

Classicalization (and saturons)

Gia Dvali

LMU-MPI

based on: *mostly*

2003.05546 [hep-th]

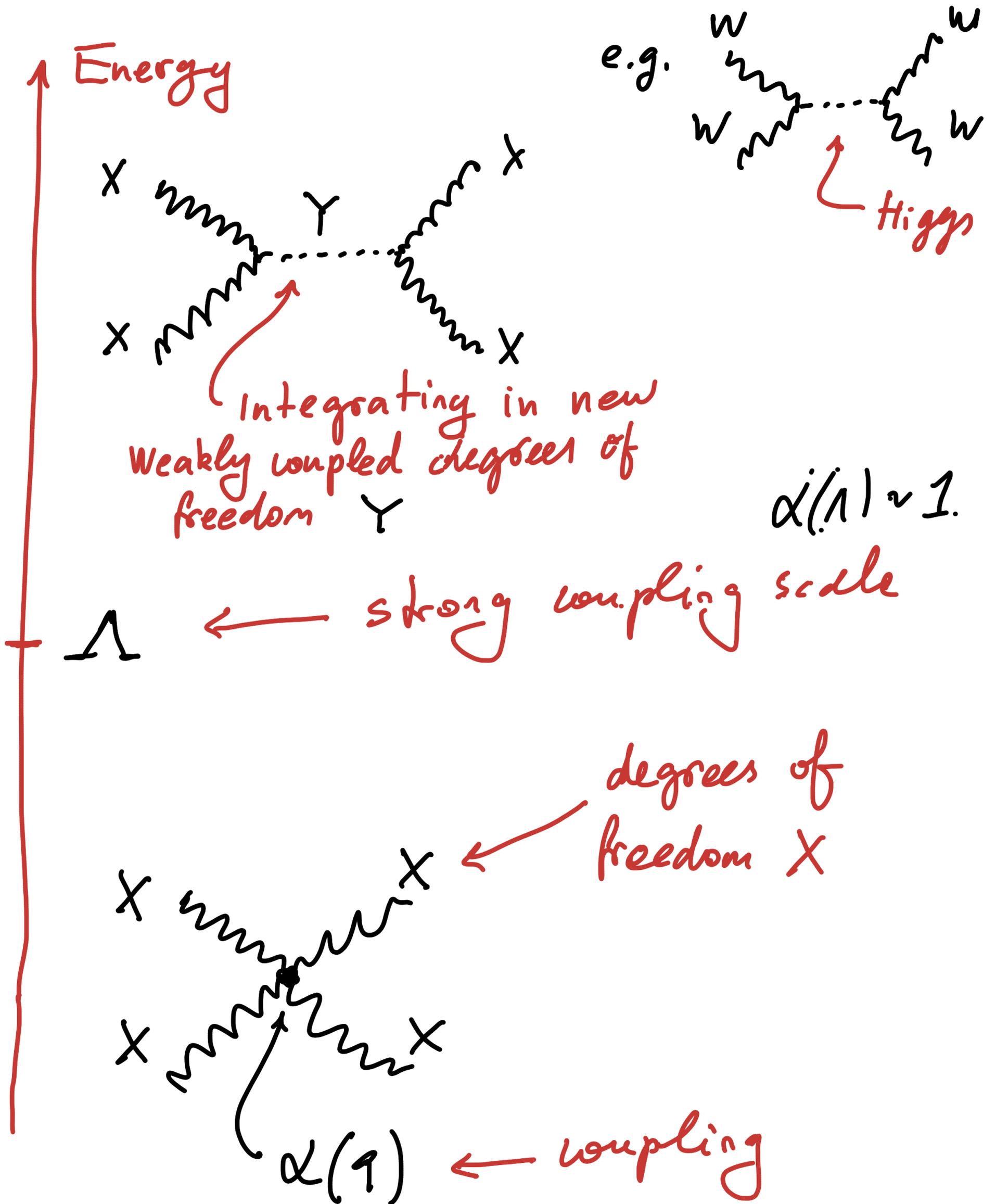
1907.07332 [hep-th]

1906.03530 [hep-th]

Original: 1010.1415 [hep-th]

with Gian Giudice, Cesar Gomez,
Alex Kehagias.

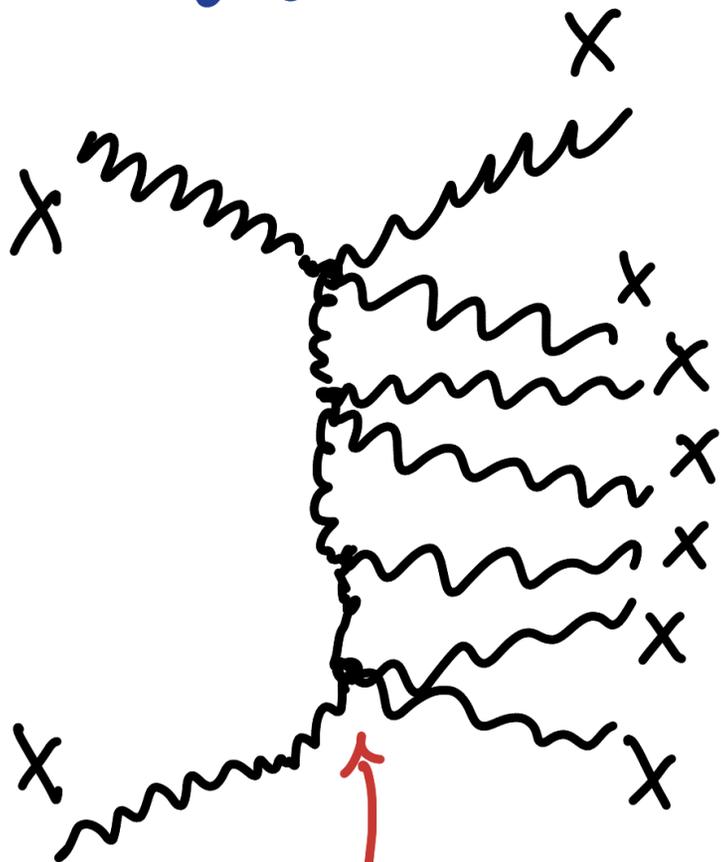
Wilsonian UV-completion



UV-completion by classicalization (self-completion)

G.D., Giudice, Gomez, Kehagias '11

↑ Energy



Unitarization by
many soft quanta

$$N \gg 1$$

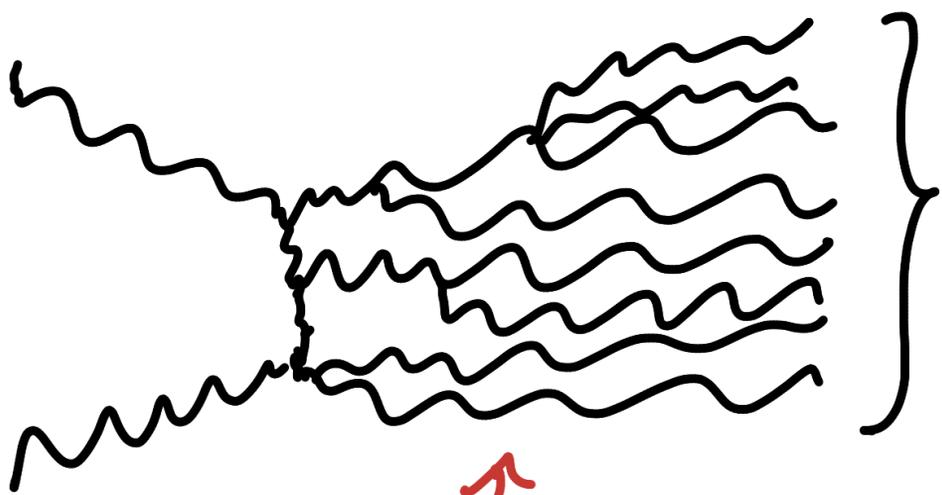
effective coupling $\alpha(\frac{E}{\Lambda}) \ll 1$

← Λ

← strong coupling scale $\alpha(\Lambda) \sim 1$



$\alpha(q)$



"classical"
state

strong field
but

Weak coupling $\alpha(\frac{E}{M}) \ll 1$

But, production of multiparticle states at $\alpha \ll 1$ is suppressed.

See, e.g., Brown '92; Voloshin '92

Argyres, Kleiss, Papadopoulos '93

Gorsky Voloshin '93; Libanov, Son,

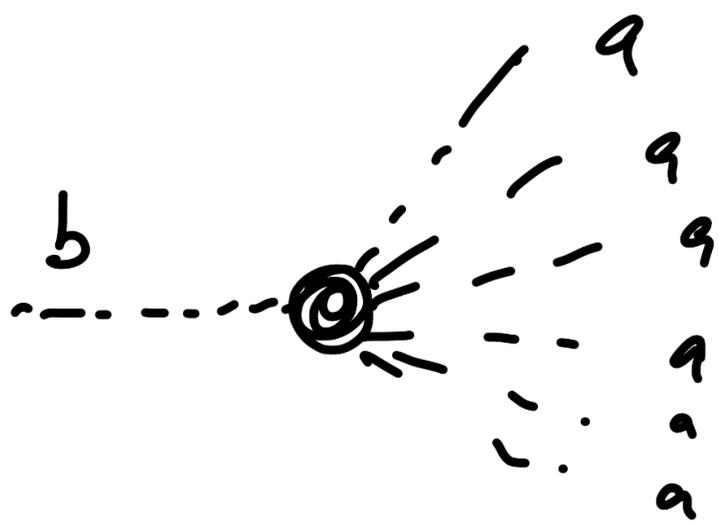
Troitsky '95; Son '96; ...

Monin '18, ...

Spin-2 $2 \rightarrow N$ scattering: G.D. Gomez,
Isermann, Lüst, Stieberger '14;
Addazi, Bianchi, Veneziano '16.

