The Harvey M. Krueger Family Center for Nanoscience and Nanotechnology

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

New Methods of β Estimation

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Why do we need another method?

- Current analysis methods of dark current only account for the mean behavior of the measurement.
- These methods do not allow for field specific measurements of β.
- These methods also require a range of field measurements to be implemented and therefore cannot be done in real-time during operation.

- In this talk I will present two new methods of β estimation that allow for field specific measurements that are faster, can be done in real time, and, theoretically, more precise than current methods.

Current Estimation Method

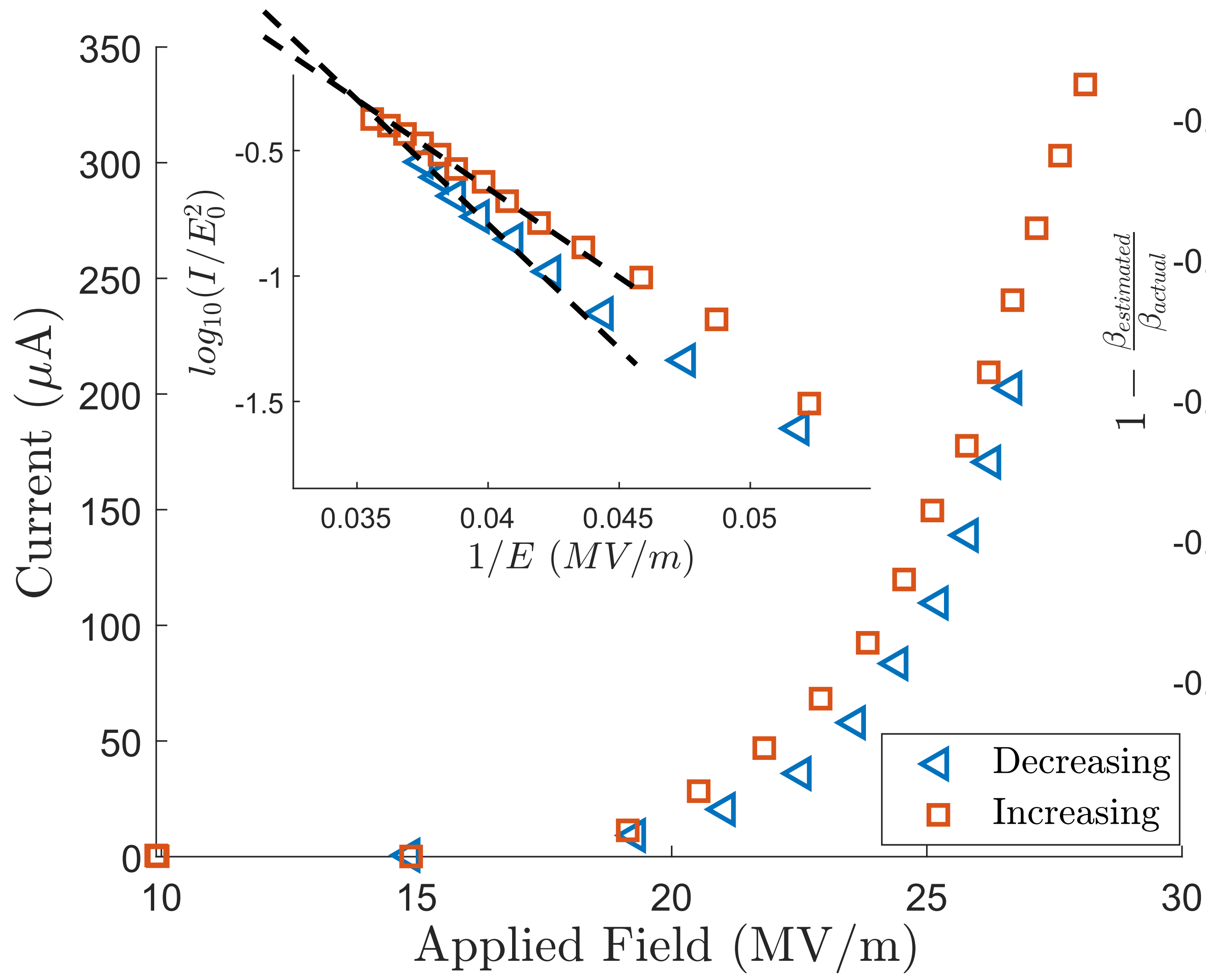
- The most commonly used estimation method is that of G. A. Loew and J. W. Wang[1]:
- In this method, the applied field and mean current are measured over a range of fields. The slope of the semi-log plot of $\log_{10} I/E^2$ as a function of $1/E$ is calculated and from that:

