Main sources of theoretical uncertainties (for discussion)

- (1) QCD Instanton rates are interesting in the regime where they become large — lower end of partonic energies 10-80 GeV. The weak coupling approximation used in the semiclassical calculation can be problematic. How to address: vary s' minimal partonic energy cutoff and note the value of alpha_s.
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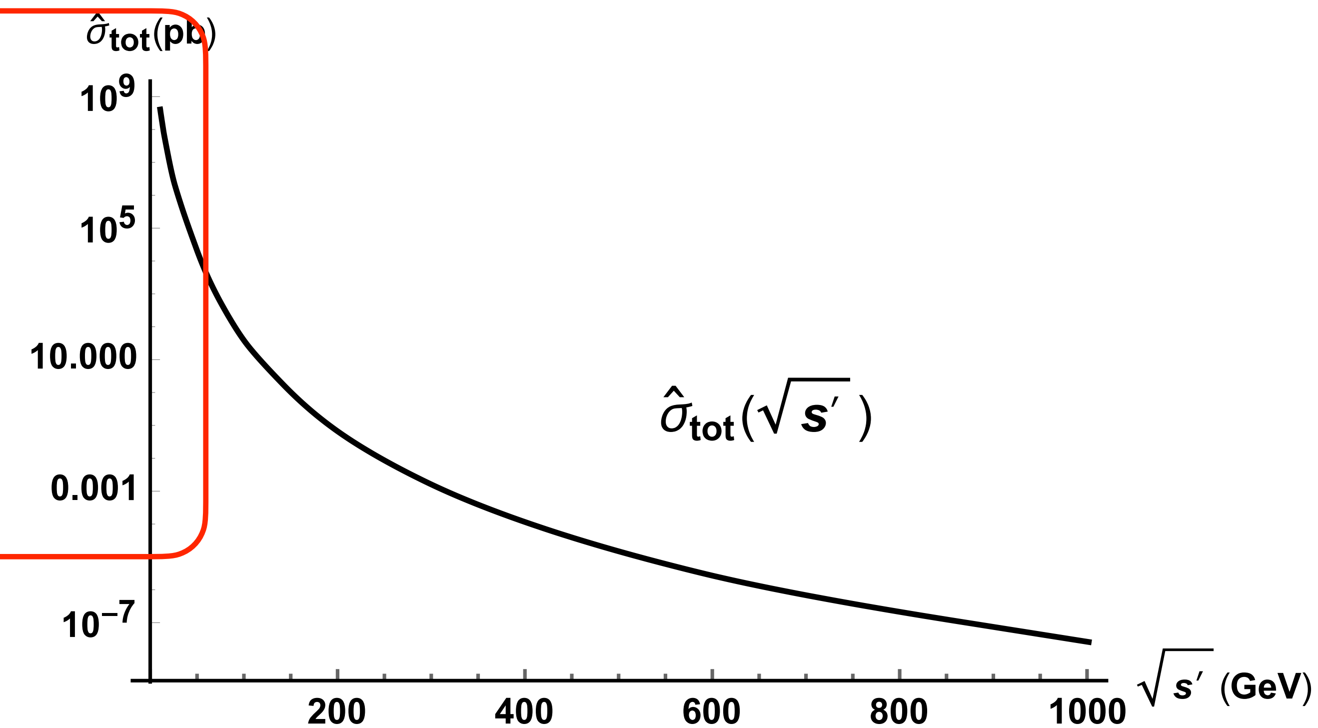
(1)

partonic cross-sections

$\sqrt{s'}$ [GeV]	$1/\rho$ [GeV]	$\alpha_S(1/\rho)$	$\langle n_g \rangle$	$\hat{\sigma}$ [pb]
10.7	0.99	0.416	4.59	$4.922 \cdot 10^9$
11.4	1.04	0.405	4.68	$3.652 \cdot 10^9$
13.4	1.16	0.382	4.90	$1.671 \cdot 10^9$
15.7	1.31	0.360	5.13	$728.9 \cdot 10^6$
22.9	1.76	0.315	5.44	$85.94 \cdot 10^6$
29.7	2.12	0.293	6.02	$17.25 \cdot 10^6$
40.8	2.72	0.267	6.47	$2.121 \cdot 10^6$
56.1	3.50	0.245	6.92	$229.0 \cdot 10^3$
61.8	3.64	0.223	7.28	$72.97 \cdot 10^3$
89.6	4.98	0.206	7.67	$2.733 \cdot 10^3$
118.0	6.21	0.195	8.25	235.4
174.4	8.72	0.180	8.60	6.720
246.9	11.76	0.169	9.04	0.284
349.9	15.90	0.159	9.49	0.012
496.3	21.58	0.150	9.93	$5.112 \cdot 10^{-4}$
704.8	29.37	0.142	10.37	$21.65 \cdot 10^{-6}$
1001.8	40.07	0.135	10.81	$0.9017 \cdot 10^{-6}$
1425.6	54.83	0.128	11.26	$36.45 \cdot 10^{-9}$
2030.6	75.21	0.122	11.70	$1.419 \cdot 10^{-9}$
2895.5	103.4	0.117	12.14	$52.07 \cdot 10^{-12}$



