result (leading order in the instanton perturbation theory) is simply:

semiclassical suppression
('t Hooft) factor by the instanton action

$$S_I = \frac{8\pi^2}{g^2} = \frac{2\pi}{\alpha_s(\mu_r)}$$

$$\mathcal{A}_{2\to\, n_g+2N_f} \;\sim\; \int d^4x_0\, d\rho\, D(\rho)\, e^{-S_I} \left[\prod_{i=1}^{n_g+2} A^{a_i\,\mathrm{inst}}_{\mathrm{LSZ}}(p_i,\lambda_i)\right] \left[\prod_{j=1}^{2N_f} \psi^{(0)}_{\mathrm{LSZ}}(p_j,\lambda_j)\right]$$

- the integrand: a product of bosonic and fermionic components of the instanton field configurations
- the factorised structure implies that emission of individual particles in the final state is uncorrelated and mutually independent.

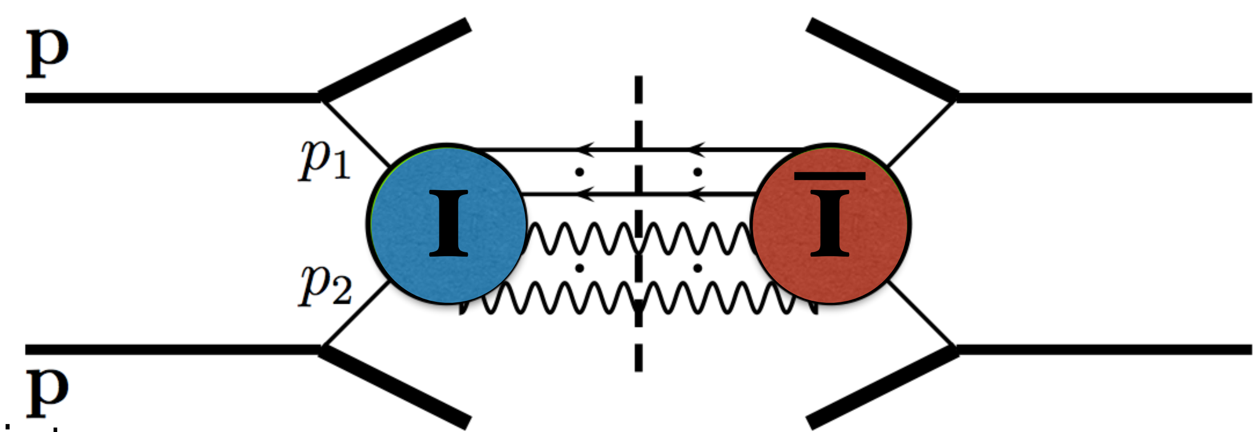


[this is correct at the LO in instanton pert. theory approximation]

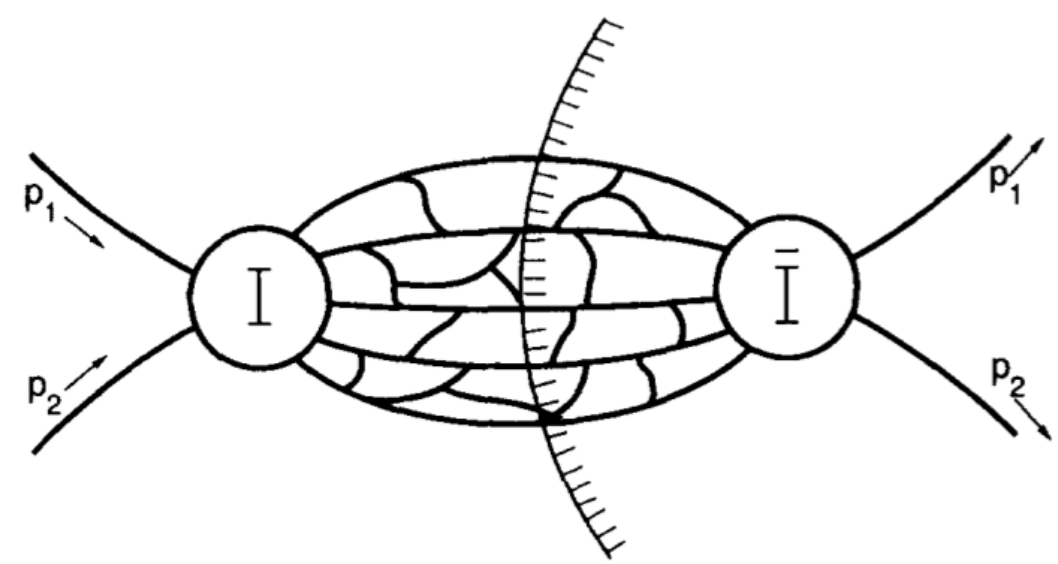
LO Instanton vertex

The Optical Theorem approach: to include final state interactions

$$\hat{\sigma}_{\rm tot}^{\rm inst} = \frac{1}{E^2}\,{\rm Im}\,\mathcal{A}_4^{I\bar{I}}(p_1,p_2,-p_1,-p_2)$$



- Cross-section is obtained by |squaring| the instanton amplitude.
- Final states have been instrumental in combatting the exp. suppression.
- Now also the interactions between the final states (and the improvement on the point-like I-vertex) are taken into account.
- Use the Optical Theorem to compute *Im* part of the 2->2 amplitude in around the Instanton-Anti-instanton configuration.
- Varying the energy E changes the dominant value of I-Ibar separation R. At R=0 instanton and anti-instanton mutually annihilate.
- The suppression of the EW instanton cross-section is gradually reduced at lower R(E).



VVK & Ringwald 1991

The Optical Theorem approach: to include final state interactions

- Instanton — anti-instanton valley configuration has Q=0; it interpolates between infinitely separated instanton—anti-instanton and the perturbative vacuum at R=0

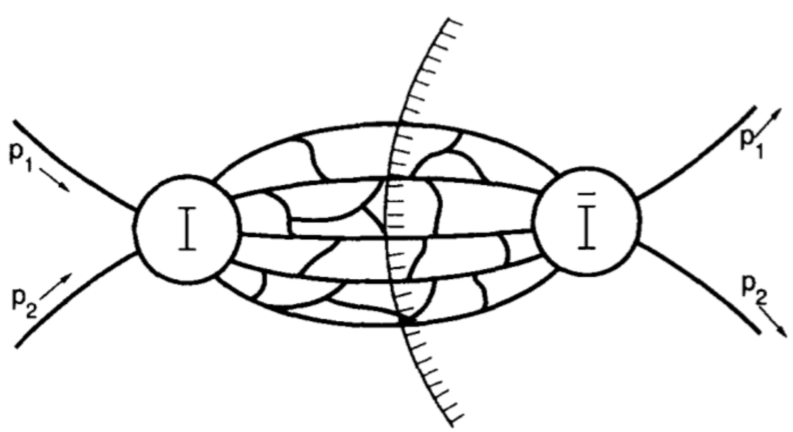
(anti)-instanton sizes

(anti)-instanton separation

$$\sigma_{\rm tot}^{\rm (cl)\, inst}$$

$$\simeq \frac{1}{s}\,{\rm Im} \int_0^\infty d\rho \int_0^\infty d\bar\rho \int d^4R \int d\Omega\; D(\rho) D(\bar\rho)\; e^{-S_{I\bar I}}\; \mathcal{K}_{\rm ferm} \times$$
$$A^{\rm inst}_{LSZ}(p_1)\, A^{\rm inst}_{LSZ}(p_2)\, A^{\overline{\rm inst}}_{LSZ}(-p_1)\, A^{\overline{\rm inst}}_{LSZ}(-p_2)\,,$$

$$S_{I\bar I}(\rho,\bar\rho,R) = \frac{4\pi}{\alpha_s(\mu_r)}\,\hat{\mathcal{S}}$$



instanton-anti-instanton action (see next slide)

- Exponential suppression is gradually reduced at lower R (Energy-dependent)
- no radiative corrections from hard initial states included in this approximation

