Sphalerons, Baryogenesis & Gravitational Waves

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The Sphaleron

At zero Temperature

\[ \Gamma \propto \exp \left( - \frac{E_{\text{sp}}}{T} \right) \sim \text{very small} \]
The Sphalerons

\[ \frac{\text{high}}{\text{EWPT}} \]

\[ \Gamma_{\text{sym}} \sim \alpha^4 T^4 \]

\[ \Gamma_{\text{broken}} \sim \alpha^4 \omega \exp \left( -\frac{\epsilon_{\text{sp}}}{T} \right) \]

\[ \Gamma_0 \sim \exp \left( -\frac{\epsilon_{\text{sp}}}{\omega} \right) \sim \exp \left( -\frac{\epsilon_{\text{P}}}{\omega} \right) \]
High-T Sphaleron

\[ \Gamma_{\text{sph}} = (8.24 \pm 0.10) \left( \frac{N}{2} \right)^5 \frac{g^2 T^2}{m_D^2} \left( \log \frac{m_D}{g^2 T} + 3.041 \right) \alpha^5 T^4 \]

In thermal equilibrium below

| \( T_{ss} \) | \( 2.4 \times 10^{13} \) GeV | strong sphaleron |
| \( T_{ws} \) | \( 1.8 \times 10^{12} \) GeV | weak sphaleron |

Affect washout rate in Leptogenesis

- strong \( q_L \leftrightarrow q_R \)
- weak \( \beta \leftrightarrow \ell \)

Boedeker, hep-ph/9801430
Moore, hep-ph/0001216

Garbrecht, PS, 1404.2915

"spectator effects"
Weak scale sphaleron

\[ \langle v^2(T) / T^2 \rangle \]

\[ \log \frac{\Gamma}{T} \]

\[ \log [\alpha H(T) / T] \]

\[ T / \text{GeV} \]

\[ \text{pure gauge} \]

\[ T / \text{GeV} \]

\[ \text{multicanonical} \]

\[ \text{standard} \]

\[ \text{fit} \]

\[ \text{perturbative} \]

exponential suppression starts

freeze-out

\[ \Gamma / H < 1 \]
Weak scale sphaleron

Relevant e.g. for low scale leptogenesis scenarios & electroweak baryogenesis

\[
\begin{align*}
\mathcal{z} &= \frac{T_{\text{sph}}}{T} \\
\text{this value of the baryon asymmetry frozen in!}
\end{align*}
\]

Baumholzer, Brdar, PS, 1806.06864
Changing $\Gamma_{\text{ sphaleron}}$

- High $T$: pure gauge theory
  - change RGE running
  - embed $SU(2)$ in larger group

- Low $T$: modify Higgs potential
  - extend Higgs sector
  - change $E_{\text{ sph}}$ → also affect zero $T$ rate
1990s

singlet extension

MSSM

Enqvist, Vilja, PLB 287

Moreno, Oaknin, Quiros, hep-ph/9605387
Figure 1: Energy of the electroweak sphaleron in the units of $4\pi v/g$. Different colors of curves correspond to different operators as shown in the legend. Solid (dashed) curves indicate that the operator coefficient is positive (negative).

On the right axis of the left panel of Fig. 1 we show the lower bound on the electroweak order parameter $v(T)/T$ from Eq. (20). We take the coefficient to be $1.07$, which is the arithmetic mean of $0.973$ and $1.16$. The washout avoidance condition can vary by as much as a few percent due to the presence of dimension-six operators while keeping $\langle - | c_i | \rangle_{\text{phys}} \sim 1$ to be consistent with collider observations.

In Fig. 2 we show how the size of the electroweak sphaleron changes with the presence of dimension-six operators. To define the sphaleron size, we calculate the energy density as $E_{\text{sph}}, 0(\ll) = (gv)\frac{3}{4}(4\pi \ll 2)^{1/3} dE_{\text{sph}}, 0/\ll$ with $E_{\text{sph}}, 0$ given by Eq. (19). Then we define a fiducial sphaleron radius $\ll_{\text{sph}}$ as the width at half-maximum, $\ll_{\text{sph}}, 0(\ll_{\text{sph}}) = (1/2) \ll_{\text{sph}}, 0(0)$.

4 Conclusion

We have allowed the Standard Model to be extended with a set of 20 dimension-six operators that are constructed from the Higgs field, isospin gauge field, and hypercharge gauge field. We have calculated analytically the effect of these operators on the field equations (Table 1), and to our knowledge such a calculation has not appeared before in the literature. We have calculated analytically the sphaleron equations of motion (Table 2) with the assumption that the hypercharge gauge field is set to zero. Using numerical methods, we solve the equations of motion and calculate the sphaleron energy.

