Topological Effects in the Standard Model: Instantons, Sphalerons and Beyond at LHC,
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Contribution to the “QCD instantons” discussion session:

Remarks on the numerical impact of potential theoretical systematics in the prediction of QCD instanton cross sections

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- no comments on the overall theoretical framework or detailed aspects of the calculations
- take for granted the approach, and just explore the impact on LHC predictions of possible sources of implicit assumptions that are nevertheless subject to systematics
- …. *for more details, see the note attached to this item on Indico*

[1] V. V. Khoze, D. L. Milne and M. Spannowsky, arXiv:2010.02287
[2] V. V. Khoze, F. Krauss and M. Schott, arXiv:1911.09726
[3] A. H. Mueller, Nucl. Phys. B 348 (1991) 310.
[4] A. H. Mueller, Nucl. Phys. B 353 (1991) 44.
[5] A. H. Mueller, Nucl. Phys. B 364 (1991) 109.

From Ref [1], neglecting overall factors like constants or wave function normalizations:

$$\mathscr{A}(2 \to n_g + 2n_f) \sim \int_0^\infty d\rho^2 \left(\rho^2\right)^{n_g+n_f-1} \boxed{e^{-\frac{2\pi}{\alpha_s(1/\rho)}}} {}^{-\frac{\alpha_s(1/\rho)}{16\pi}E^2\rho^2\log(E^2\rho^2)}$$

To reach this expression, the relation ρ μ_R = 1 has been imposed between the renormalization scale μ_R and the instanton radius ρ

Remark on energy dependence:

b_0=11–2/3 n_f

$$\mathscr{A} \propto E^{-2(n_g+n_f)} \boxed{\left(\frac{\Lambda}{E}\right)^{b_0}}$$

These b_0 powers of Λ_{QCD} are a reflection of the non-perturbative nature of the instanton action, and are an intrinsic feature of the transition amplitude

These powers of E will be matched in the calculation of the cross section by powers of E from the phase-space and from the normalization of the external states

Saddle-point evaluation of the amplitude:

$$\boxed{\mathscr{A} \sim \int_0^\infty d\rho^2 \, e^{f(\rho^2)} \; = \; e^{f(\bar\rho)} \sqrt{\frac{2\pi}{-f''(\bar\rho)}}}$$

with $\bar\rho$ defined by $\frac{\partial f}{\partial \rho^2}|_{\rho=\bar\rho} = 0$

$$f(\rho^2) = (n_g + n_f - 1 + b_0/2) \, \log \rho^2 \; - \; \frac{\alpha_s(1/\rho)}{16\pi} E^2\rho^2 \log(E^2\rho^2)$$

$\frac{\partial f}{\partial \rho^2}|_{\rho=\bar\rho} = 0$ **implies**

$$\boxed{\frac{1}{\bar\rho^2} = \frac{E^2}{16\pi\, A} \, \alpha_s(1/\bar\rho) \; \log(E^2\bar\rho^2) + \mathscr{O}(\alpha_s^2)}$$

