# Nucleon axial structure from lattice QCD: Controlling pion pole enhanced excited states

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based on JHEP 05 (2020) 126

December 3, 2020, Dublin

# Motivation

Many experiments sensitive to axial and (induced) pseudoscalar form factors (FF):

- $\beta$ decay — $g_A$
- $\nu$ scattering — $G_A(Q^2)$
- $\pi$ electroproduction — $G_A(Q^2)$
- radiative muon capture (RMC) — $g_P^\star$
- ordinary muon capture (OMC) — $g_P^\star$
- (low energy) $\pi N$ scattering — $g_{\pi NN}$
- (low energy) $NN$ scattering — $g_{\pi NN}$
- $\pi^- p$ and $\pi^- d$ pionic atoms — $g_{\pi NN}$

$$g_{\tilde{P}}^\star = \frac{m_\mu}{2m} \tilde{G}_P(0.88 m_\mu^2) \qquad g_{\pi NN} = \lim_{Q^2 \to -m_\pi^2} \frac{m_\pi^2 + Q^2}{4 m F_\pi} \tilde{G}_P(Q^2)$$

# Correlation functions

On the lattice, we calculate correlation functions...

$$C_{2\text{pt}}^{\mathbf{p}}(t) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} P_+^{\alpha\beta} \langle O_N^\beta(\mathbf{x},t)\ \bar{O}_N^\alpha(\mathbf{0},0)\rangle$$

$$C_{3\text{pt},\Gamma}^{\mathbf{p}',\mathbf{p},\mathcal{O}}(t,\tau) = a^6 \sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{p}'\mathbf{x}+i(\mathbf{p}'-\mathbf{p})\mathbf{y}} \Gamma^{\alpha\beta} \langle O_N^\beta(\mathbf{x},t)\ \mathcal{O}(\mathbf{y},\tau)\ \bar{O}_N^\alpha(\mathbf{0},0)\rangle$$

- Fourier transform to fix three-momenta
- (Wuppertal-)smeared three-quark interpolating currents at source and sink
- Current insertion $\mathcal{O}$ is $\mathcal{A}_\mu = \bar{q}\gamma_\mu\gamma_5 q$ or $\mathcal{P} = \bar{q}\gamma_5 q$ with flavor structure $\bar{u}u - \bar{d}d$
- Projection matrices $P_+ = (\mathbb{1} + \gamma_0)/2$ and $\Gamma$ (in our case $\Gamma = P_+\gamma^i\gamma_5$, $i = 1, 2, 3$)

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$\Rightarrow$ insert complete sets of states to perform spectral decomposition

- use translational properties to shift currents to the origin
- evaluate FT (trivial)

to arrive at ...

# Spectral decomposition

$$C^{\mathbf{p}}_{2\mathrm{pt}}(t) = \sum_{\sigma} P_+^{\alpha\beta} \langle 0|O_N^\beta|N_\sigma^{\mathbf{p}}\rangle\langle N_\sigma^{\mathbf{p}}|\bar{O}_N^\alpha|0\rangle \frac{e^{-E_{\mathbf{p}}t}}{2E_{\mathbf{p}}} + \dots$$

$$C^{\mathbf{p}',\mathbf{p},\mathcal{O}}_{3\mathrm{pt},\Gamma}(t,\tau) = \sum_{\sigma,\sigma'} \Gamma^{\alpha\beta} \langle 0|O_N^\beta|N_{\sigma'}^{\mathbf{p}'}\rangle\langle N_{\sigma'}^{\mathbf{p}'}|\mathcal{O}|N_\sigma^{\mathbf{p}}\rangle\langle N_\sigma^{\mathbf{p}}|\bar{O}_N^\alpha|0\rangle \frac{e^{-E_{\mathbf{p}'}(t-\tau)}e^{-E_{\mathbf{p}}\tau}}{4E_{\mathbf{p}'}E_{\mathbf{p}}} + \dots$$

- up to excited states hidden in the $+\dots$
- $\langle 0|O_N^\beta|N_\sigma^{\mathbf{p}}\rangle = \sqrt{Z_{\mathbf{p}}}\, u^\beta_{\mathbf{p},\sigma}$ determines the ground-state overlap
- FFs appear in the decomposition of the nucleon-nucleon matrix elements $\langle N_{\sigma'}^{\mathbf{p}'}|\mathcal{O}|N_\sigma^{\mathbf{p}}\rangle = \bar{u}_{\mathbf{p}',\sigma'} J[\mathcal{O}] u_{\mathbf{p},\sigma}$

$$J[\mathcal{A}_\mu] = \gamma_\mu\gamma_5 G_A(Q^2) + \frac{q_\mu}{2m}\gamma_5 \tilde{G}_P(Q^2) \qquad q = p' - p$$

$$J[\mathcal{P}] = \gamma_5 G_P(Q^2) \qquad Q^2 = -q^2$$

But not all FFs are independent in the continuum (due to PCAC):

$$\partial_\mu \mathcal{A}^\mu = 2im_\ell \mathcal{P} \qquad \Rightarrow \qquad \frac{m_\ell}{m} G_P(Q^2) = G_A(Q^2) - \frac{Q^2}{4m^2}\tilde{G}_P(Q^2)$$

## Spectral decomposition

$$C^{\mathbf{p}}_{2\text{pt}}(t) = \sum_{\sigma} P_{+}^{\alpha\beta} \langle 0|O_N^{\beta}|N_\sigma^{\mathbf{p}}\rangle\langle N_\sigma^{\mathbf{p}}|\bar{O}_N^{\alpha}|0\rangle \frac{e^{-E_{\mathbf{p}}t}}{2E_{\mathbf{p}}} + \dots$$

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Next use the identity $\sum_\sigma u_{\mathbf{p},\sigma}\bar{u}_{\mathbf{p},\sigma} = \not{p} + m$

to arrive at ...

# Extracting the ground state contribution

$$C^{\mathbf{p}}_{2\mathrm{pt}}(t) = Z_{\mathbf{p}}\,\mathrm{Tr}\Big\{P_{+}(\not{p}+m)\Big\}\frac{e^{-E_{\mathbf{p}}t}}{2E_{\mathbf{p}}} + \ldots = Z_{\mathbf{p}}\frac{E_{\mathbf{p}}+m}{E_{\mathbf{p}}}e^{-E_{\mathbf{p}}t} + \ldots$$

$$C^{\mathbf{p}',\mathbf{p},\mathcal{O}}_{3\mathrm{pt},\Gamma}(t,\tau) = \sqrt{Z_{\mathbf{p}'}}\sqrt{Z_{\mathbf{p}}}\,\mathrm{Tr}\Big\{\Gamma(\not{p}'+m)J[\mathcal{O}](\not{p}+m)\Big\}\frac{e^{-E_{\mathbf{p}'}(t-\tau)}e^{-E_{\mathbf{p}}\tau}}{4E_{\mathbf{p}'}E_{\mathbf{p}}} + \ldots$$

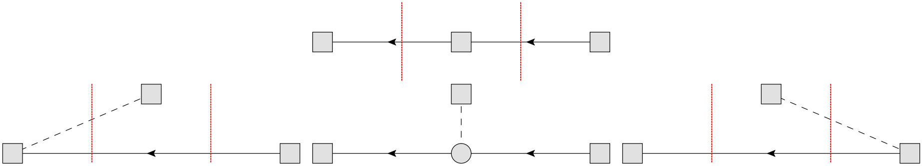
- to determine the FFs we perform a simultaneous fit to the 2-pt and 3-pt functions
- the FFs (hidden in $J[\mathcal{O}]$) are directly used as fit parameters
  $\rightarrow$ no overdetermined system of Eqs., etc.

Traditional ansatz:

- use multi-exponential fits (taking into account some excited states)
- assume that the excited states in two- and three-point function are the same

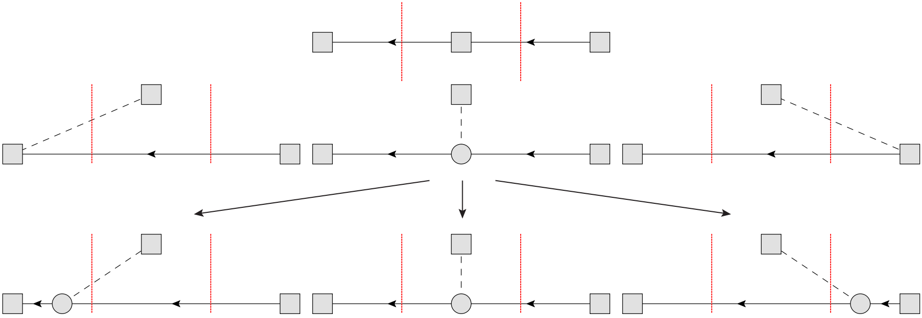
In some situations this does not work very well. Why?

