Sp($2N_c$) gauge theories for Beyond the Standard Model Physics

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Outline

- Introduction
- Motivations for studying $\mathrm{Sp}(2N_c)$ gauge groups
- Lattice methods and results
- Conclusion

The Higgs particle and the SM

Despite its outstanding success, culminated with the discovery of the Higgs boson with,

$$m_{\mathrm{H}} = 125.18(16)\ \mathrm{GeV}, \quad v_{\mathrm{H}} \simeq 246\ \mathrm{GeV}\ ,$$

whose rôle is to "give" mass to the gauge bosons and to the SM fermions

$$m_{\mathrm{W}} = \frac{g}{2} v_{\mathrm{H}} = m_{\mathrm{Z}} \cos\vartheta_W , \quad m_f = \frac{\lambda_f}{\sqrt{2}} v_{\mathrm{H}}\ ,$$

the SM cannot be the full story:

- Gravity
- Dark Matter
- Origin of ElectroWeak Symmetry Breaking (EWSB)
- ...

there must be some new physics above an energy scale Λ_{SM}.

With an elementary Higgs and $\Lambda_{\mathrm{SM}} \gg v_{\mathrm{H}}$, understanding EWSB would imply a tremendous amount of fine tuning Δ between the unrelated contributions from SM and BSM,

$$\Delta \geq \frac{\delta_{\mathrm{SM}} m_{\mathrm{H}}^2}{m_{\mathrm{H}}^2} \simeq \left(\frac{\Lambda_{\mathrm{SM}}}{500\ \mathrm{GeV}} \right)^2 .$$

Several ways to protect m_{H} from upper scales have been proposed: we will focus on Composite Higgs Models (CHM).

Composite Higgs Models

To stabilize m_{H}, we could mimick QCD with a new composite sector.

Let G_{HC} the gauge group of the composite sector, defined at some Λ_{UV}. At scale $f \ll \Lambda_{\mathrm{UV}}$, the strong coupling induces the breaking of global symmetry $\mathcal{G}$,

$$\mathcal{G} \longmapsto \mathcal{H}_1 \ ,$$

by condensation of the fermionic degrees of freedom. Then:

- A number $n = dim(\mathcal{G}) - dim(\mathcal{H}_1)$ of massless Nambu-Goldstone bosons (NGb) π^A that parametrize the manifold $\mathcal{G}/\mathcal{H}_1$ of equivalent vacua.
- A potential for the π^A is forbidden by shift symmetry.

If $\mathcal{H}_0 \subset \mathcal{G}$ is gauged by external vector bosons, with $\mathcal{H} = \mathcal{H}_1 \cap \mathcal{H}_0$,

- The original symmetry $\mathcal{G}$ is explicitly broken, a (model dependent) vacuum expectation value (VEV) v for π^A is generated radiatively.
- $n_0 = dim(\mathcal{H}_0) - dim(\mathcal{H})$ become the longitudinal components of the gauge bosons
- $n - n_0$ become pseudo NGb whose mass is controlled by the VEV of π^A, i.e. by the misalignement between $\mathcal{H}_0$ and $\mathcal{H}_1$.

Kaplan and Georgi 1984

Composite Higgs models

The spectrum is composed by:

- A Higgs field with a mass $m_h \sim g_0 v$, where g_0 is a generic gauge coupling.
- Resonances of mass scale $m_\rho \sim g_\rho f$ with g_ρ of order 1.

The parameter

$$\xi = \left(\frac{v}{f}\right)^2$$

controls the deviations from the SM.

- $\xi = 0$ we obtain the SM with an elementary Higgs particle.
- $\xi \sim 1$ we obtain a TechniColor (TC) model.
- $\xi \ll 1$ we have vacuum misalignement, the Higgs stays light and resonances decouple.