A simple argument: G.D., '18

transition $|1\rangle_a \rightarrow |n\rangle_b$



effective
S-matrix operator

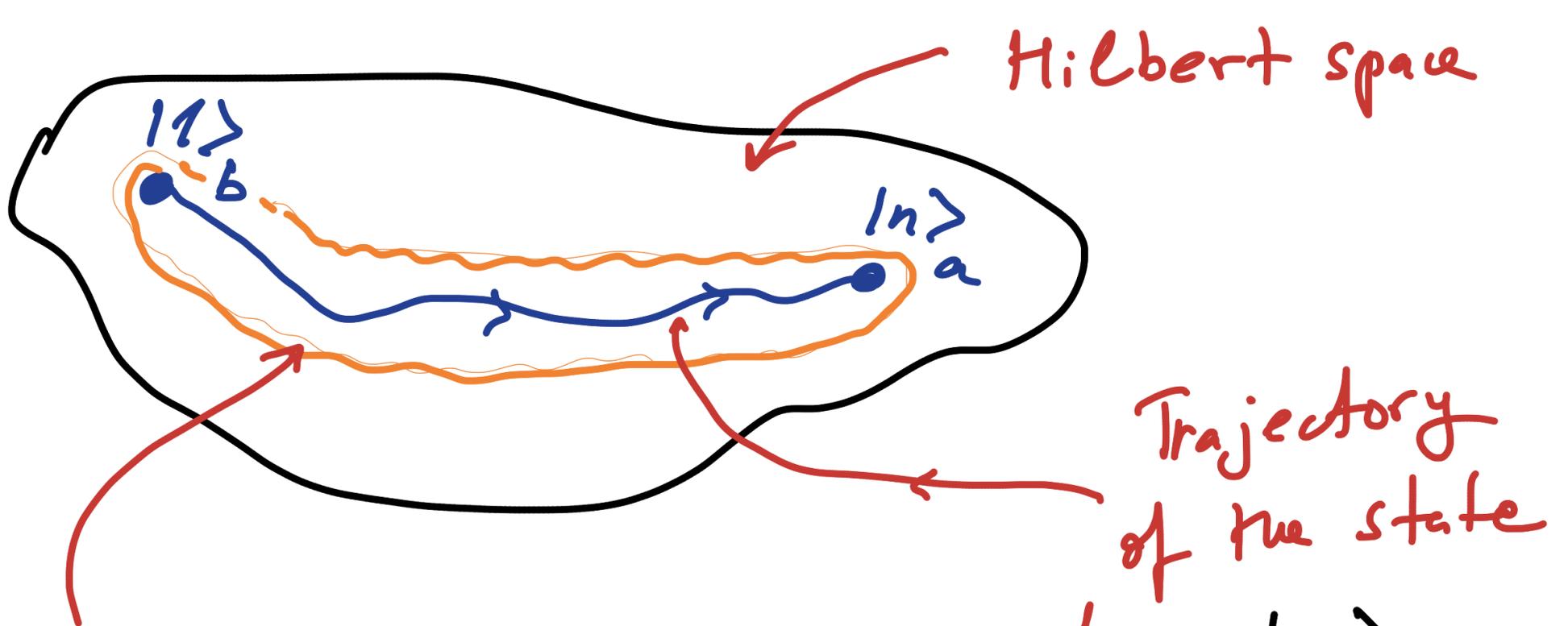
$$\hat{S} = \mathcal{H}(\hat{a}^\dagger)^n \hat{b}$$

bound

$$\mathcal{H} \lesssim n^{\frac{n}{2}}$$

Argument: On any state $|4\rangle$ that is physically close, we must have:

$$|\langle 4 | \hat{S} | 4 \rangle|^2 < 1$$



"Physically close"

states

$$|4\rangle$$



take

$$|4\rangle = e$$

$$\sqrt{n}(\hat{a}^\dagger - \hat{a}) + (\hat{b}^\dagger - \hat{b})$$

$$|0\rangle$$

Then, $|\langle \psi | \hat{S} | \psi \rangle|^2 < 1$

$$\hat{S} = \kappa (\hat{a}^\dagger)^n \hat{b}$$

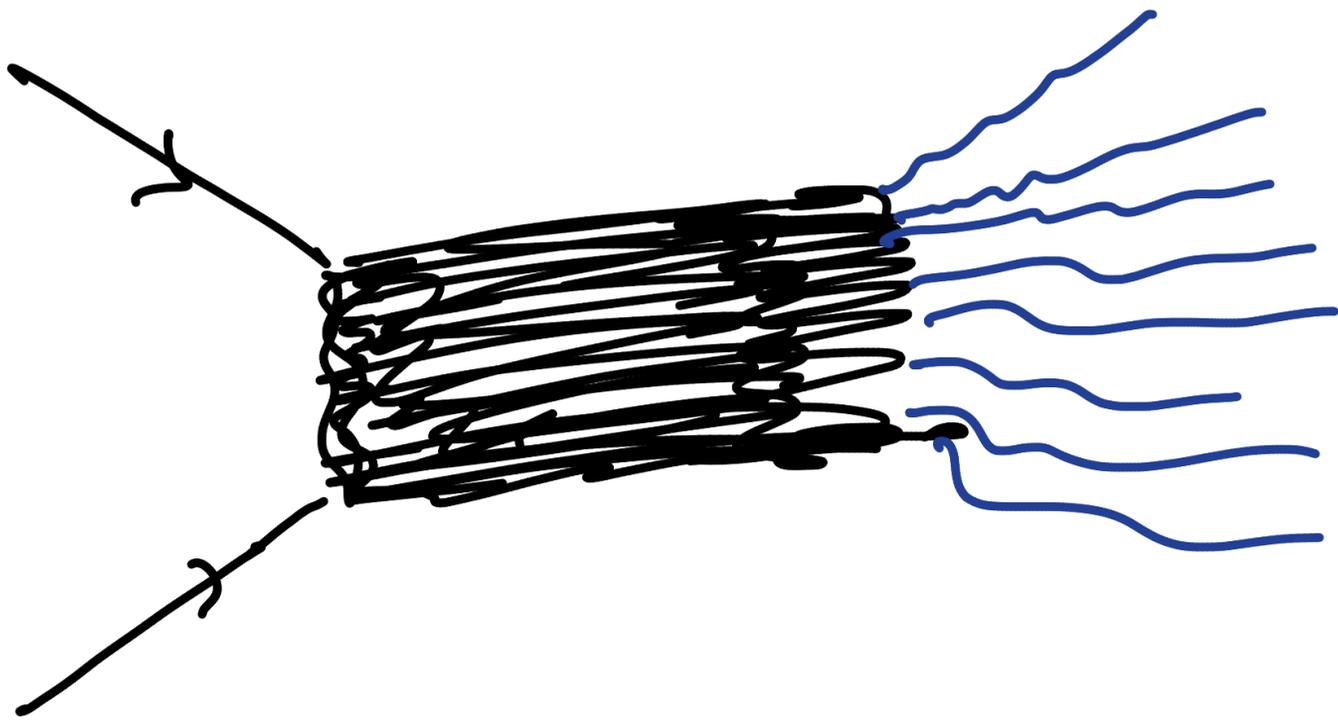
$\kappa < n^{-\frac{n}{2}}$

Thus,

$$|\langle_a \kappa | \hat{S} | 1 \rangle_b|^2 < \kappa! n^{-n} \sim e^{-n}$$

In order to compensate this suppression, the object must have a maximal micro-state entropy compatible with unitarity.

I call such objects "Saturns".

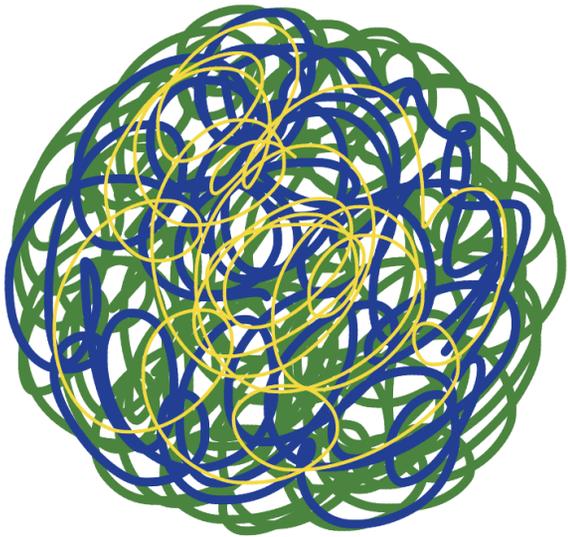


What is this maximal entropy?

The original entropy bound,
and the only one considered
until recently, is the bound
from Bekenstein

$$S \leq S_{\text{Bek}} = 2\pi E R$$

Maximal entropy
carried by object
of energy E
and radius R



← $2R$ →

And, until recently, the only objects discussed as saturating the bound, were black holes

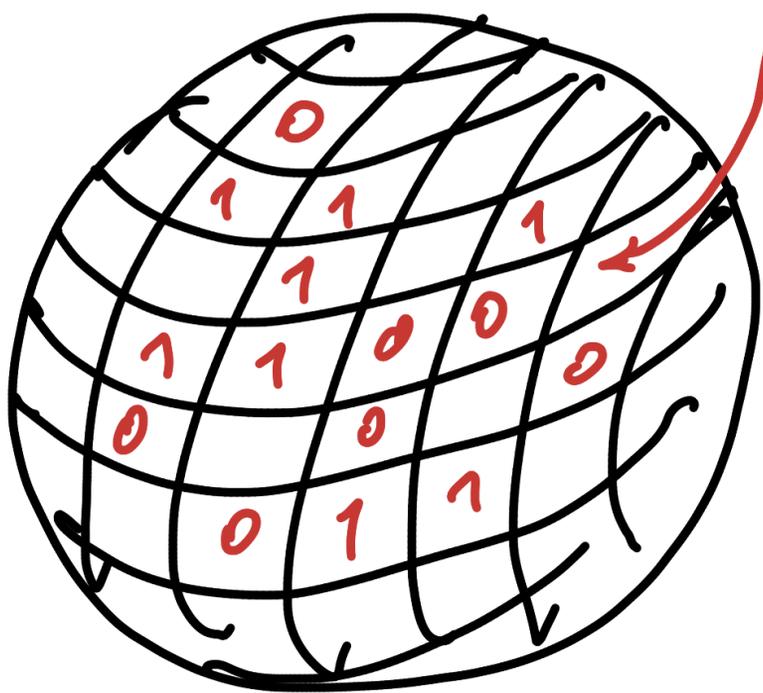
Bekenstein-Hawking entropy:

$$S_{\text{BH}} = \frac{\text{Area}}{4G} = \frac{4\pi R^2}{4G} = 2\pi ER = S_{\text{Bek}}$$

$$E = M_{\text{BH}} = \frac{R}{2G}$$

Planck area pixels

$$L_p^2 \equiv G \equiv 1/M_p^2$$

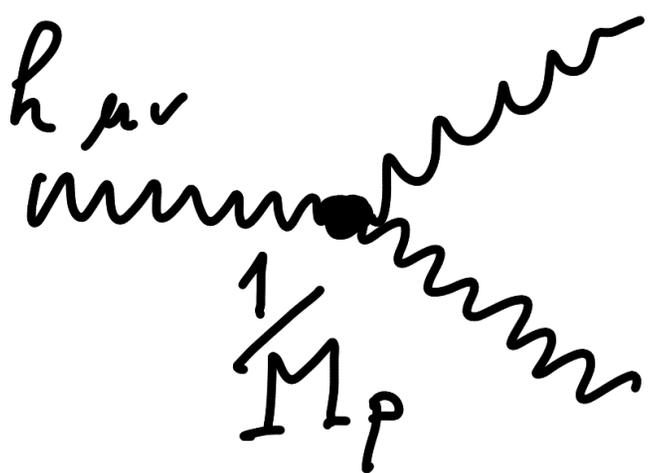


$$S_{\text{BH}} \sim \frac{R^2}{L_p^2} \sim (RM_p)^2$$



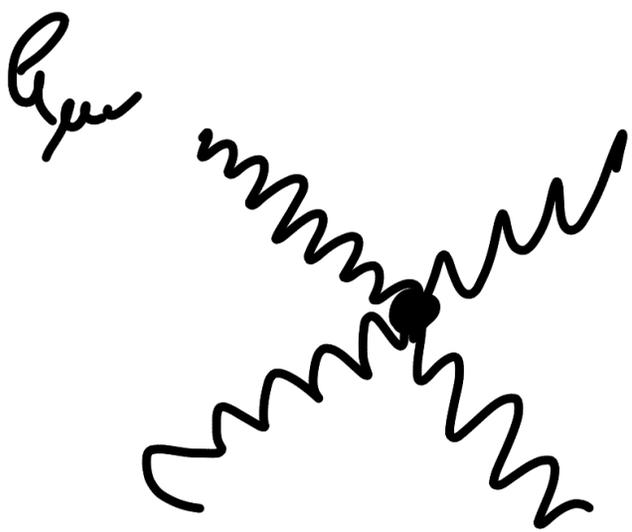
Alice

Particle physics meaning

$$h_{\mu\nu} \frac{1}{M_P}$$


$$\frac{h_{\mu\nu}}{M_P} T^{\mu\nu}$$

graviton decay constant = M_P

$$g_{\mu\nu}$$


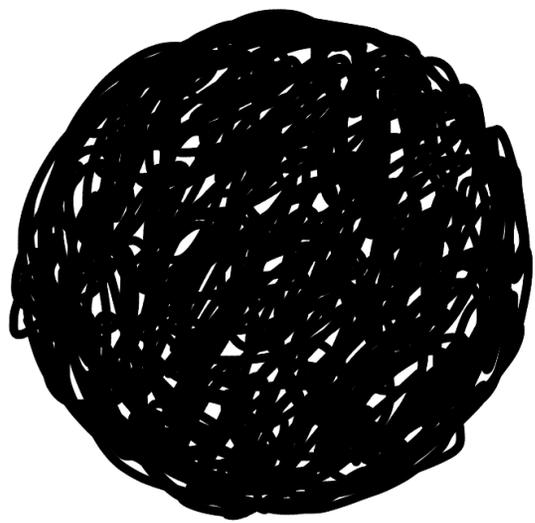
$$\alpha = \frac{g^2}{M_P^2} = \frac{L_P^2}{R^2}$$

graviton coupling at

momentum-transfer $g \equiv \frac{1}{R}$

Thus, $S_{BH} = \frac{1}{\alpha} = \frac{L_p^2}{R^2}$

$\leftarrow \sim R \rightarrow$



Breaks Poincaré
symmetry spontaneously

Order parameter = M_{Pl}

$$S_{BH} = (R f)^2$$

Goldstone decay constant

$$f = M_{Pl}$$

Thus, for a black hole

$$S = ER = \frac{1}{\alpha} = (Rf)^2$$

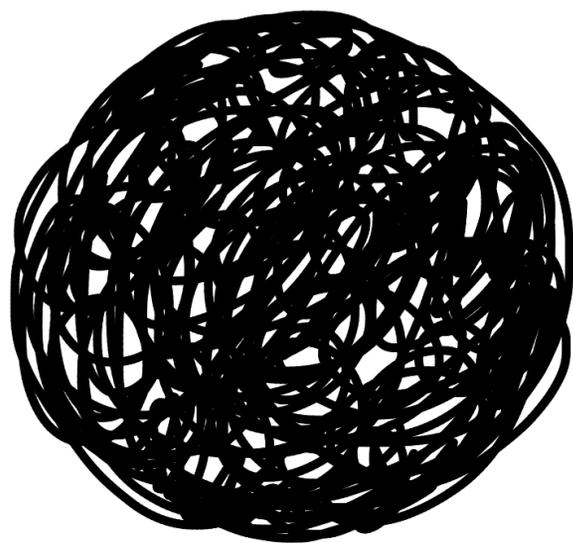
Due to this, black holes must (and do) saturate unitarity

$2 \rightarrow \text{Anything}$

$2 \rightarrow \text{BH}$

Exists number of micro-states

$$N_{st} = e^{S_{BH}} = e^{\frac{1}{\alpha}}$$



S_{BH}

$$|BH\rangle = |0, 1, 1, \dots, 0, 1\rangle$$

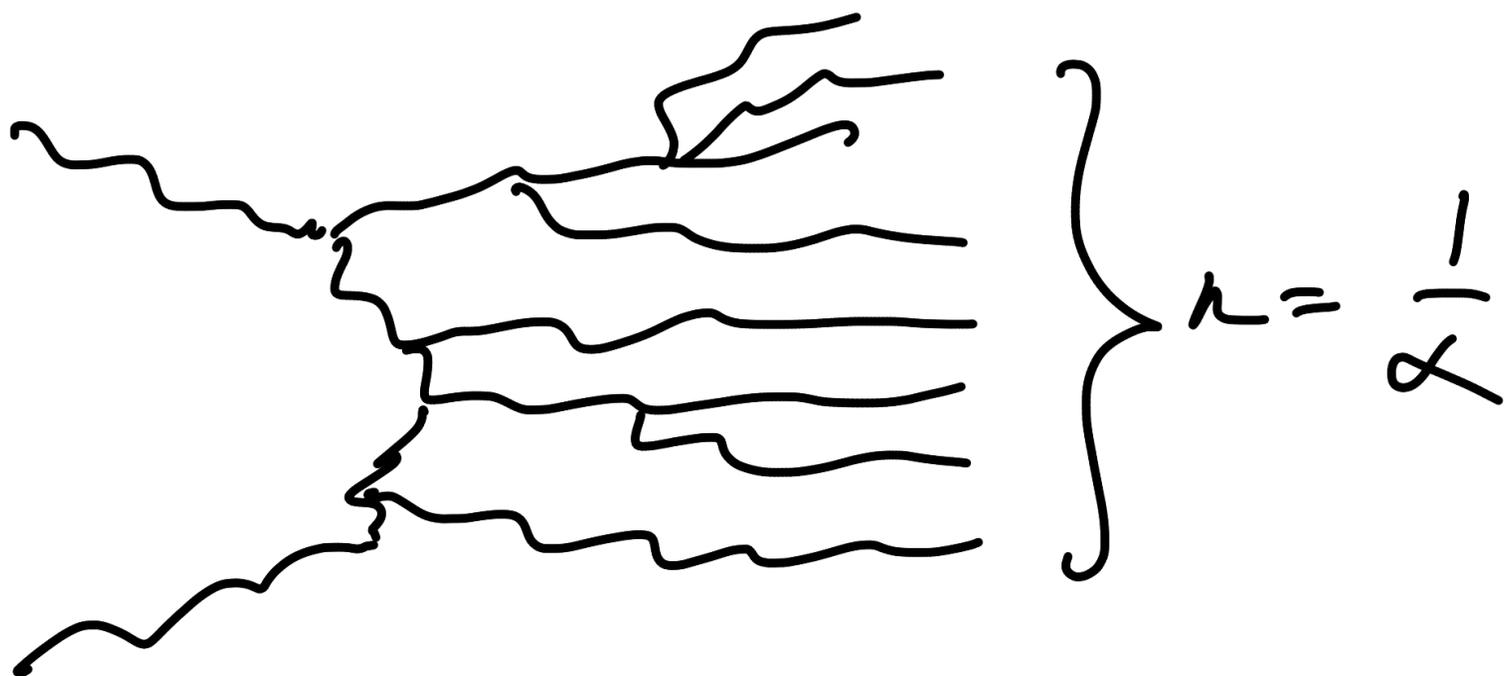


Production of black holes in
scattering at $E \gg M_{\text{Pl}}$ is

old ideas: 't Hooft '87;

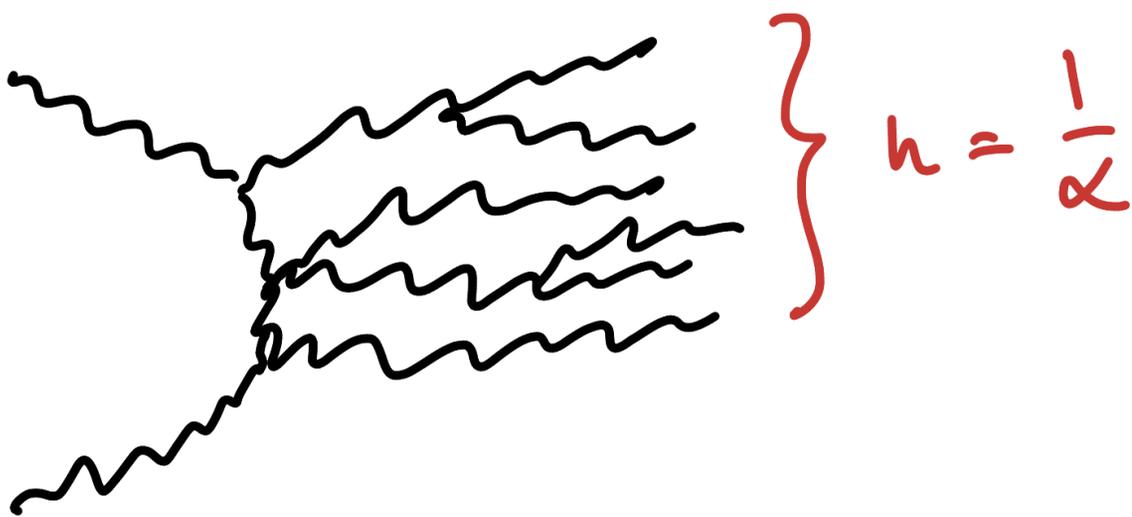
Amati, Ciafaloni and Veneziano '87
Gross, Mende '87,
.....