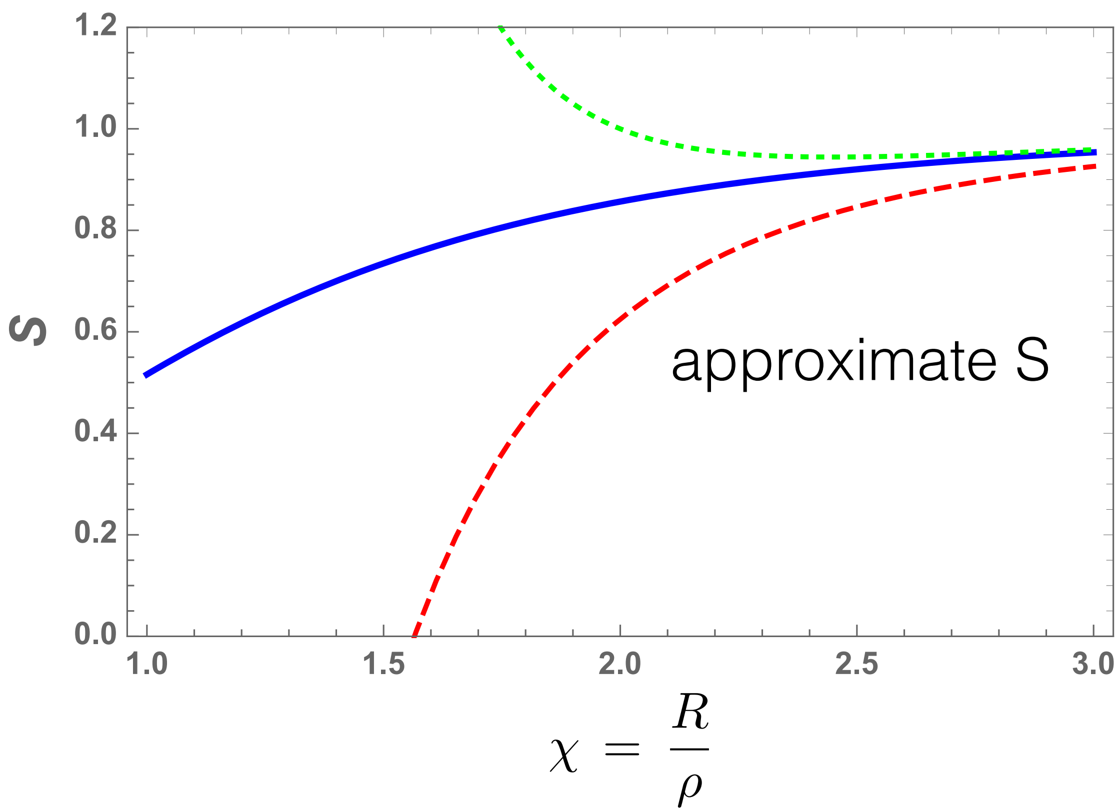
$$\sigma_{\rm tot}^{\rm (cl)\,inst} = \frac{1}{s}\,{\rm Im}\,\mathcal{A}_4^{I\bar{I}}(p_1,p_2,-p_1,-p_2)$$

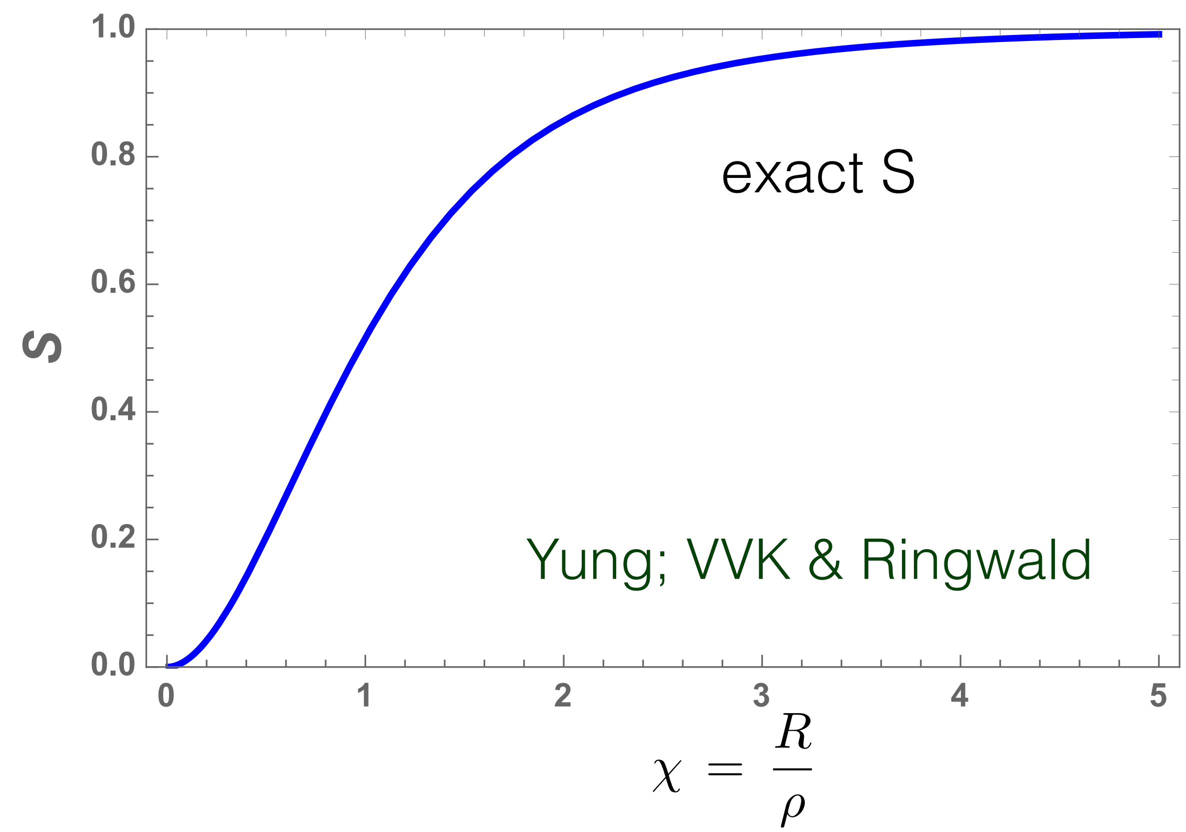
$$\simeq \frac{1}{s}\,{\rm Im}\int_0^\infty d\rho\int_0^\infty d\bar\rho\,\int d^4R\int d\Omega\;D(\rho)D(\bar\rho)\;e^{-S_{I\bar{I}}}\;\mathcal{K}_{\rm ferm}\times$$

$$A_{LSZ}^{\rm inst}(p_1)\,A_{LSZ}^{\rm inst}(p_2)\,A_{LSZ}^{\overline{\rm inst}}(-p_1)\,A_{LSZ}^{\overline{\rm inst}}(-p_2)\,,$$

$$S_{I\bar{I}}(\rho,\bar\rho,R) = \frac{4\pi}{\alpha_s(\mu_r)}\,\mathcal{S}$$

$$\mathcal{S}(\chi) \simeq 1 - 6/\chi^4 + 24/\chi^6 + \ldots \qquad \chi = \frac{R}{\rho}$$





- Exponential suppression is gradually reduced at lower and lower $\chi = \frac{R}{\rho}$
- no radiative corrections from hard initial states included in this approximation

$$D(\rho,\mu_r) \;=\; \kappa\,\frac{1}{\rho^5}\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^6\,(\rho\mu_r)^{b_0}$$

$$\sigma_{\rm tot}^{\rm (cl)\,inst} \;=\; \frac{1}{s}\,{\rm Im}\,\mathcal{A}_4^{I\bar I}(p_1,p_2,-p_1,-p_2)$$
$$\simeq\; \frac{1}{s}\,{\rm Im}\int_0^\infty d\rho\int_0^\infty d\bar\rho\int d^4R\int d\Omega\; D(\rho)D(\bar\rho)\; e^{-S_{I\bar I}}\;\mathcal{K}_{\rm ferm}\times$$
$$A^{\rm inst}_{LSZ}(p_1)\,A^{\rm inst}_{LSZ}(p_2)\,A^{\overline{\rm inst}}_{LSZ}(-p_1)\,A^{\overline{\rm inst}}_{LSZ}(-p_2)\,,$$

fermion prefactor
from Nf qq-bar pairs

$$A^{\rm inst}_{LSZ}(p_1)\,A^{\rm inst}_{LSZ}(p_2)\,A^{\overline{\rm inst}}_{LSZ}(-p_1)\,A^{\overline{\rm inst}}_{LSZ}(-p_2) \;=\; \frac{1}{36}\left(\frac{2\pi^2}{g}\rho^2\sqrt{s'}\right)^4\, e^{iR\cdot(p_1+p_2)}$$

$$\exp\left(R_0\sqrt{s}\;-\;\frac{4\pi}{\alpha_s(\mu_r)}\,\hat{\mathcal{S}}(z)\right)$$

But the instanton size has not been stabilised.
In QCD - **rho** is a ***classically flat direction —***
*need to **include and re-sum quantum corrections**!*

in the EW theory:

$$G_{4\,\mathrm{Eucl}} \sim \int d^4R\, d\rho_I d\rho_{\bar{I}} \ldots \exp\left[i(p_1+p_2)\cdot R - S_{I\bar{I}}(z) - \pi^2 v^2(\rho_I^2+\rho_{\bar{I}}^2)\right]$$