Gan, Long, Wang, 1708.03061
\[ \frac{c_i}{\Lambda^2} (H^+H)^3 \]

For \( 500 \text{ GeV} < \Lambda < 800 \text{ GeV} \)

EWPT is first order

Grojean, Servant, Wells, hep-ph/0407019

Gan, Long, Wang, 1708.03061
GW's in SM + H^6

Figure 6: Example of gravity wave spectrum produced during the EW phase transition both by turbulence (left peak) and collision effects (right peak slightly emerging from the tail of the turbulence spectrum). This plot is for m_h = 115 GeV and f' = 600 GeV where \( \tau = 0.51, \) /H = 89 and T_n = 39 GeV. Note that suitable values of \( \tau \), \( /H \) to get a strong signal always imply a small nucleation temperature (\(< 100 \text{ GeV}\)) due to important overcooling effects that drag the peak below the lower bound of the space-based detectors frequency band (\( \approx 10^{-4} \text{ Hz} \)), making the gravity waves delicate to observe.

The phase transition today if the latent heat energy released is large and the emission lasts a long time. This can be understood easily by recalling that the power spectrum is given by the square of the quadrupole moment of the source which in turns scales as the kinetic energy over the time of emission \[29\]. In other words, typically \( \tau \) has to be \( O(1) \) and \( /H \) as small as \( O(100) \) to get a sufficiently high energy density \( \Omega_{GW} h^2 \).

Relying on our effective (nonrenormalizable) potential approach, we find that generically the dynamics of the first order EWPT beyond the SM generate too weak gravity waves to observe except for a tiny region of the parameter space. Namely, by looking closely at Figs. 5 one can see that for a Higgs mass slightly above the LEP2 bound, m_h \( \gg 115 \text{ GeV} \), and a relatively low scale, \( f' \ll 650 \text{ GeV} \), we get at best \( \tau \ll 0.5 \) and \( /H \ll 100 \). The corresponding nucleation temperature in this region is about 50 GeV, according to Fig 3. For such a temperature scale, only LISA and BBO will be sensitive to the emitted spectrum of gravity waves, according to the results presented in Figs. 3 and 4 of \[22\]. Its detectability is probably beyond the capability of LISA. This result is in qualitative agreement with the results of \[30\]. Indeed LISA requires at least values of \( \tau > 0.6 \) for \( /H \ll 100 \) in order to see the characteristic peak from turbulence while the collision peak starts to be probed for \( \tau > 0.8 \). On the other hand, BBO should be able to observe both peaks if \( \tau \) is around 0.3 (keeping \( /H \ll 100 \)). Thus it seems that one will have to wait until the launching of the second generation of space-based interferometers to really study the EWPT through gravity wave detectors within this framework. Moreover this would be possible only in the maximizing case where the Higgs mass is close to its current experimental bound and the composite scale of the

Delaunay, Grojean, Wells, 0711.2511
GW - $E_{\text{sp}}$ correlation?

\[
\frac{1}{\Lambda^2} (H^+H)^3
\]

singlet with $\langle \phi \rangle = \nu_s$

GW signal detectability @ LISA

(\text{SNR} > 10 (50))

Zhou, Bian, Guo, 1910.00234
Also: Composite Higgs

\[ V(H) = V_0 + m_H^2 H^\dagger H + (H^\dagger H)^2 \left( -\lambda + \beta \log \left[ \gamma + \frac{2H^\dagger H}{\phi_0^2} \right] \right) \]
Figure 7: Relic abundance of gravitational waves from the collision of cosmic bubbles in the LISA C2 regime. For $T_e = 1$ keV and $T_e = 10$ MeV, the sensitivity of LISA and other detectors is shown.

Baratella, Pomarol, Rompineve, 1812.06996

see also Bruggisser, von Harling, Matsedonskyi, Servant, ...
Collider probes

\[ \frac{1}{\Lambda^2} \ H^6 \] double Higgs production

\[ \alpha = \lambda_{3SM} + O \left( \frac{v^2}{\Lambda^2} \right) \]
Collider probes

- singlet extension
  - di-photon, etc.

- composite warped
  - modified Higgs couplings \((1 - \frac{u^2}{f^2})\)
  - light dilaton, radion

sphaleron directly ... unclear
Instantons

- unsuppressed at scales $> \text{few GeV}$
- axion couples to derivative of CS-current
  \[ \frac{a}{f} \bar{G}G \]
- Any effect on instanton rates
  - what if we are in a coherent axion bath ground?
Axion & GW's

Superradiance (Arvanitaki et al.)

GW's from axion coupled to dark photon
Yeah, I learned about it when I was researching anomalous electroweak sphaleron transition baryogenesis.

Cool.

My hobby: Collecting really satisfying-sounding five-word technical phrases.

Topology is cool - let's find something