1st Approach: VVK, Krauss, Schott

(1)

hadronic total cross-section

$$\sigma_{pp\to I}\left(\hat{s}>\hat{s}_{\rm min}\right)=\int_{\hat{s}_{min}}^{s_{pp}} dx_1 dx_2 \quad f\left(x_1,Q^2\right) f\left(x_2,Q^2\right) \hat{\sigma}\left(\hat{s}=x_1 x_2 s_{pp}\right)$$

practical approach: vary minimal E

$E_{\rm min}$ [GeV]	50	100	150	200	300	400	500
$\sigma_{p\bar{p}\to I}$ $\sqrt{s_{p\bar{p}}}$=1.96 TeV	2.62 μb	2.61 nb	29.6 pb	1.59 pb	6.94 fb	105 ab	3.06 ab
$\sigma_{pp\to I}$ $\sqrt{s_{pp}}$=14 TeV	58.19 μb	129.70 nb	2.769 nb	270.61 pb	3.04 pb	114.04 fb	8.293 fb
$\sigma_{pp\to I}$ $\sqrt{s_{pp}}$=30 TeV	211.0 μb	400.9 nb	9.51 nb	1.02 nb	13.3 pb	559.3 fb	46.3 fb
$\sigma_{pp\to I}$ $\sqrt{s_{pp}}$=100 TeV	771.0 μb	2.12 μb	48.3 nb	5.65 nb	88.3 pb	4.42 pb	395.0 fb

2nd Approach: VVK, Milne, Spannowsky

(2)

$$\hat{\sigma}_{\text{tot}}^{\text{inst}} \simeq \frac{1}{s'}\,\text{Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}_{\text{ferm}}$$

$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\;\exp\left(R_0\sqrt{s'}-\frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z)-\boxed{\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)}\right)$$

Effect of higher order corrections to Mueller's term in the exponent ??

Only initial-initial quantum corrections are included which is the correct approach for optical theorem.

Final-final and initial-final corrections already accounted for in the instanton-anti-instanton interactions.
[Was explicitly verified for final-final corrections by Mueller.]

Initial-final effects … related to next point in (3)

Even higher order in alpha_s corrections?
Non-exponentiated additive corrections?

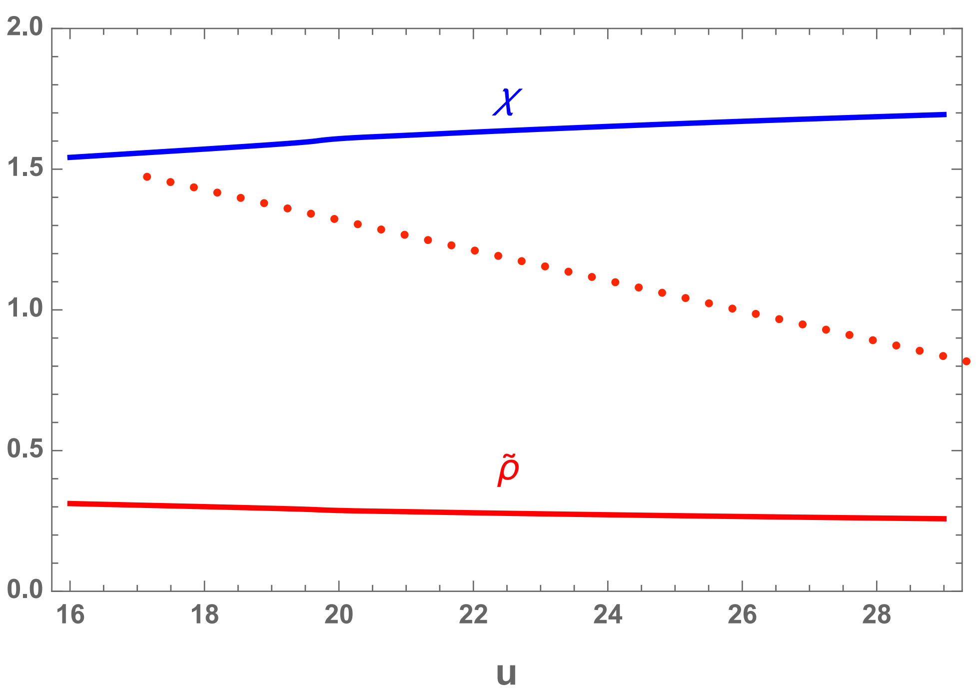
VVK, Krauss, Schott
VVK, Milne, Spannowsky