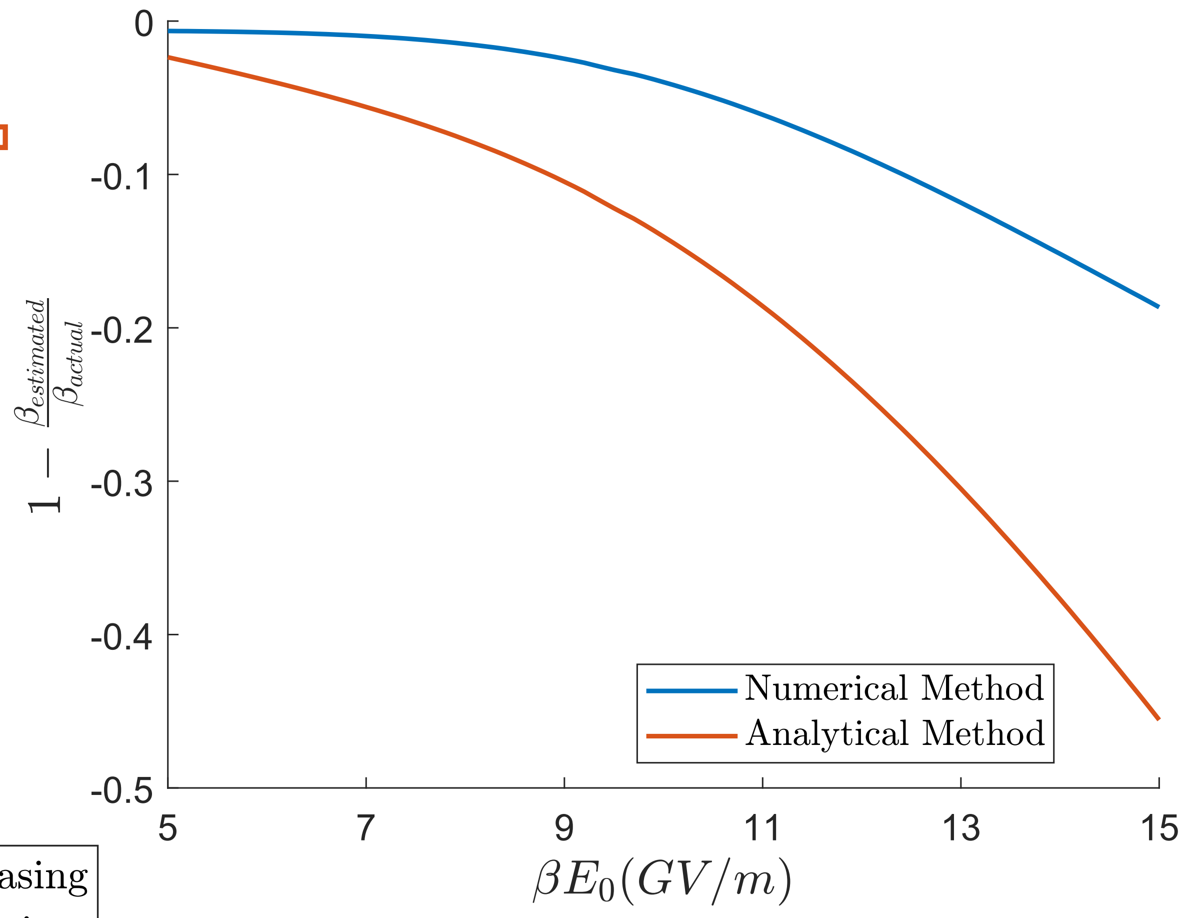
$$\frac{d\log_{10}(I/E^2)}{d(1/E)} = -\frac{G\phi^{3/2}}{\beta}; \quad G = 2.84 \times 10^9 \left(\frac{V/m}{eV^{3/2}}\right)$$

- ϕ is the work function of the electrode.

[1]G. A. Loew and J. W. Wang. Progress report on high-gradient rf studies in copper accelerator structures, Sep 1990

Current Estimation Method





Data courtesy of Iaroslava Profatilova

New Methods

- The new methods that I propose both allow for the field specific estimation of β.
- These methods both rely on statistical analysis of the dark current, are faster to implement and, theoretically, more precise.
- One of these methods requires minimal modification of the experimental setup.
- The other requires a higher time resolution to implement.

Field Variation

- The first, and more convenient, method is the field variation method.
- This method relies on a small variation of the applied field during the operation of the system.

