Bekenstein bound from the Pauli principle

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Contents

Basic Ideas

Bekenstein Bound and Xons

Bekenstein's argument that a black hole (BH) reaches the maximal entropy at disposal of a physical system[1] led to two main proposals:

- The degrees of freedom (dof) responsible for the BH entropy have to take into account both matter and spacetime and hence must be of a new, more fundamental nature than the dof we know, here we call such dof "Xons" [2]
- The Hilbert space $\mathcal{H}$ of the Xons of a given BH is necessarily finite dimensional[3]

[1] J. D. Bekenstein, Phys. Rev. D **23**, 287 (1981).
[2] G. Acquaviva, A. Iorio and M. Scholtz, Ann. Phys. **387**, 317 (2017).
[3] N. Bao, S. M. Carroll and A. Singh, Int. J. Mod. Phys. D **26**, no. 12, 1743013 (2017).

Bekenstein Bound from the Pauli principle

Here[4] we reverse that logic. We suppose that.

- In a BH only free Xons exist
- They are *finite* in number and *fermionic* in nature. This amounts to have a finite dimensional $\mathcal{H}$

Then, one can show that BH evaporation is a dynamical mechanism producing a maximal entanglement entropy $\mathcal{S}_{max}$:

$$\dim \mathcal{H} \; = \; e^{\mathcal{S}_{max}}$$

where $\mathcal{S}_{max}$ is equal to the initial *BH entropy*.

[4] G. Acquaviva, A. Iorio and L. S., Phys. Rev. D **102**, 106002 (2020).

Black hole evaporation and Thermofield dynamics

TFD: Thermal average and thermal vacuum

Thermal average of a quantum operator A is $\langle A \rangle = \mathrm{Tr}[A\,\rho]/\mathrm{Tr}[\rho]$ where ρ is the density matrix. In the canonical ensemble $\rho = e^{-\beta H}$, with $\beta = 1/T$. Thermal vacuum is defined by[5].

$$\langle A \rangle = \frac{1}{\mathcal{Z}(\beta)} \sum_n e^{-\beta E_n} \langle n|A|n\rangle = \langle 0(\beta)|\, A\, |0(\beta)\rangle$$

$\mathcal{Z}(\beta) \equiv \mathrm{Tr}\rho$ is the partition function and $|n\rangle$ are energy eigenstates ($H|n\rangle = E_n|n\rangle$). The solution is:

$$|0(\beta)\rangle \;=\; \frac{1}{\sqrt{\mathcal{Z}(\beta)}} \sum_n e^{\frac{-\beta E_n}{2}} |n\rangle \otimes |\tilde{n}\rangle$$

[5]Y. Takahashi and H. Umezawa, Collective Phenomena **2**, 55 (1975).

TFD: Tilde modes

$|\tilde{n}\rangle$ can be seen as reservoir modes (or holes): The price of purification is the doubling of degrees of freedom.

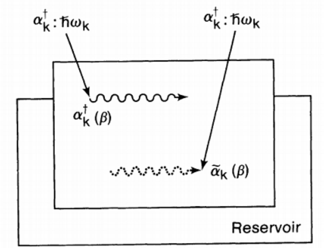


Figure 1: Taken from H. Umezawa, H. Matsumoto and M. Tachiki, *Thermo Field Dynamics And Condensed States*, (North-Holland, Amsterdam, 1982).

We define

$$c(\beta, \mathbf{k}) = \sqrt{1 + \varepsilon\, f(\beta, \mathbf{k})} \qquad d(\beta, \mathbf{k}) = \sqrt{f(\beta, \mathbf{k})}$$

where $\varepsilon = 1$ (-1) for bosons (fermions) and $f(\beta, \mathbf{k}) = 1/\left(e^{\beta\omega_{\mathbf{k}}} - \varepsilon\right)$.