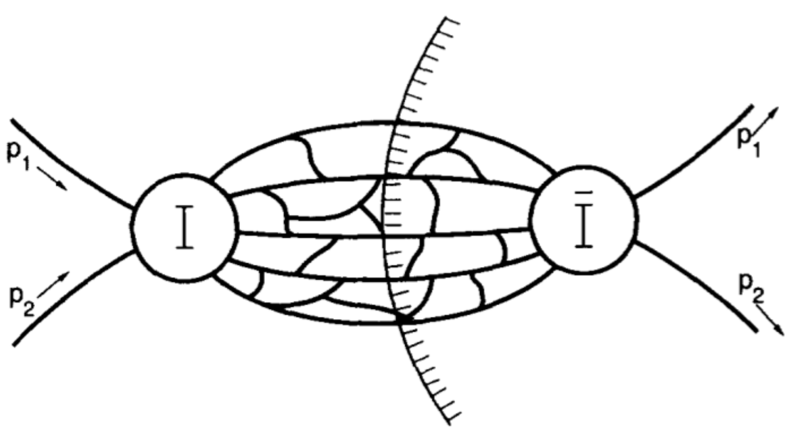
instanton separation

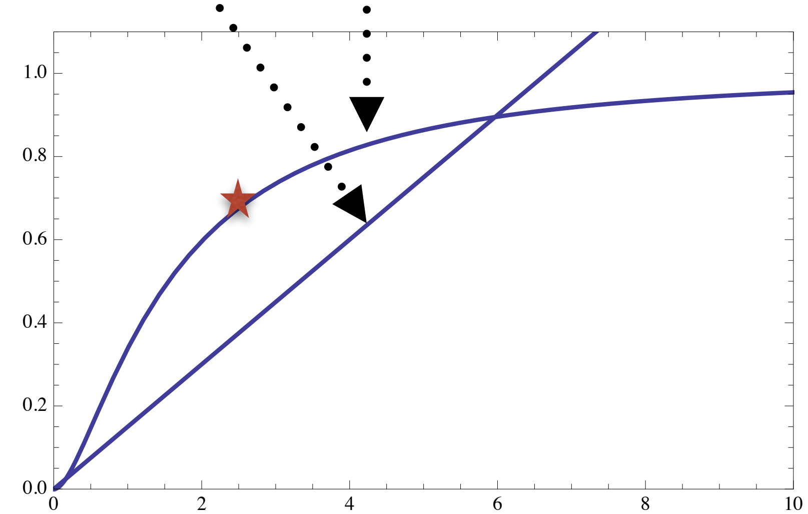
instanton sizes

$$z \sim \frac{R^2+\rho_I^2+\rho_{\bar{I}}^2}{\rho_I \rho_{\bar{I}}}$$

Higgs vev:
EW theory - **not QCD!**

$$\sigma_{B+L} \sim \mathrm{Im} \int d^4R\, d\rho_I d\rho_{\bar{I}} \ldots \exp\left[ER - S_{I\bar{I}}(R) - \pi^2 v^2(\rho_I^2+\rho_{\bar{I}}^2)\right]$$





Higgs vev cuts-off large instantons

- Exponential suppression is gradually reduced with energy [in the EW theory]

In QCD:

$$G_{4\,\mathrm{Eucl}} \sim \int d^4R\, d\rho_I d\rho_{\bar I} \ldots \exp\left[i(p_1+p_2)\cdot R - S_{I\bar I}(z) \text{ - new in QCD}\right]$$

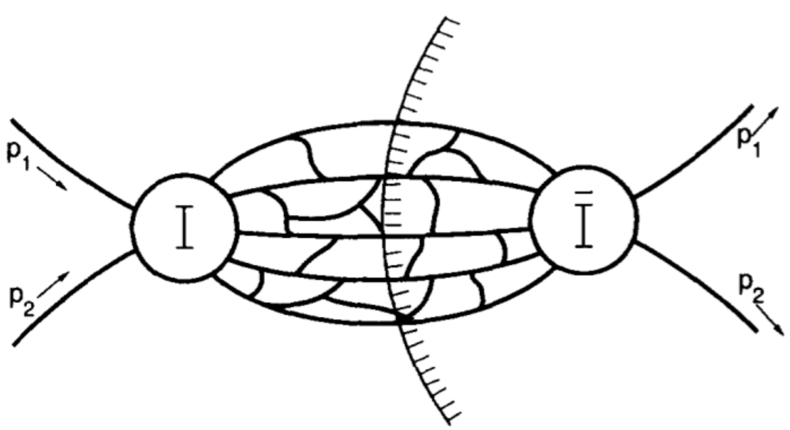
instanton separation

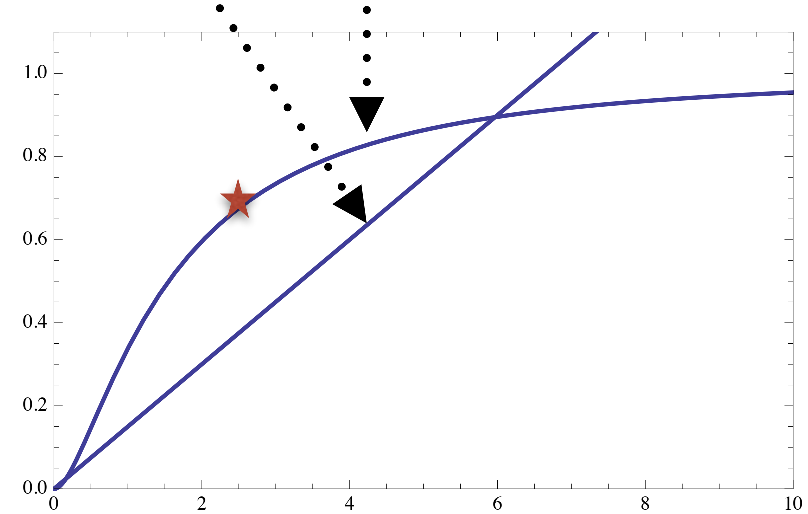
instanton sizes

$$z \sim \frac{R^2+\rho_I^2+\rho_{\bar I}^2}{\rho_I \rho_{\bar I}}$$

Quantum effects to cut-off Instanton size integrations

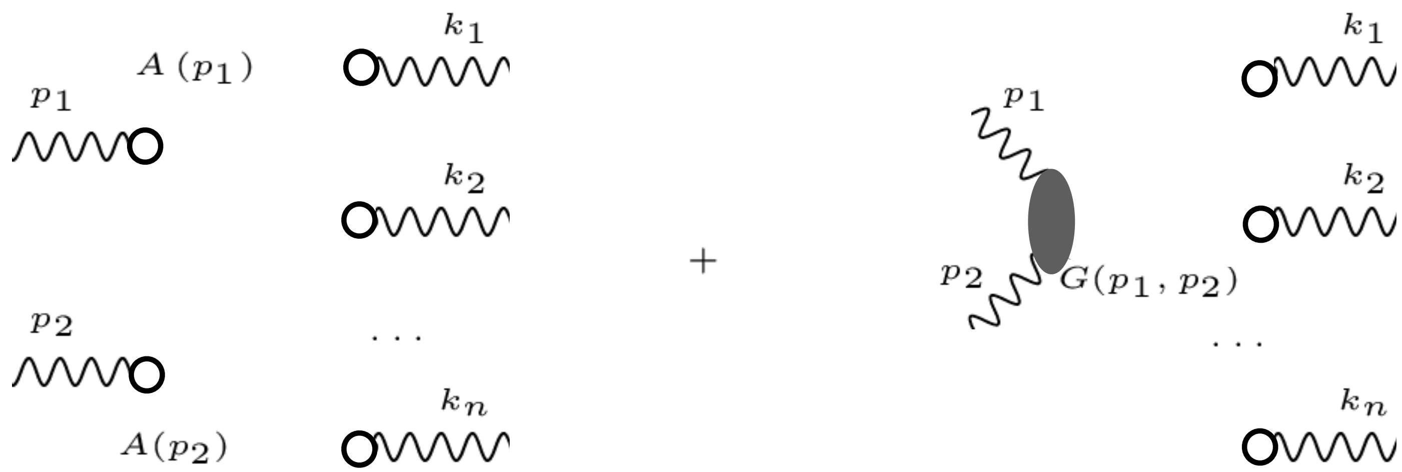
$$\sigma_{B+L} \sim \mathrm{Im} \int d^4R\, d\rho_I d\rho_{\bar I} \ldots \exp\left[ER - S_{I\bar I}(R) - \text{new in QCD}\right]$$





Initial state interactions in the instanton approach

LO instanton process NLO instanton process



propagator in the instanton background

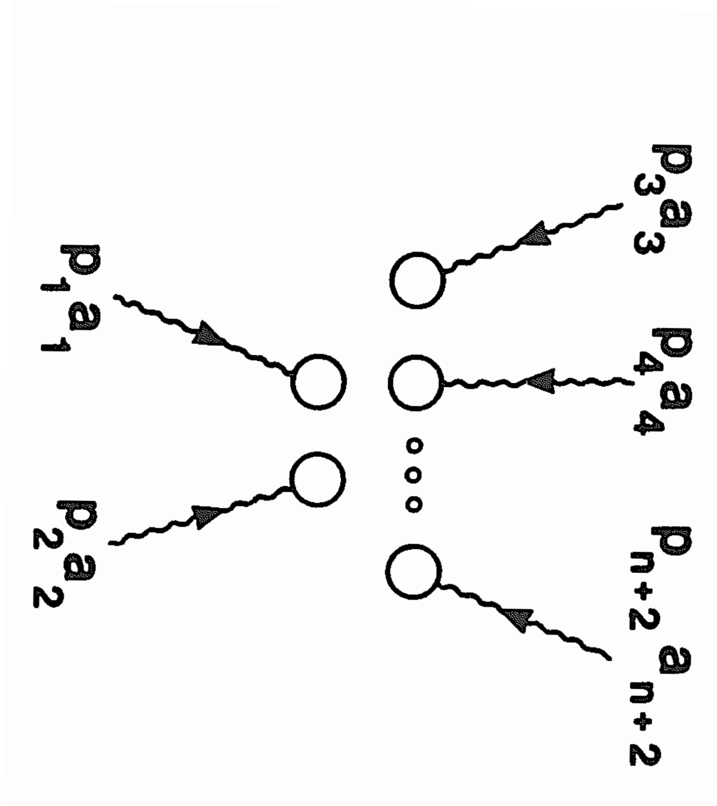
$$\boxed{G^{ab}_{\mu\nu}(p_1, p_2) \to -\frac{g^2\rho^2 s}{64\pi^2}\log(s)\, A^a_\mu(p_1)\, A^b_\nu(p_2)}$$

$$p_1^2 = 0 = p_2^2, \quad 2p_1p_2 = s \gg 1/\rho^2$$

Include now higher order corrections in the high-energy limit:

$$\boxed{\sum_{r=1}^{N} \frac{1}{r!}\left(-\frac{g^2\rho^2 s}{64\pi^2}\log(s)\right)^r A^a_\mu(p_1)\, A^b_\nu(p_2)}$$

