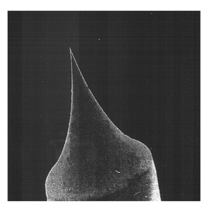

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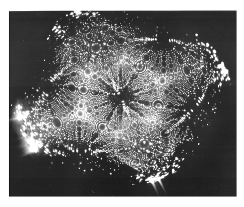

COMMENTS ON THE PRESENTATION OF FIELD EMISSION THEORY IN THE SLAC REPORT

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SUMMARY

This Poster forms part of a quiet campaign to bring the theory of field electron emission (FE), as used in technological contexts, into a common form based on: (a) the "new" rules for writing scientific equations introduced as part of the 1970s reforms associated with the introduction of SI units; and (b) a modern "21st Century" formulation of the 1956 FE theory of Murphy and Good (MG). Hopefully, establishment of a common form could provide a common background for further scientific development of FE theory and (more important) related procedures for analyzing experimental data. This Poster comments on how the presentation of FE theory in the SLAC report might be updated.

1. BACKGROUND TO MODERN FIELD ELECTRON EMISSION THEORY

The zero-temperature theory of ***field electron emission (FE)***, as developed by Murphy and Good (MG) in 1956, corrected both a conceptual error in the original 1928 treatment of Fowler and Nordheim (FN) and a mathematical error in a related 1928 paper by Nordheim. The last 20 years have seen significant additional mathematical developments in MG FE theory, and I now (in 2021) prefer to present this MG "mainstream" FE theory in the following way [1].

The so-called ***"Extended Murphy-Good (EMG) FE equation"*** [2] for the ***local emission current density (LECD)*** J^{EMG} can be written, in terms of the local work function ϕ and the magnitude F of the local barrier field, as

$$J^{\mathrm{EMG}} = \lambda a \phi^{-1} F^2 \exp[-\mathrm{v}(f)\cdot b\phi^{3/2}/F] \,, \qquad (1)$$

where a [≅ 1.541434 μA eV V^{-2}] and b [≅ 6.830890 eV$^{3/2}$ V nm^{-1}] are the FN constants. The factor λ replaces the factor t^{-2} in 1956 MG theory as applied at 0 K; λ is an **uncertainty factor** that *formally* takes account of various physical effects (notably atomic-wave-function effects) disregarded in 1956 MG theory.

The exponent correction factor $\mathrm{v}(f)$ is a particular value of the ***principal field emission special mathematical function*** $\mathrm{v}(x)$ [1], and is obtained by setting the mathematical ***Gauss variable x*** equal to the ***scaled field f for a Schottky-Nordheim (SN) barrier of zero-field height*** ϕ. This parameter f is a modelling parameter for the SN barrier of height ϕ, and is defined by

$$f \equiv c^2\phi^{-2}F \cong \{1.439965 \text{ eV}^2 \text{ (V/nm)}^{-1}\}\, \phi^{-2}F \,, \qquad (2)$$

where c [$\equiv(e^3/4\pi\varepsilon_0)^{1/2}$] is a universal constant called the ***Schottky constant.***

Use of the variable x and the parameter f has replaced use of the Nordheim parameter y [$=\sqrt{f}=\sqrt{x}$], on the grounds that: (a) it is best procedurally to separate the pure mathematics of $\mathrm{v}(x)$ from issues relating to the modelling of the SN barrier; (b) in the mathematics the variable x is more appropriate than y; and (c) f is better than y for interpreting FE experiments (see [1]).

For $\mathrm{v}(x)$ we now have: (a) an exact series expansion; (b) a "high precision" approximation [1]; and (c) a "simple good approximation", effective over the range $0 \leq x \leq 2$ (maximum error in $0 \leq x \leq 1$ is 0.33%), with the form

$$\mathrm{v}(x) \approx 1 - x + (1/6)x\ln x. \qquad (3)$$

Many older approximations for $\mathrm{v}(x)$ also exist, most of them now obsolete.

2. THE CURRENT SITUATION

Murphy-Good (1956) FE theory is not a physically accurate theory, because it is based on an underlying (1920s) physical model that disregards the existence of atoms and uses a Sommerfeld-type free-electron approach to model the emitter as having a smooth planar surface of large lateral extent. I call this approach ***smooth planar metal-like emitter (SPME) methodology.***

In due course, there will be a need to move on from SPME methodology and MG FE theory. In principle, there are many different corrections (some very fundamental) that need to be explored and implemented, and the time-scale needed to do this is very long (many years). The most immediate needs appear to be to correct for: (a) non-planar emitter shape, when evaluating expressions for emission current; (b) the effect of emitter curvature on the evaluation of electron transmission probabilities, particularly for highly curved emitters; and (c) the effect of atomic-level wave-functions on the emission process.