Entropy operator

Then, thermal vacuum can be rewritten as

$$|0(\beta)\rangle \;=\; \exp\left(-\frac{S}{2}\right)\,\exp\left(\sum_{\mathbf{k}} a_{\mathbf{k}}^{\dagger}\,\tilde{a}_{\mathbf{k}}^{\dagger}\right)\,|0\rangle\otimes|\tilde{0}\rangle$$

where $H|0\rangle = E_0|0\rangle$ and the same for tilde. Entropy operator was defined:

$$S \;=\; -\sum_{\mathbf{k}}\left(a_{\mathbf{k}}^{\dagger}\,a_{\mathbf{k}}\,\ln d^2(\beta,\mathbf{k})\,-\,\varepsilon\,a_{\mathbf{k}}\,a_{\mathbf{k}}^{\dagger}\,\ln c^2(\beta,\mathbf{k})\right)$$

S permits to quantify entanglement entropy of $|0(\beta)\rangle$:

$$\langle 0(\beta)|S|0(\beta)\rangle \;=\; -\sum_{n} w_n\,\ln w_n\,, \qquad w_n \;=\; \frac{e^{-\beta\,E_n}}{\mathcal{Z}(\beta)}$$

The system is entangled with its *double*.

Euclidean Schwarzschild vs Rindler metric

Schwarzschild metric:

$$\mathrm{d}s^2 \;=\; -f(r)\,\mathrm{d}t^2 \,+\, \frac{\mathrm{d}r^2}{f(r)} \,+\, r^2\,\mathrm{d}\Omega^2$$

$f(r) = \left(1 - \frac{r_s}{r}\right)$, $r_s = 2M$. Near the horizon (outside)

$$\mathrm{d}s^2 \;=\; -\rho^2\,\mathrm{d}\eta^2 \,+\, \mathrm{d}\rho^2 \,+\, r_s^2\,\mathrm{d}\Omega^2$$

$\eta = \kappa\, t$ and $\mathrm{d}\rho = \mathrm{d}r/\sqrt{f}$. $\kappa = 1/2r_s$ is the surface gravity. This is equivalent to $\mathcal{R}_2$ (Rindler) $\times\mathcal{S}_2$. After Wick rotation ($\theta = i\eta$)

$$\mathrm{d}s_E^2 \;=\; \rho^2\,\mathrm{d}\theta^2 \,+\, \mathrm{d}\rho^2 \,+\, r_s^2\,\mathrm{d}\Omega^2$$

This is euclidean metric in four dimensions $\left(\mathcal{R}_2^E \times \mathcal{S}_2 = \mathcal{M}_4^E\right)$.

Rindler vs Minkowski Hilbert space

The (right wedge) Rindler Hilbert space is

$$\mathcal{H}_R \equiv \{\Psi[\phi_R] \mid \phi_R(X) \equiv \phi(X > 0)\}$$

and the same for left wedge ($X < 0$) Hilbert space $\mathcal{H}_L$. Here $X = \rho \cosh \eta, T = \rho \sinh \eta$. The Minkowski Hilbert space is:

$$\mathcal{H}_{\mathcal{M}} = \mathcal{H}_L \otimes \mathcal{H}_R$$

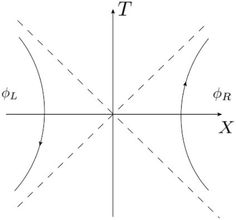


Figure 2: Taken from H. Liu, https://ocw.mit.edu/courses/physics/8-821-string-theory-and-holographic-duality-fall-2014/

Minkowski vacuum as thermal vacuum

Minkowski vacuum wave functional:

$$\Psi_0[\phi_L, \phi_R] = \int_{\phi(\theta=-\pi)=\phi_L}^{\phi(\theta=0)=\phi_R} \mathcal{D}\phi \, e^{-S_E[\phi]}$$

With respect to η

$$\Psi_0[\phi_L, \phi_R] = \langle \phi_L | e^{-i\,(-iH_R)} | \phi_R \rangle = \sum_n e^{-\pi\, E_n} \, \chi_n[\phi_L] \, \chi_n[\phi_R]$$

H_R is the generator of η translations. Minkowski vacuum state:

$$|0\rangle_{\mathcal{M}} \propto \sum_n e^{-\pi\, E_n} \, |n\rangle_L \otimes |n\rangle_R$$

thermal vacuum with $T = 1/(2\pi)$ (Unruh effect)[6].

[6] W.G, Unruh, Phys. Rev. D **14**, 870 (1976).

BH temperature and BH entropy

In the BH case (respect to t), we have Hartle–Hawking vacuum, with temperature $T_{BH} = \kappa/(2\pi)$. BH (Bekenstein–Hawking) entropy[7]:

$$\mathcal{S}_{BH} = \int_0^M \frac{\mathrm{d}E}{T_{BH}} \; = \; 4\,\pi\,M^2 = \frac{\mathcal{A}}{4}$$

$\mathcal{A}$ is the area of the event horizon.

