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Comments for the QCD instanton session

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Abstract: Remarks on potential numerical systematics in the evaluation of instanton-induced multiparton production

1 Introduction

This note addresses the papers in ref. [1, 2], presenting calcuations of QCD-instanton induced multi-parton final states at hadron colliders. The note is a contribtion to a discussion session of the Workshop on "Topological Effects in the Standard Model: Instantons, Sphalerons and Beyond at LHC", https://indico.cern.ch/event/965112/. I will not address here the underlying formalism used in refs. [1, 2]. I may have a few minor issues to raise, likely due to my own ignorance, but they tend to be technical and not suitable for discussion at the Workshop. They are also rather independent of the remarks presented below, which focus on possible systematics that should still be attached to the results in [1, 2], even accepting as correct the overall theoretical formulation. This is perhaps more interesting for the Workshop discussions, since it puts in perspective the interpretation of the LHC searches for final states induced by QCD instantons.

2 Results and remarks

The amplitude for the instanton-induced production of n_g gluons and n_f quark-antiquark pairs is given in ref. [1] as:

$$\mathcal{A}(2 \to n_g + 2n_f) \sim \int_0^\infty d\rho^2 \left(\rho^2\right)^{n_g+n_f-1} e^{-\frac{2\pi}{\alpha_s(1/\rho)} - \frac{\alpha_s(1/\rho)}{16\pi} E^2\rho^2 \log(E^2\rho^2)} \tag{1}$$

where ρ is the instanton radius, and $E = \sqrt{\hat{s}}$ is the partonic CM energy. As in ref. [1], we neglect overall constants, wave function normalization factors, etc. The second term in the exponent, proportional to $\rho^2 \log(E^2\rho^2)$, reflects the Mueller's form-factor, discussed in refs. [3–5].

Remark 1. From dimensional analysis, the amplitude given above scales with E as follows:

$$\mathcal{A} \propto E^{-2(n_g+n_f+b_0/2)} \propto E^{-2(n_g+n_f)} \left(\frac{\Lambda}{E}\right)^{b_0} . \tag{2}$$

The second expression highlights the fact that, while $(n_f + n_g)$ powers of $1/E^2$ are matched by the energy dependence of the wave function normalization of the external states and by the final phase-space integration, leading to dim(σ)=-2, the b_0 powers of $1/E$ are matched instead by the QCD scale Λ, which therefore must appear in the amplitude expression. This is the consequence of the power suppressed nature of this non-perturbative amplitude, embodied by the contribution $\exp(-2\pi/\alpha_s(1/\rho))$. This means that the amplitude has an intrinsic Λ^{b_0} dependence. More on this later.

As indicated in ref. [1], the amplitude can be evaluated in the saddle-point approximation, where, leaving out again constant numerical factors:

$$\mathcal{A} \sim \int_0^\infty d\rho^2 e^{f(\rho^2)} = e^{f(\bar{\rho})} \sqrt{\frac{1}{-f''(\bar{\rho})}} \tag{3}$$

where

$$f(\rho^2) = (n_g + n_f - 1 + b_0/2) \log \rho^2 - \frac{\alpha_s(1/\rho)}{16\pi} E^2\rho^2 \log(E^2\rho^2) , \tag{4}$$

The saddle point $\bar{\rho}$ is defined through:

$$\partial f(\rho^2)/\partial\rho^2 = \frac{A}{\rho^2} - \frac{E^2}{16\pi}\alpha_s(1/\rho) \log(E^2\rho^2) + \mathcal{O}(\alpha_s^2) = 0 \quad \text{at} \quad \rho = \bar{\rho} \tag{5}$$

where $A = n_g + n_f - 1 + b_0/2$. We used the a-posteriori knowledge that $E^2\rho^2 >> 1$, to neglect in the derivative a term of order 1 w.r.t. $\log(E^2\rho^2)$.

Remark 2. Notice that we neglected here terms formally of higher powers of α_s. Some arise by including the term proportional to $\partial\alpha_s(1/\rho)/\partial\rho^2 \sim b_0\alpha_s^2(1/\rho)$ in the derivative of $f(\rho)$, others would arise in taking the NLO beta-fuction rather than the LO one, others, unknown, arise from NLO corrections to the function $f(\rho)$ itself. Since we cannot control the exact form of these higher order terms, we stick to the strict LO expression, but must keep in mind their existence for the assessment of the systematics of the final result.