There are already a fair number of published theoretical investigations into these and other corrections, and into how to use them to predict emission current densities and currents. However, there are few investigations, as yet, into how to use these corrected theories to interpret experimental results. Thus, as yet, we do **not** know how to reliably carry out convincing scientific checks on the *quantitative* validity of these "corrected" theories, although they do exhibit expected qualitative trends. Thus, it seems to the author that SPME methodology and EMG theory will continue to be useful for some time to come−−I would guess at least the next 5–10 years, maybe longer.

In practice, the biggest current problems seem to be (a) the extensive confusion that exists in FE technological literature and (b) the plethora of different variants of SPME methodology. A useful step would be to persuade all who use SPME methodology to use the same or similar versions, thus providing a common background for further scientific development. This Poster discusses the application of this thinking to the FE theory in the SLAC Report [3].

3. THE FE THEORY IN THE SLAC REPORT

Although more-sophisticated FE theory [4] has been used in some recent simulations, earlier discussions of FE in the context of vacuum breakdown in accelerators have on several occasions used the FE theory in the so-called "SLAC Report" [3], from the Stanford Linear Accelerator Centre. The quantitative results of this "SLAC Report FE theory" are acceptable in SPME methodology, although they are not necessarily the best in all circumstances (see Section 5. below). The remarks here primarily concern the *format* in which the SLAC Report FE theory is presented.

In the SLAC report, the expression given for the MG (1956) FE local emission current density (j_F in the SLAC report) is:

$$j_F = [1.54\times10^{-6}\, E^2/t^2(s)\,\phi]\, \exp[-6.83\times10^{9}\, \phi^{1.5}\, v(s)/E] \,, \qquad (4)$$

where E is stated to be the surface electric field measured in V/m, ϕ is stated to be in eV, and

$$s = 3.79\times10^{-5}\, E^{0.5}/\phi \,, \qquad (5)$$

$$v(s) = 0.956 - 1.062\, s^2 \,. \qquad (6)$$

By substituting these into (4) and letting $\mathrm{t}^2(\mathrm{s})\approx1$, the SLAC report derives (after further algebraic manipulation) the "all-in-one" approximate formula

$$j_F = [1.54\times10^{-6}\times10^{(4.52/\sqrt{\phi})}E^2/\phi]\, \exp[-6.53\times10^{9}\phi^{1.5}/E] \;\; (\mathrm{A/m^2}). \qquad (7)$$

4. COMMENTS ON the SLAC REPORT PRESENTATION

Comparisons of Sections 1 and 3 yield the following comments.

(a) Eqns (4), (5) & (7) are in 1960s-style "numerical" format. In consequence, although these equations are entirely comprehensible, they do not technically conform to the new rules for writing scientific equations introduced as part of the 1970s reforms, and are dimensionally inconsistent. It would be better practice, and closer to what is now taught in universities, to put constants in the equations and give values for the constants separately. In this case (as required by the 1970s reforms) there is no need to state the units in which quantities in the main equation are measured. Also, when numerical equations are used, then the underlying physics is lost, at least to some extent.

(b) In some papers that use the SLAC Report theory, only eq. (7) is given. This practice of *only* giving the "all-in-one" equation is non-transparent, and makes it difficut for the non-expert to disentangle what the underlying physics is, and what mathematical approximations have been used. It is clearer to state the LECD equation in a form that contains the correction factors "v" and "t" (or λ), with the approximations used for "v" (and, if appropriate, "t") given separately.

(c) Even if an all-in-one formula is used, the term $10^{(4.52/\sqrt{\phi})}$ looks to be more complicated than necessary. In terms of approximation (6) and the constants b and c used in Section 1, dimensionally consistent alternatives for this term are:

$$\exp[1.062\, bc^2\phi^{-1/2}] \cong \exp[10.45\, (\mathrm{eV}/\phi)^{1/2}] \,. \qquad (8)$$

(d) The SLAC Report uses the Nordheim parameter as the barrier modelling parameter but denotes this by s rather than the conventional symbol y. If the Nordheim parameter is to be used, then it would be better to use the standard symbol y and the dimensionally consistent post-reform definition

$$y \equiv c\phi^{-1}E^{1/2} \cong \{3.794686\times10^{-5}\ \mathrm{eV\ (V/m)^{-1/2}}\}\, \phi^{-1}E^{1/2} \,. \qquad (9)$$

However, the author's view (as stated earlier) is that using the scaled field f is the best approach, with approximation (6) above rewritten in the form

$$v(f) \approx 0.956 - 1.062\, f \,. \qquad (10)$$

This approximation (but stated in terms of y) was in fact developed in 1962 by Charbonnier and Martin [5]; the SLAC Report does not record this.