New point here is understanding
in terms of $2 \rightarrow n = \frac{1}{\alpha}$ graviton
amplitudes



$$S = ER = \frac{1}{\alpha} = (Rf)^2$$

Is correlated to saturation of
unitarity by $2 \rightarrow n = \frac{1}{\alpha}$ graviton (closed string)
amplitudes



$$\sigma = \bullet n! \alpha^n e^S = e^{-\frac{1}{\alpha} + S}$$

G.D, Gomer, Isermann, Lüst,
Stieberger '14;
Addazi, Bianchi, Veneziano '16.

Are black holes unique in this?

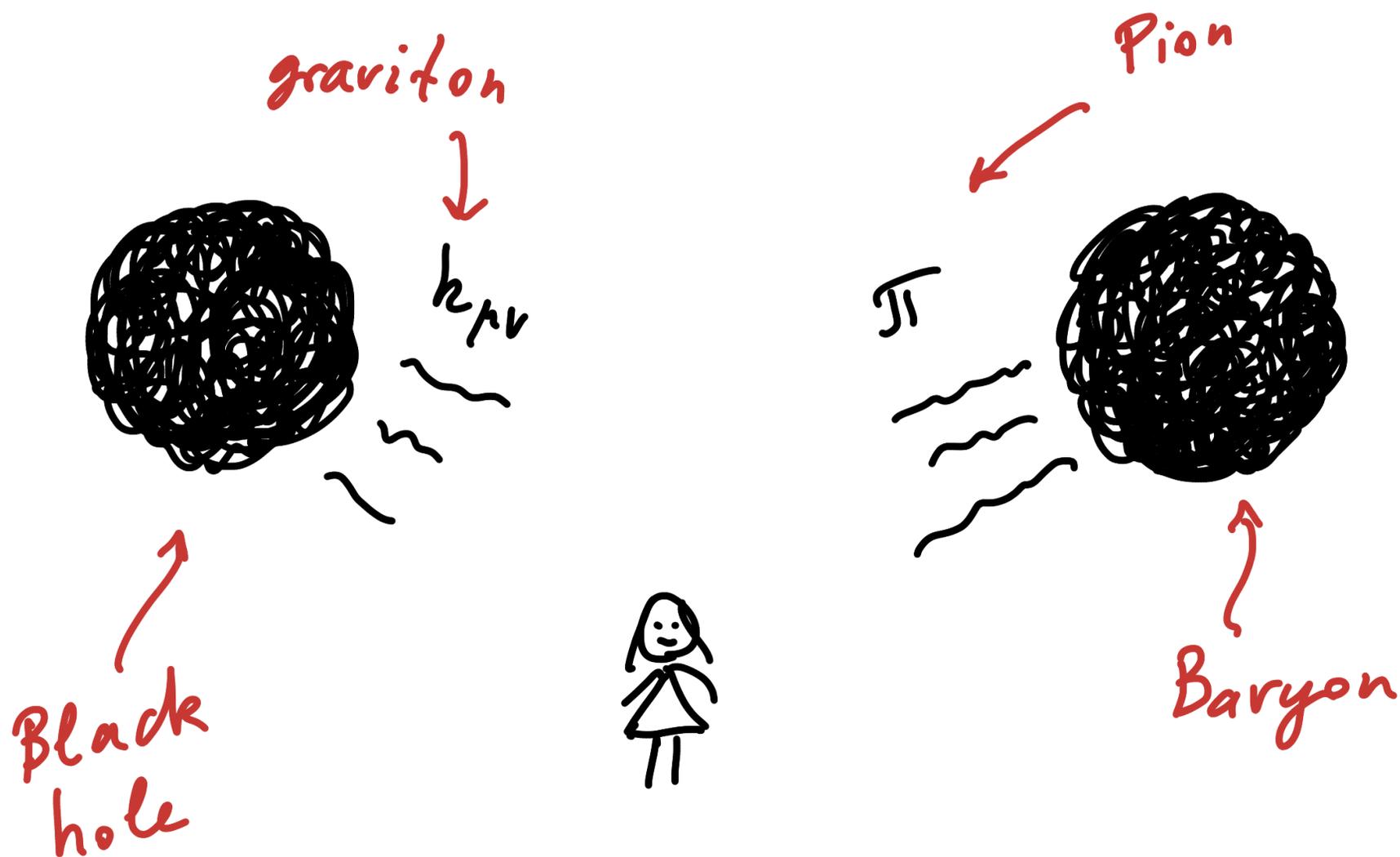
Saturation of unitarity

$$S^u = S_{\text{max}}^d = \frac{1}{\alpha} = (Rf)^2 = ER$$

This question could have been asked long ago, but was not.

Here is what we have discovered:

All QFT objects (such as solitons, baryons, instantons or simply lumps of classical fields) at the saturation point behave like black holes.

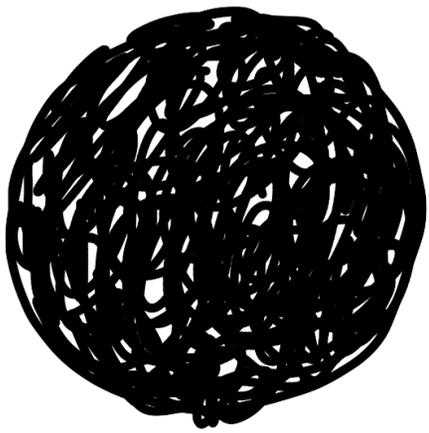


Unitarity imposes the following upper bounds on entropy of any QFT object of radius R :

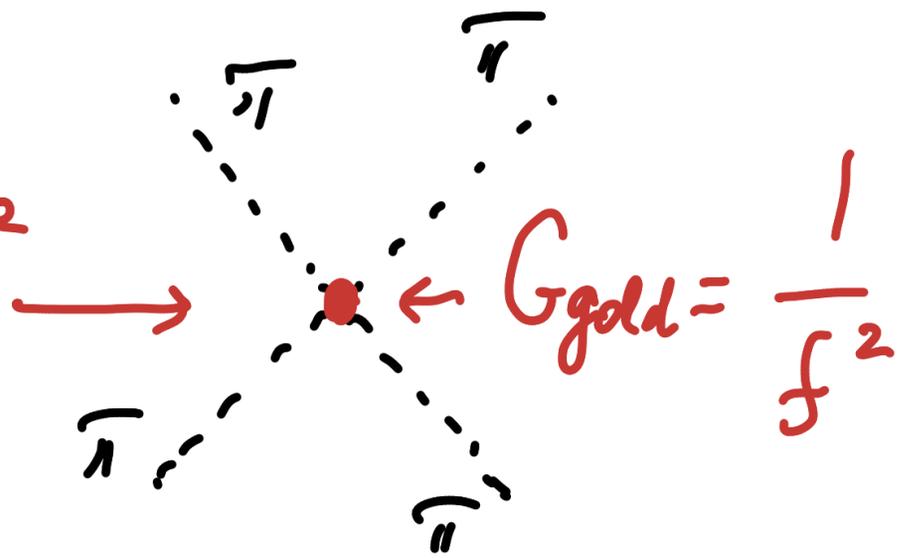
① Area-law bound:

$$S_{\max} = \frac{\text{Area}}{G_{\text{gold}}} = \frac{R^2}{f^2}$$

$\leftarrow R \rightarrow$



↑
Goldstone
coupling



Goldstone boson of spontaneously broken symmetry: ① Space-translation; ② Internal.