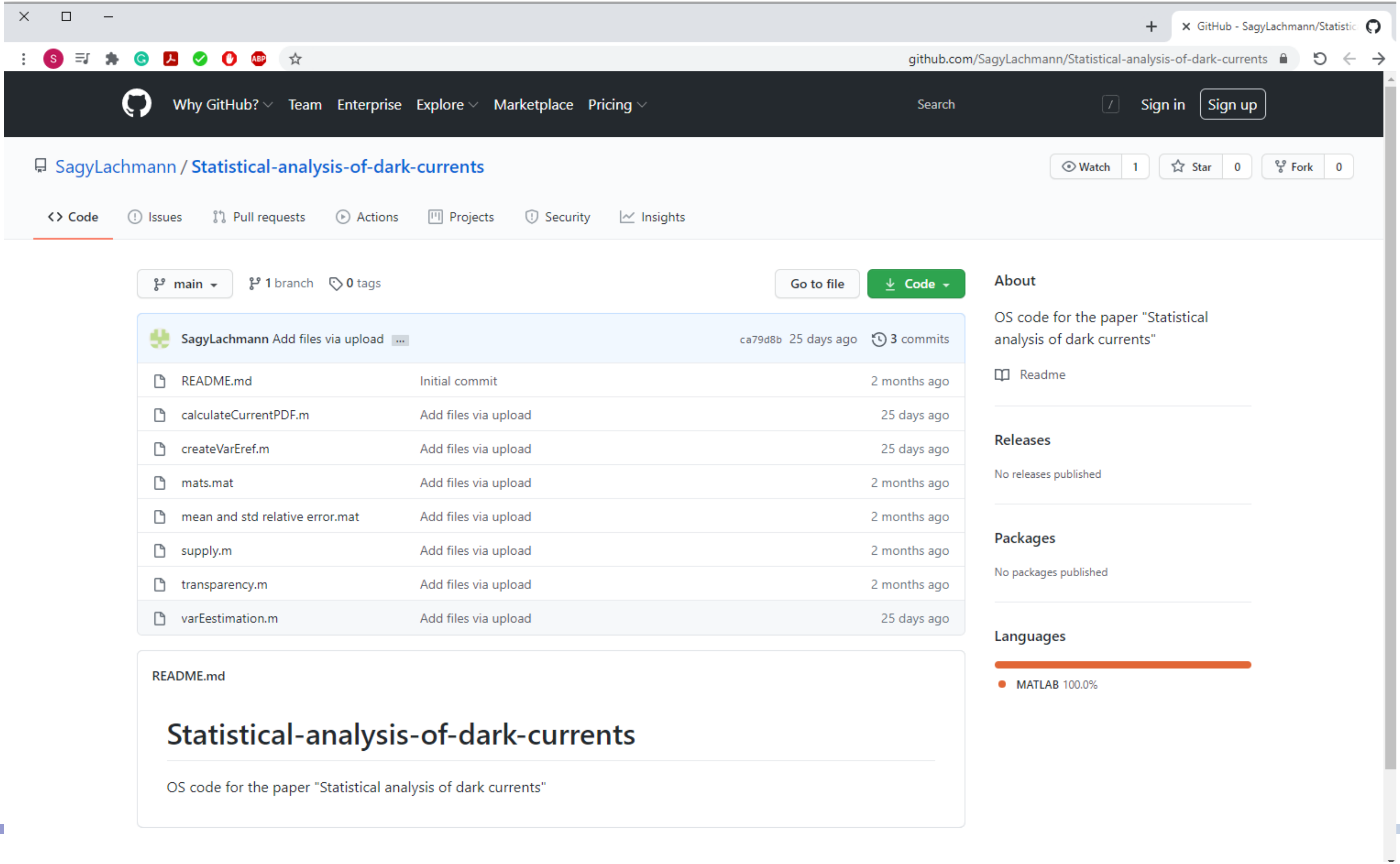
$$\sigma_I = \frac{\partial I}{\partial E_0}\sigma_{E_0} = \beta\frac{\partial I}{\partial E}\sigma_{E_0} \rightarrow \eta(E) = \frac{1}{I}\frac{\sigma_I}{\sigma_{E_0}} = \frac{\beta}{I}\frac{\partial I}{\partial E}$$