# Correlation functions from EFT point of view



- second line: e.g., for the vector current such contributions are not possible
- right and left: contribute to excited state contaminations
- center: contributes to everything

# Correlation functions from EFT point of view



- second line: e.g., for the vector current such contributions are not possible
- right and left: contribute to excited state contaminations
- center: contributes to everything
- lower center: this is a ground state contribution (PPD contribution to $\tilde{G}_P$ and $G_P$)
- lower left and lower right are the “bad guys”

see also PRD **99** (2019) 054506, PRD **100** (2019) 054507 (O. Bär)

# EFT calculation: ingredients

Propagators:

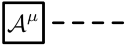
$$S_N(x) = i \int \frac{d^4q}{(2\pi)^4} e^{-iqx} \frac{\not{q} + m}{q^2 - m^2 + i\epsilon}\,,$$

$$S_\pi^{ab}(x) = \delta^{ab} S_\pi(x) = i \int \frac{d^4q}{(2\pi)^4} e^{-iqx} \frac{\delta^{ab}}{q^2 - m_\pi^2 + i\epsilon}\,.$$

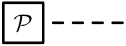
Insertion operator vertices (at lowest order):


$= g_A \gamma^\mu \gamma_5 \sigma^3$


$= 0$ $\qquad B_0 \equiv \dfrac{m_\pi^2}{2m_\ell}$

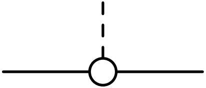



$= -2F_\pi \partial^\mu \delta^{a3}$




$= -2iF_\pi B_0 \delta^{a3}$

Interaction vertex:



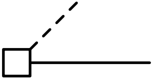
$= -i \dfrac{g_A}{2F_\pi} \not{\partial} \gamma_5 \sigma^a$ (derivative acts on pion propagator)

# EFT calculation: smearing dependence

Source/sink operator vertices:

 $= \sqrt{Z'}\bar{\Psi}_N$

 $= \sqrt{Z}\Psi_N$

 $= \sqrt{\tilde{Z}'}\frac{i}{2F_\pi}\bar{\Psi}_N\gamma_5\sigma^a$

 $= \sqrt{\tilde{Z}}\frac{i}{2F_\pi}\gamma_5\sigma^a\Psi_N$

- nucleon isospinor $\Psi_N$: $\Psi_p = (1,0)^T$ and $\Psi_n = (0,1)^T$ (here: $\bar{\Psi}_p\sigma^3\Psi_p = 1$)
- in general $Z$, $Z'$, $\tilde{Z}$, $\tilde{Z}'$ are momentum-dependent
- also dependent on masses, smearing method, smearing radii
- for local currents (at lowest order)

$$a' \equiv \frac{\sqrt{\tilde{Z}'}}{\sqrt{Z'}} = 1 \qquad\qquad a \equiv \frac{\sqrt{\tilde{Z}}}{\sqrt{Z}} = 1$$

heuristic arguments: also holds for smeared currents with $r_{\text{sm}} \ll \lambda_\pi = 1.41\,\text{fm}$

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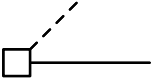
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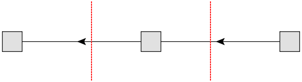
**We will not make this assumption! Instead we will test it.**

# EFT calculation: warm up

insertion: $\mathcal{A}^\mu$



$$\sqrt{Z'}\sqrt{Z}\int d^3x\, e^{-i\mathbf{p}'\cdot\mathbf{x}}\int d^3y\, e^{-i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{y}} S_N(x-y) g_A\gamma^\mu\gamma_5 S_N(y) =$$

$$= -\sqrt{Z'}\sqrt{Z}\int\frac{dE_2}{2\pi}e^{-iE_2(t-\tau)}\int\frac{dE_1}{2\pi}e^{-iE_1\tau}\frac{(\gamma_0E_2-\boldsymbol{\gamma}\cdot\mathbf{p}'+m)g_A\gamma^\mu\gamma_5(\gamma_0E_1-\boldsymbol{\gamma}\cdot\mathbf{p}+m)}{(E_2^2-\mathbf{p}'^2-m^2+i\epsilon)(E_1^2-\mathbf{p}^2-m^2+i\epsilon)}$$

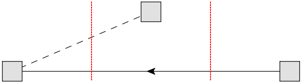
$$= \frac{\sqrt{Z'}\sqrt{Z}}{2E'2E}e^{-iE'(t-\tau)}e^{-iE\tau}(\not{p}'+m)g_A\gamma^\mu\gamma_5(\not{p}+m)$$

- FT fixes three-momenta in propagators $\Rightarrow$ only two integrations over energies remain
- similar result as before (after $t\to -it$, $\tau\to -i\tau$, and taking the the trace with $\Gamma$)
- $g_A$ instead of $G_A(Q^2)$ makes sense (we work at leading order in low-energy EFT)
- where is $\tilde{G}_P(Q^2)$?

$$p=\begin{pmatrix}E\\ \mathbf{p}\end{pmatrix}\qquad E=\sqrt{\mathbf{p}^2+m^2}\qquad p'=\begin{pmatrix}E'\\ \mathbf{p}'\end{pmatrix}\qquad E'=\sqrt{\mathbf{p}'^2+m^2}$$

# EFT calculation: additional pion at the sink

insertion: $\mathcal{A}^\mu$



$$\sqrt{\tilde{Z}'}\sqrt{Z}\int d^3x\, e^{-i\mathbf{p}'\cdot\mathbf{x}}\int d^3y\, e^{-i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{y}}\left(\frac{i}{2F_\pi}\gamma_5\right)\left(-2F_\pi\frac{\partial}{\partial y_\mu}\right)S_\pi(x-y)S_N(x) =$$

$$= -\sqrt{\tilde{Z}'}\sqrt{Z}\int\frac{dE_2}{2\pi}e^{-iE_2(t-\tau)}\int\frac{dE_1}{2\pi}e^{-iE_1 t}\frac{\begin{pmatrix}E_2\\ \mathbf{q}\end{pmatrix}^\mu}{E_2^2-\mathbf{q}^2-m_\pi^2+i\epsilon}\frac{\gamma_5(\gamma_0 E_1-\boldsymbol{\gamma}\cdot\mathbf{p}+m)}{E_1^2-\mathbf{p}^2-m^2+i\epsilon}$$

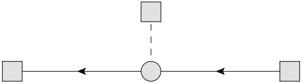
$$= +\frac{\sqrt{\tilde{Z}'}\sqrt{Z}}{2E\,2E_\pi}e^{-iE_\pi(t-\tau)}e^{-iEt}r_+^\mu\gamma_5(\not{p}+m)$$

- calculation similar as on last slide
- exponentials: nucleon from source to sink + pion from insertion to sink
- the pion has momentum $\mathbf{q}$, the nucleon has momentum $\mathbf{p}$
- simple Dirac structure

$$q = \begin{pmatrix}E'-E\\ \mathbf{p}'-\mathbf{p}\end{pmatrix} \qquad r_\pm = \begin{pmatrix}E_\pi\\ \pm(\mathbf{p}'-\mathbf{p})\end{pmatrix} \qquad E_\pi = \sqrt{(\mathbf{p}'-\mathbf{p})^2+m_\pi^2}$$