(e) Three small differences between Sections 1 and 3 are matters of author choice. **(i)** The electrostatic field E^{ES} at a field electron emitter surface is in fact negative, but convention demands that positive field-quantities be inserted into FE LECD equations. For clarity, the author prefers to use F (rather than E) to denote the (positive) absolute magnitude of the surface electrostatic field. **(ii)** The author typesets the symbols "v" and "t" upright, on the grounds that these symbols (like "**sin**") represent special mathematical functions. **(iii)** For information, the author gives values of universal constants to 7 significant figures (but expects these to be appropriately rounded in practice).

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5. ACCURACY OF THE CHARBONNIER-MARTIN APPROXIMATION

To discuss the mathematics of the Charbonnier-Martin (CM) approximation used in the SLAC Report, I now prefer to write it as a function of the Gauss variable $\boldsymbol{x}$, as

$$\mathbf{v}(\boldsymbol{x}) \approx 0.956 - 1.062\,\boldsymbol{x}\ . \qquad (11)$$

The CM approximation, despite its early date (1962), is one of the better of the many (about 15–20) 20th-Century approximations for $\mathbf{v}(\boldsymbol{x})$. Matters of interest here are the mathematical accuracy of the CM approximation, and its performance as compared with the "simple good approximation" ("F06 approximation") (3).

Comparisons of this type are now easy, because the high-precision approximation [1] (which has a maximum error-magnitude of less than 10^{-9} in the range $0 \leq \boldsymbol{x} \leq 1$) has been coded into an EXCEL spreadsheet.

There is a question: over what range of $\boldsymbol{x}$–values is it best to make comparisons? With FE point electron sources of work function 4.5 eV, emitters usually operate [6] within the scaled-field range $0.20 \leq \boldsymbol{f} \leq 0.35$. However, I find it more appropriate to make comparisons over the range $0.15 \leq \boldsymbol{f} \leq 0.45$. This corresponds to the PASS range of the orthodoxy test [7], and (obviously) corresponds to the Gauss-variable range $0.15 \leq \boldsymbol{x} \leq 0.45$. Over this range, the precise (to 5 decimal places) value of $\mathbf{v}(\boldsymbol{x})$ varies smoothly from 0.80021 down to 0.48861.

A comparison of different approximations was presented some years ago [8]. This Poster re-presents relevant comparisons and draws additional conclusions.

Figure 1 shows the errors in the "simple good approximation" (F06) and in the CM approximation, as functions of $\boldsymbol{x}$, as compared with the high-precision (HP) approximation. It is seen that the error in the CM approximation is small and negative for $\boldsymbol{x}$ in the range $0.20 \leq \boldsymbol{x} \leq 0.30$ and just outside this, but that this error increases rapidly *in magnitude* (gets increasingly negative) as $\boldsymbol{x}$ approaches and exeeds 0.35. By contrast, the error in the F06 approximation is a slowly varying function of *x*.

Figure 2 shows the difference in the error magnitudes for the two approximations. In the range $0.20 \leq \boldsymbol{x} \leq 0.30$, and just outside this, the CM approximation is the better approximation, but for $\boldsymbol{x}$-values well outside this range the F06 approximation is better.

What this analysis brings out is that the SLAC Report uses an approximation for $\mathbf{v}(\boldsymbol{x})$ that was originally designed by CM for use with field emitters intended as electron sources for electron microscopes. It is arguable that in vacuum breakdown contexts it might be better to use the F06 approximation [eq. (3) above], which performs reasonably well across the whole range $0 \leq \boldsymbol{x} \leq 1$]. In the unlikely event that even better performance is required, then the 9-term "high-precision approximation" [1] (with maximum error magnitude less than 10^{-9}) is available. [Also note that the derivation of Murphy-Good type zero-temperature FE equations breaks down *physically* for $\boldsymbol{f}$-values greater than about 0.8.]