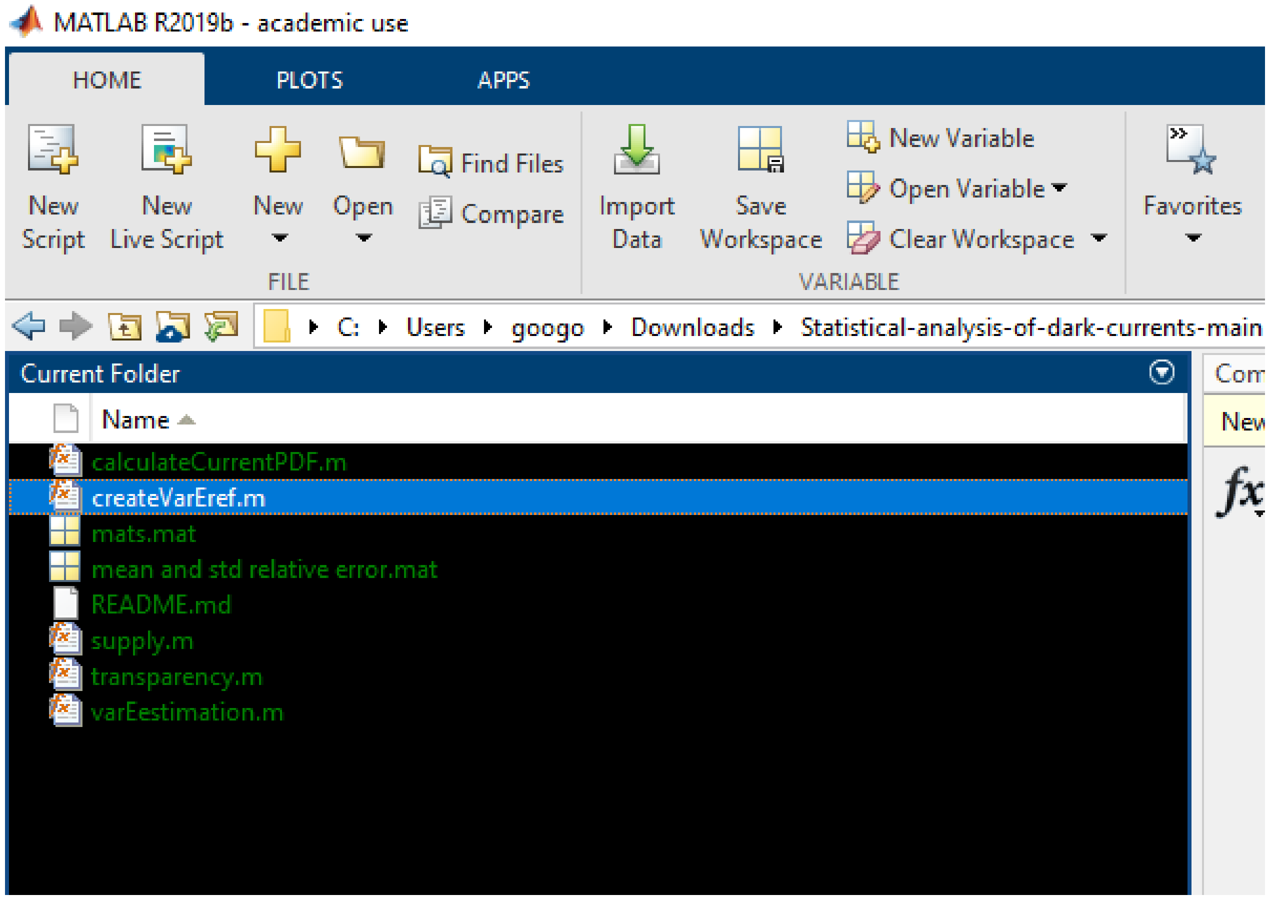
- The estimated β is therefore

$$\beta = \frac{E(\eta)}{E_0}$$

- The code needed to implement this method is freely available at https://github.com/SagyLachmann/Statistical-analysis-of-dark-currents