# EFT calculation: the non-trivial part

insertion: $\mathcal{A}^\mu$

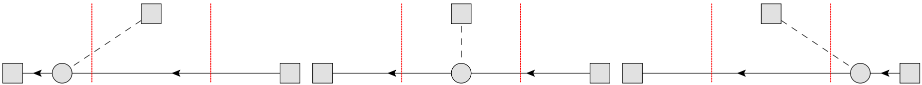


$$\sqrt{Z'}\sqrt{Z}\int d^3x\, e^{-i\mathbf{p}'\cdot\mathbf{x}}\int d^3y\, e^{-i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{y}}\int d^4z$$
$$\times S_N(x-z)\left[\left(-i\frac{g_A}{2F_\pi}\gamma_\nu\gamma_5\frac{\partial}{\partial z_\nu}\right)\left(-2F_\pi\frac{\partial}{\partial y_\mu}\right)S_\pi(z-y)\right]S_N(z)=$$
$$=g_A\sqrt{Z'}\sqrt{Z}\int\frac{dE_2}{2\pi}e^{-iE_2(t-\tau)}\int\frac{dE_1}{2\pi}e^{-iE_1\tau}$$
$$\times\frac{\left(\begin{smallmatrix}E_2-E_1\\ \mathbf{q}\end{smallmatrix}\right)^\mu\left(\begin{smallmatrix}E_2-E_1\\ \mathbf{q}\end{smallmatrix}\right)^\nu}{(E_2-E_1)^2-\mathbf{q}^2-m_\pi^2+i\epsilon}\,\frac{(\gamma_0E_2-\boldsymbol{\gamma}\cdot\mathbf{p}'+m)\gamma_\nu\gamma_5(\gamma_0E_1-\boldsymbol{\gamma}\cdot\mathbf{p}+m)}{(E_2^2-\mathbf{p}'^2-m^2+i\epsilon)(E_1^2-\mathbf{p}^2-m^2+i\epsilon)}\,.$$

- calculation a bit more intricate ($E_1$ and $E_2$ integrations are entangled)
- at first: two poles in each integration
- **BUT:** one pole in $E_2$ occurs when the poles in $E_1$ collapse to a double pole
- analysis of the double pole case in $E_1$ $\rightarrow$ no pole in $E_2$ anymore

$\Rightarrow$ 3 poles in total

# EFT calculation: the non-trivial part



$$\dots = -\frac{g_A\sqrt{Z'}\sqrt{Z}}{2E'\,2E} e^{-iE'(t-\tau)} e^{-iE\tau} q^\mu q^\nu \frac{(\not{p}'+m)\gamma_\nu\gamma_5(\not{p}+m)}{q^2-m_\pi^2}$$
$$-\frac{g_A\sqrt{Z'}\sqrt{Z}}{2E\,2E_\pi} e^{-iE_\pi(t-\tau)} e^{-iEt} r_+^\mu r_+^\nu \frac{(\not{p}+\not{r}_+ +m)\gamma_\nu\gamma_5(\not{p}+m)}{(p+r_+)^2-m^2}$$
$$-\frac{g_A\sqrt{Z'}\sqrt{Z}}{2E'\,2E_\pi} e^{-iE't} e^{-iE_\pi\tau} r_-^\mu r_-^\nu \frac{(\not{p}'+m)\gamma_\nu\gamma_5(\not{p}'+\not{r}_- +m)}{(p'+r_-)^2-m^2}$$

- the three poles correspond to 1 ground state and 2 excited state contributions
- in the ground state contribution we can simplify

$$q^\mu q^\nu \frac{(\not{p}'+m)\gamma_\nu\gamma_5(\not{p}+m)}{q^2-m_\pi^2} = \frac{2mq^\mu}{q^2-m_\pi^2}(\not{p}'+m)\gamma_5(\not{p}+m)$$

**this yields the leading ground state contribution to $\tilde{G}_P(Q^2)$**

# EFT calculation: the ground state result

After matching to the form factor decomposition one finds:

$$G_A = g_A + \text{higher order}$$

$$\tilde{G}_P = g_A \frac{4m^2}{Q^2 + m_\pi^2} + \text{higher order}$$

$$G_P = g_A \frac{m}{m_\ell} \frac{m_\pi^2}{Q^2 + m_\pi^2} + \text{higher order}$$

- **important:** we do not impose any of this
- we use the standard form factor decomposition for the ground state!
- it is obvious that higher orders in $Q^2$ are missing in the tree-level ChPT calculation
- $\Rightarrow$ it is consistent to replace $g_A \to G_A(Q^2)$ everywhere (also in the $N\pi$ part)
- this replacement yields the pion pole dominance (PPD) ansatz for $\tilde{G}_P$
- we will show later on that this is the superior choice (by far)

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# EFT calculation: the full three-point function

$\Rightarrow$ take trace with $\Gamma = P_+^i = \frac{(1+\gamma_0)}{2}\gamma^i\gamma_5$ and rotate to $t \to -it$, $\tau \to -i\tau$:

$$C^{\mathbf{p}',\mathbf{p},\mathcal{A}^\mu}_{3\text{pt},P_+^i} = +\frac{\sqrt{Z'}\sqrt{Z}}{2E'\,2E} e^{-E'(t-\tau)} e^{-E\tau} \times \Bigg[ B^{\mathbf{p}',\mathbf{p}}_{P_+^i,A^\mu} \left(1 + B_{10} e^{-\Delta E'(t-\tau)} + B_{01} e^{-\Delta E\tau} + B_{11} e^{-\Delta E'(t-\tau)} e^{-\Delta E\tau}\right)$$
$$+ e^{-\Delta E'_{N\pi}(t-\tau)} \frac{E'}{E_\pi} r_+^\mu \left(c' p^i + d' q^i\right) + e^{-\Delta E_{N\pi}\tau} \frac{E}{E_\pi} r_-^\mu \left(c\, p'^i + d\, q^i\right)\Bigg]$$

$$C^{\mathbf{p}',\mathbf{p},\mathcal{P}}_{3\text{pt},P_+^i} = +\frac{\sqrt{Z'}\sqrt{Z}}{2E'\,2E} e^{-E'(t-\tau)} e^{-E\tau} \times \Bigg[ B^{\mathbf{p}',\mathbf{p}}_{P_+^i,P} \left(1 + B_{10} e^{-\Delta E'(t-\tau)} + B_{01} e^{-\Delta E\tau} + B_{11} e^{-\Delta E'(t-\tau)} e^{-\Delta E\tau}\right)$$
$$+ e^{-\Delta E'_{N\pi}(t-\tau)} \frac{E'}{E_\pi} \frac{m_\pi^2}{2m_\ell} \left(c' p^i + d' q^i\right) - e^{-\Delta E_{N\pi}\tau} \frac{E}{E_\pi} \frac{m_\pi^2}{2m_\ell} \left(c\, p'^i + d\, q^i\right)\Bigg]$$

- **this is incredibly cool!**
- standard GS contribution:
  $B^{\mathbf{p}',\mathbf{p}}_{P_+^i,A^\mu} = \mathrm{tr}\left\{P_+^i(\not{p}' + m)\left[\gamma^\mu\gamma_5 G_A + \frac{q^\mu}{2m}\gamma_5 \tilde{G}_P\right](\not{p} + m)\right\}$
- usual excited states: the $B_{ij}$ depend on channel, polarization and momenta
- excited state energies obtained from two-point functions

# EFT calculation: the full three-point function

$\Rightarrow$ take trace with $\Gamma = P_+^i = \frac{(1+\gamma_0)}{2}\gamma^i\gamma_5$ and rotate to $t \to -it$, $\tau \to -i\tau$:

$$C^{\mathbf{p}',\mathbf{p},\mathcal{A}^\mu}_{3\text{pt},P_+^i} = +\frac{\sqrt{Z'}\sqrt{Z}}{2E'\,2E} e^{-E'(t-\tau)} e^{-E\tau} \times \Bigg[ B^{\mathbf{p}',\mathbf{p}}_{P_+^i,A^\mu} \Big( 1 + B_{10} e^{-\Delta E'(t-\tau)} + B_{01} e^{-\Delta E\tau} + B_{11} e^{-\Delta E'(t-\tau)} e^{-\Delta E\tau} \Big) + e^{-\Delta E'_{N\pi}(t-\tau)} \frac{E'}{E_\pi} r_+^\mu \Big( c' p^i + d' q^i \Big) + e^{-\Delta E_{N\pi}\tau} \frac{E}{E_\pi} r_-^\mu \Big( c\, p'^i + d\, q^i \Big) \Bigg]$$

$$C^{\mathbf{p}',\mathbf{p},\mathcal{P}}_{3\text{pt},P_+^i} = +\frac{\sqrt{Z'}\sqrt{Z}}{2E'\,2E} e^{-E'(t-\tau)} e^{-E\tau} \times \Bigg[ B^{\mathbf{p}',\mathbf{p}}_{P_+^i,P} \Big( 1 + B_{10} e^{-\Delta E'(t-\tau)} + B_{01} e^{-\Delta E\tau} + B_{11} e^{-\Delta E'(t-\tau)} e^{-\Delta E\tau} \Big) + e^{-\Delta E'_{N\pi}(t-\tau)} \frac{E'}{E_\pi} \frac{m_\pi^2}{2m_\ell} \Big( c' p^i + d' q^i \Big) - e^{-\Delta E_{N\pi}\tau} \frac{E}{E_\pi} \frac{m_\pi^2}{2m_\ell} \Big( c\, p'^i + d\, q^i \Big) \Bigg]$$