- $L - R$ entanglement entropy can be computed as VEV of entropy operator[8]
- Same considerations hold for AdS BHs[9]

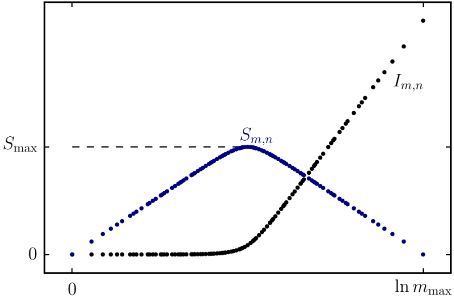
[7] J. D. Bekenstein, Lett. Nuovo Cim. **4**, 737 (1972); S.W. Hawking, Nature **248**, 30 (1974).

[8] A. Iorio, G. Lambiase and G. Vitiello, Ann. Phys. **309**, 151 (2004).

[9] J. M. Maldacena, JHEP **0304**, 021 (2003).

Information loss paradox: Page curve

BH evaporation ⇒ BH in pure state can end up a mixed state[10]: Loss of unitarity?. Page[11] studied the bipartite BH/radiation system in a random pure state ⇒ Entanglement entropy $S_{m,n}$ and Information $I_{m,n}$ as function of $\ln m$, with $m \equiv \dim \mathcal{H}_{rad}$, $n \equiv \dim \mathcal{H}_{BH}$:



System starts and ends up in a pure state ⇒ Unitary evolution.

[10] S.W. Hawking, Phys. Rev. D **14**, 2460 (1976).

[11] D. N. Page, Phys. Rev. Lett. **71**, 3743 (1993).

The Hilbert space of quantum gravity and Xons

Bekenstein Bound

Bekenstein suggested[12] that the entropy of a physical system with energy E, enclosed in a sphere of radius R, should be bounded from above: Bekenstein Bound (BB)

$$\mathcal{S} \leq 2\pi \, R \, E$$

In general relativity, it must hold:

$$R \geq 2E$$

$(r \geq r_s)$. Then[13]

$$\mathcal{S} \leq \frac{\mathcal{A}}{4}$$

This last is called holographic bound. Both are saturated by a BH.

[12] J. D. Bekenstein, Phys. Rev. D **23**, 287 (1981).

[13] P.F. González-Díaz, Phys. Rev. D **27**, 3042 (1983).

Bekenstein Bound and finiteness of QG Hilbert space (1)

- In QFT, an infinite number of dof, in a system in a finite region R, could be entangled with an infinite number of external dof $\Rightarrow$ infinite entanglement entropy
- When gravity is involved, only a finite number of dof, N, can occupy R: over a certain energy they collapse in a BH $\Rightarrow$ maximum entropy in R
- Collapse of more energy will just increase the size of the BH, over R

But $N = \dim \mathcal{H}_R \Rightarrow$ locally finite dimensional Hilbert space[14]

$$\dim \mathcal{H}_R \; \leq \; e^{\mathcal{S}_{BH}}$$

[14] N. Bao, S. M. Carroll and A. Singh, Int. J. Mod. Phys. D **26**, 1743013 (2017).

Bekenstein Bound and finiteness of QG Hilbert space (2)

This means that the Hilbert space of the universe can be factorized as[15]

$$\mathcal{H}_{uni} \; = \; \bigotimes_{\alpha} \mathcal{H}_{\alpha} \, , \quad \mathrm{dim}\mathcal{H}_{\alpha} < \infty$$

In one region, physics is described by reduced density matrix:

$$\rho_R \; \equiv \; \mathrm{tr}_{\bar{R}} \rho_{uni}$$

The mentioned dof should describe, at an emergent level, both matter/field and spacetime on which fields are defined[16].

[15] S. M. Carroll and A. Singh, [arXiv:1801.08132 [quant-ph]].

[16] G. Acquaviva, A. Iorio and M. Scholtz, Ann. Phys. **387**, 317 (2017).