For this construction to be realistic:

- The $\mathcal{H}_{\mathrm{EW}} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_L$ group must be embeddable in $\mathcal{H}_1$.
- The Higgs multiplet must be embeddable in $\mathcal{G}/\mathcal{H}_1$.

So far:

- Not very hard to produce phenomenologically viable models for EW and Higgs physics.
- Very hard to generate SM fermions masses.

SM fermions masses

We might expect the fermion masses to be generated at scale Λ_{UV} by terms of type

$$\frac{\lambda_t}{\Lambda_{UV}^{d-1}} \bar{q}_L \mathcal{O}_S^c t_R + \frac{\lambda_b}{\Lambda_{UV}^{d-1}} \bar{q}_L \mathcal{O}_S b_R + \text{h.c.}$$

where $q_L = (t_L, b_L)^T$, t_R and b_R are SM fermions, and O_S is a Lorentz scalar of energy dimension d, composed of fields of the composite sector or of some additional strong sector.

However:

- If $\Lambda_{\text{UV}} \gg f$ and $d \simeq 1$, it is difficult to generate large Yukawa couplings from underlying perturbative theory.
- By lowering Λ_{UV} to make the Yukawa couplings large, we might incur in phenomenologically prohibited FCNC.

To bypass these problems, one can try to implement Fermion Partial Compositeness (FPC).

Kaplan 1991

Partial Fermion Compositeness

Introduce resonances $\mathcal{O}^f_{L,R}$ of the (extended) composite sector, with quantum numbers that allow them to couple linearly with the SM fermions,

$$\frac{\lambda_{t_L}}{\Lambda_{UV}^{d^f_L-5/2}}\,\bar{q}_L\mathcal{O}^L_f + \frac{\lambda_{t_R}}{\Lambda_{UV}^{d^f_R-5/2}}\,\bar{t}_R\mathcal{O}^R_f + \text{h.c.}$$

As a consequence:

- Not hard to define operators with $d^f_{L,R} \simeq 5/2$. Flavor hierarchies can be reproduced without fine-tuning.
- The physical states are superpositions of SM fermions with the composite resonances

$$|Phys.\rangle = \alpha|SM_i\rangle + \beta|Composite_i\rangle$$

For example, with an extended composite sector that include additional elementary (colored) Weyl fermions χ and χ^c,

$$\mathcal{O}^{f,\alpha ab} = q^a\chi^\alpha q^b, \quad \mathcal{O}^{c,f,ab}_\alpha = q^a\chi^c_\alpha q^b$$

where α is a color index and a, b indices in the 2-index anti-symmetric representation of G_{HC}.

Barnard, Gherghetta, and Ray 2014

UV completions for CHMs

What are the possible UV completions of a phenomenologically viable CHM where FPC can be implemented?

Ferretti and Karateev 2014

Consider Left Handed (LH) Weyl fermions in a representation,

$$n_1 R_1 \oplus n_2 R_2 \oplus \cdots \oplus n_p R_p \ ,$$

of G_{HC} where n_i indicates how many times R_i is repeated. Then,

$$\mathcal{G} = SU(n_1) \otimes SU(n_2) \otimes \cdots \otimes SU(n_p) \oplus U(1)^{p-1} \ .$$

The groups G_{HC}, $\mathcal{G}$, $\mathcal{H}_1$ and $\mathcal{H}_0$ should be chosen so that:

- G_{HC} is free of gauge (global) anomalies if $G_{HC} = SU(N_c)$ (if $G_{HC} = Sp(2N_c)$). $\mathcal{H}_0$ free of 't Hooft anomalies.
- The gauge theory should be Asymptotically Free.
- The Breaking $\mathcal{G} \longmapsto \mathcal{H}_1 \supset \mathcal{H}_{\text{cus.}} \supset \mathcal{H}_{\text{EW}}$ should be possible
- $\mathcal{G}/\mathcal{H}_1$ can accomodate at least one Higgs multiplet.
- Composite states can be used as partners to SM fermions.