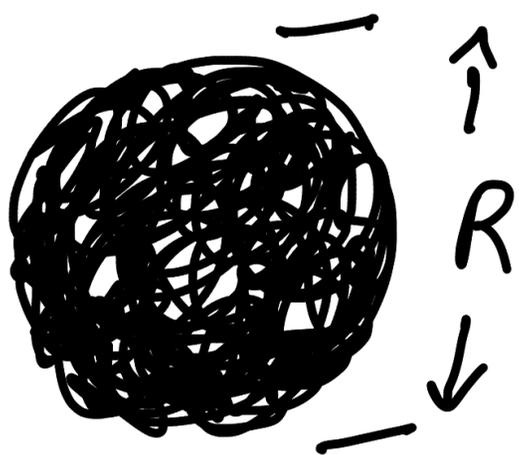
(ii)

Inverse-coupling bound:

$$S_{\max} = \frac{1}{\alpha(q)}$$

running coupling at $q = \frac{1}{R}$

for interaction of
range R



Notice, for Goldstone:

$$\alpha_{\text{Gold}}(q) = \frac{q^2}{f^2} \rightarrow \frac{1}{\alpha_{\text{Gold}}\left(\frac{1}{R}\right)} = R^2 f^2$$

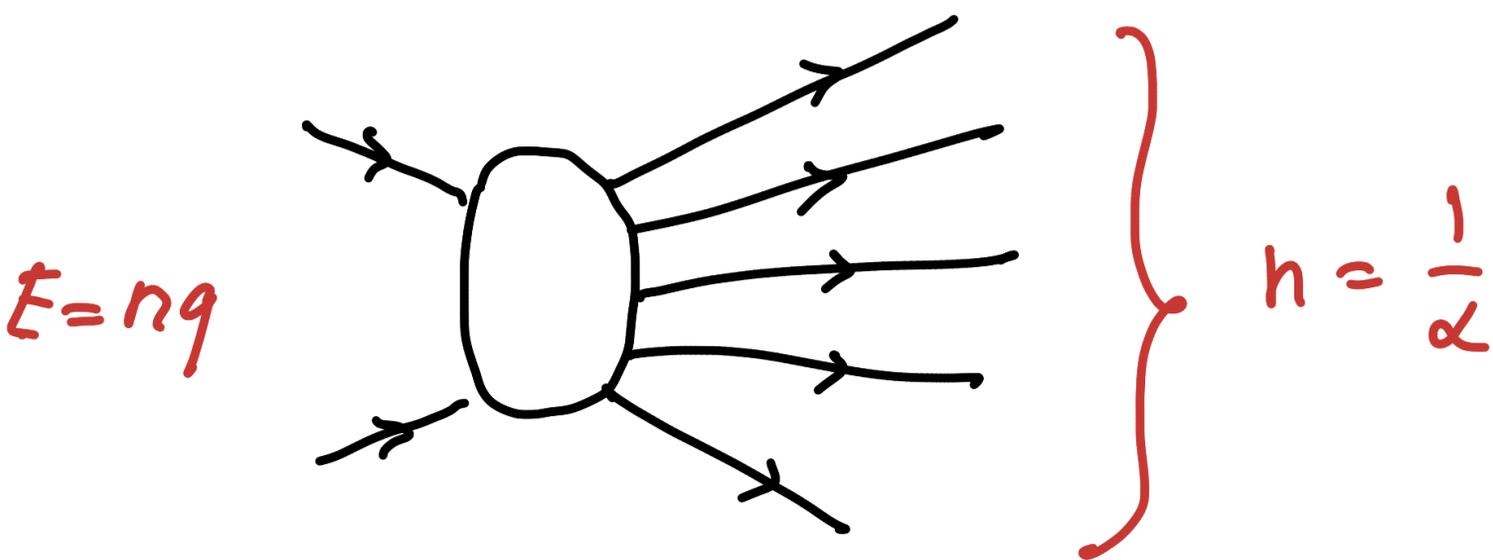
Area

Saturation of these entropy bounds is in one-to-one correspondence with saturation of unitarity by

$2 \rightarrow n$ scattering amplitudes

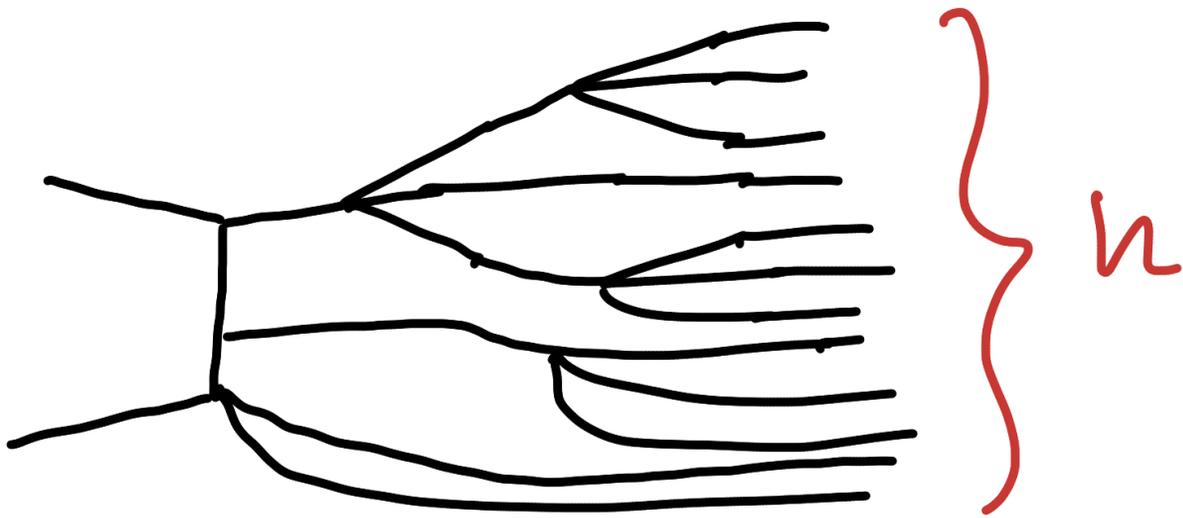
for $n = \frac{1}{\alpha}$

The point of optimal truncation.



momentum-transfer $q = \frac{1}{R}$

Not to be confused with
"fake" saturation of unitarity
for large n !

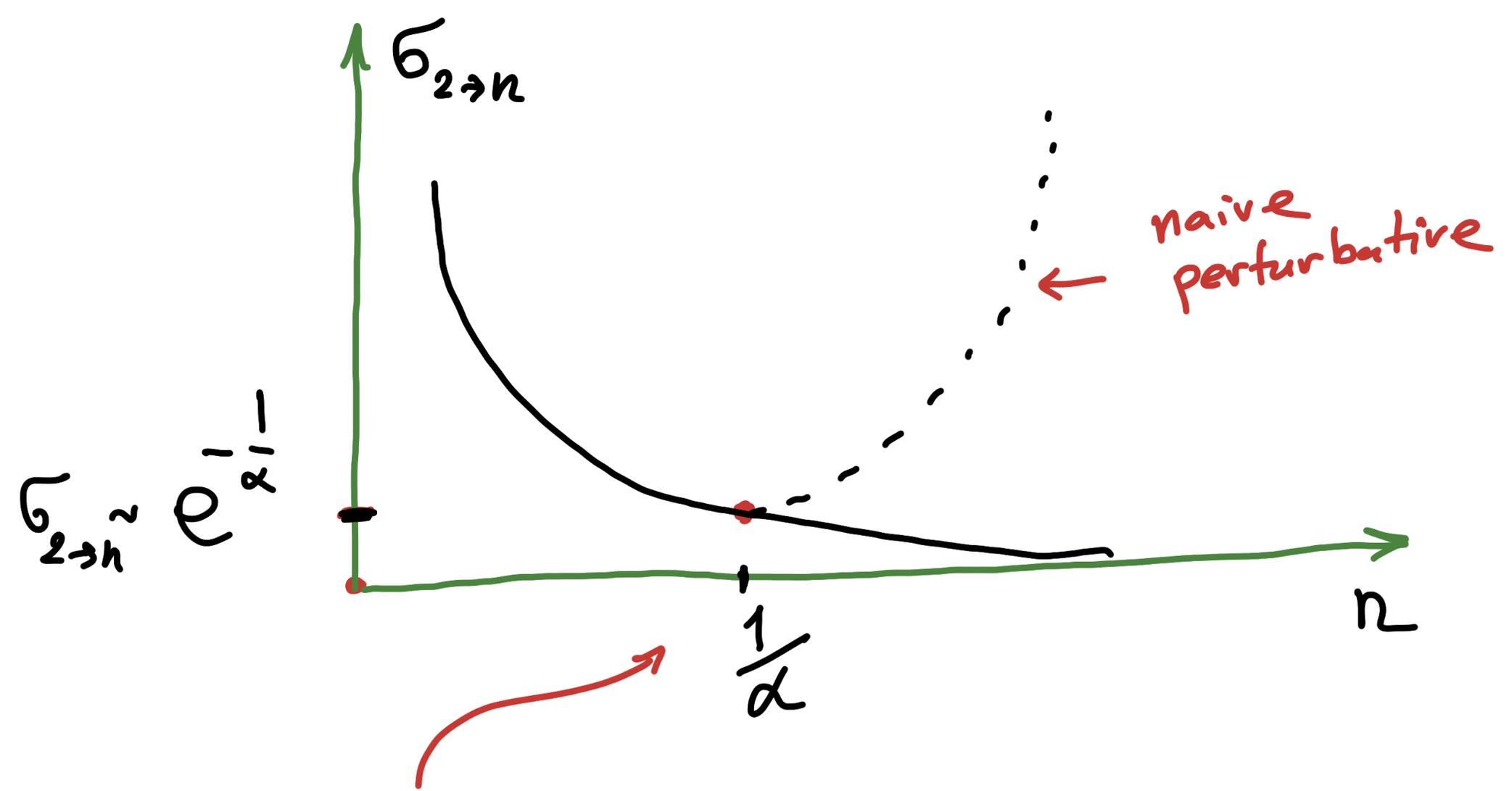


The factorial growth of cross section
due to multiplicity of Feynman
diagrams is well-known

Cornwall '90;
Goldberg '90; ...

$$\sigma_{2-n} \propto n! \alpha^n$$

But this cannot be trusted
for $n > \frac{1}{\alpha}$!



Optimal truncation:

$$\sigma_{2 \rightarrow n = \frac{1}{\alpha}} \sim e^{-\frac{1}{\alpha}}$$

Non-perturbative arguments indicate that for $n > \frac{1}{\alpha}$,

$$\sigma_{2 \rightarrow n} < e^{-n}$$

(too technical, won't be reproduced here).