Towards Xons: main assumptions

- Validity of BB $\Rightarrow$ Finite dimensional Hilbert spaces for bounded regions $\Rightarrow$ Finite number of dof

$$\Sigma \;=\; \dim \mathcal{H}$$

- N could counts dof more fundamental than gravity and matter:

Gravity and matter/fields emerge from such fundamental dof: Xons

- For a BH, Xons are fully excited:

$$\dim \mathcal{H}_{BH} = e^{\mathcal{S}_{BH}}$$

Emergent spacetime and matter

Xons must rearrange to form spacetime and matter. It should exist a map

$$|g^{(a)}\rangle \;=\; P_G|\psi\rangle\,, \qquad |\phi\rangle \;=\; P_F|\psi\rangle$$

from $\mathcal{H}$ (Xons Hilbert space) into $\mathcal{H}_F$ (field/matter modes Hilbert space) and $\mathcal{H}_G$ (geometric modes Hilbert space). Consider a number N_T of subspaces $T_{(i)} \subset \mathcal{H}$ of states with specific distribution of dof between field and spacetime. We can decompose $\mathcal{H}$

$$T_{(i)} = \mathcal{H}_G^{p_i} \otimes \mathcal{H}_F^{q_i} \quad \text{and} \quad \mathcal{H} = \bigoplus_{i=1}^{N_T} (\mathcal{H}_G^{p_i} \otimes \mathcal{H}_F^{q_i})$$

Here p_i $(q_i) \equiv \dim \mathcal{H}_G^{p_i} (\dim \mathcal{H}_F^{q_i})$.

Re-shuffling of dof: loosing unitarity

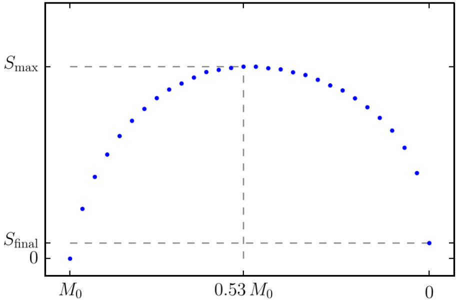
- Unitary evolution of states in $\mathcal{H}$, at the Xons level
- Many configurations in $\mathcal{H}$ correspond to the same emergent geometry: many microstates compatible with the same macrostate
- dof in $\mathcal{H}$ are re-shuffled during evolution

Consequences:

Two equivalent geometries can differ by the number of dof available for the quantum fields $\Rightarrow$ even though fundamental evolution is unitary, emergent spacetime and matter get entangled

Geometry–Matter entanglement entropy

Geometry–Matter entanglement entropy in BH evaporation, as a function of BH mass:



Non-zero final entanglement entropy is due to the presence of more than one microscopic realization of the same emergent geometry[17].

[17] G. Acquaviva, A. Iorio and M. Scholtz, Ann. Phys. **387**, 317 (2017).

Bekenstein Bound from the Pauli principle

Fermionic Xon model of BH evaporation

- We assume our system has only a finite number N of quantum levels (slots) to be filled (e.g. Planck cells)
- We assume that Xons are fermions $\Rightarrow$ Each quantum level can be filled by no more than one fermion $\Rightarrow$ $\dim\mathcal{H} < \infty$.
- Before evaporation, BH state is described by free Xons, which fill all slots.
- Evaporation consists of steady process: $N \to (N-1) \to (N-2) \to \cdots$. That is, the number of *free* Xons steadily decreases
- During evaporation Xons rearrange into quasi-particles and the spacetime they live in $\Rightarrow$ Intrinsic notion of interior (BH) and exterior (environment)[18].

[18] G. Acquaviva, A. Iorio and L.S., Phys. Rev. D **102**, 106002 (2020).

Occupation number representation for Xons

Creation/annihilation operators of Xons

$$\left\{a_n\,,\,a^\dagger_{n'}\right\} \;=\; \left\{b_n\,,\,b^\dagger_{n'}\right\} \;=\; \delta_{nn'} \qquad n=1,\ldots,N$$

a-modes=environment, b-modes=BH. Initial state:

$$|0,N\rangle \;\equiv\; |0,0,\ldots,0\rangle_{\text{I}} \otimes |1,1,\ldots,1\rangle_{\text{II}}$$

with $|1,1,\ldots,1\rangle_{\text{II}} \equiv b^\dagger_1 b^\dagger_2 \ldots b^\dagger_N \,|0,0,\ldots,0\rangle_{\text{II}}$. Final state:

$$|N,0\rangle \;\equiv\; |1,1,\ldots,1\rangle_{\text{I}} \otimes |0,0,\ldots,0\rangle_{\text{II}}$$

where $|1,1,\ldots,1\rangle_{\text{I}} \equiv a^\dagger_1 a^\dagger_2 \ldots a^\dagger_N \,|0,0,\ldots,0\rangle_{\text{I}}$.