Mueller 1991



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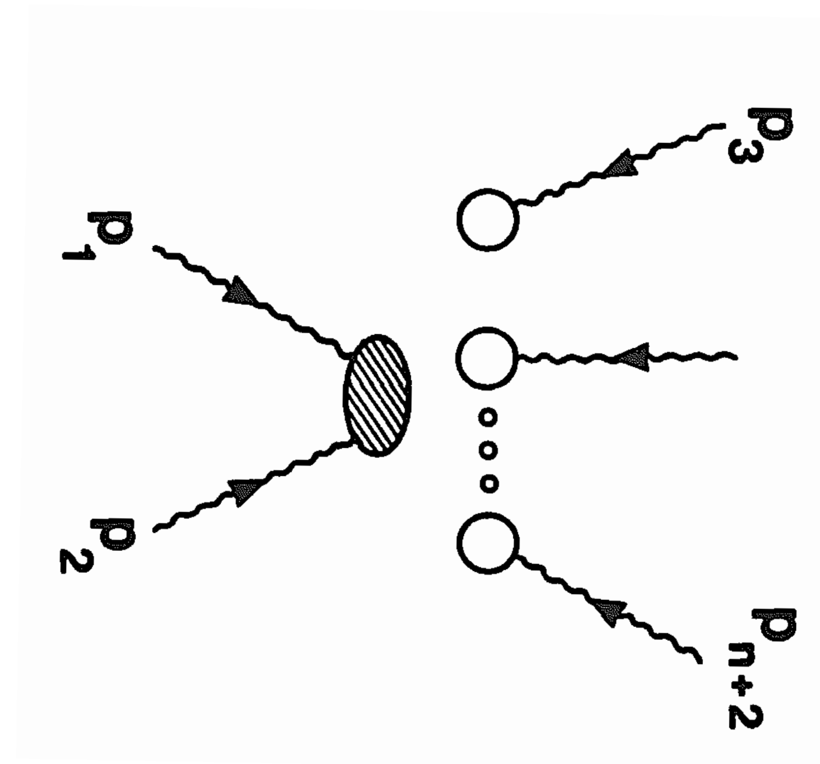

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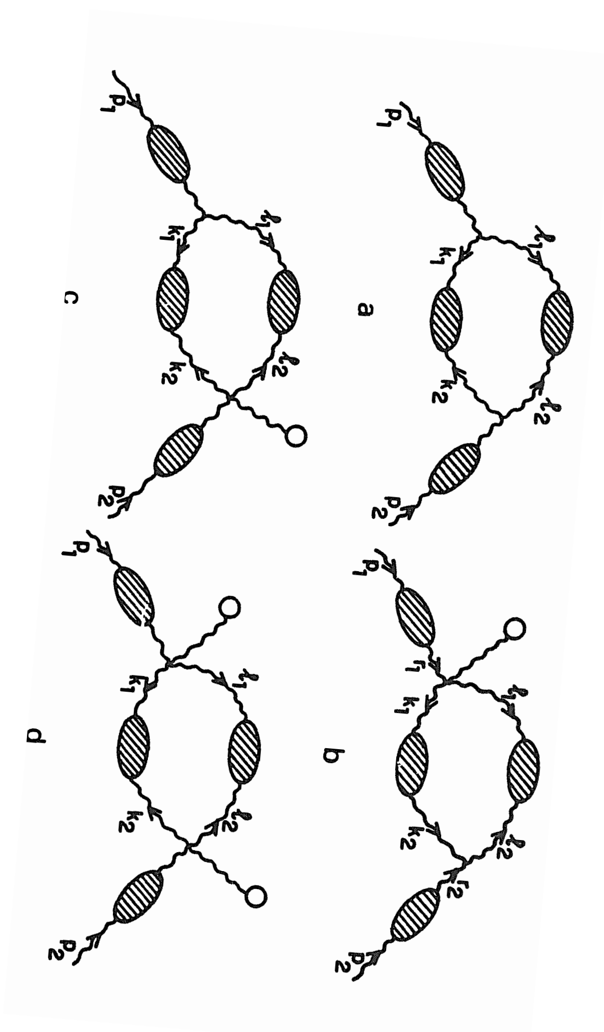
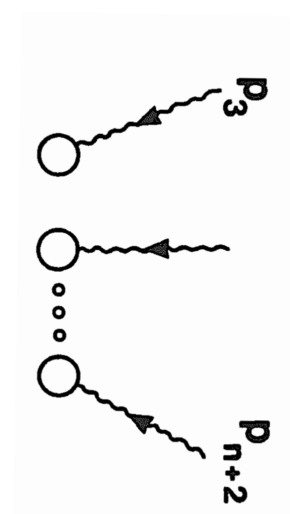




$+ \;\dots$

$$\boxed{e^{-(\alpha_s(\mu_r)/16\pi)\,\rho^2 E^2 \log E^2/\mu_r^2}}$$

Mueller 1991

Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}_{\rm tot}^{\rm inst} \simeq \frac{1}{s'}\,{\rm Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}_{\rm ferm}$$
$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\,\exp\left(R_0\sqrt{s'}-\frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z)-\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)\right)$$

Instanton size is cut-off by $\sim\sqrt{s}$
this is what sets the
effective QCD sphlarenon scale

Mueller's result for
quantum corrections
due to in-in states
interactions

Basically, in QCD one can never reach the effective
sphaleron barrier — it's hight grows with the energy.

=> Among other things, no problems with unitarity.

This is the main idea of the approach:

[1] VVK, Krauss, Schott
[2] VVK, Milne, Spannowsky

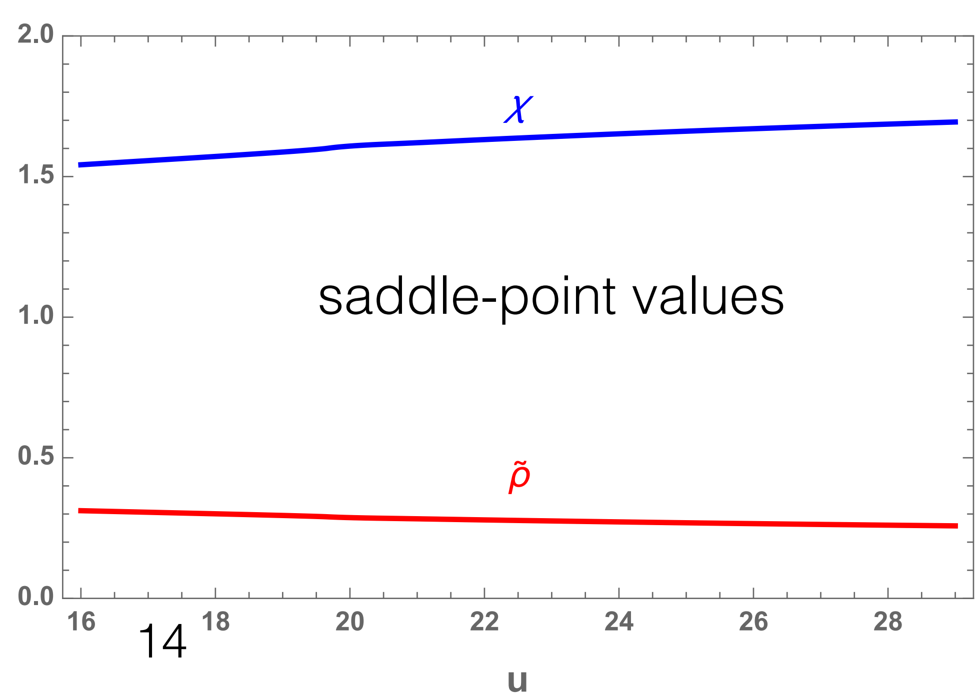
Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}_{\rm tot}^{\rm inst} \simeq \frac{1}{s'}\,{\rm Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}_{\rm ferm}$$
$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\;\exp\left(R_0\sqrt{s'}-\frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z)-\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)\right)$$

1. Extermise the holy-grail function in the exponent by finding a saddle-point in variables:

$$\mathcal{F}\,=\,\rho\chi\sqrt{s}-\frac{4\pi}{\alpha_s(\rho)}\,\mathcal{S}(\chi)-\frac{\alpha_s(\rho)}{4\pi}\,\rho^2 s\,\log(\sqrt{s}\rho)$$

$$\tilde\rho\,=\,\frac{\alpha_s(\rho)}{4\pi}\sqrt{s}\rho\,,\qquad \chi\,=\,\frac{R}{\rho}$$



$$u=\sqrt{s}/\mu=\sqrt{s}\rho$$

Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}^{\rm inst}_{\rm tot} \simeq \frac{1}{s'}\,{\rm Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}_{\rm ferm}$$
$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\ \exp\left(R_0\sqrt{s'}-\frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z)-\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)\right)$$