Fig. 1 Errors in v(F06)(blue) and in v(CM)(red)

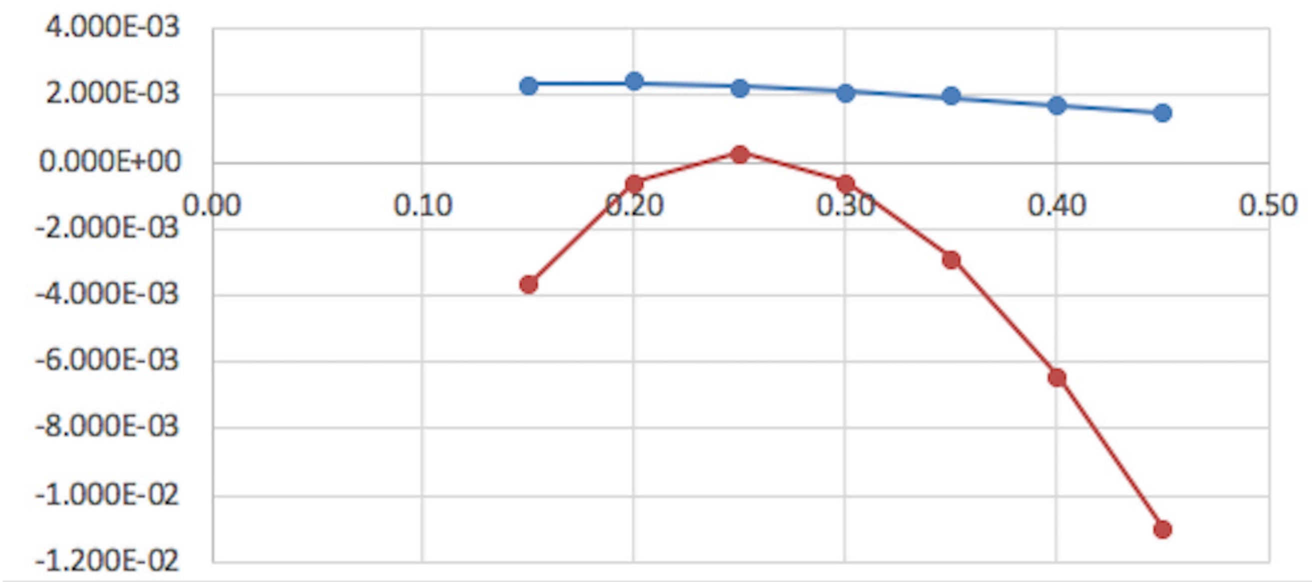
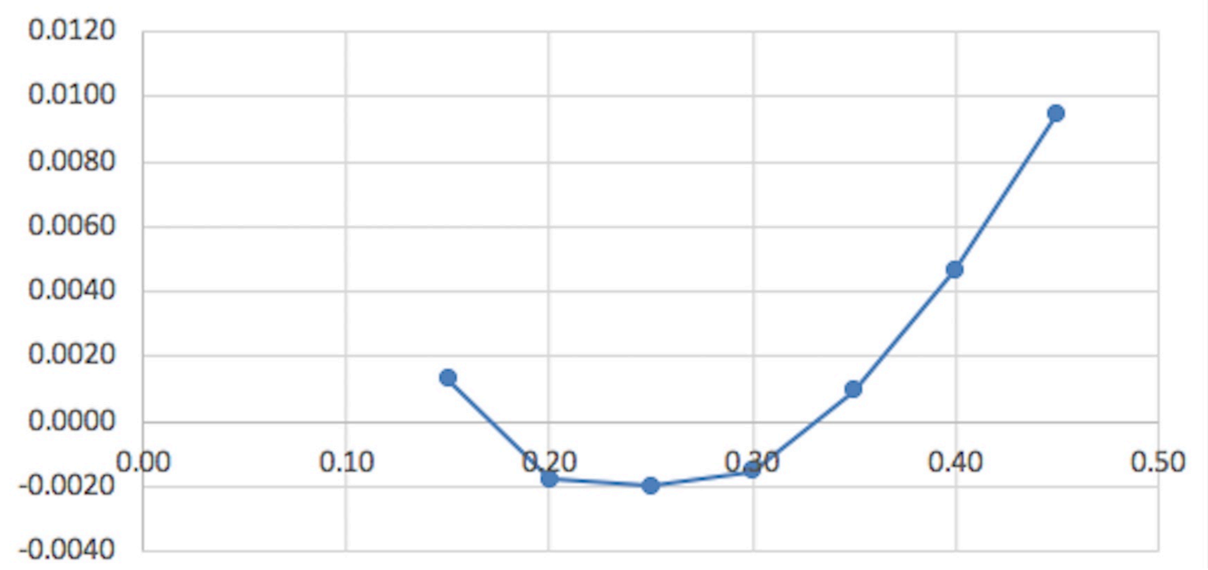


Fig. 2 Error-magnitude (E) Difference [E(CM)-E(F06)]



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