How to use the code




MATLAB R2019b - academic use
HOME
PLOTS
APPS
New Script
New Live Script
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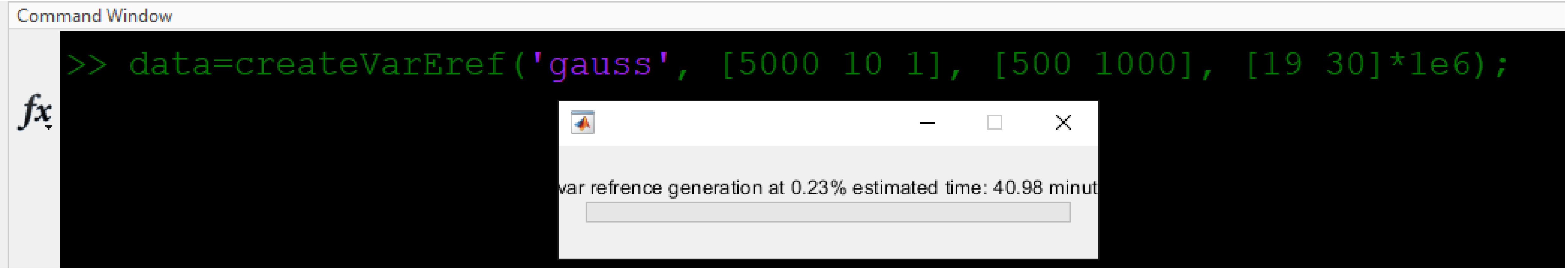
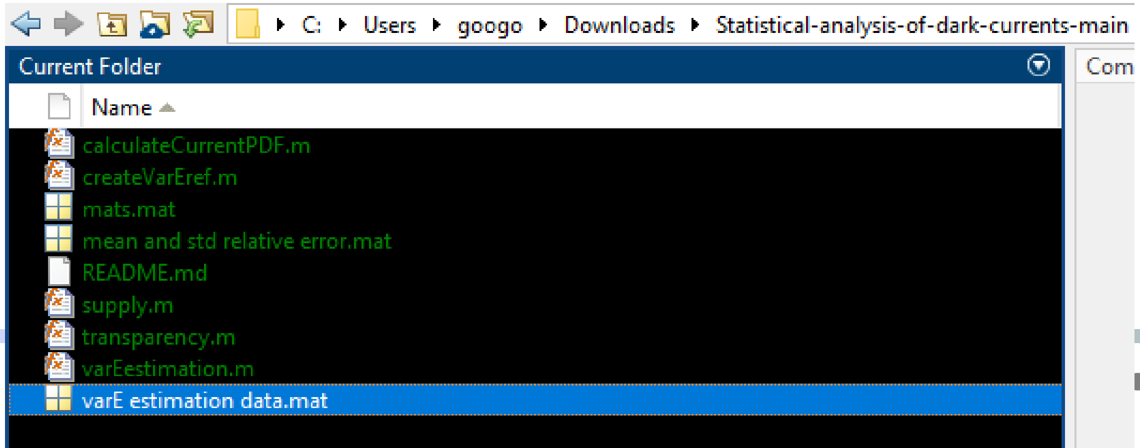
Open
Find Files
Compare
FILE
Import Data
Save Workspace
New Variable
Open Variable
Clear Workspace
VARIABLE
Favorites
C: Users googo Downloads Statistical-analysis-of-dark-currents-main
Current Folder
Name
calculateCurrentPDF.m
createVarEref.m
mats.mat
mean and std relative error.mat
README.md
supply.m
transparency.m
varEestimation.m

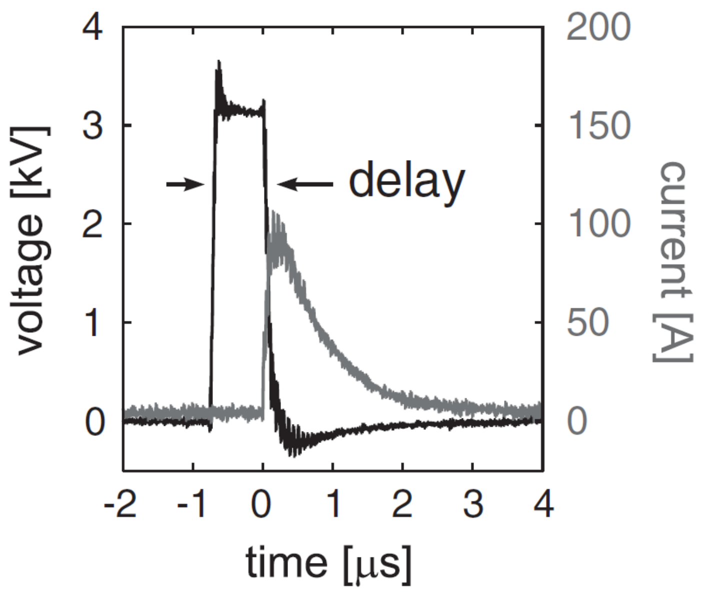


- type=gauss/saw/sine
- params=[σ_{E_0} amplitude, # of points, integration time] (Volts/m, #, seconds)
- beta_range=range of expected β values
- E0_range=range of applied fields that will be used (Volts/m)