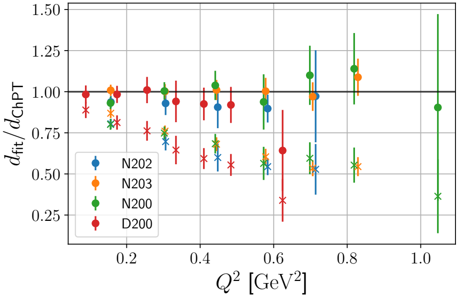
- the $N\pi$ excited state energies are fixed
- its contribution to different channels and polarizations is related
- we allow $c$, $d$, $c'$, $d'$ to be momentum-dependent fit parameters

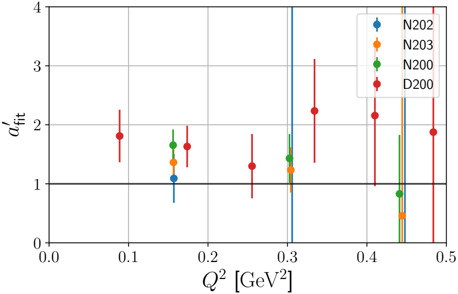
  $\Rightarrow$ we can compare to the EFT prediction

$$\left[ r_\pm = \begin{pmatrix} E_\pi \\ \pm\mathbf{q} \end{pmatrix} \right]$$

$$c' = 2a' - 2G_A \frac{2mE_\pi + 2p\cdot r_+ + m_\pi^2}{(p+r_+)^2 - m^2} \qquad c = 2a - 2G_A \frac{2mE_\pi + 2p'\cdot r_- + m_\pi^2}{(p'+r_-)^2 - m^2} \qquad \leftarrow \text{sensitive to } a,\ a'$$

$$d' = G_A \frac{4m(m+E)}{(p+r_+)^2 - m^2} \qquad d = -G_A \frac{4m(m+E')}{(p'+r_-)^2 - m^2} \qquad \leftarrow \text{parameter-free}$$




**left:**

- ratio of fitted result for $d$ obtained from the fit and its EFT prediction circles: using $G_A(Q^2)$; crosses: using $g_A$
- it is absolutely clear that $g_A \to G_A(Q^2)$ is the correct generalization to $Q^2 \neq 0$

**right:**

- no clear signal for $a'$ (only small contribution to the correlator)
- remember: $a' = 1$ is the leading order ChPT prediction ignoring smearing
- if anything, the smearing seems to increase the overlap with the $N\pi$ state slightly
- fun fact: for our kinematics this slightly decreased the overall $N\pi$ contribution (because the smearing-independent part has opposite sign)

**Next:** perform simultaneous fits to two-point functions and the ratios

$$R^{\mathbf{p}',\mathbf{p}}_{\Gamma,\mathcal{O}}(t,\tau) = \frac{C^{\mathbf{p}',\mathbf{p},\mathcal{O}}_{3\text{pt},\Gamma}(t,\tau)}{C^{\mathbf{p}'}_{2\text{pt},P_+}(t)}$$

superior to the usual choice

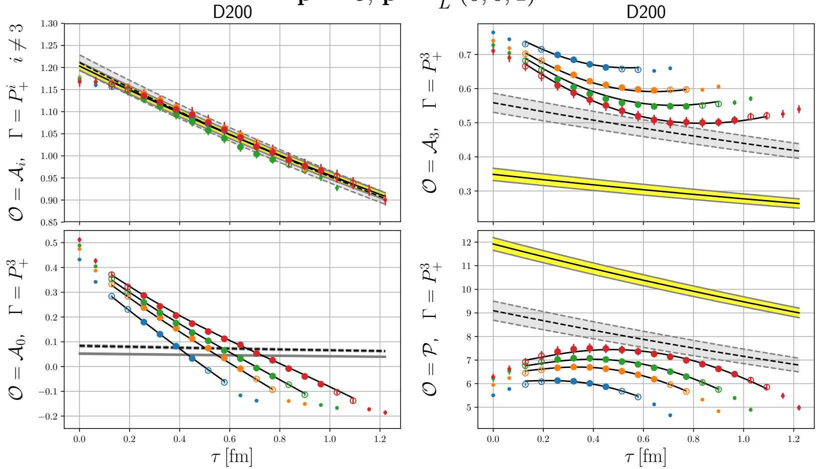
$$\frac{C^{\mathbf{p}',\mathbf{p},\mathcal{O}}_{3\text{pt},\Gamma}(t,\tau)}{C^{\mathbf{p}'}_{2\text{pt},P_+}(t)} \sqrt{\frac{C^{\mathbf{p}'}_{2\text{pt},P_+}(\tau) C^{\mathbf{p}'}_{2\text{pt},P_+}(t) C^{\mathbf{p}}_{2\text{pt},P_+}(t-\tau)}{C^{\mathbf{p}}_{2\text{pt},P_+}(\tau) C^{\mathbf{p}}_{2\text{pt},P_+}(t) C^{\mathbf{p}'}_{2\text{pt},P_+}(t-\tau)}}$$

for various reasons:

1. maximal cancellation of correlations
   (source and sink currents have same spacetime positions and phase factors)
2. no additional excited states from two-point functions at small distances $\tau$ or $t-\tau$
3. avoids negative values under the square root due to statistical fluctuations

**price to pay:** ground state contribution not flat anymore for $\mathbf{p}' \neq \mathbf{p}$

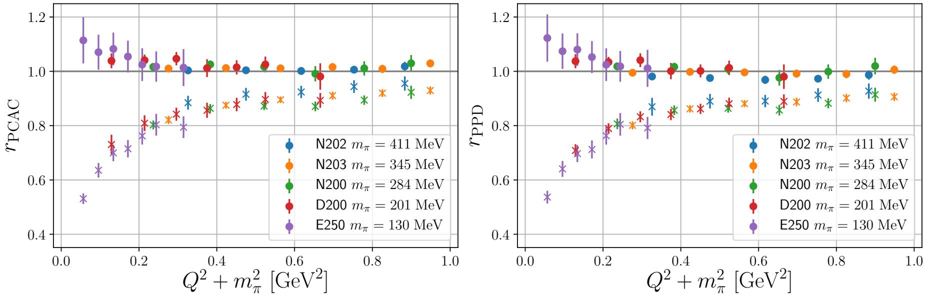
$$\mathbf{p}' = \mathbf{0},\ \mathbf{p} = \frac{2\pi}{L}(0,0,1)^T$$




yellow, solid: ground state from our fit; gray, dashed: traditional ex. state fit

- top left: responsible for $G_A$; does not give a damn about the $N\pi$ state
- lower left: obvious strange behaviour perfectly fitted
- right: huge effect; traditional fits look reasonable but are completely off

# PCAC and PPD (approximately) recovered at data level




$$r_{\text{PCAC}} = \frac{\frac{m_\ell}{m} G_P(Q^2) + \frac{Q^2}{4m^2} \tilde{G}_P(Q^2)}{G_A(Q^2)} \stackrel{?}{=} 1 \qquad r_{\text{PPD}} = \frac{(m_\pi^2 + Q^2)\tilde{G}_P(Q^2)}{4m^2 G_A(Q^2)} \stackrel{?}{=} 1$$

- standard fit (crosses $\hat{=}$ gray bands on last slide) $\Rightarrow$ huge deviations at small $m_\pi^2$
- new ansatz (circles $\hat{=}$ yellow bands on last slide) $\Rightarrow$ both approximately satisfied
- remaining deviations from PCAC due to $a$ effects
- note: PPD does not have to be satisfied... but experiment says otherwise