These requests restrict the possible values of N_c and n_i.

UV completions for CHMs

The realistic case with the minimal value of p is $p = 2$, for which we have,

$$\mathcal{G} = \mathrm{SU}(n_1) \otimes \mathrm{SU}(n_2)$$

with $n_1 \geq 4$ and $n_2 \geq 6$. The possible theories are then,

G_{HC}	R_1	R_2	Restrictions
$\mathrm{Sp}(2N_c)$	$5 \times Ad$	$6 \times F$	$2N_c \geq 12$
$\mathrm{Sp}(2N_c)$	$5 \times A_2$	$6 \times F$	$2N_c \geq 4$
$\mathrm{Sp}(2N_c)$	$4 \times F$	$6 \times A_2$	$2N_c \leq 36$
$\mathrm{SO}(N_c)$	$5 \times S_2$	$6 \times F$	$N_c \geq 55$
$\mathrm{SO}(N_c)$	$5 \times Ad$	$6 \times F$	$N_c \geq 15$
$\mathrm{SO}(N_c)$	$5 \times F$	$6 \times Spin$	$N_c = 7, 9, 10, 11, 13, 14$
$\mathrm{SO}(N_c)$	$5 \times F$	$6 \times F$	$N_c = 7, 9$
$\mathrm{SO}(N_c)$	$4 \times F$	$6 \times F$	$N_c = 11, 13$

The minimal solution are Sp(2) and Sp(4) gauge theories with $4 \times F$ Weyl fermions and $6 \times A_2$ Weyl fermions, with breaking

$$\mathrm{SU}(4) \otimes \mathrm{SU}(6) \longmapsto \mathrm{Sp}(4) \otimes \mathrm{SO}(6)\ .$$

This is the model we are approaching on the lattice, it has been explored (analytically) by Barnard, Gherghetta, and Ray 2014 and the Sp(2) studied extensively on the lattice (see Cacciapaglia, Pica, and Sannino 2020).

Insights from Gauge/Gravity dualities

Even outside of conformality, the paradigm introduced by the AdS/CFT correspondence has opened the way to new ways to obtain informations about gauge theories, albeit in a non-conformal setting:

- Partial compositeness can be implemented in higher dimensional gravity theories, and their dual gauge theories studied employing the dictionnary between the two. The results for $Sp(4)$ are found to be close to lattice findings.

  Erdmenger et al. 2020
- The bosonic fluctuations of a family of gravity duals to four-dimensional gauge theories are studied. They correspond to glueball in the latter. Specific features of the spectrum are then found to agree with lattice results

  Elander, Piai, and Roughley 2019

The quest for FPC

A great effort is devoted to finding satisfactory realizations of models of CH, especially regarding implementations of the FPC paradigm.

- An UV complete explicit realization of the idea of FPC can be obtained in a model with $\mathrm{Sp}(2N_c)$ gauge symmetry in the presence of four fermion operators, known to drive chiral symmetry breaking in the Nambu-Jona Lasinio. This is the model we are approaching on the lattice!

  Barnard, Gherghetta, and Ray 2014
- The SM and HC gauge groups are *partially unified* and four fermions interaction, and thus FPC, are mediated by (massive) gauge bosons produced by the breaking of a unified gauge symmetry in the so-called *Techni-Pati-Salam* model. The final ("unbroken") gauge group is Sp(4).

  Cacciapaglia, Vatani, and Zhang 2019

Dark Matter candidates

Natural candidates emerge whenever we enlarge the SM with additional sectors, where it is natural to look for DM candidates.

- The SIMP miracle: the $2 \to 2$ scattering producing WIMP miracle is supplemented by a $3 \to 2$ (strong) annihilation. This is capable of producing Tev mass DM and both credible and (nearly) testable predictions with $\mathrm{Sp}(2N_c)$ gauge group.