1st Approach: VVK, Krauss, Schott

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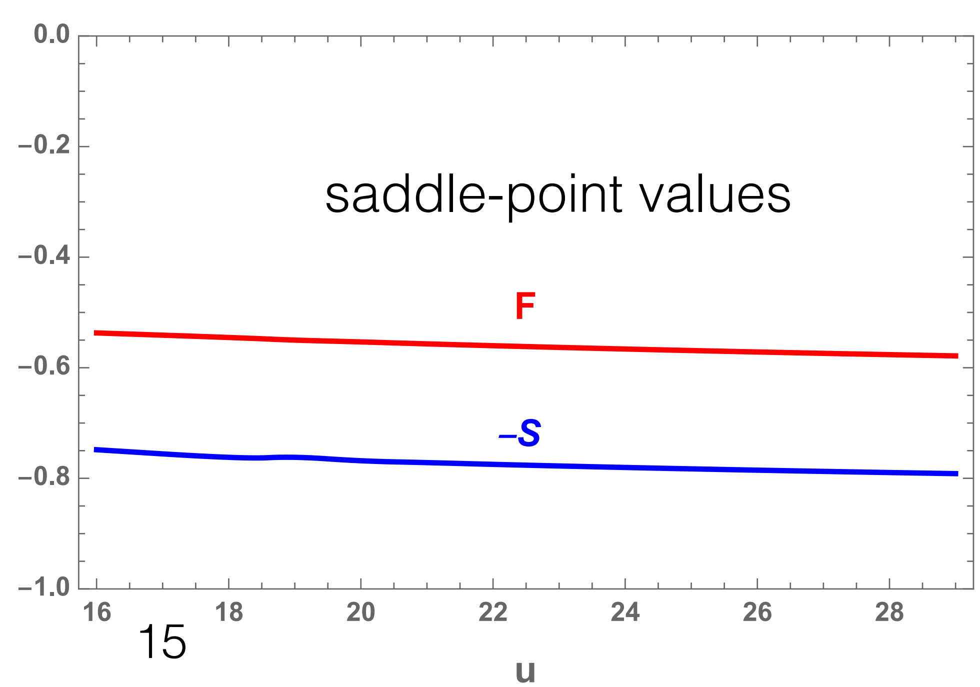
$$\tilde\rho\,=\,\frac{\alpha_s(\rho)}{4\pi}\sqrt{s}\rho\,,\qquad \chi\,=\,\frac{R}{\rho}$$

- II action



The holy-grail

in units of $\frac{4\pi}{\alpha_s}$

$u = \sqrt{s}/\mu = \sqrt{s}\rho$

Combined effect of initial and final states interactions in QCD

$$\hat{\sigma}_{\rm tot}^{\rm inst} \simeq \frac{1}{s'}\, {\rm Im}\, \frac{\kappa^2\pi^4}{36\cdot 4} \int \frac{d\rho}{\rho^5} \int \frac{d\bar\rho}{\bar\rho^5} \int d^4R \int d\Omega \left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14} (\rho^2\sqrt{s'})^2 (\bar\rho^2\sqrt{s'})^2\, \mathcal{K}_{\rm ferm}$$

$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\, \exp\left(R_0\sqrt{s'} - \frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z) - \frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\, s'\, \log\left(\frac{s'}{\mu_r^2}\right)\right)$$

1st Approach: VVK, Krauss, Schott

1. Extermise the holy-grail function in the exponent by finding a saddle-point in variables:

$$\mathcal{F} = \rho\chi\sqrt{s} - \frac{4\pi}{\alpha_s(\rho)}\mathcal{S}(\chi) - \frac{\alpha_s(\rho)}{4\pi}\rho^2 s\, \log(\sqrt{s}\rho)$$

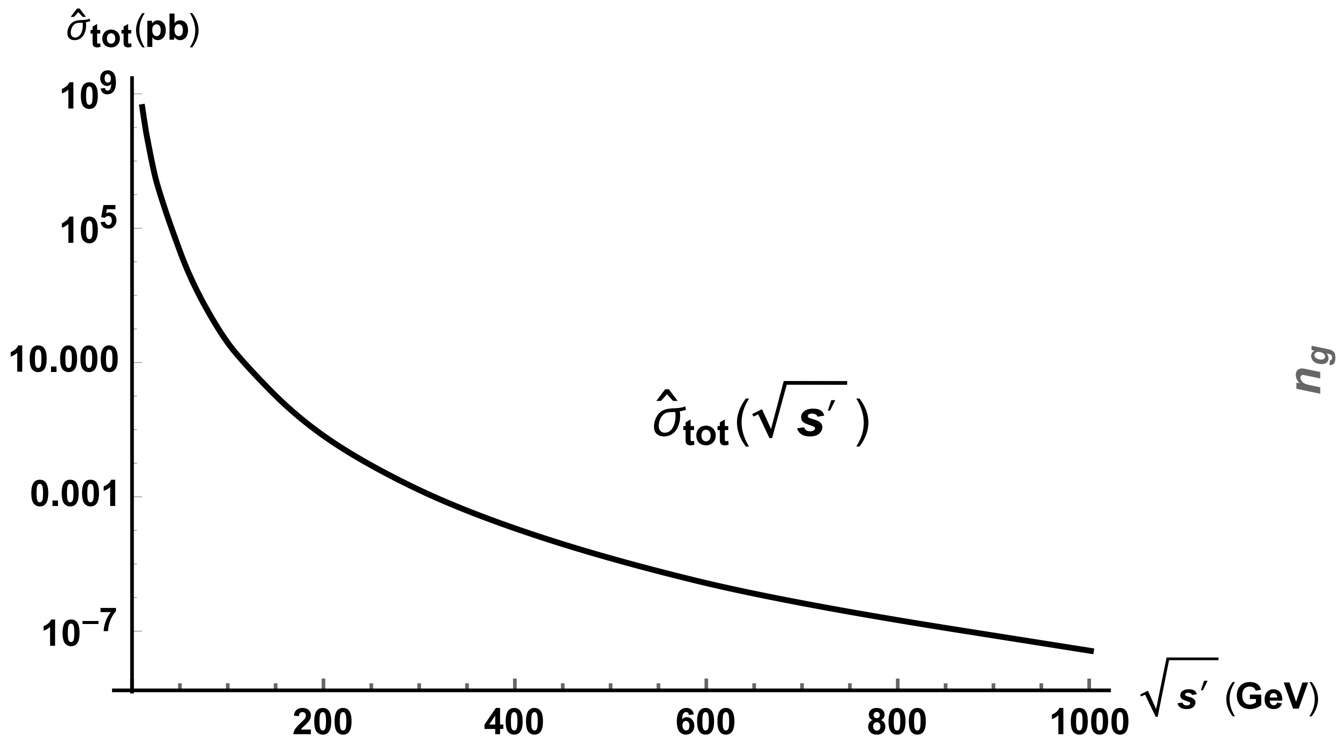
$$\tilde\rho = \frac{\alpha_s(\rho)}{4\pi}\sqrt{s}\rho\,, \qquad \chi = \frac{R}{\rho}$$

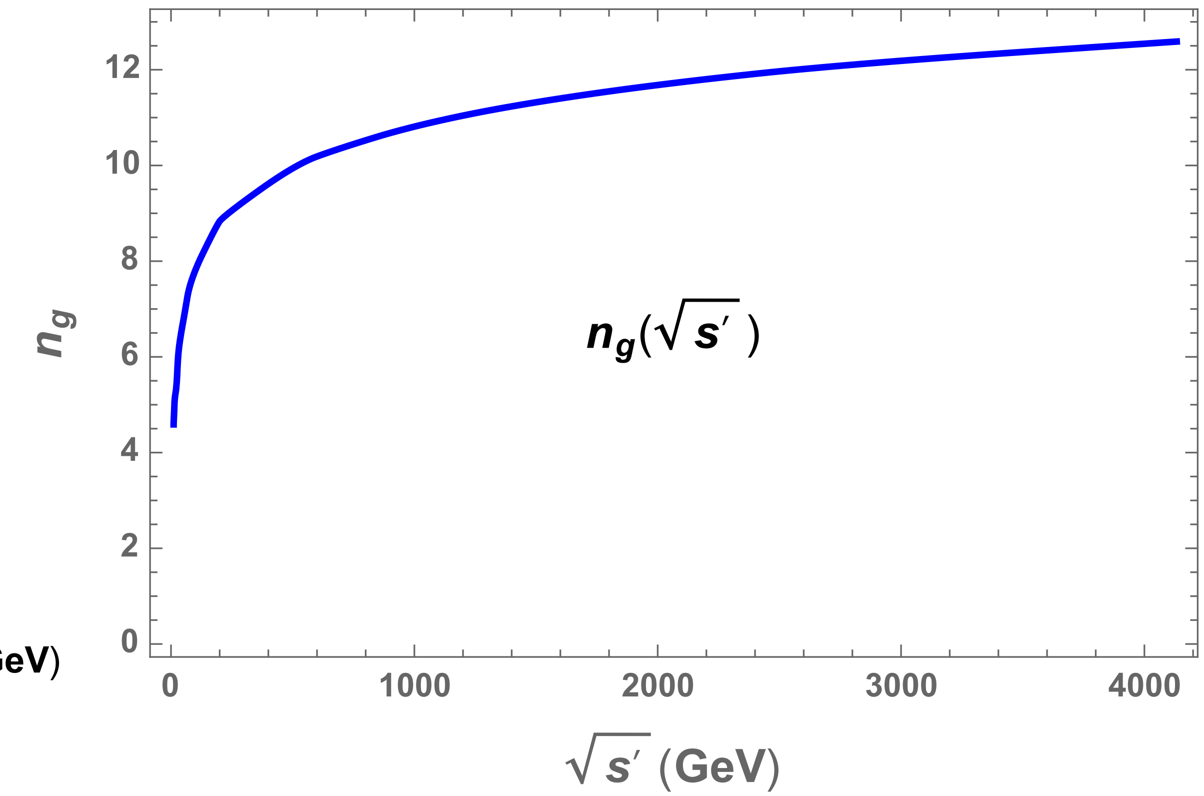
2. Carry out all integrations using the steepest descent method evaluating the determinants of quadratic fluctuations around the saddle-point solution