Entropy enhancement of
the cross section:

$$\tilde{\sigma} = \sum_{\text{mer.-states}} \tilde{\sigma}_{2 \rightarrow n} = \tilde{\sigma}_{2 \rightarrow n} e^S$$

at optimal truncation $n = \frac{1}{\alpha}$,

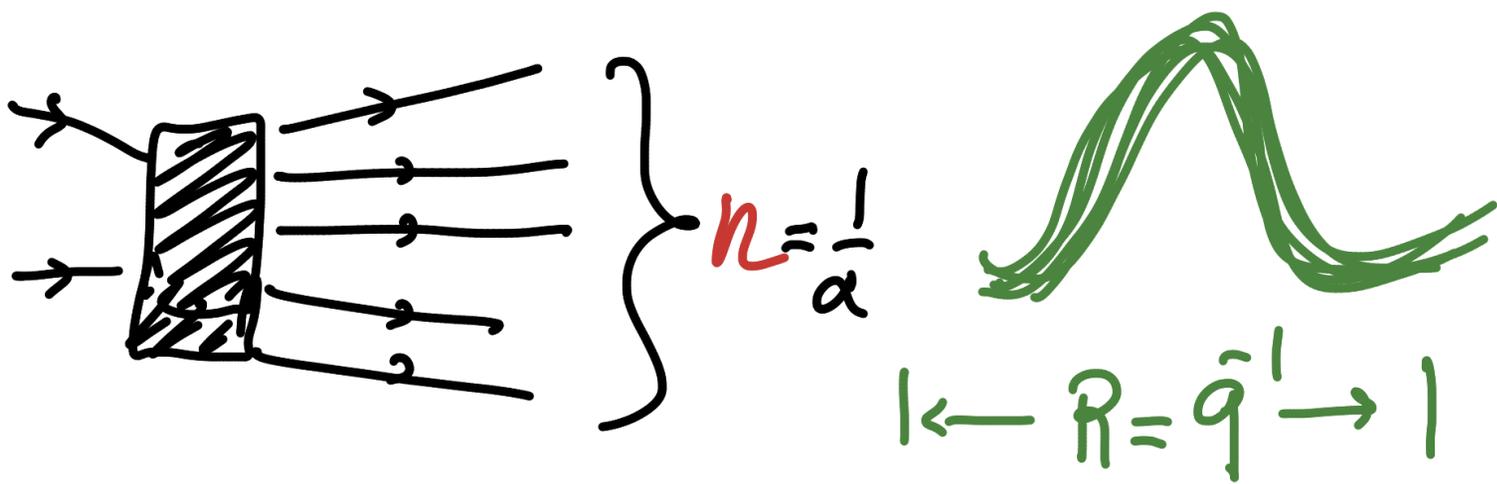
$$\tilde{\sigma}_{2 \rightarrow n} = e^{-\frac{1}{\alpha}}$$

$$\hookrightarrow \tilde{\sigma} = e^{-\frac{1}{\alpha} + S}$$

Entropy bound from unitarity:

$$S_{\text{MAX}} = \frac{1}{\alpha}$$

The final n -particle state
breaks spontaneously translation
symmetry



Lump \nearrow

Goldstone decay constant

$$f = \sqrt{n} q = \frac{1}{\sqrt{a} R}$$

$$S_{\text{MAX}} = \frac{1}{a} = R^2 f^2 = \frac{\text{Area}}{f^2}$$

Area-law!

Entropy of an n -particle state
with internal quantum number

$$j = 1, 2, \dots, N$$

↑ "N-ality"

Number of degenerate micro-states:

$$N_{St} \approx \binom{n+N}{N} = \frac{(n+N)!}{n! N!}$$

Notations:

⊛ Collective coupling:

$$\lambda_c \equiv \alpha n \quad \leftarrow \begin{array}{l} \text{parameter} \\ \text{of state} \end{array}$$

⊛ 't Hooft coupling:

$$\lambda_t \equiv \alpha N \quad \leftarrow \begin{array}{l} \text{parameter} \\ \text{of theory} \end{array}$$

Then,

$$N_{st} \approx C_N \left(\left(1 + \frac{\lambda_t}{\lambda_c}\right)^{\lambda_c} \left(1 + \frac{\lambda_c}{\lambda_t}\right)^{\lambda_t} \right)^{\frac{1}{\alpha}}$$

For $n = \frac{1}{\alpha} \rightarrow \lambda_c = 1$

$$N_{st} \approx \left[(1 + \lambda_t) \left(1 + \frac{1}{\lambda_t}\right)^{\lambda_t} \right]^{\frac{1}{\alpha}}$$

↓ entropy

$$\vec{S} = \frac{1}{\alpha} \ln \left[(1 + \lambda_t) \left(1 + \frac{1}{\lambda_t}\right)^{\lambda_t} \right]$$

Saturates unitarity bound for

$$\lambda_t \approx 0.54$$

Exact for $N, n \rightarrow \infty$ $\lambda_t, \lambda_c = \text{fixed}$

$$\text{Error} \sim \frac{\ln N}{N}$$

Also, the bounds

$$S_{\max} = \frac{1}{\alpha} = (Rf)^2$$

can be applied to Euclidean objects, such as instantons for which the energy (and thus Bekenstein bound) cannot be defined.

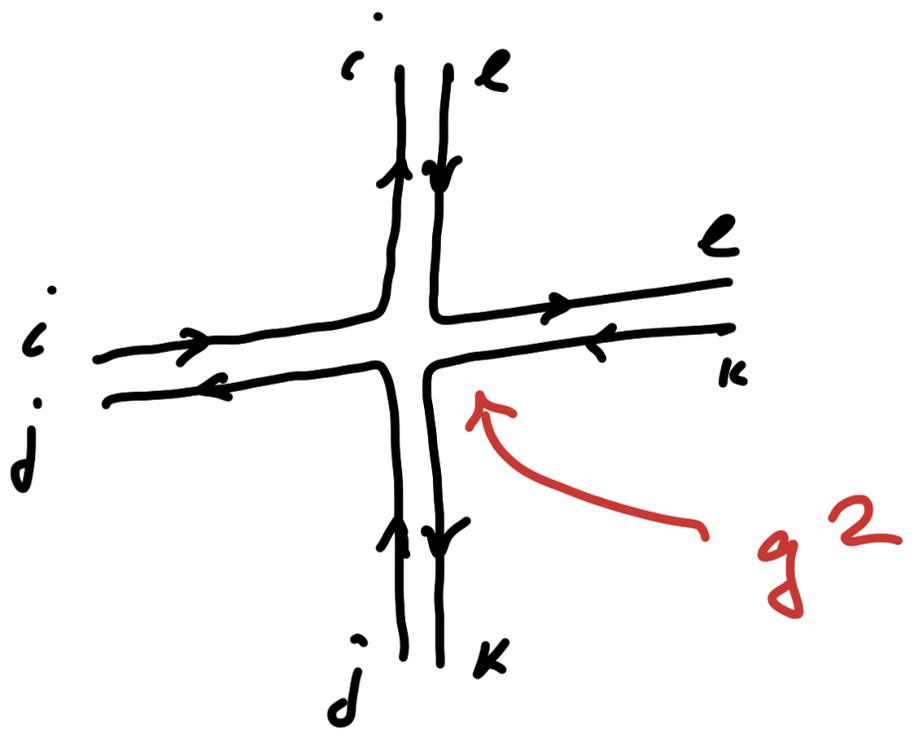
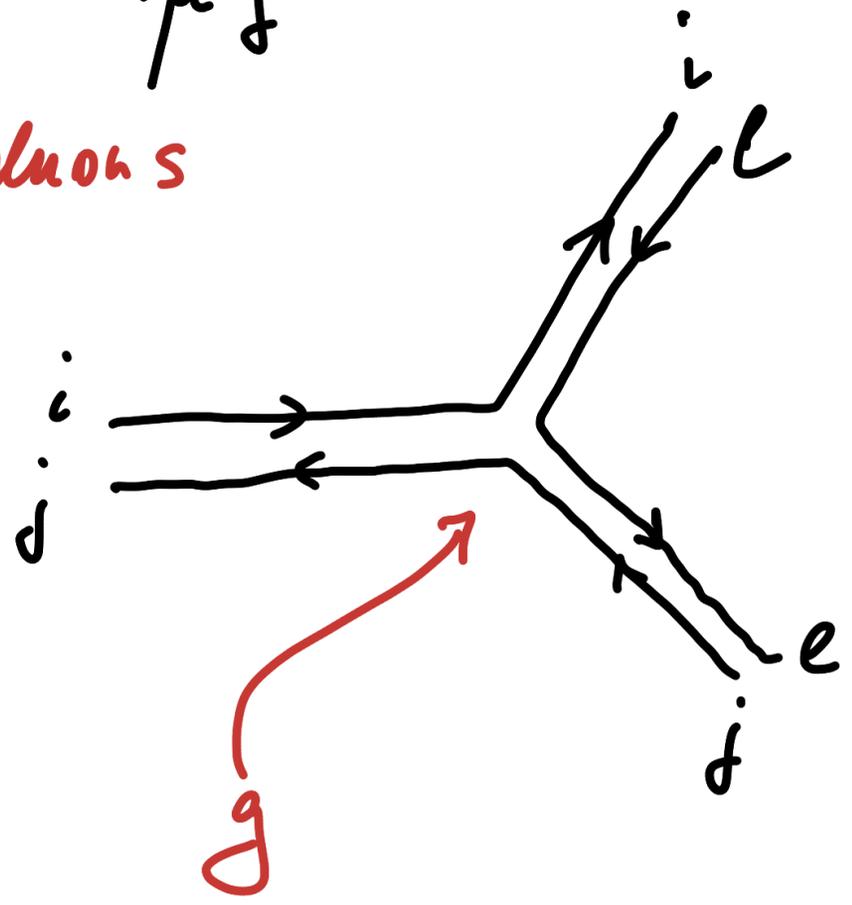
$$S_{\text{inst}} \approx \frac{1}{\alpha} \ln \left[(1 + \lambda_t) \left(1 + \frac{1}{\lambda_t}\right)^{\lambda_t} \right]$$

Saturates bound for $\lambda_t \sim 1$

SU(N) gauge theory. Pure glue.