- previous subtraction method: only PCAC was recovered, while PPD was still violated

# subtraction method revisited

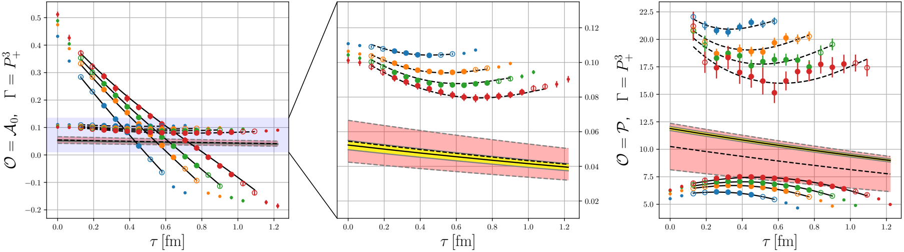
previous method: [PLB **789** (2019) 666]

- choose subtraction such that $\bar{p}_\mu \langle N|\mathcal{A}^\mu|N\rangle = 0$ is fulfilled
- choose corresponding subtraction for $\mathcal{P}$
- **can we combine this with our new method?**

# subtraction method revisited

previous method: [PLB **789** (2019) 666]

- choose subtraction such that $\bar{p}_\mu \langle N|\mathcal{A}^\mu|N\rangle = 0$ is fulfilled
- choose corresponding subtraction for $\mathcal{P}$
- **can we combine this with our new method?** $\rightarrow$ yes, but it is not advantageous




- subtraction method removes the strange linear behaviour in $\mathcal{A}_0$
- traditional fit + subtraction method: excited state effect overestimated in $\mathcal{P}$
- EFT ansatz + subtraction method (red band): leads to consistent results
- **But:** removal of clear $N\pi$ excited state signal in $\mathcal{A}_0$ $\rightarrow$ larger errors

## special case: zero momentum transfer, but $\mathbf{p}' = \mathbf{p} \neq \mathbf{0}$ (on D201)

our formula predicts that

- $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$ are not affected at all by the $N\pi$ state
- $\mathcal{A}_0$ gets an excited state contribution $\propto \exp\big(-(E_N + m_\pi/2)t\big)\cosh\big(m_\pi(\tau - t/2)\big)$

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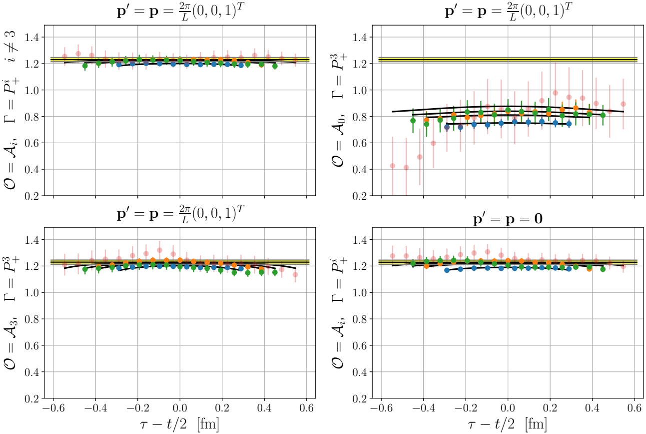
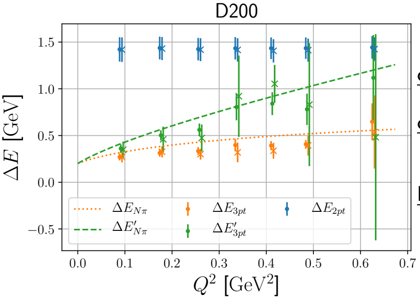



Ratio normalized such that the ground state contribution directly corresponds to $g_A$

# $N\pi$ energies from the fit




crosses: our ansatz with $\Delta E_{N\pi}$ as free parameter

circles: standard ansatz with one excited state
+ different energies in 3pt function

lines: $E_{N\pi} = E_\pi(\mathbf{q}) + E_N(\mathbf{0})$
$E'_{N\pi} = E_\pi(\mathbf{q}) + E_N(-\mathbf{q})$

- $N\pi$ energies from the fit agree with EFT prediction
- we fix the $N\pi$ energies to the EFT prediction (fits are more stable)

alternative method: [proposed in PRL **124** (2020) 072002 by Jang, et. al]

- determine excited state energy from fit to $A_0$ → apply it in other channels
- really cool idea! also recovers approximate PPD and PCAC
- difference: does not make use of the excited state structure
  → larger statistical errors
  → determination of $G_A$ is affected

# Form factor parametrization

$$G_A \equiv A(Q) \qquad \tilde{G}_P \equiv \frac{4m^2}{Q^2 + m_\pi^2} \tilde{P}(Q) \qquad G_P \equiv \frac{m}{m_\ell} \frac{m_\pi^2}{Q^2 + m_\pi^2} P(Q)$$

- now you can parametrize $X(Q)$ $(X = A, \tilde{P}, P)$ according to your wishes
- you have to use **the same** parametrization for all of them! otherwise your parametrization itself will violate PCAC and PPD
- many possible parametrizations; dipole and $z$-expansion most common

$$X(Q) = \frac{g_X}{\left(1 + Q^2/M_X^2\right)^2} \quad \text{(dipole, 2P)} \qquad X(Q) = \sum_{n=0}^{N} a_n^X z(Q)^n \quad (z\text{-exp})$$

$$z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}} \qquad t_{\text{cut}} = 9m_\pi^2,\ t_0 \text{ tuneable param.}$$

- to get correct asymptotic behaviour in $z$-exp $\Rightarrow$ 4 params. fixed $\rightarrow$ call it $z^{4+(N-3)}$

# We can do better than that!

PCAC exact in continuum → obtain $P$ from $A$ and $\tilde{P}$

$$P(Q) = \left(1 + \frac{Q^2}{m_\pi^2}\right) P_1(Q) - \frac{Q^2}{m_\pi^2} P_2(Q)$$

- $P_1(Q) = A(Q)$ and $P_2(Q) = \tilde{P}(Q)$ corresponds to exact PCAC
- additional asymptotic constraint: $g_A M_A^4 = g_{\tilde{P}} M_{\tilde{P}}^4$
- **idea:** assume $P_1(Q)$ and $P_2(Q)$ as independent FF at $a \neq 0$
  enforce $P_1(Q) = A(Q)$ and $P_2(Q) = \tilde{P}(Q)$ at $a = 0$
- also possible when using $z$-exp
- we call fits using this technique !2P, $!z^{4+3}$, etc.

# Continuum, volume, and chiral extrapolation

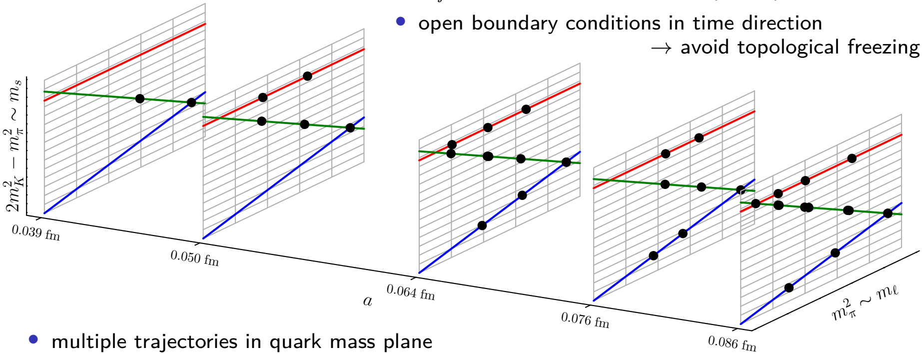
for $x \in g_X$, $M_X$, $a_n^X$ we make the generic ansatz

$$
\begin{aligned}
x &= x^{\not a} x^{a} \\
x^{\not a}(m_\pi, m_K, L) &= c_1^x + c_2^x \bar{m}^2 + c_3^x \delta m^2 \\
&\qquad + c_4^x \frac{m_\pi^2}{\sqrt{m_\pi L}} e^{-m_\pi L} + c_5^x \frac{m_K^2}{\sqrt{m_K L}} e^{-m_K L} + c_6^x \frac{m_\eta^2}{\sqrt{m_\eta L}} e^{-m_\eta L}, \\
x^{a}(a, m_\pi, m_K) &= 1 + a^2 \left( d_1^x + d_2^x \bar{m}^2 + d_3^x \delta m^2 \right)
\end{aligned}
$$