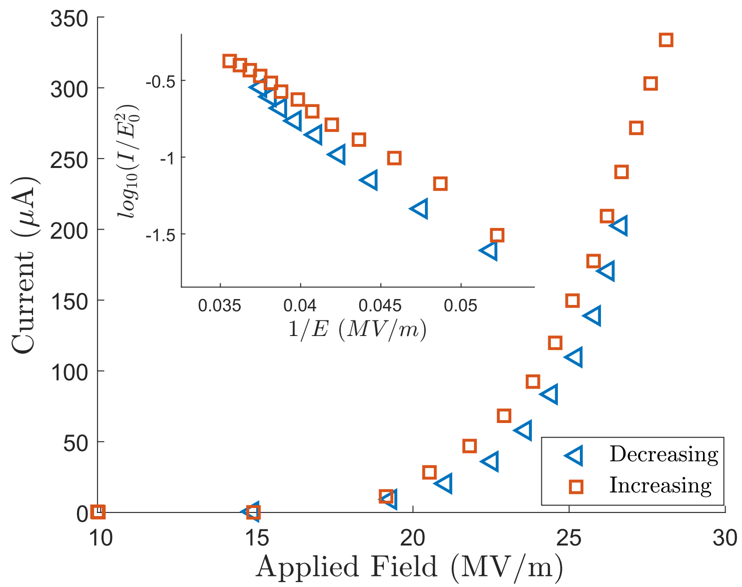
- Save the generated structure 'data'

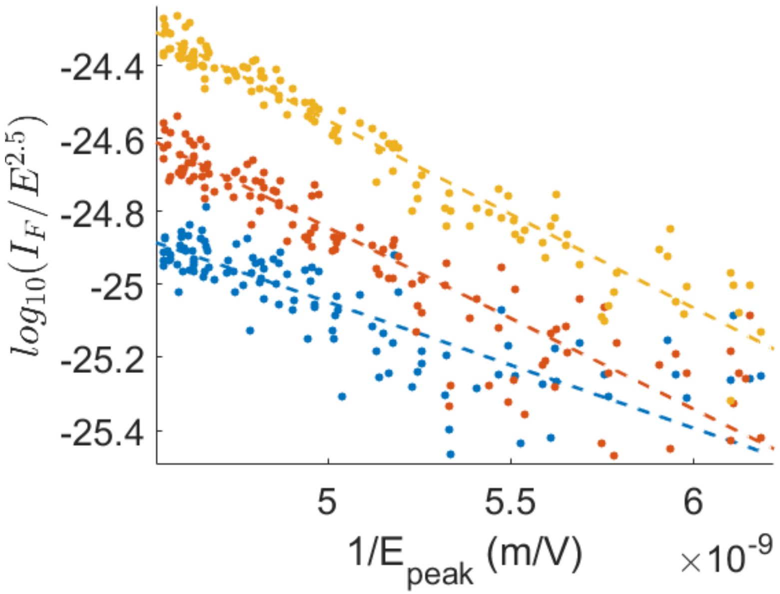


MEASURE!



A. Descoeudres et. al, Investigation of the dc vacuum breakdown mechanism, 2009





J. Paszkiewicz, Temporal and Spatial Measurement of Field Emission, Mini MeVArc 2018

MEASURE!