  Hochberg et al. 2015

- The lightest glueballs are the natural Dark Matter candidates when the (vector) quarks of the composite are heavy.

  Yamanaka et al. 2019

Not only CHMs!

Sp($2N_c$) gauge theories are also interesting for:

- Thermodynamics of gauge theories: as Sp($2N_c$) only have pseudoreal representations, the phase space can be explored without a sign problem.
- The center of the group is always $\mathbb{Z}_2$, useful to test the Svetitsky-Yaffe conjecture
  Holland, Pepe, and Wiese 2004
- Large-N Physics: Sp($2N_c$) symmetric gauge theories can be shown to have the same $N \to \infty$ of the SU(N_c) and SO(N_c) cases, with a $O(N_c^{-1})$ difference in the approach to $N = \infty$,

$$\frac{m}{\sqrt{\sigma}}(N_c) = \begin{cases} \frac{m}{\sqrt{\sigma}}(N_c = \infty) + \frac{c_{N_c}}{N_c}, & \mathrm{Sp}(2N_c), \mathrm{SO}(N_c) \\ \frac{m}{\sqrt{\sigma}}(N_c = \infty) + \frac{c_{N_c}}{N_c^2}, & \mathrm{SU}(N_c) \end{cases}$$

  Lovelace 1982
- Quantities related to the glueball spectrum may be computed in alternative to SU(N_c) and SO(N_c) gauge theories: Casimir scaling and the ratio of the tensor to glueball masses.
  E. Bennett, Holligan, et al. 2020; Hong et al. 2017

Checkpoint

The $Sp(2N_c)$ group

Definition and main properties

The symplectic group $\mathrm{Sp}(2N_c)$ can be defined as a subgroup of $\mathrm{SU}(2N_c)$,

$$\mathrm{Sp}(2N_c) = \left\{ U \in \mathrm{SU}(2N_c) \mid \Omega U \Omega^T = U^* \right\}$$

where Ω is the Symplectic matrix,

$$\Omega = \begin{bmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{bmatrix}$$

As direct consequences of the definition:

- $\mathrm{Sp}(2) \simeq \mathrm{SU}(2)$.
- The center of the group is $\mathbb{Z}_2$ for every N_c
- All representations are pseudo-real and Charge conjugation is trivial.
- The block structure of Ω is inherited by $\mathrm{Sp}(2N_c)$ matrices,

$$U = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix}, \quad \left\{ \begin{array}{l} A^\dagger A + B^\dagger B = \mathbb{I}, \\ A^T B = B^T A, \end{array} \right. \quad A, B \in \mathbb{C}^{N \times N} .$$

The breaking of SU(4)

The Lagrangian for 2 Dirac fermions coupled to a Sp(4) gauge field is,

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\ V_{\mu\nu}V^{\mu\nu} \ + \ i\bar{Q}^i_a\gamma^\mu(D_\mu Q^i)^a - m\bar{Q}^i_a Q^{ia} \ , \quad a \text{ color indices, } i = 1, 2$$

with $D_\mu = \partial_\mu + igV^A_\mu T^A_R$, and with the usual definition for $V_{\mu\nu}$ and its kinetic part.

From the pseudo-real nature of the gauge group,

$$\Omega U \Omega^T = U^\star \longrightarrow \Omega T^A_R \Omega = -\left(T^A_R\right)^\star \ ,$$

one can recast the above action in terms of LH Weyl fermions q^{ia},

$$Q^{ia} = \begin{bmatrix} q^{ia} \\ \Omega^{ab}(-\tilde{C}q^{i+2,b\star}) \end{bmatrix} \ ,$$

and one obtains

$$\mathcal{L} = iq^{ak\dagger}\bar{\sigma}^\mu(D_\mu q^k)^a - m\Omega_{kn}\Sigma^{kn} \ , \quad \Sigma^{kn} = \Omega_{ab}\left(q^{kbT}\tilde{C}q^{na}\right) \ ,$$

where now $k, n = 1, \ldots, 4$.