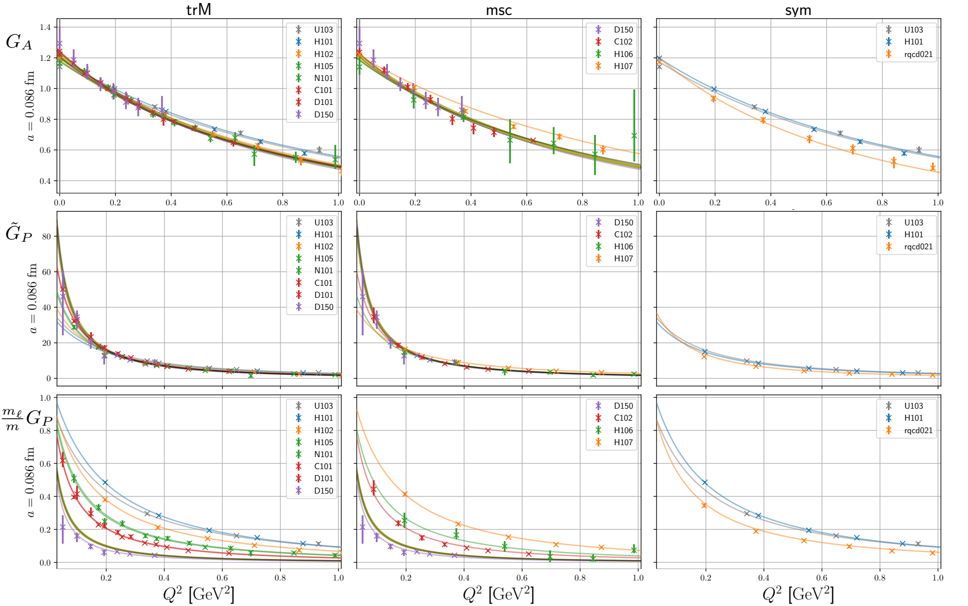
- $m_\eta^2 = (4 m_K^2 - m_\pi^2)/3$ from GMOR; $\delta m^2 = m_K^2 - m_\pi^2$; $\bar{m}^2 = (2 m_K^2 + m_\pi^2)/3$
- constraints from last slide can be implemented easily
- particular definition of $A$, $\tilde{P}$, and $P$
  $\rightarrow$ we can use the same parametrization for all FF

# Coordinated Lattice Simulations gauge ensembles

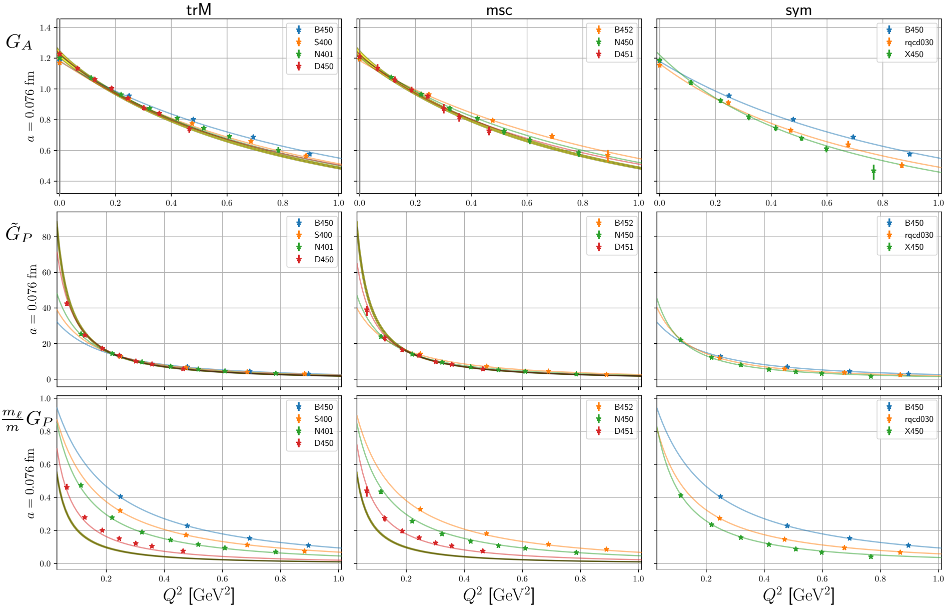
- over 40 ensembles (37 are used here)
- $N_f = 2 + 1$ dynamical Wilson (clover) fermions
- open boundary conditions in time direction
  $\rightarrow$ avoid topological freezing




- multiple trajectories in quark mass plane
- wide range of lattice spacings $0.039\,\text{fm} \leq a \leq 0.086\,\text{fm}$
- large volumes (almost all ensembles have $m_\pi L > 4$)




- simultaneous !$z^{4+3}$ fit to all ensembles ($\chi^2$/d.o.f. $= 0.83$)
- dipole (!2P) fit looks similarly convincing (even better $\chi^2$/d.o.f. $= 0.71$)

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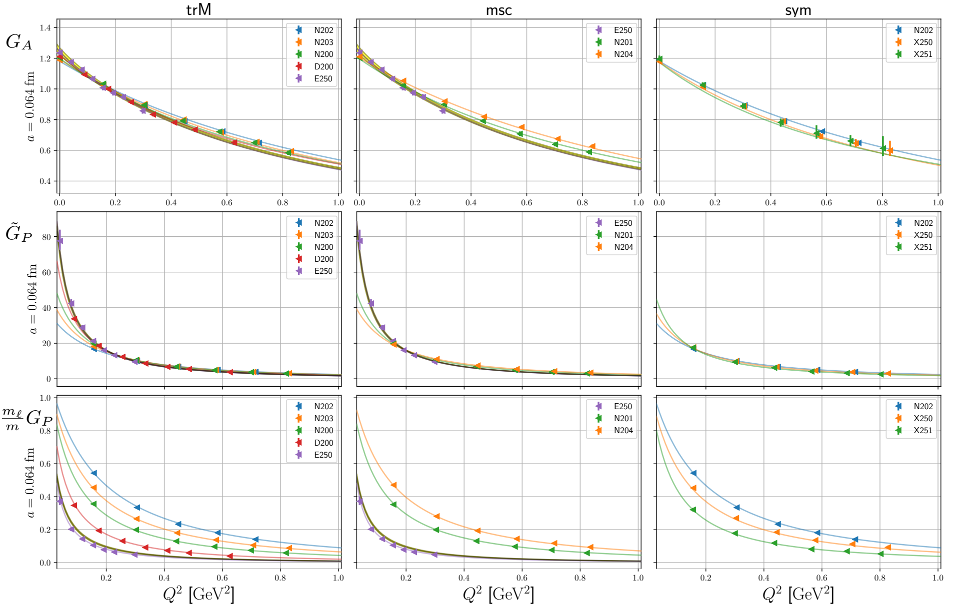


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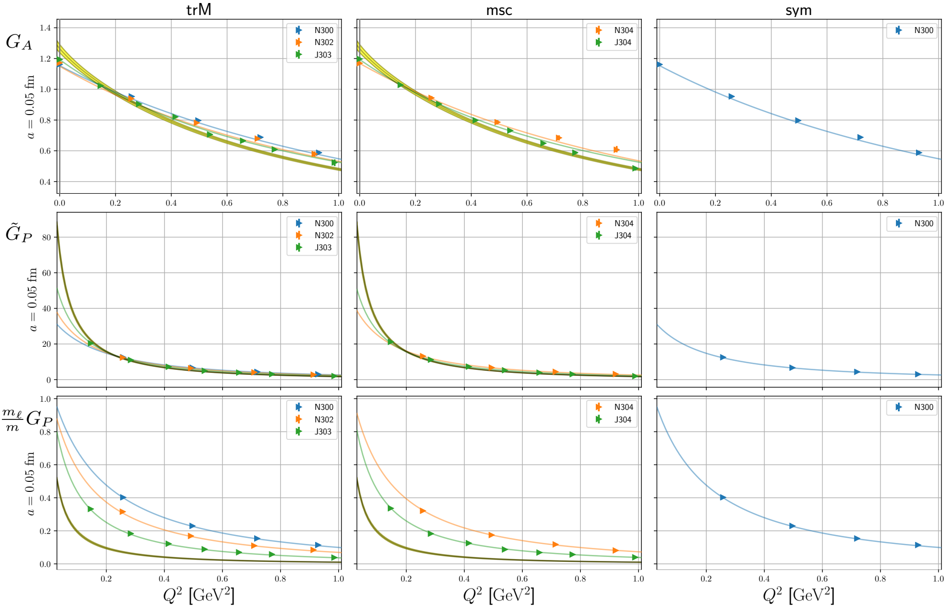