$A_{\mu j}^i$ color index $i, j = 1, 2, \dots, N$

gluons



QCD coupling $\alpha \equiv \frac{g^2}{4\pi}$

1st 't Hooft coupling

$$\lambda_t \equiv \alpha N$$

't Hooft limit

$$d \rightarrow 0, N \rightarrow \infty, \lambda_t = \text{finite}$$

Running 't Hooft coupling

$$\frac{d\lambda_t}{d\ln(q^2)} = -\frac{11}{12\pi} \lambda_t^2 + \dots$$

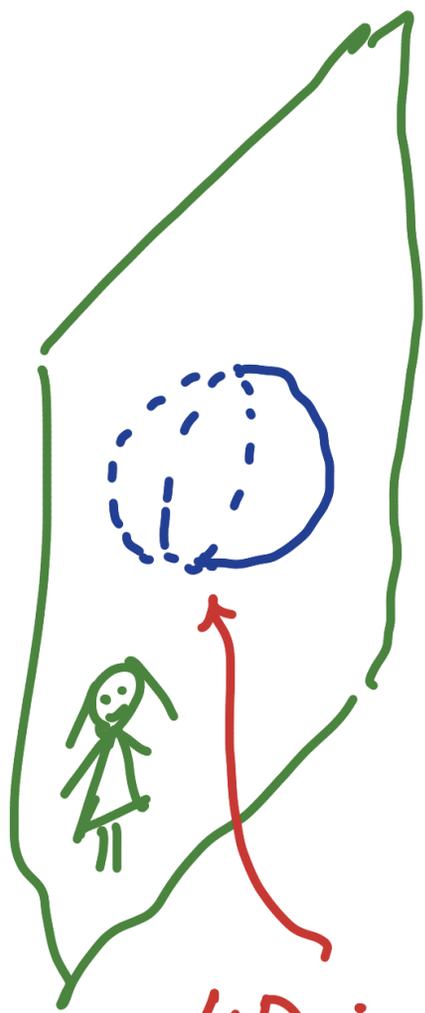
QCD scale

$$\Lambda_{\text{QCD}} = \text{finite}$$

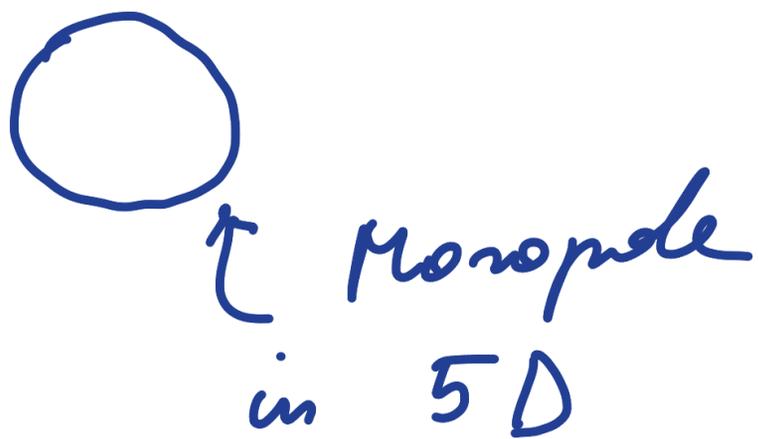
Theory becomes confining:

Asymptotic states are colorless.

Several ways to think about instanton entropy. Lift it in one dimension higher: $SU(N)$ monopole in 5D



$$A_\mu^a = \frac{2}{g_5} \eta^a \frac{x^\nu}{x^2 + R^2}$$



4D instanton = Tunneling-through 5D monopole

$$S_{inst} = S_{mon}$$

5D 't Hooft coupling

$$\lambda_t^5 \equiv N(g_5^2 g) \rightarrow \alpha_5 \equiv g_5^2 g$$

Goldstone decay constant

$$\bar{f}^3 = g_5^2 R^2$$

Area Law entropy!

$$S_{\text{non}} = (Rf)^3 = \frac{1}{g_5}$$

↖ Area in 5D

This allows to give a well defined meaning to instanton

entropy:

$$S_{\text{inst}} = \binom{\frac{1}{g^2} + N}{N} = \frac{1}{g^2} = \underline{\underline{\text{Action}}}$$

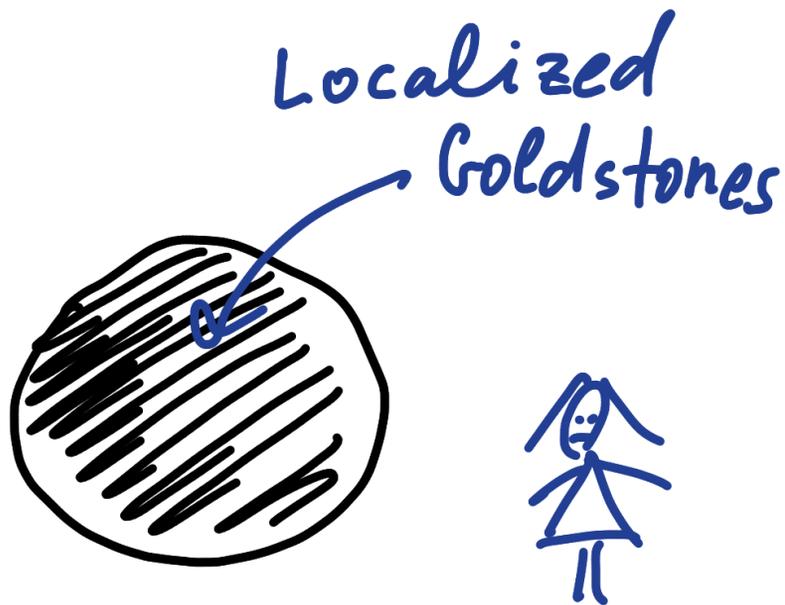
for $\lambda_t \sim 1$

Think of it as Goldstone phenomenon:

Monopole (instanton) breaks $SU(N)$ global symmetry spontaneously.

Order parameter:

$$N_{\text{mon}} \equiv R/g_5^2$$



Goldstone vacuum is degenerate:

$$n_{\text{st}} = \binom{N_{\text{mon}} + 4N}{N_{\text{mon}}} \leftarrow \text{micro states}$$

Entropy: $\mathcal{S} = \ln(n_{\text{st}})$

Saturates bound:

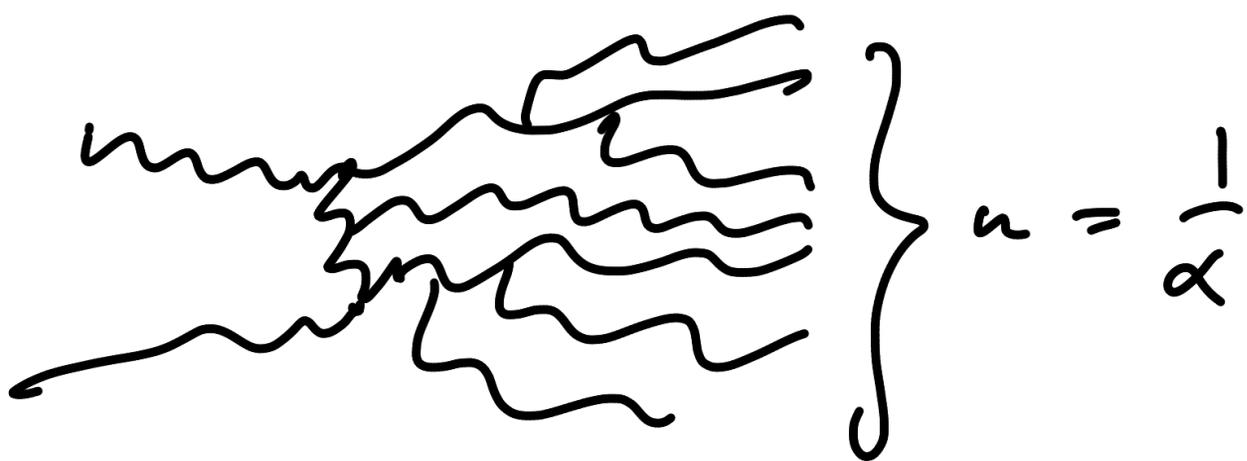
$$\mathcal{S} = \mathcal{S}_{\text{max}} = M_{\text{mon}} R = \frac{R}{g_5^2} \equiv N$$

Per 4D $\lambda_+ \sim 1$!

The goal here is not a
computation of instanton
amplitudes (see many excellent
talks at this workshop)

but to correlate saturation
of entropy bound by instantons
to saturation of entropy
and unitarity by multi-gluon
amplitudes.

$$S_{\text{Int}} \rightarrow \frac{1}{\alpha}$$

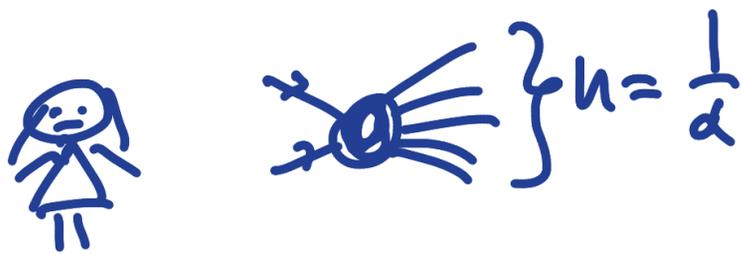


Understanding confinement as
consequence of entropy bound

$$S_{\max} = \frac{1}{\alpha} = (Rf)^2$$

and unitarity by colored states.

Assume that theory never
confines. Equivalently, saturation scale
 $R = \bar{q}^{-1}$ is arbitrarily shorter than the
scale of confinement L_{conf}



$$\leftarrow R = \bar{q}^{-1} \rightarrow$$

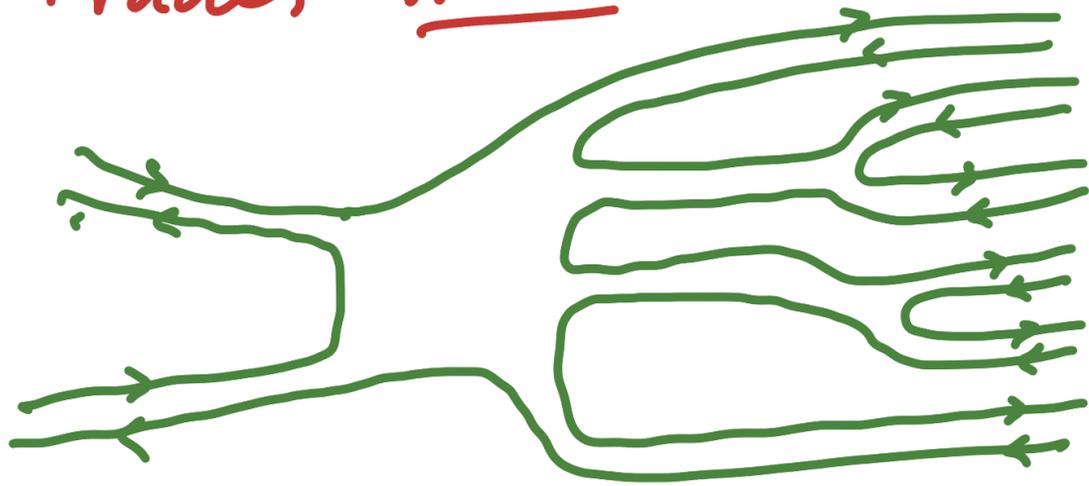


Alice would use colored states as asymptotic S -matrix states.