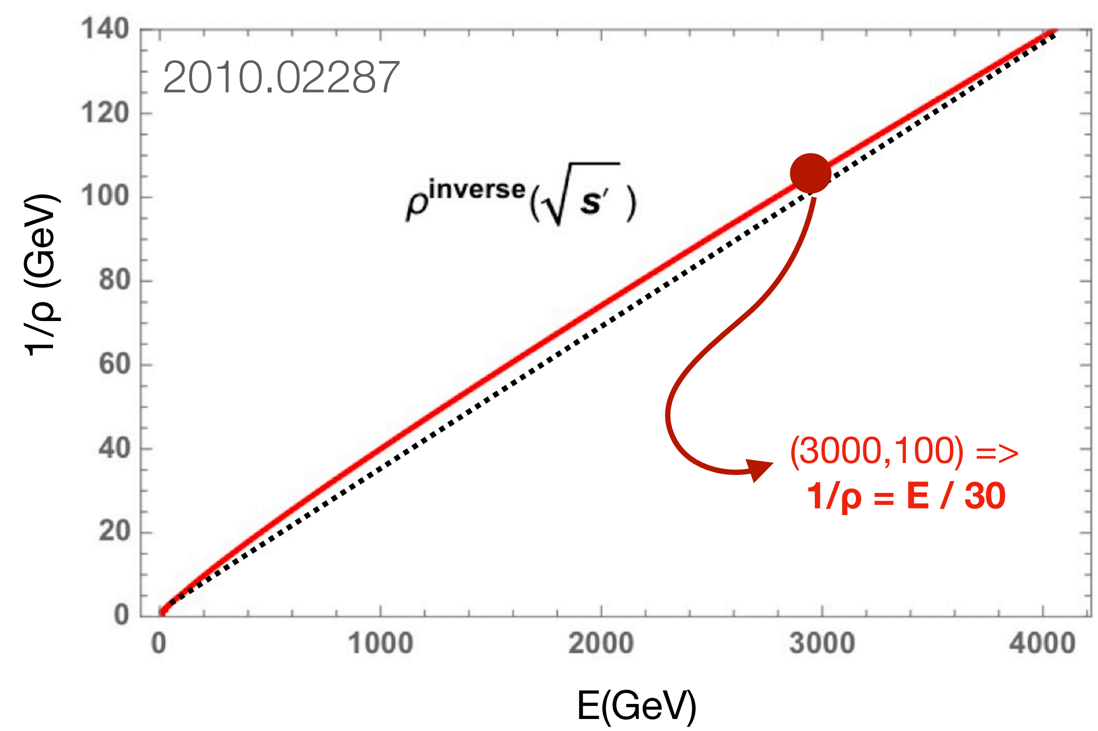
$$A = n_g + n_f - 1 + \frac{b_0}{2}$$

Remark on O(αS):

Corrections of higher order in α_S arise by taking the derivative of f(ρ) (eg $\partial\alpha/\partial\rho^2 \sim \beta(\alpha)\alpha^2$). It makes no sense to keep these, since there is no full control of all other sources of O(α_S) corrections in f(ρ)

Approximate solution of the saddle point condition (within 10% of the exact numerical solution)

$$\boxed{\frac{1}{\bar{\rho}^2} = \left[\eta\ \alpha_s(\eta E^2)\ \log(1/\eta)\right]\ E^2 + \mathcal{O}(\alpha_s^2)}$$



where:

$$\eta = \frac{1}{16\pi A} \sim \left(\frac{1}{30}\right)^2 \quad \text{for} \quad (n_g, n_f) = (10{,}5)$$

Remark

$$\alpha_s(1/\bar{\rho}^2)\ \bar{\rho}^2 E^2\ \sim\ \left[\eta\ \log(1/\eta)\right]^{-1} =\ \text{const}$$

The relative suppression from the Mueller's factor is only very slowly dependent on E… the dramatic drop of cross section at high-E is mostly due to the $1/E^n$ terms

Final result, up to overall constants:

$$\mathscr{A} \sim \left(\frac{\Lambda}{E}\right)^{b_0} \left(\frac{1}{E}\right)^{n_g+n_f} \left[\frac{1}{\alpha_s(\eta E^2)[1+O(\alpha_s)]}\right]^{n_g+n_f+b_0/2}$$

Remarks

- Which numerical value for Λ?
  - $\alpha_S^{1\text{-loop}}(M_Z, \Lambda) = 0.120 \Rightarrow \Lambda = 100$ MeV
  - $\alpha_S^{2\text{-loop}}(M_Z, \Lambda) = 0.120 \Rightarrow \Lambda = 260$ MeV
  - $\Rightarrow \sigma \sim |\mathscr{A}|^2$ can vary in a range $2^{\pm b0}\ \sigma_0 \sim [0.04 — 250]\ \sigma_0$
- $O(\alpha_S)$ effects:
  - $n_g+n_f+b_0/2 > 10$ for $n_g>1$
  - If $O(\alpha_S)\sim\pm 20\% \Rightarrow \Delta\mathscr{A} / \mathscr{A} \sim (1.2/0.8)^{ng+nf+b0/2} \sim 50$
- More in general:
  - if we had set $\rho\ \mu_R = \lambda$ instead of 1, with $\lambda \in [0.5,2]$, variations of size similar to those listed above would have arisen
- Less clear what are the potential systematics in the modeling of the detailed final-state structure: impact on event simulation, kinematical distributions, bg suppression, differentiation of possible signals from BSM sources of “soft-bombs-like” final states, … Maybe discussed on **Friday**?

Conclusions

- Potential for large sources of systematics, leading to uncertainties covering several orders of magnitude.
- This does not remove interest in the search for such final states, but a possible lack of evidence does not lead to the immediate conclusion that instantons ``do not exist'', but simply that their actual production rate is unfortunately on the lower end of the systematics, wrt to the central baseline rates