The broken phase

$$\mathcal{L} = iq^{ka\dagger}\bar{\sigma}^{\mu}(D_\mu q^k)^a - m\Omega_{kn}\Sigma^{kn} \ , \quad \Sigma^{kn} = \Omega_{ab}\left(q^{kbT}\tilde{C}q^{na}\right)$$

For $m \to 0$, the Lagrangian above enjoys a SU(4) symmetry. The condensation,

$$\langle\Sigma\rangle = \Omega \neq 0 \ ,$$

drives the breaking SU(4) $\longmapsto$ Sp(4).

The NGbs $\pi^A(x)$, transforming in the 5 representation of Sp(4), parametrize the coset SU(4)/Sp(4),

$$\Sigma(x) = e^{\frac{2i}{f}\pi^A\hat{T}^A}\Omega \ ,$$

where $\hat{T}^A$ broken generators and f has the dimensions of energy.

At the Leading Order (LO), the Lagrangian density of the EFT is

$$\mathcal{L}_{p^2} = \frac{f^2}{4}\mathrm{Tr}\left\{\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right\} - \frac{v^3}{4}\mathrm{Tr}\left\{M\Sigma + \Sigma^\dagger M^\dagger\right\} \ ,$$

where v^3 is the magnitude of the condensate and the transformation properties of Σ and of the spurion $M = m\Omega$ are

$$M \to U^\star M U^\dagger, \quad \Sigma \to U\Sigma U^T \ , \quad U \in \mathrm{SU}(4) \ .$$

The Low Energy Constants

The above EFT treatment can be extended:

- Include "light" mesons, like the ρ and the a_1, along the lines of Hidden Local Symmetry (HLS).
- To consider fermions un multiple representations to implement FPC.
- In all the cases above, the NLO approximation can be computed.

$$
\begin{aligned}
M_\rho^2 &= \frac{g_\rho^2}{4(1+\kappa+my_3)}(bf^2+F^2+2mv_1) \\
M_{a_1}^2 &= \frac{g_\rho^2}{4(1-\kappa-my_4)}(bf^2+F^2+2mv_1) + \frac{g_\rho^2}{(1-\kappa-my_3)}(bf^2+m(v_2-v_1)) \\
f_\rho^2 &= \frac{1}{2}(bf^2+F^2+2mv_1) \\
f_{a_1}^2 &= \frac{(bf^2-F^2+2m(v_1-v_2))^2}{2((b+4)f^2+F^2-2mv_1+fmv_2)} \\
f_0^2 &= F^2+(b+2c)f^2
\end{aligned}
$$

with $f_\pi^2 = f_0^2 - f_\rho^2 - f_{a_1}^2$.

The Low Energy Constants (LEC) f, F, b, c, and g_ρ can be computed on the lattice and used for phenomenology.

Lattice setup

The HiRep code[1] was suitably modified to accomodate the $\mathrm{Sp}(2N_c)$ gauge group.

In particular:

- The Cabibbo-Marinari technique was adapted to $\mathrm{Sp}(2N_c)$.
- Fermions in multiple representations of $\mathrm{Sp}(2N_c)$
- Cooling, re-projection on the group, smearing, variational methods...

So far:

- Glueball spectrum of $\mathrm{Sp}(2N_c)$ for $N_c \leq 4$

  Bennett et al. 2020
- Meson spectrum for quenched fermions at $N_c = 2$ in multiple representations.

  E. Bennett, Hong, Lee, Chi-Jen David Lin, et al. 2020
- Meson spectrum at $N_c = 2$, for $N_f = 2$ fundamental fermions.

  E. Bennett, Hong, Lee, C.-J. David Lin, et al. 2019

Several lattice sizes $L_t \times L_s^3$, couplings β and bare fermion masses m_0 were probed,

- To avoid bulk phase transitions.
- To have control of Finite Volume Effects.
- To extrapolate to the continuum limit (also using Wilson χPT for fermion simulations)
- The scale was set with ω_0 or $\sqrt{\sigma}$.