BH state in Xon model

A toy-model corresponding to such boundary conditions is defined by the entangled state

$$|\Psi(\sigma)\rangle \;=\; \prod_{i=1}^{N} \sum_{n_i=0,1} C_i(\sigma)\,\left(a_i^\dagger\right)^{n_i}\,\left(b_i^\dagger\right)^{1-n_i}\,|0\rangle_{\mathrm{I}} \otimes |0\rangle_{\mathrm{II}}$$

with

$$C_i = (\sin\sigma)^{n_i}\,(\cos\sigma)^{1-n_i}$$

σ is an interpolating parameter, which describes the evolution of the system, from $\sigma = 0$ till $\sigma = \pi/2$. $\mathcal{H}$ has dimension

$$\Sigma \;=\; 2^N\,.$$

TFD entropy as von Neumann entropy

We define the entropy operator for environment modes as in TFD

$$S_{\rm I}(\sigma) \;=\; -\sum_{n=1}^{N} \left(a_n^\dagger\, a_n\, \ln\sin^2\sigma + a_n\, a_n^\dagger\, \ln\cos^2\sigma\right)$$

and the entropy operator for BH modes

$$S_{\rm II}(\sigma) \;=\; -\sum_{n=1}^{N} \left(b_n^\dagger\, b_n\, \ln\cos^2\sigma + b_n\, b_n^\dagger\, \ln\sin^2\sigma\right)$$

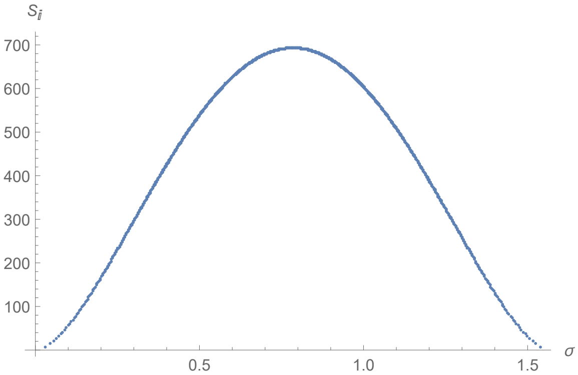
Taking the expectation value of $\mathcal{S}_{\rm I/II}(\sigma) \equiv \langle S_{\rm I/II}(\sigma)\rangle_\sigma$:

$$\mathcal{S}_{\rm I}(\sigma) = -N\left(\sin^2\sigma\,\ln\sin^2\sigma + \cos^2\sigma\,\ln\cos^2\sigma\right) = \mathcal{S}_{\rm II}(\sigma)$$

where $\langle \ldots \rangle_\sigma \equiv \langle \Psi(\sigma)| \ldots |\Psi(\sigma)\rangle$. This is the von Neumann entropy of $|\Psi(\sigma)\rangle$.

Entanglement entropy in BH evaporation

Entropy as a function of σ, in the case $N = 1000$:



It describes a unitary evolution: Page-like behavior.

Maximum entropy and Hilbert space dimension

The maximum entropy value is

$$\mathcal{S}_{max} \;=\; N\,\ln 2 \;=\; \ln\Sigma$$

Then

$$\Sigma \;=\; e^{\mathcal{S}_{max}}$$

- $\mathcal{S}_{\text{I}} \;=\; \mathcal{S}_{max}$ when the modes have *half* probability to be inside and *half* probability to be outside the BH $\Rightarrow$ the largest amount of bits are necessary to describe the system.
- Intrinsic way to know how big is $\mathcal{H}$, which is related to the initial size of the BH $\Rightarrow \mathcal{S}_{max}$ tells how big was BH before evaporation $\Rightarrow \mathcal{S}_{max} = \mathcal{S}_{max}(\mathcal{M}_0, \mathcal{J}_0, \mathcal{Q}_0)$. This is BB in this picture

Number operators

Number operators:

$$\hat{N}_{\mathrm{I}} \ = \ \sum_{n=1}^{N} a_n^\dagger \, a_n \, , \qquad \hat{N}_{\mathrm{II}} \ = \ \sum_{n=1}^{N} b_n^\dagger \, b_n$$