An approximate solution of the saddle-point condition leads to the relation:

$$\frac{1}{\bar{\rho}^2} \;=\; \eta\alpha_s(\eta E^2)\;\log(1/\eta)\;E^2 + \mathcal{O}(\alpha_s^2) \tag{6}$$

where $\eta = 1/(16\pi A)$. This approximate solution to eq. 5 agrees to within 10% with the exact solution, leading to a value of the inverse instanton radius of $\epsilon = 1/\bar{\rho} = \gamma E$, with γ in the range of $1/20 - 1/30$ for $E \sim 100 - 3000$ GeV, consistent with the findings of Fig. 4 of ref. [2].

Evaluating $f''(\rho)$ at the saddle point gives

$$f''(\bar{\rho}) = -\frac{A}{\bar{\rho}^4}(1 + \mathcal{O}(\alpha_s)) \tag{7}$$

and, putting things back into eq. 3 and neglecting overall constant factors, we obtain:

$$\mathcal{A} \sim \left(\frac{\Lambda}{E}\right)^{b_0}\;\left(\frac{1}{E}\right)^{n_g+n_f}\;\left[\frac{1}{\alpha_s(\eta E^2)[1+O(\alpha_s)]}\right]^{n_g+n_f+b_0/2}\;;. \tag{8}$$

Remark 3. I see here two separate sources of potential systematics.

1. The first one is the Λ^{b_0} term upfront. On one side this inherits the intrinsic 1% uncertainty on $\alpha_s(M_Z)$. But $\Delta\Lambda/\Lambda \sim \Delta\alpha_s/\alpha_s \times \log(M_Z/\Lambda) \sim 6\%$, leading to $\Delta\mathcal{A}/\mathcal{A} \sim \pm 60\%$, which is negligible overall. On the other hand, the choice of the perturbative order at which Λ is estimated is not well defined, and the difference between LO and NLO Λ is large. For example, to obtain $\alpha_s(M_Z) = 0.12$ from the 1-loop evolution we get $\Lambda \sim 100$ MeV, while at 2-loop we get $\Lambda \sim 260$ MeV. So, in the cross section $\sigma \propto \mathcal{A}^2$ there is a potential systematics factor in the range of $2^{\pm b_0} \sim [4 \times 10^{-3} - 250]$.

2. There is an independent uncertainty arising from the $O(\alpha_s)$ corrections indicated in eq. 8. These are independent of the LO vs NLO issue raised in the previous point: in the previous point we dealt with the order at which the leading power-suppressed instanton action, $\exp(-2\pi/\alpha_s)$ is calculated. Here we are dealing with higher-order corrections to Mueller's form factor. It is reasonable to expect these $O(\alpha_s)$ uncertainties to be limited to a $\pm 20\%$, but when raised to the power of $(n_g + n_f + b_0/2)$ this can become an overall factor of $(1.2/0.8)^{(n_g+n_f+b_0/2)} \geq 50$ for the amplitude, and greatly more for the cross sections.

More in general, the relation between renormalization scale μ_r and the instanton radius, $\mu_r\rho = 1$, which is chosen in ref. [1] to arrive at the initial expression of eq. 1, is subject to the usual factor of 2 uncertanity. The legitimate choice of $\mu_r\rho = [0.5 - 2]$ would lead to a factor of $[0.5 - 2]$ rescaling of Λ in the argument of α_s, leading again to a systematics similar to what discussed at point 1 above.

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3 Conclusions

In conclusion, it appears that there could be large sources of systematics associated to the predictions for instanton-induced QCD processes at the LHC. If the analysis reported in this note is correct, it is fair to admit that these uncertainties cover several orders of magnitude. This does not remove interest in the search for such final states, but a possible lack of evidence does not lead to the immediate conclusion that instantons "do not exist", but simply that their actual production rate is unfortunately on the lower end of the systematics, wrt to the central baseline rates discussed in refs. [1, 2].

References

[1] V. V. Khoze, D. L. Milne and M. Spannowsky, *Searching for QCD Instantons at Hadron Colliders*, arXiv:2010.02287 [hep-ph].

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