[1] Del Debbio, Patella, and Pica 2010

Lattice discretizations

The discretized action used to perform simulations was

$$S\left[U,\bar{\psi},\psi\right] = S_g\left[U\right] + S_f\left[U,\bar{\psi},\psi\right]$$

where:

- For the gauge part, we used the Wilson action,

$$S_g(U) = \beta \sum_{x,\,\mu>\nu} \left(1 - \frac{1}{2N} \Re\,\mathrm{Tr}\, P_{\mu\nu}(x)\right) \ ,$$

  where $P_{\mu\nu}$ is the elementary plaquette operator and $\beta = 2N/g^2$.
- For the fermion part, we used

$$S_f\left[U,\bar{\psi},\bar{\psi}\right] = a^4 \sum_x \bar{\psi}(x) D_m \psi(x) \ ,$$

  where D_m is the unimproved Dirac operator,

$$D_m\psi(x) = \left(\frac{4}{a} + m_0\right)\psi(x) - \frac{1}{2a}\sum_\mu \{(1-\gamma_\mu)U_\mu(x)\psi(x+\mu)$$
$$+(1+\gamma_\mu)U_\mu(x-\mu)\psi(x-\mu)\} \ ,$$

  where a is the lattice spacing and m_0 the bare lattice mass.

Meson spectrum

The masses and decay constants are obtained on the lattice from the the large-time euclidean behaviour of

$$C_{\mathcal{O}_M}(t) = \sum_{\vec{x}} \langle 0|\mathcal{O}_M(\vec{x},\, t)\mathcal{O}_M^\dagger(\vec{0},\, 0)|0\rangle \longmapsto_{t\to\infty} \frac{|\langle 0|\mathcal{O}_M|M\rangle|^2}{2m_M} e^{-m_M t} \ ,$$

where operators $\mathcal{O}_M = \bar{\psi}_1 \Gamma_M \psi_2$ have the desired quantum numbers.

Label (M)	$\mathcal{O}_M$	Meson	J^P	Sp(4)
PS	$\bar{\psi}_1\gamma_5\psi_2$	π	0^-	5
S	$\bar{\psi}_1\psi_2$	a_0	0^+	5
V	$\bar{\psi}_1\gamma_\mu\psi_2$	ρ	1^-	10
T	$\bar{\psi}_1\gamma_0\gamma_\mu\psi_2$	ρ	1^-	10
AV	$\bar{\psi}_1\gamma_5\gamma_\mu\psi_2$	a_1	1^+	5
PT	$\bar{\psi}_1\gamma_5\gamma_0\gamma_\mu\psi_2$	b_1	1^+	10

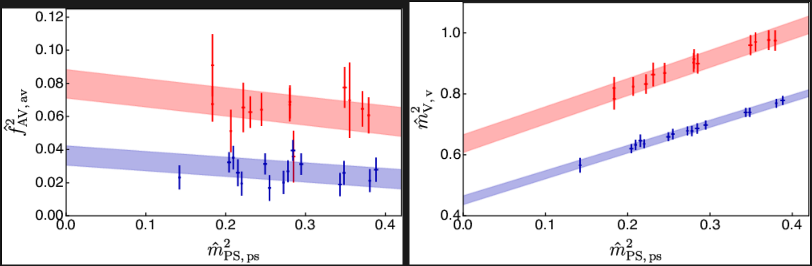
- We use $\mathbb{Z}_2 \times \mathbb{Z}_2$ single time-slice stochastic wall sources.
- The matrix elements at finite lattice spacing a have been renormalized via one-loop perturbative matching,

$$f_M^{\text{bare}} = \langle 0|\mathcal{O}_M|M\rangle \ , \quad f_M = Z_M f_M^{\text{bare}} \ .$$

- Calculations have been performed in the regime $m_{PS,ps}L \geq 7.5$.