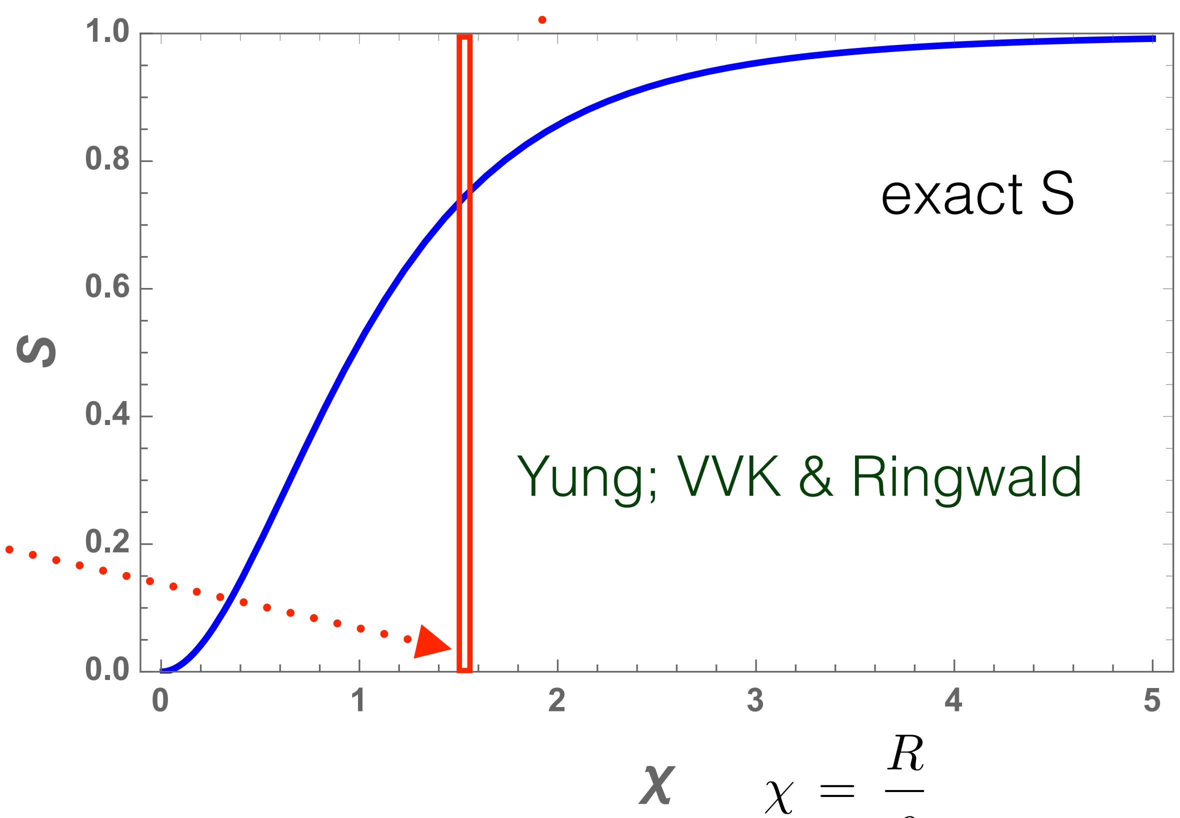
(3)

$$\hat{\sigma}^{\rm inst}_{\rm tot} \simeq \frac{1}{s'}\,{\rm Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar\rho}{\bar\rho^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}(\rho^2\sqrt{s'})^2(\bar\rho^2\sqrt{s'})^2\,\mathcal{K}$$
$$(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}\,\exp\left(R_0\sqrt{s'}-\boxed{\frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z)}-\frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\,s'\,\log\left(\frac{s'}{\mu_r^2}\right)\right)$$

Effect of corrections to our
instantoin-anti-instanton
interactions model ??