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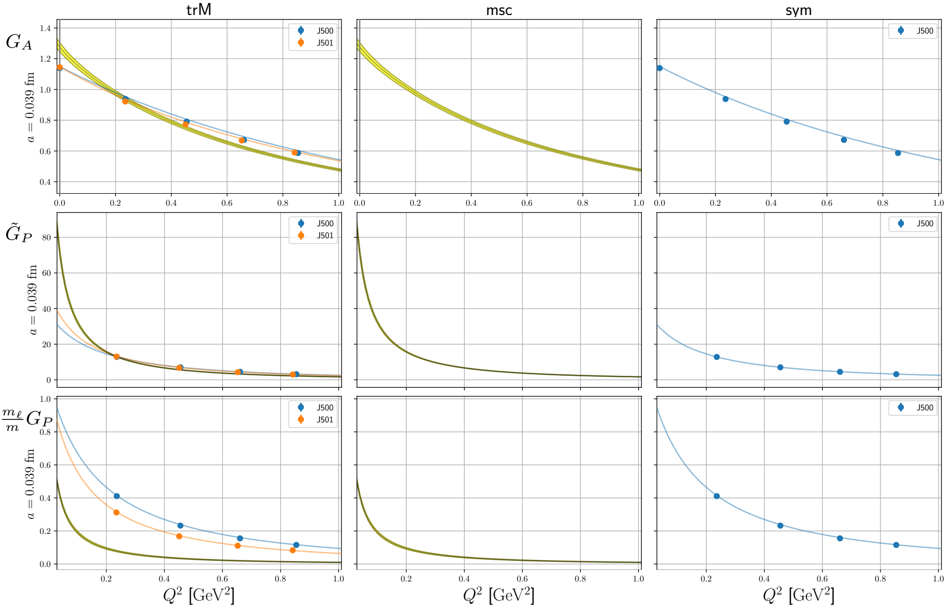


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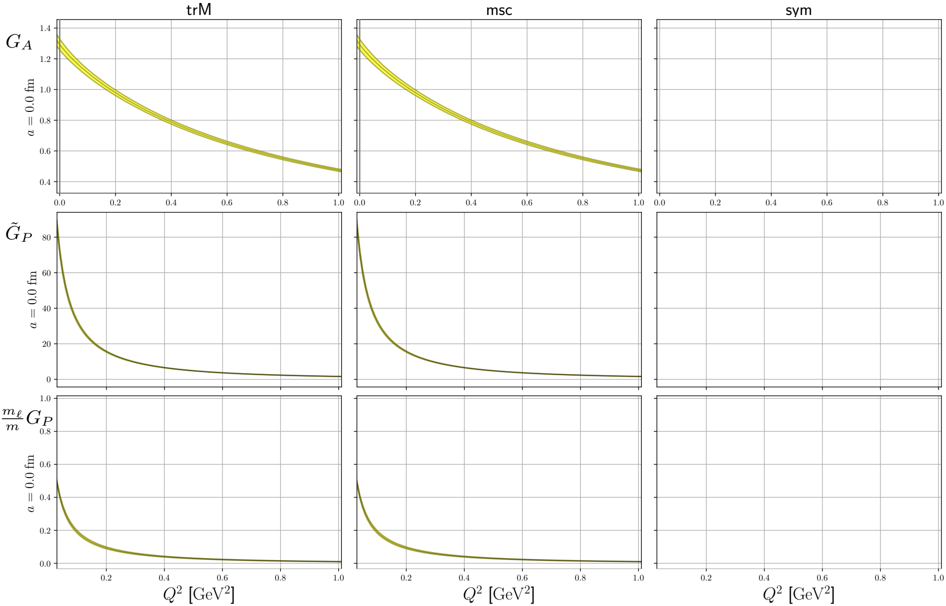


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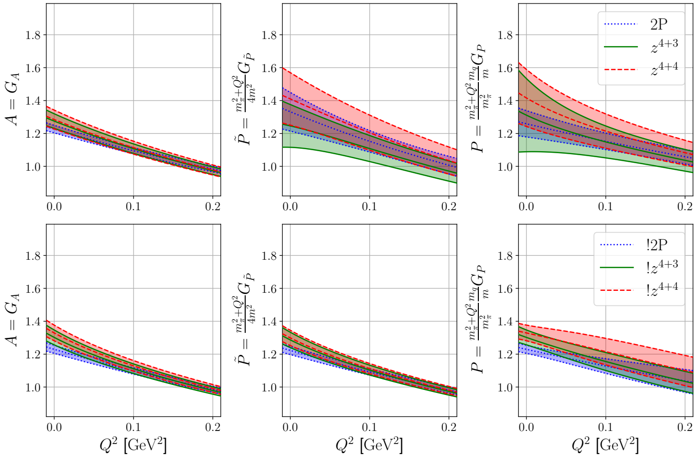
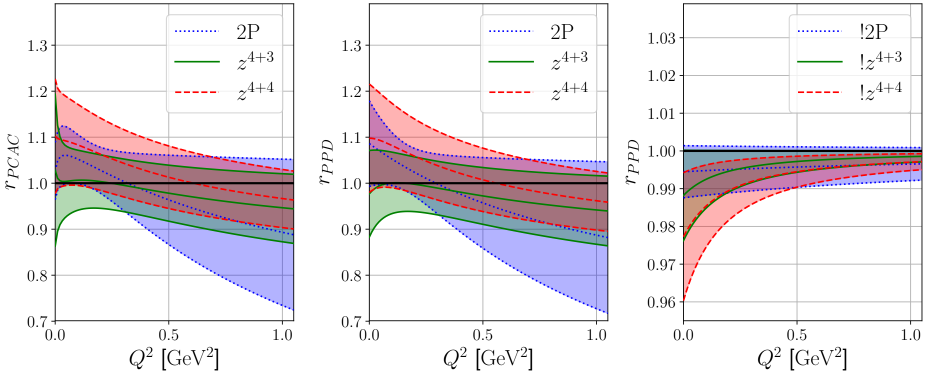
# Continuum results: form factors




- note that $A$, $\tilde{P}$, and $P$ have the same scale (perfect choice of prefactors...)
- only statistical errors
- fits with PCAC in continuum $\rightarrow$ error much smaller

# Continuum results: PCAC and PPD

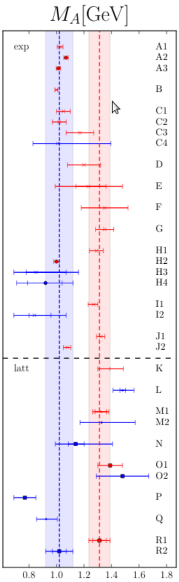



<u>fits without enforced PCAC in continuum:</u>

- PCAC (left) and PPD (center) are satisfied within the (large) statistical errors

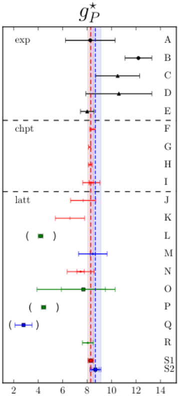
<u>fits with exact PCAC in continuum:</u>