Quenched fermions in multiple representations

Sample results



- Masses and decay constants in the 2-index anti-symmetric representations are considerably larger than in the fundamental. In the massless limit:

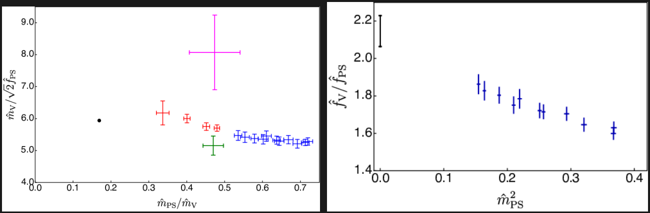
$$\frac{\hat{f}_{av}}{\hat{f}_{AV}} = 2.7 \pm 1.1 \;, \qquad \frac{\hat{m}_s}{\hat{m}_S} = 1.18 \pm 0.13 \;.$$

- The difference with dynamical calculation should $\sim 25\%$ for $\hat{m}_{PS}$.

We can check the validity of the KRSF relations,

$$g_{\rm VPP} = \frac{m_{\rm V}}{\sqrt{2} f_{\rm PS}} \ , \quad f_{\rm V} = \sqrt{2} f_{\rm PS} \ .$$

to the same quantities in real-world QCD and in other lattice models.



E. Bennett, Hong, Lee, C.-J. David Lin, et al. 2019

- In the case of $Sp(4)$ gauge theory, the first KSRF relation is not satisfied while the second holds,

$$\hat{f}_{\rm V}/\hat{f}_{\rm PS} \sim 2.1 \neq \sqrt{2} \ , \quad \hat{m}_{\rm V}/\sqrt{2}\hat{f}_{\rm PS} = 5.72(18) \simeq 6.0(4) = g_{\rm VPP} \ .$$

- $\hat{m}_{\rm V}/\sqrt{2}\hat{f}_{\rm PS}$ can be computed on the lattice for $SU(2)$ (purple), $SU(3)$ (red), $SU(4)$ (green) and compared to $Sp(4)$ (blue).

Glueballs spectrum

The Glueball correlators are particularly noisy, and we perform a (automatized) variational calculation, looking for the $v_i \in \mathbb{C}$ in

$$\Phi(t) = \sum_i v_i O_i^{R^P}(t) \ ,$$

which minimize the effective mass in each of the (Poincaré) symmetry channels $R^P = A_1^{\pm}$, $A_2^{\pm}$, $E^{\pm}$, $T_1^{\pm}$, $T_2^{\pm}$.

The mass $m(R^P)$ can be obtained from the correlator of the optimal operators at large t,

$$\tilde{C}(t) = \langle 0|\Phi(t)\Phi^\dagger(0)|0\rangle \longmapsto_{t\to\infty} |\tilde{c}|^2 e^{-m(R^P)t} \ .$$

Once the Finite Volume Effects are under control, i.e. $m(R^P)L \geq 3$, the continuum limit can be obtained from discretization errors,