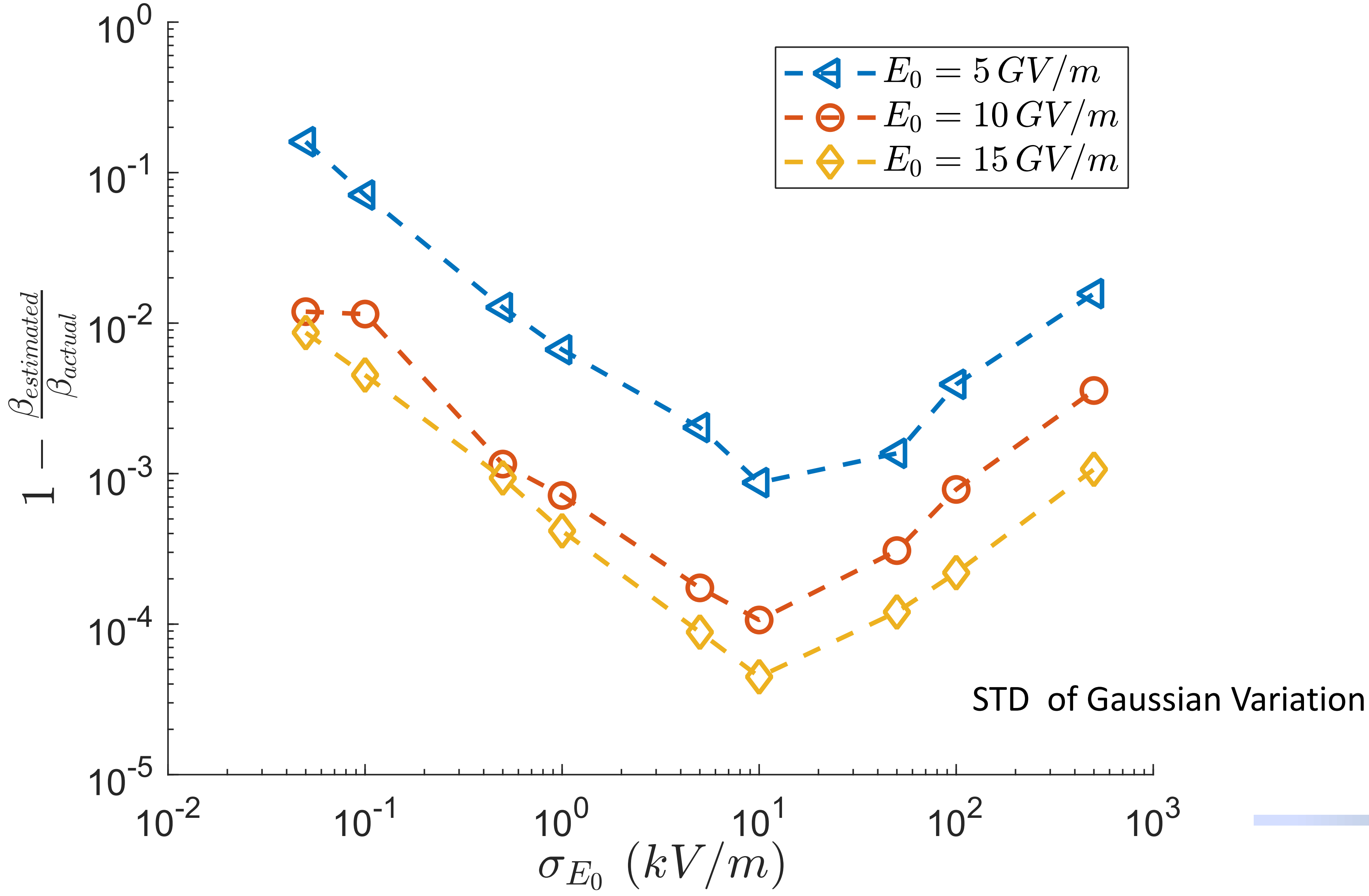
```
>> m=10;
E=(randn(1,m)*50+1.95e5)*100;
for j=1:m
temp=calculateCurrentPDF('Cu', E(j)/100*540, 293, 1,1);%returns 2.5e4 points
temp_I=sum(temp,2);
I(j)=mean(temp_I);
end
```

$$\beta_{actual} = 540, \beta_{estimated} = 540.5 \pm 106.7$$
$$error \cong 0.09\%$$

```
>> m=10;
E=(randn(1,m)*50+2.44e5)*100;
for j=1:m
temp=calculateCurrentPDF('Cu', E(j)/100*797, 293, 1,1);%returns 2.5e4 points
temp_I=sum(temp,2);
I(j)=mean(temp_I);
```

$$\beta_{actual} = 797, \beta_{estimated} = 795.8 \pm 167.9$$
$$error \cong 0.15\%$$

Choose Your Variation Carefully!



Shot noise estimation

$$N(W) = \frac{4\pi m k_B T}{h^3} L\left(\frac{W-\mu}{kT}\right); \; L(x) = ln(1+e^{-x})$$

$$D(W) = \left\{1+\exp\left[\frac{4\sqrt{2}}{3}\left(\frac{E\hbar^4}{m^2 e^5}\right)^{-1/4} y^{-3/2} v(y)\right]\right\}^{-1} =$$