Their expectation values are:

$$N_{\mathrm{I}}(\sigma) \ \equiv \ \langle \hat{N}_{\mathrm{I}} \rangle_\sigma = N \sin^2 \sigma$$

$$N_{\mathrm{II}}(\sigma) \ \equiv \ \langle \hat{N}_{\mathrm{II}} \rangle_\sigma = N - N_{\mathrm{I}}(\sigma) = N \ \cos^2 \sigma$$

It is clear that σ could be discretized:

$$\sigma(N_{\mathrm{I}}) = \arcsin \sqrt{\frac{N_{\mathrm{I}}}{N}}$$

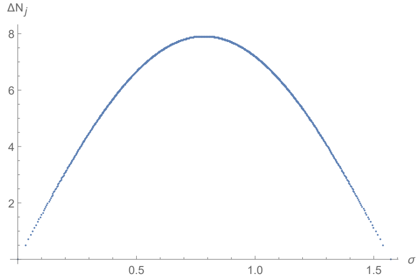
N_I is constrained to be an integer $N_I = m \Rightarrow \sigma(N_{\mathrm{I}}) = \sigma_m$

Number fluctuations and entanglement

Standard deviation of $\hat{N}_j$ on $|\Psi(\sigma)\rangle$:

$$\Delta N_{\text{I}}(\sigma) \;=\; \Delta N_{\text{II}}(\sigma) \;=\; \frac{\sqrt{N}\,\sin(2\sigma)}{2}$$

$\Delta N_j \equiv \sqrt{\langle \hat{N}_j^2\rangle_\sigma - \langle \hat{N}_j\rangle_\sigma^2}$. This is a measure of the entanglement[19]:



[19] A.A Klyachko, B. Öztop and A.S. Shumovsky, Phys. Rev. A **75**, 032315 (2007).

BH entropy and environment entropy

One could think that the physical Hilbert spaces of the two subsystems have to take into account only the number of modes truly occupied, at any stage of the evaporation $\Rightarrow$ the actual dimensions would be $2^{N_{\rm I}(\sigma)}$, and $2^{N_{\rm II}(\sigma)}$ and we can decompose:

$$2^N = 2^{N_{\rm II}(\sigma)} \times 2^{N_{\rm I}(\sigma)} \equiv n \times m$$

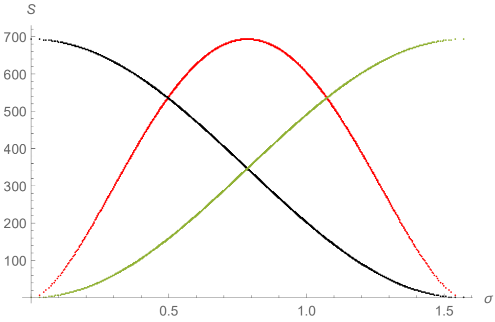
$n = 2^N, 2^{N-1}, \ldots, 1$, and $m = 1, \ldots, 2^{N-1}, 2^N$.

We define the Bekenstein/environment entropy as

$$\mathcal{S}_{BH} \equiv \ln n = N \ln 2 \cos^2 \sigma \, , \quad \mathcal{S}_{env} \equiv \ln m = N \ln 2 \sin^2 \sigma \, .$$

Entropy comparison

Plot of $\mathcal{S}_{\mathrm{I}}$, $\mathcal{S}_{BH}$ and $\mathcal{S}_{env}$ for $N = 1000$:



These satisfy

$$\mathcal{S}_{\mathrm{I}} \;\leq\; \mathcal{S}_{BH} + \mathcal{S}_{env} \;=\; \mathcal{S}_{max}$$

Wheeler–DeWitt equation

$|\Psi(\sigma)\rangle$ can be written as

$$|\Psi(\sigma)\rangle \quad \equiv \quad e^{-i\,\sigma\,G}\,|\Psi(0)\rangle$$

where

$$G \;=\; i\sum_{n=1}^{N}\left(a_n^\dagger\, b_n \;-\; b_n^\dagger\, a_n\right)$$

$|\Psi(\sigma)\rangle$ solves *Wheeler–DeWitt* equation[20]

$$H\,|\Psi(\sigma)\rangle \;=\; 0$$

$H \equiv i\,\partial_\sigma - G$. This constrains $\mathcal{K}$ to $\mathcal{H}$.

[20]C. Rovelli, *Quantum gravity*, (Cambridge University Press, Cambridge, 2004).