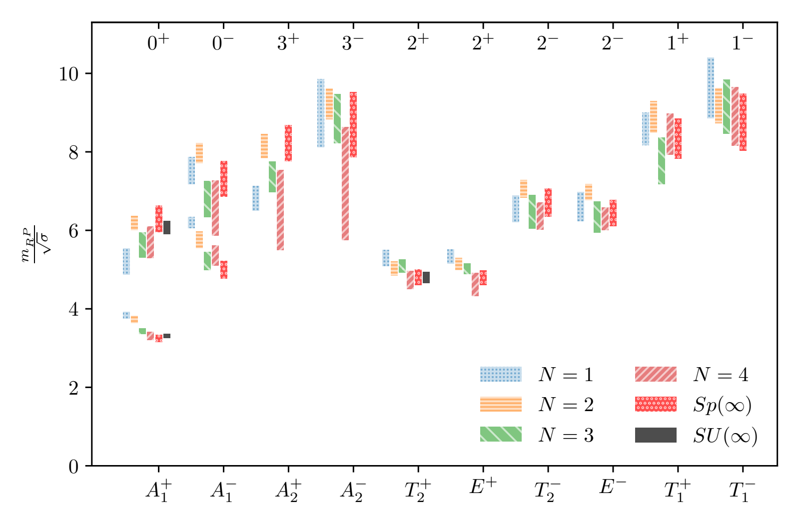
$$\frac{m(R^P)}{\sqrt{\sigma}}(a) = \frac{m(R^P)}{\sqrt{\sigma}}(0) + c_0(R^P)\sigma a^2 \ ,$$

where σ is measured from torelon propagation and Effective String Theory, and the large-N_c limit can be obtained from[2]

$$\frac{m(R^P)}{\sqrt{\sigma}}(N_c) = \frac{m(R^P)}{\sqrt{\sigma}}(\infty) + \frac{c_1(R^P)}{N_c} \ .$$

[2] Note that this is different from $SU(N_c)$!

The spectrum of Sp($2N_c$) glueballs

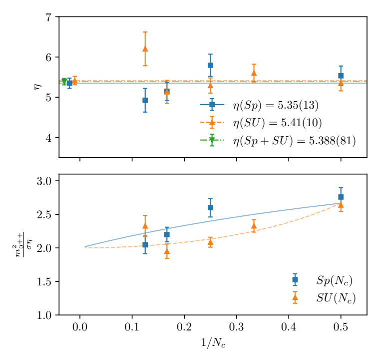


Casimir Scaling

Scaling arguments suggest that

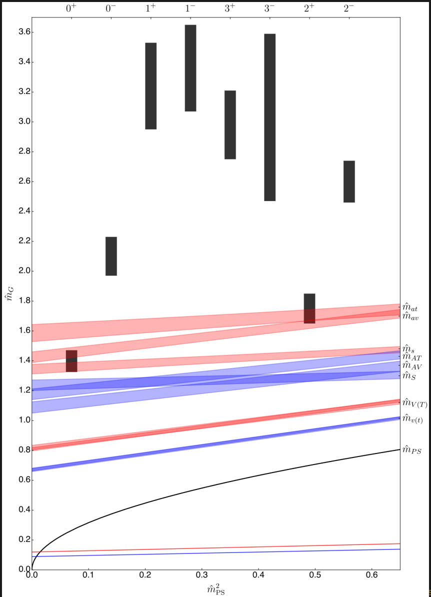
$$\eta = \frac{m_{0^{++}}}{\sqrt{\sigma}} \frac{C_2(F)}{C_2(A)}\,, \quad \text{with} \quad \frac{C_2(F)}{C_2(A)} = \begin{cases} \frac{2N^2}{N^2-1} SU(N) \\ \frac{2(N-2)}{N-1} SO(N) \\ \frac{4(N+1)}{2N+1} Sp(2N) \end{cases}$$

is a universal quantity. Hong et al. 2017



Summary of meson and glueball spectrums

In units of ω_0



Conclusion

To summarize:

- $Sp(2N_c)$ gauge theories describe some of the possible UV completions of CHMs.
- They can be useful to better understand thermodynamics and large-N physics.
- Lattice studies have been performed for the pure gauge theory, for quenched mesons in multiple representations, and for $N_f = 2$ fundamental fermions.

WIP:

- Dynamical Sp(4) fermions in multiple representations.
- Finite temperature and topology of pure gauge theory.

Future work:

- (Quenched?) Meson spectrum of $Sp(2N_c)$ gauge theories
- Thermodynamics with dynamical fermions starting with Sp(4)

THANK YOU

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