But, such states with occupation number $n = \frac{1}{\alpha}$ violate the entropy bound for $\lambda_t \gg 1$:

$$S \approx \frac{1}{\alpha} \ln(e \lambda_t)$$

Correspondingly $2 \rightarrow n = \frac{1}{\alpha}$ gluon amplitudes would violate unitarity!



cross section

$$\sigma \approx (\lambda_t)^{\frac{1}{\alpha}} \rightarrow \infty$$

Of course, this cannot happen because theory is asymptotically-free and thus consistent in UV.

Thus, theory must prevent itself from entering the strong Λ_t regime with colored states.

The options are:

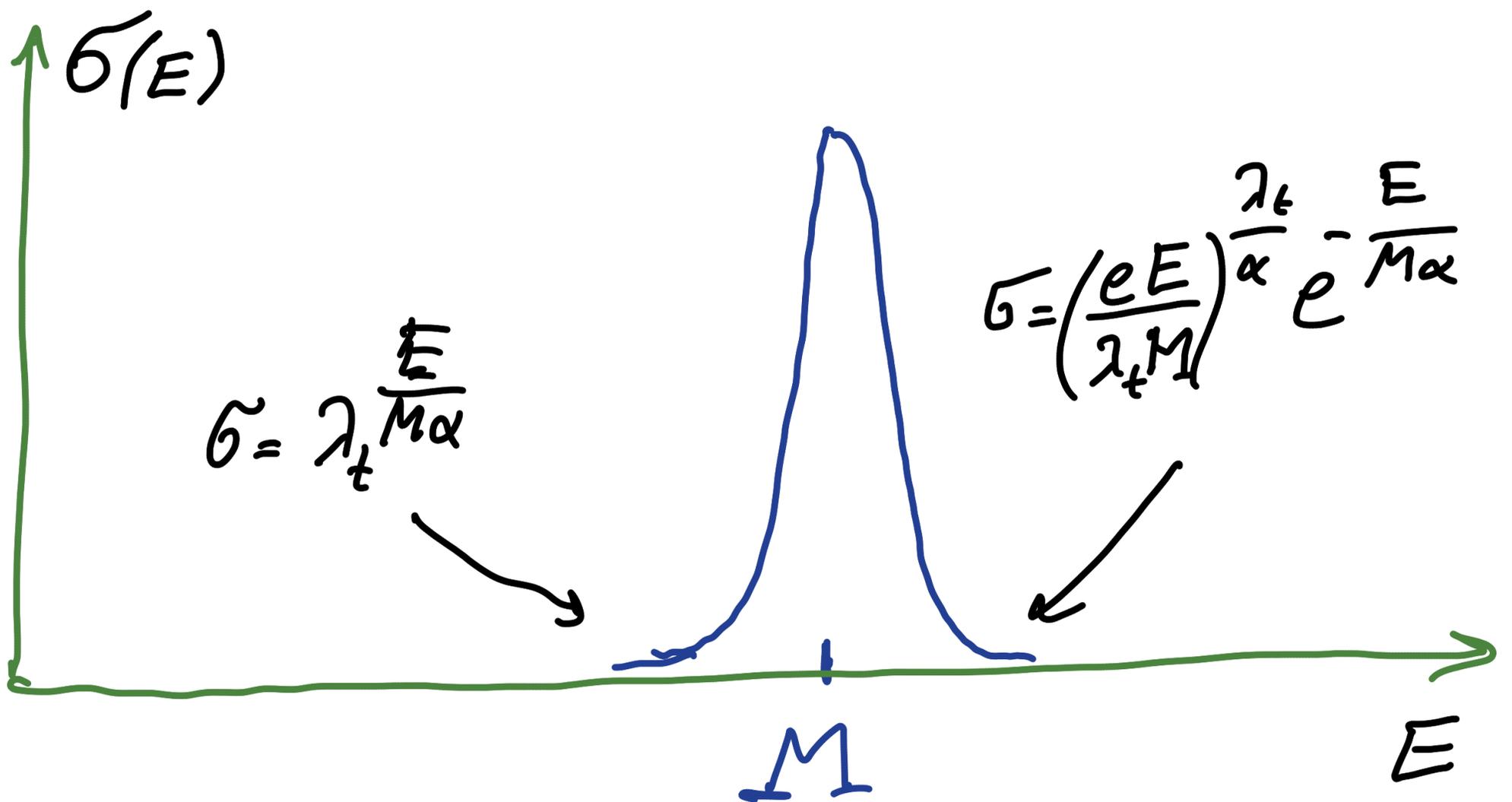
- ① Hit IR fixed point;
- ② Generate a mass gap via Higgsing;
- ③ Confinement.

In pure glue the only viable option is ③:

Theory must confine!

This said, no free lunch.

For fixed q and saturation mass $M = \frac{q}{\alpha}$,
the dependence of σ on energy $E = nq$



Saturation window: $\frac{\Delta E}{M} \sim \alpha$

We pay price in preparing
initial state!

Scanning with $\lambda_+(q)$ while
fixing $\lambda_c = n\alpha = 1$

$$\frac{d \ln(\sigma)}{dq^2} \simeq -N \ln \left((1 + \lambda_+) \bar{e} \right)^{\frac{1}{\lambda_+}} \frac{d \ln(\lambda_+)}{dq^2}$$

around the saturation value
 $\lambda_+(q) \simeq 0.54$ ($\simeq 1$)

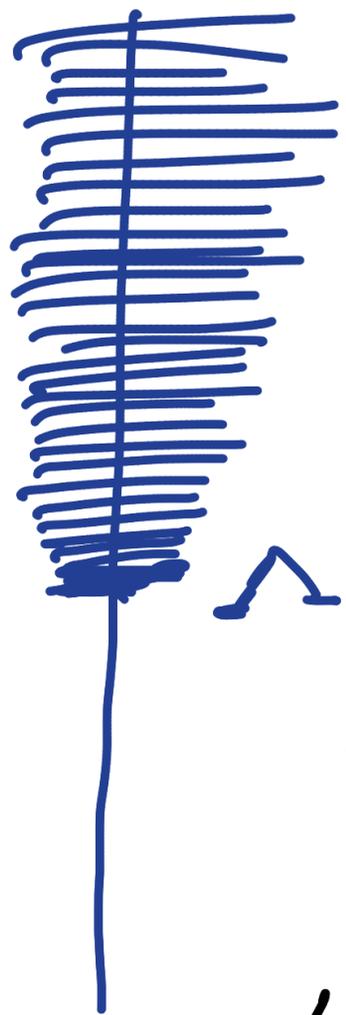
$$\frac{d \ln(\sigma)}{d \ln(\lambda_+)} \simeq N$$

in asymptotically free theory
(away from a fixed point)

$$\frac{d \ln(\lambda_+)}{d \ln(q^2)} \simeq \lambda_+$$

Then, $\sigma \rightarrow 0$ exponentially in UV!

Lesson: In order to UV complete theory by classicalization, above certain scale Λ , we need a continuum of saturons in the spectrum!



← Saturon spectrum $\alpha(q)h=1$

← This is what happens in gravity due to black holes. Black hole fixed point:

$$\alpha(q)h=1, \quad q=1/R$$

G.D., Gomez '11.

Other fixed points?

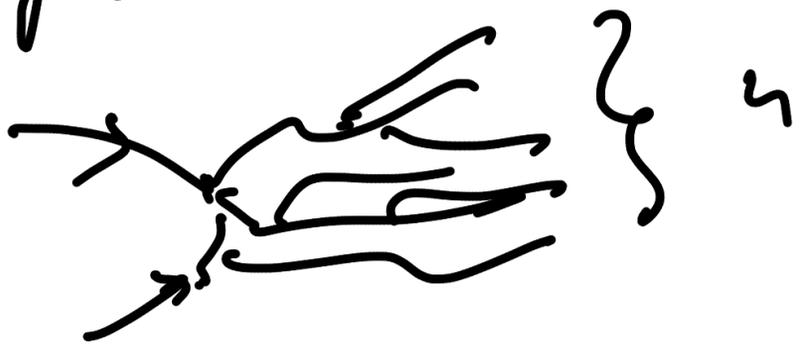
Summary:

⊛ Regardless of renormalizability or gravity, all objects that saturate entropy bound behave as black holes (true for, solitons, instantons, baryons, ...)

⊛ $S = S_{\text{max}} = \frac{1}{2} = (Rf)^2 = ER$

Correlated with saturation of unitarity

by $2 \rightarrow n = \frac{1}{\alpha}$ amplitudes



⊛ In QCD, instantons saturate entropy bound for $\lambda_t \sim 1$.

(*) Confinement can be viewed as self-defense mechanism against violation of entropy bounds by colored states and violation of unitarity by $2 \rightarrow n = \frac{1}{\alpha}$ gluon amplitudes



(*) Isolated saturons can exist in renormalizable theories.

(*) UV-completion by classicalization requires a fixed point

$$d(q)k = 1$$

⊛ In classicalization there is no strong coupling in UV.

Instead, there are many soft quanta

$$n = \frac{1}{\alpha} \gg 1$$

⊛ But, $d \sim 1$ for $nr \sim 1 \leftarrow$ strong coupling.

⊛ Saturons in BSM.

⊛ Stabilizing Higgses must via classicalization.

⊛ Saturon tower (similar to black hole tower)

~~_____~~ } n

$$n = \frac{1}{\alpha(E/n)}$$