$$= \left[1+\exp\left(H(U-W)^{3/2} v(y) E^{-1}\right)\right]^{-1}$$

Numerical Gaussian Distribution

$$\mu_W = SD(W)N(W)$$

$$\sigma_W = Sdt^{-1/2}\sqrt{(1-D(W))D(W)N(W)}$$

Integration by energy

$$\mu = S\int D(W)N(W)dW$$

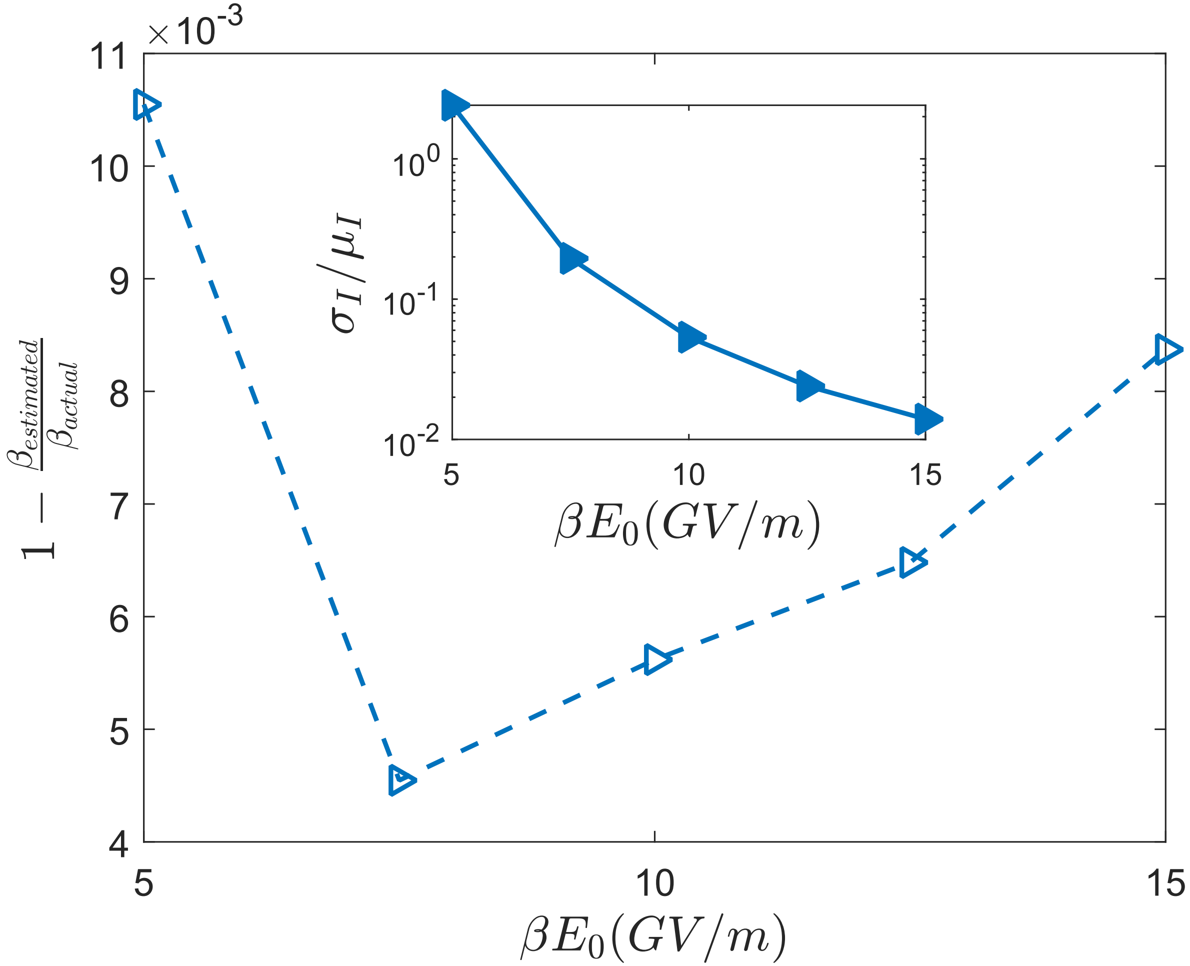
$$\sigma = Sdt^{-1/2}\sqrt{\int (1-D(W))D(W)N(W)dW}$$

Shot noise estimation

$$\mu = S\int D(W,E)N(W)dW$$

$$\sigma = Sdt^{-1/2}\sqrt{\int \big(1 - D(W,E)\big)D(W,E)N(W)dW}$$

$$\widetilde{\boldsymbol{D}}(\boldsymbol{E}) = \mathbf{1} - \frac{\boldsymbol{\sigma}}{\boldsymbol{\mu}}$$



Conclusions

- The current method of β estimation has several limitations.
- The methods presented offer new possibilities for the monitoring and characterization of high field systems.
- The field variation method is ready for implementation with the online code https://github.com/SagyLachmann/Statistical-analysis-of-dark-currents
- For shot noise implementation you are welcome to contact me at sagy.lachmann@mail.huji.ac.il .

THANK YOU!