- PCAC fulfilled automatically
- much smaller errors on $r_{\mathrm{PPD}}$ (right)
- possible deviation at small $Q^2$ up to some percent (depending on parametrization)




| id | ref. | description |
|---|---|---|
| A | [24] | reanalysis of experimental data (year $\leq$ 1999) |
| A1 | | $\nu$ scattering; various targets; world avg. year $\leq$ 1990 |
| A2 | | $\pi$ electroproduction; world avg. year $\leq$ 1999 |
| A3 | | $\pi$ electroproduction; world avg. year $\leq$ 1999; HBChPT corrected |
| B | [110] | $\nu$ scattering; reanalysis of ANL, BNL, FNAL, CERN, and IHEP data; various targets; RFG model; dipole ansatz |
| C | [12] | reanalysis of $\nu$ scattering data (from BNL [111], ANL [112], FNAL [113]) |
| C1 | | BNL data; dipole ansatz |
| C2 | | ANL data; dipole ansatz |
| C3 | | FNAL data; dipole ansatz |
| C4 | | combined analysis of BNL, ANL, and FNAL data; $z$-exp |
| D | [114] | $\nu$ scattering; K2K (SciFi); oxygen target; dipole ansatz |
| E | [115] | $\nu$ scattering; MINOS; iron target; dipole ansatz |
| F | [116] | $\nu$ scattering; MiniBooNE; carbon target; assuming RFG model; dipole ansatz |
| G | [117] | reanalysis of [116]; RFG model and spectral function model; dipole ansatz |
| H | [105] | reanalysis of MiniBooNE [116] and $\pi$ electroproduction data |
| H1 | | MiniBooNE [116] data; dipole ansatz |
| H2 | | $\pi$ electroproduction data (from refs. [118–122]); dipole ansatz |
| H3 | | MiniBooNE [116] data; $z$-exp |
| H4 | | $\pi$ electroproduction data (from refs. [118–122]); $z$-exp |
| I | [123] | analysis of MiniBooNE [124] $\bar{\nu}$ scattering data |
| I1 | | dipole ansatz |
| I2 | | $z$-exp |
| J | [125] | reanalysis of MiniBooNE data [116] |
| J1 | | LFG model; dipole ansatz |
| J2 | | LFG model + multi-nucleon reactions + RPA, etc., see [126] |
| K | [32] | $N_f = 2+1$ DWF; RBC/UKQCD; $a = 0.114$ fm |
| L | [52] | $N_f = 2+1$ Wilson (clover) fermions; $a = 0.114$ fm |
| M | [53] | $N_f = 2$ Wilson (clover) fermions; ETMC; $a = 0.0938$ fm |
| M1 | | dipole ansatz |
| M2 | | $z$-exp |
| N | [54] | $N_f = 2$ Wilson (clover) fermions; CE |
| O | [55] | $N_f = 2+1+1$ Wilson (clover-on-HISQ) fermions; PNDME; CE |
| O1 | | dipole ansatz |
| O2 | | $z$-exp |
| P | [60] | $N_f = 2$ Wilson (clover) fermions; RQCD; subtraction method; CE; $z$-exp |
| Q | [73] | $N_f = 2+1+1$ Wilson (clover-on-HISQ) fermions; PNDME; $a = 0.0871$ fm; takes into account $N\pi$ state; $z$-exp |
| R | | **This work;** $N_f = 2+1$ Wilson (clover) fermions; RQCD; full resolution of $N\pi$ state; CE |
| R1 | | dipole ansatz |
| R2 | | $z$-exp |

$$M_A \equiv \sqrt{\frac{12}{r_A^2}} = \sqrt{-2\frac{G_A(0)}{G'_A(0)}}$$




| id | ref. | description |
|---|---|---|
| A | [13] | RMC on calcium; $g_P^\star = 6.5(1.6) g_A$; point in plot obtaind by multiplying with $g_A = 1.27$ |
| B | [14, 15] | RMC on hydrogen; TRIUMF; updated value from [16] |
| C | [16] | OMC world avg. (year $\leq$ 1981) |
| D | [17] | OMC in hydrogen; Saclay; updated value from [16] |
| E | [18, 19] | OMC in hydrogen gas; MuCap |
| F | [22] | HBChPT; $M_A$ from $\nu$ scattering; assuming $g_{\pi NN} = 13.31$ |
| G | [23] | HBChPT; $M_A$ from $\pi$ electroproduction [5, 121, 122]; assuming $g_{\pi NN} = 13.0$ |
| H | [24] | HBChPT; $M_A$ from $\nu$ scattering; assuming $g_{\pi NN} = 13.10$ |
| I | [25] | covariant BChPT (EOMS); $M_A$ from $\nu$ scattering; assuming $g_{\pi NN} = 13.21$ [131] |
| J | [30] | $N_f = 2$ DWF; $a = 0.116$ fm; dipole ansatz |
| K | [32] | $N_f = 2+1$ DWF; RBC/UKQCD; $a = 0.114$ fm; dipole ansatz |
| L | [40] | $N_f = 2$ Wilson (clover) fermions; RQCD; CE; EFT ansatz corrected by missing factor of 2 |
| M | [52] | $N_f = 2+1$ Wilson (clover) fermions; $a = 0.114$ fm; $z$-exp |
| N | [53] | $N_f = 2$ Wilson (clover) fermions; ETMC; $a = 0.0938$ fm; dipole ansatz |
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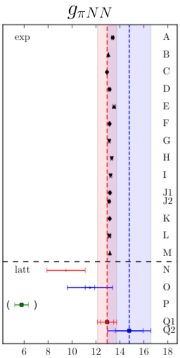
| S1 | | dipole ansatz |
| S2 | | $z$-exp |

$$g_{\tilde{P}}^\star = \frac{m_\mu}{2m} \tilde{G}_P(0.88 m_\mu^2)$$




| id | ref. | description |
|---|---|---|
| A | [134] | $\pi N$ scattering; PWA |
| B | [135–137] | $np$, $pp$ scattering; PWA |
| C | [138, 139] | $\pi N$ scattering; PWA |
| D | [140] | $\pi N$ scattering; PWA; GMO |
| E | [141] | $np$ backward cross section |
| F | [142] | $\pi N$ scattering; PWA; DR |
| G | [143] | $\pi^- p$ and $\pi^- d$ pionic atoms; GMO |
| H | [144] | $\pi^- p$ and $\pi^- d$ pionic atoms; GMO |
| I | [131] | $\pi^- p$ and $\pi^- d$ pionic atoms; GMO |
| J | [145] | $\pi N$ scattering; DR; |
| J1 | | CERN data |
| J2 | | TRIUMF data |
| K | [146] | $\pi N$ scattering; PWA; DR |
| L | [147–149] | $\pi^- p$ and $\pi^- d$ pionic atoms; GMO; including third-order ChPT corrections |
| M | [150] | $np$, $pp$ scattering; PWA |
| N | [32] | $N_f = 2+1$ DWF; RBC/UKQCD; $a = 0.114$ fm; dipole ansatz |
| O | [52] | $N_f = 2+1$ Wilson (clover) fermions; $a = 0.114$ fm; $z$-exp |
| P | [55] | $N_f = 2+1+1$ Wilson (clover-on-HISQ) fermions; PNDME; CE; EFT ansatz |
| Q | | **This work;** $N_f = 2+1$ Wilson (clover) fermions; RQCD; full resolution of $N\pi$ state; CE |
| Q1 | | dipole ansatz |
| Q2 | | $z$-exp |

$$g_{\pi NN} = \lim_{Q^2 \to -m_\pi^2} \frac{m_\pi^2 + Q^2}{4mF_\pi} \tilde{G}_P(Q^2)$$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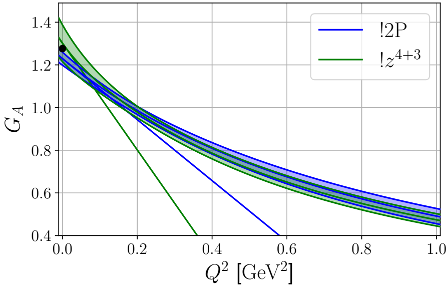
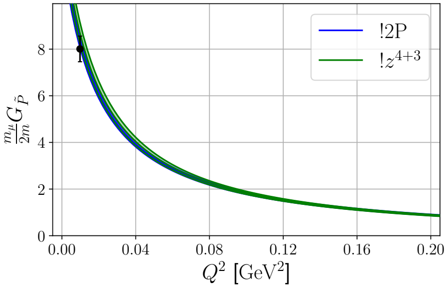
# Summary




- understanding of pion pole enhanced excited states using EFT
- $\to$ reliable extraction of axial and pseudoscalar form factors
- improved continuum limit by implementation of PCAC at $a = 0$
- we find the PPD ansatz to be fulfilled (possible deviations up to $\sim 1\%$ at small $Q^2$)
- $g_P^\star$ consistent with MuCap experiment (OMC) and ChPT determination
- we can add to the smaller *vs* larger $M_A$ controversy (but not resolve it)