Duality with a TFD-like model (1)

Consider the canonical transformation (non-connected with the identity)

$$A_n \ = \ a_n \, , \qquad B_n \ = \ b_n^\dagger$$

Vacua in the new representation are defined as

$$A_n \, |0\rangle_A \ = \ B_n \, |0\rangle_B \ = \ 0$$

One can check that

$$|0\rangle_A = |0\rangle_{\text{I}} \, , \quad |0\rangle_B = |1_1 \, 1_2 \, \ldots \, 1_N\rangle_{\text{II}}$$

Duality with a TFD-like model (2)

In this representation

$$G \;=\; i \sum_{n=1}^{N} \left(A_n^\dagger \, B_n^\dagger \,-\, B_n \, A_n \right)$$

Therefore $|\Psi(\sigma)\rangle$ has a TFD-vacuum-like structure

$$|\Psi(\sigma)\rangle = e^{-\frac{1}{2} S_A(\sigma)} \, |I\rangle = e^{-\frac{1}{2} S_B(\sigma)} \, |I\rangle \tag{1}$$

$|I\rangle \;=\; \exp\left(\sum_{n=1}^{N} A_n^\dagger \, B_n^\dagger \right) |0\rangle_A \,\otimes\, |0\rangle_B$, and

$$\begin{aligned} S_A &= -\sum_{n=1}^{N} \left(A_n^\dagger \, A_n \, \ln \sin^2 \sigma + A_n \, A_n^\dagger \, \ln \cos^2 \sigma \right) \\ S_B &= -\sum_{n=1}^{N} \left(B_n^\dagger \, B_n \, \ln \sin^2 \sigma + B_n \, B_n^\dagger \, \ln \cos^2 \sigma \right) \end{aligned}$$

Formation of matter and geometry

- When the system evolves, a pair of A and B particles is created. The B-modes enter into the BH, annihilating BH modes, while the A-modes form the environment
- Xons do not discern fields and spacetime dof. However some dof are indeed responsible for the reduction of the BH's horizon area during the evaporation and annihilators of geometric modes can be defined. As first (linear) approximation, we can decompose A_n as

$$A_n = \sum_k \left(g_{k,n} \, A^k_{G,n} \otimes \mathbb{1}_{F,n} + f_{k,n} \, \mathbb{1}_{G,n} \otimes A^k_{F,n} \right)$$

where k labels the emergent modes.

Identification of parameters

A quantum of BH horizon area measures $\Delta A \; = \; \alpha \, l_P^2$. Then

$$\mathcal{S}_{BH} \; = \; \frac{\alpha \, N}{4}$$

N is the number of Planck cells = number of slots. In our model

$$\alpha \;\; = \;\;\; 4 \, \ln 2$$

This value agrees with the condition $\alpha = 4 \ln k$, with k integer, which was proposed to constrain the number of microstates to an integer[21]. A comparison with BH formula, when $\mathcal{Q}_0 = 0 = \mathcal{J}_0$, gives

$$N \; = \; \frac{4 \, \pi \, \mathcal{M}_0^2}{l_P^2 \; \ln 2}$$

[21] S. Hod, Phys. Rev. Lett. **81**, 4293 (1998).

Conclusions and Perspectives

Conclusions

- BB implies that Hilbert space of every local system is finite dimensional
- This Hilbert space describes fundamental modes (Xons)
- Matter/fields and spacetime emerge from Xons dynamics
- Reversing the point of view, and starting from fermionic Xons dynamics, one can prove that a maximum entropy should exist, for the BH/environment system, during BH evaporation (Page-like behavior)[22]
- The maximum entropy corresponds to the entropy of initial BH $\Rightarrow$ Xons formulation of BB
- Our model of Xons dynamics can be mapped into a TFD-like model, where Xons particle/hole pair are created on vacuum
- Prediction of $\Delta A/l_P^2$ ratio

[22] G. Acquaviva, A. Iorio and L.S., Phys. Rev. D **102**, 106002 (2020).

Perspectives

- Research of complete mapping from Xons to emergent spacetime and fields
- Connection with specific theories of quantum gravity
- Comparison with other approaches to Page-curve[23] [24]
- Research of phenomenological implications
- Emergence of different phases[25]

[23] A. Almheiri, R. Mahajan, J. Maldacena and Y. Zhao,JHEP **03**, 149 (2020).
[24] F. F. Gautason, L. Schneiderbauer, W. Sybesma and L. Thorlacius, JHEP **05**, 091 (2020).
[25] G. Acquaviva, A. Iorio and L.S., in preparation.

Thank you for the attention!