Should not be large since chi ~1.5





$$\chi = \frac{R}{\rho}$$

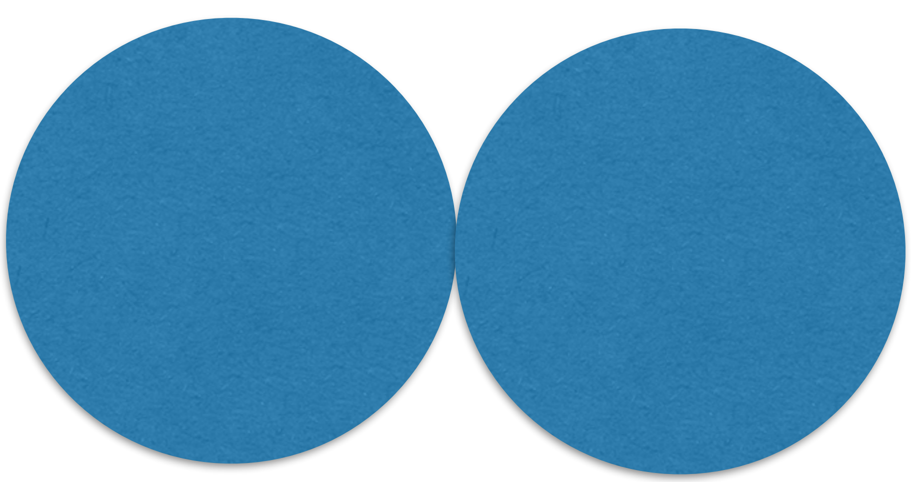
(4) Corrections due to non-factorisation of instanton-anti-instanton determinants at chi ~1.5 ??

$$\hat{\sigma}_{\rm tot}^{\rm inst} \simeq \frac{1}{s'}\, {\rm Im}\, \frac{\kappa^2\pi^4}{36\cdot 4} \int \frac{d\rho}{\rho^5} \int \frac{d\bar\rho}{\bar\rho^5} \int d^4R \int d\Omega \boxed{\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14} (\rho^2\sqrt{s'})^2 (\bar\rho^2\sqrt{s'})^2} \mathcal{K}$$
$$\boxed{(\rho\mu_r)^{b_0}(\bar\rho\mu_r)^{b_0}} \exp\left(R_0\sqrt{s'} - \frac{4\pi}{\alpha_s(\mu_r)}\hat{\mathcal{S}}(z) - \frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar\rho^2)\, s' \log\left(\frac{s'}{\mu_r^2}\right)\right)$$

These are the determinants of quadratic fluctuation operators in the instanton-anti-instanton background = 1-loop effects.
They were computed on far separated instanton and anti-instanton, but in fact R/rho ~1.5 - 2

The effect can be significant.

R/rho=2



(5)

Fixing the RG scale mu at 1/rho:

Notice that the instanton integrand contains the factor:

$$(\rho\mu_r)^{b_0}(\bar{\rho}\mu_r)^{b_0}\, e^{-\frac{4\pi}{\alpha_s(\mu_r)}} = e^{-\frac{2\pi}{\alpha_s(1/\rho)}-\frac{2\pi}{\alpha_s(1/\bar{\rho})}}\,, \tag{2.30}$$

where $(\rho\mu_r)^{b_0}$ and $(\bar{\rho}\mu_r)^{b_0}$ come from the instanton and the anti-instanton measure $D(\rho)$ and $D(\bar{\rho})$, and the factor $e^{-\frac{4\pi}{\alpha_s(\mu_r)}}$ accounts for the instanton and the anti-instanton action contributions in the dilute limit.

=> Standard instanton RG prescription: mu=1/rho

In the calculations leading to the first paper we checked that varying the RG prescription did not lead to massive changes in the results.

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- [This is by default a non-perturbative semiclassical computation in a (moderately) strongly interacting theory and in the regime where quantum corrections exponentiate. This is not a few % uncertainty in perturbative calculations. Can expect an overall factor of a ~100.]