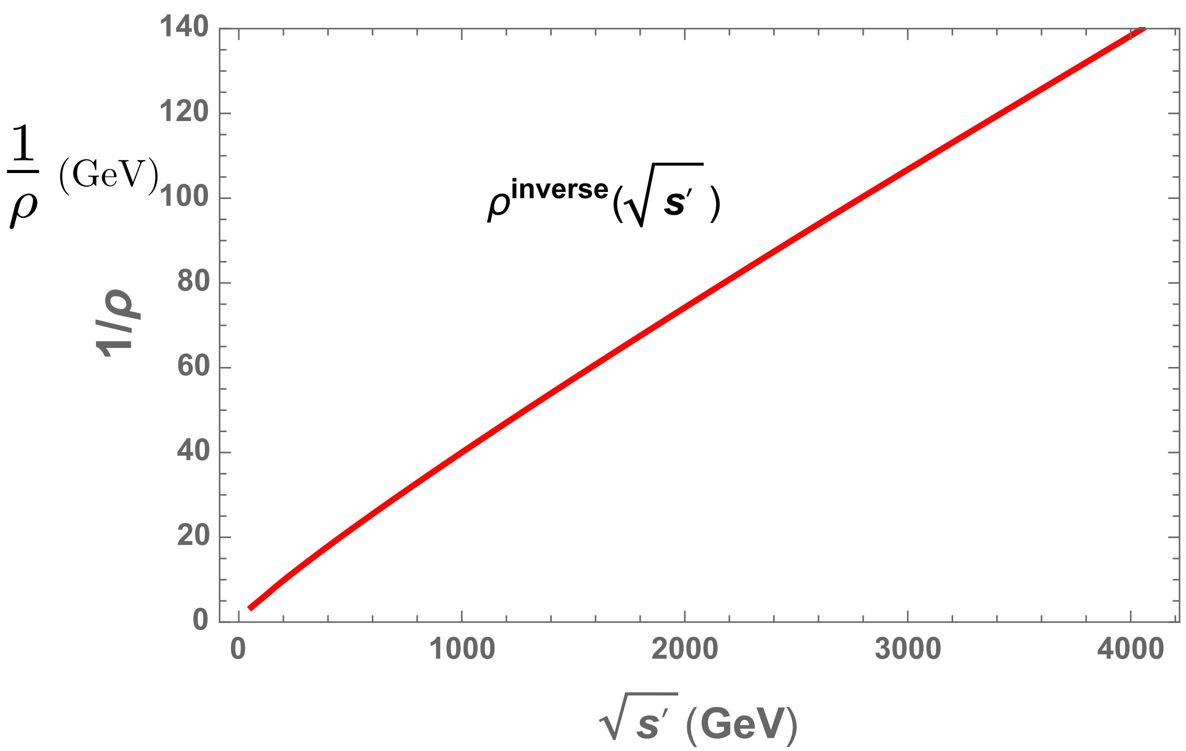
3. Pre-factors are very large — they compete with the semiclassical exponent which is very small!

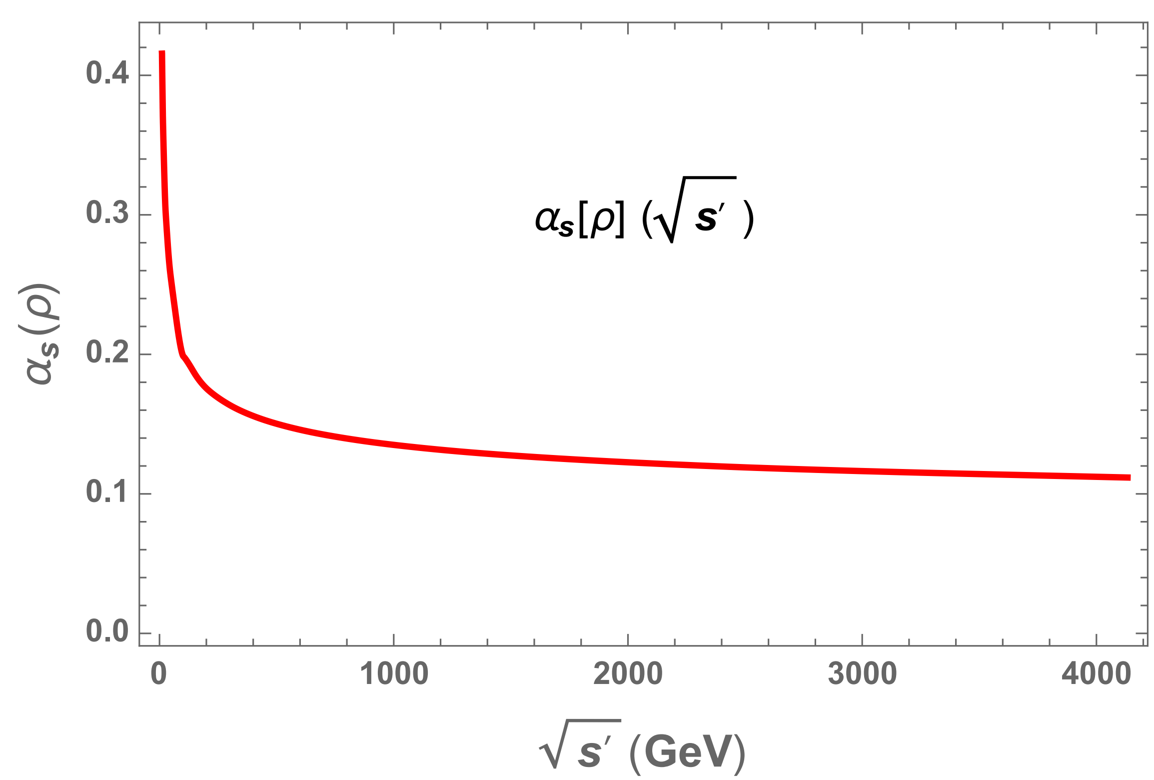
Results

1st Approach: VVK, Krauss, Schott



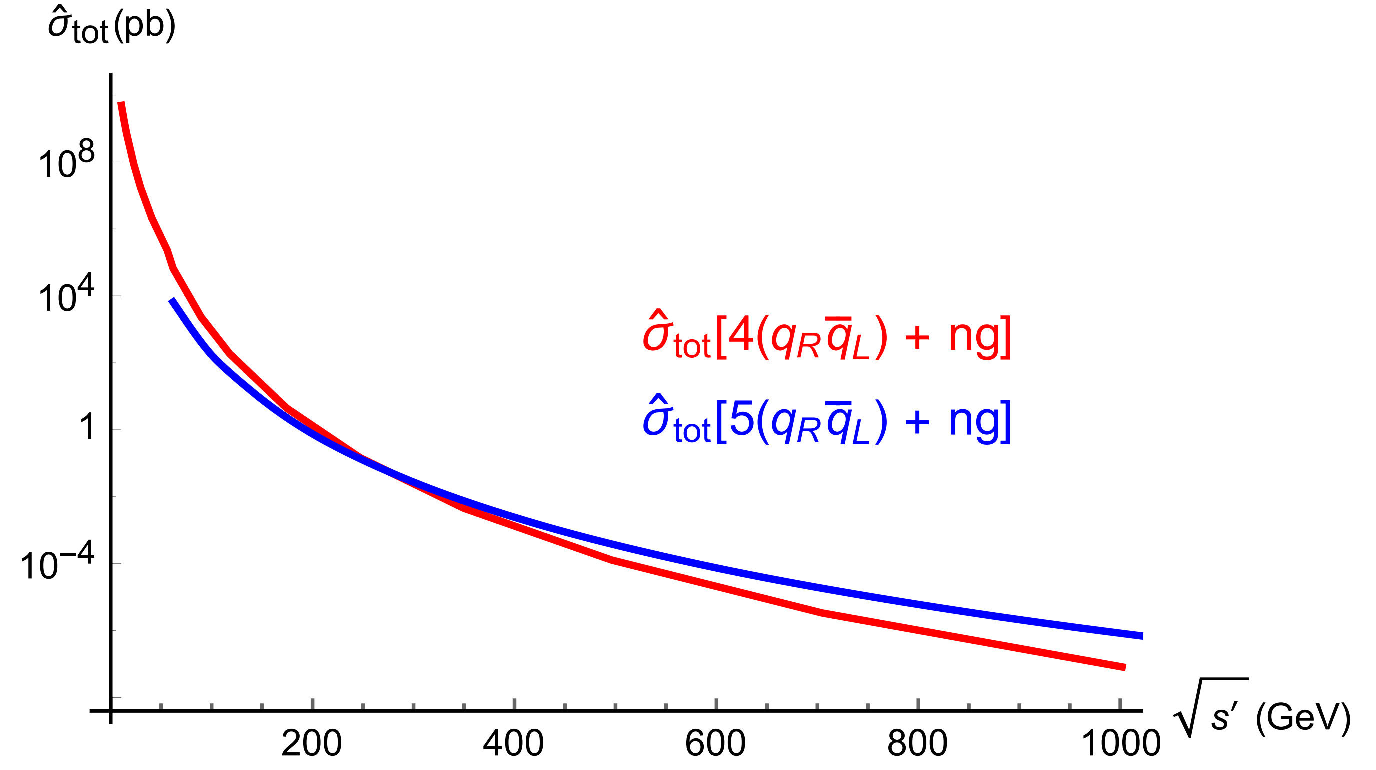






Results and partonic cross-section

$\sqrt{s'}$ [GeV]	$1/\rho$ [GeV]	$\alpha_S(1/\rho)$	$\langle n_g \rangle$	$\hat{\sigma}$ [pb]
10.7	0.99	0.416	4.59	$4.922 \cdot 10^9$
11.4	1.04	0.405	4.68	$3.652 \cdot 10^9$
13.4	1.16	0.382	4.90	$1.671 \cdot 10^9$
15.7	1.31	0.360	5.13	$728.9 \cdot 10^6$
22.9	1.76	0.315	5.44	$85.94 \cdot 10^6$
29.7	2.12	0.293	6.02	$17.25 \cdot 10^6$
40.8	2.72	0.267	6.47	$2.121 \cdot 10^6$
56.1	3.50	0.245	6.92	$229.0 \cdot 10^3$
61.8	3.64	0.223	7.28	$72.97 \cdot 10^3$
89.6	4.98	0.206	7.67	$2.733 \cdot 10^3$
118.0	6.21	0.195	8.25	235.4
174.4	8.72	0.180	8.60	6.720
246.9	11.76	0.169	9.04	0.284
349.9	15.90	0.159	9.49	0.012
496.3	21.58	0.150	9.93	$5.112 \cdot 10^{-4}$
704.8	29.37	0.142	10.37	$21.65 \cdot 10^{-6}$
1001.8	40.07	0.135	10.81	$0.9017 \cdot 10^{-6}$
1425.6	54.83	0.128	11.26	$36.45 \cdot 10^{-9}$
2030.6	75.21	0.122	11.70	$1.419 \cdot 10^{-9}$
2895.5	103.4	0.117	12.14	$52.07 \cdot 10^{-12}$



1st Approach: VVK, Krauss, Schott

HOW QCD instantons address criticism of EW sphaleron production in high-E collisions:

The sphaleron is a semiclassical configuration with

$$\boxed{\text{Size}_{\text{sph}} \sim m_W^{-1}\,, \quad E_{\text{sph}} = \text{few} \times m_W/\alpha_W \simeq 10\,\text{TeV}.}$$

It is 'made out' of $\sim 1/\alpha_W$ particles (i.e. it decays into $\sim 1/\alpha_W$ W's, Z's, H's).

$$\boxed{2_{\text{initial hard partons}} \;\to\; \text{Sphaleron} \;\to\; (\sim 1/\alpha_W)_{\text{soft final quanta}}}$$

The sphaleron production out of 2 hard partons is unlikely.

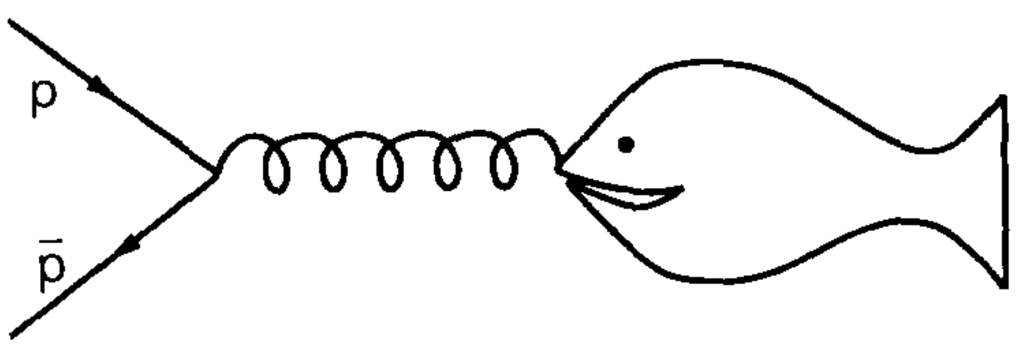


Fig. 3. "You can't make a fish in a $p\bar{p}$ collider."

from Mattis PRpts 1991

But in QCD instantons are small

[A `small fish' compared to the EW case]

This criticism does not apply to our QCD calculation

2nd more direct approach: VVK, Milne, Spannowsky

compute all (but one) collective coordinate integrals numerically using numerical techniques

introduce dimensionless integration variables,

$$r_0 = R_0 E\,, \qquad r = |\vec{R}|E\,,$$
$$y = \rho\bar{\rho}\,E^2\,, \qquad x = \rho/\bar{\rho}\,,$$

numerically evaluate (no saddle point approximation needed):

$$G(r_0,E) = \frac{\kappa^2\pi^4}{2^{17}}\sqrt{\frac{\pi}{3}}\int_0^\infty r^2\,dr\ \int_0^\infty \frac{dx}{x}\int_0^\infty \frac{dy}{y}\left(\frac{4\pi}{\alpha_s}\right)^{21/2}\left(\frac{1}{1-\mathcal{S}(z)}\right)^{7/2}$$
$$\mathcal{K}_{\rm ferm}(z)\ \exp\left(-\frac{4\pi}{\alpha_s}\mathcal{S}(z) - \frac{\alpha_s}{4\pi}\frac{x+1/x}{4}\,y\log y\right).$$

and use this to compute the final integral (in the saddle-point approximation to get Im):

$$\hat{\sigma}^{\rm inst}_{\rm tot}(E) = \frac{1}{E^2}\,{\rm Im}\int_{-\infty}^{+\infty} dr_0\, e^{r_0}\, G(r_0,E)\,,$$

Results and partonic cross-section

$\sqrt{\hat{s}}$ [GeV]	50	100	150	200	300	400	500
$\langle n_g \rangle$	9.43	11.2	12.22	12.94	13.96	14.68	15.23
$\hat{\sigma}^{\rm inst}_{\rm tot}$ [pb]	207.33×10^3	1.29×10^3	53.1	5.21	165.73×10^{-3}	13.65×10^{-3}	1.89×10^{-3}

$$G(r_0, E) \;=\; \frac{\kappa^2 \pi^4}{2^{17}} \sqrt{\frac{\pi}{3}} \;\; \int_0^\infty r^2\, dr \int_0^\infty \frac{dx}{x} \int_0^\infty \frac{dy}{y} \left(\frac{4\pi}{\alpha_s}\right)^{21/2} \left(\frac{1}{1-\mathcal{S}(z)}\right)^{7/2} \mathcal{K}_{\rm ferm}$$
$$\sum_{n_g=0}^{\infty} \frac{1}{n_g!}\, (U_{\rm int})^{n_g}\, \exp\left(-\frac{4\pi}{\alpha_s} - \frac{\alpha_s}{4\pi}\, \frac{x+1/x}{4}\, y \log y\right).$$

$$\langle n_g \rangle \;=\; \langle U_{\rm int} \rangle$$

2nd Approach: VVK, Milne, Spannowsky

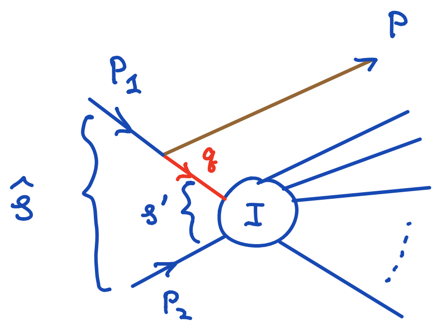
hadronic total cross-section

$$\sigma_{pp\to I}\left(\hat{s}>\hat{s}_{\min}\right)=\int_{\hat{s}_{min}}^{s_{pp}} dx_1 dx_2 \quad f\left(x_1,Q^2\right) f\left(x_2,Q^2\right)\hat{\sigma}\left(\hat{s}=x_1x_2s_{pp}\right)$$

$E_{\min}$ [GeV]	50	100	150	200	300	400	500
$\sigma_{p\bar{p}\to I}$ $\sqrt{s_{p\bar{p}}}$=1.96 TeV	2.62 μb	2.61 nb	29.6 pb	1.59 pb	6.94 fb	105 ab	3.06 ab
$\sigma_{pp\to I}$ $\sqrt{s_{pp}}$=14 TeV	58.19 μb	129.70 nb	2.769 nb	270.61 pb	3.04 pb	114.04 fb	8.293 fb
$\sigma_{pp\to I}$ $\sqrt{s_{pp}}$=30 TeV	211.0 μb	400.9 nb	9.51 nb	1.02 nb	13.3 pb	559.3 fb	46.3 fb
$\sigma_{pp\to I}$ $\sqrt{s_{pp}}$=100 TeV	771.0 μb	2.12 μb	48.3 nb	5.65 nb	88.3 pb	4.42 pb	395.0 fb

2nd Approach: VVK, Milne, Spannowsky

If the instanton is recoiled by a high pT jet emitted from one of the initial state gluons => hadronic cross-section is tiny



$Q^2 = -q^2 = \sqrt{\hat{s}}\, p_T$

$s' = (q+p_2)^2 = \hat{s} - 2Q^2$

A virtual log

$\Rightarrow e^{-Q\rho}$ ← form factor.

↑ cuts-off low-energy range.

Mueller corr-s cuts-off high-energy range (as before.)

$$\exp\left(-Q(\rho+\bar{\rho})\right) = \exp\left(-\frac{Q}{E}\sqrt{y\,(x+1/x+2)}\right)$$

$\sqrt{\hat{s}}$ [GeV]	310	350	375	400	450	500
$\hat{\sigma}_{\rm tot}^{\rm inst}$ [pb]	3.42×10^{-23}	1.35×10^{-18}	1.06×10^{-17}	1.13×10^{-16}	9.23×10^{-16}	3.10×10^{-15}

Table 3. The instanton partonic cross-section recoiled against a hard jet with $p_T = 150$ GeV emitted from an initial state and calculated using Eq. (3.7). Results for the cross-section are shown for a range of partonic C.o.M. energies $\sqrt{\hat{s}}$.

$\sqrt{\hat{s}}$ [GeV]	100	150	200	300	400	500
$\hat{\sigma}_{\rm tot}^{\rm inst}$ [pb]	1.68×10^{-7}	1.20×10^{-9}	3.24×10^{-11}	1.84×10^{-13}	4.38×10^{-15}	2.38×10^{-16}

Table 4. The cross-section presented for a range of partonic C.o.M. energies $\sqrt{\hat{s}} = E$ where the recoiled p_T is scaled with the energy, $p_T = \sqrt{\hat{s}}/3$.

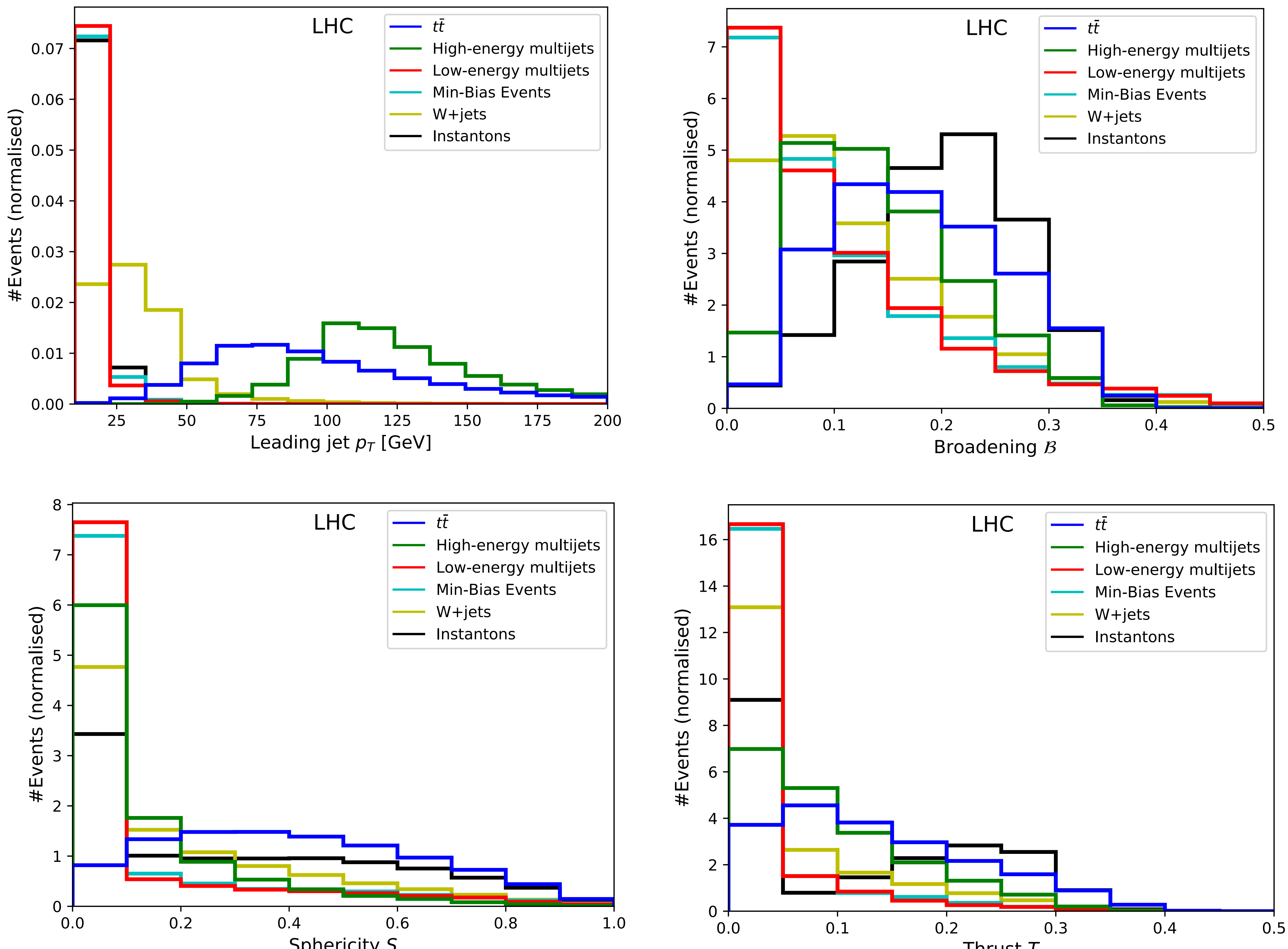
2nd Approach: VVK, Milne, Spannowsky

Phenomenology summary

- QCD instanton cross-sections can be very large at hadron colliders.
- Instanton events are isotropic multi-particle final states. Their event topology is very distinct (see Event shape observables next slide)
- Particles with large pT are rare. Especially hard to produce them at low partonic energies (for obvious kinematic reasons). Do not pass triggers.
- At higher (partonic) energies instanton events can pass triggers but have suppressed cross-sections.
- Propose to examine data collected with minimum bias trigger
- [Note: large theoretical uncertainty at low partonic energies (strong coupling in the IR) where the semiclassical approximation becomes suspect/invalid. Best would be to constrain theory prediction / normalisation with data.]

VVK, Milne, Spannowsky

LHC jet reconstruction: anti-kT, R=0.4, pT>10 GeV



Event shape observables are good. Triggering based on high pT pose problems:

Example: High-luminosity LHC

Missing transverse energy higher-level triggers require at least $E_{T\text{mis}} \geq 70$ GeV while single jet triggers are as high as $p_{T,j} \geq 360$ GeV

If one of the instanton-induced partons has a transverse momentum to pass the single-jet trigger requirements, the centre-of-mass energy of the instanton $\sqrt{s'}$ has to be at least of $\mathcal{O}(700)$ GeV.

=> Cross-section is too small.

Thus, one would have to resort to multijet triggers, either with four jets of $p_{T,j} \geq 85$ GeV or six jets of $p_{T,j} \geq 45$ GeV. Both such trigger requirements result in for instantons fairly high partonic centre-of-mass energies of $\mathcal{O}(300)$ GeV. Generating 100000 signal events

=> none of the events passed trigger.

Need to be creative: low-luminosity LHC with minimal bias triggers does much better.

Can one incorporate event-shape observables into trigger strategies?

VVK, Milne, Spannowsky

Main sources of theoretical uncertainties (for discussion)

- QCD Instanton rates are interesting in the regime where they become large — lower end of partonic energies 10-80 GeV. The weak coupling approximation used in the semiclassical calculation can be problematic. How to address: vary s' minimal partonic energy cutoff.

- What is the role of higher-order corrections to the Mueller's term in the exponent?

- Possible corrections to the instanton-anti-instanton interaction at medium instanton separations in the optical theorem approach.

- Non-factorisation of the determinants in the instanton-anti-instanton background in the optical theorem. (Instanton densities D(rho) do not factorise at finite R/rho ~1.5 - 2.)

- Choice of the RG scale = 1/rho. (can vary by a factor of 2 or use other prescriptions to test. In Ref. [1] we checked that)

- A practical point for future progress is to test theory normalisation of predicted QCD instanton rates with data. [The unbiased and un-tuned theory prediction is promising.]