



# New Methods of $\beta$ Estimation

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# Why do we need another method?

- Current analysis methods of dark current only account for the mean behavior of the measurement.
- These methods do not allow for field specific measurements of  $\beta$ .
- These methods also require a range of field measurements to be implemented and therefore cannot be done in real-time during operation.
- In this talk I will present two new methods of  $\beta$  estimation that allow for field specific measurements that are faster, can be done in real time, and, theoretically, more precise than current methods.



# Current Estimation Method

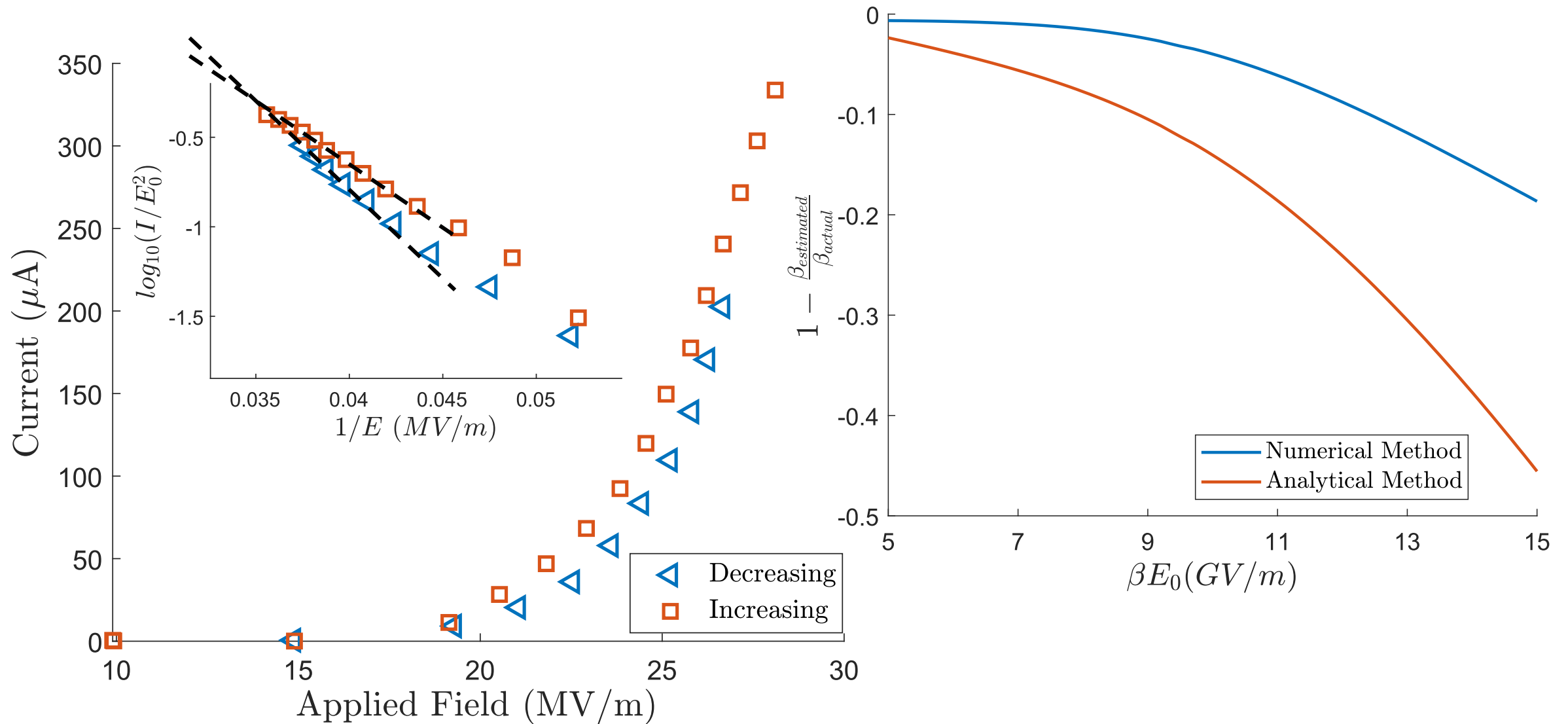
- The most commonly used estimation method is that of G. A. Loew and J. W. Wang<sup>1</sup>:
- In this method, the applied field and mean current are measured over a range of fields. The slope of the semi-log plot of  $\log_{10} I/E^2$  as a function of  $1/E$  is calculated and from that:

$$\frac{d \log_{10}(I/E^2)}{d(1/E)} = -\frac{G\phi^{3/2}}{\beta}; \quad G = 2.84 \times 10^9 \left( \frac{V/m}{eV^{3/2}} \right)$$

- $\phi$  is the work function of the electrode.



# Current Estimation Method



# New Methods

- The new methods that I propose both allow for the field specific estimation of  $\beta$ .
- These methods both rely on statistical analysis of the dark current, are faster to implement and, theoretically, more precise.
- One of these methods requires minimal modification of the experimental setup.
- The other requires a higher time resolution to implement.



# Field Variation

- The first, and more convenient, method is the field variation method.
- This method relies on a small variation of the applied field during the operation of the system.

$$\sigma_I = \frac{\partial I}{\partial E_0} \sigma_{E_0} = \beta \frac{\partial I}{\partial E} \sigma_{E_0} \rightarrow \eta(E) = \frac{1}{I} \frac{\sigma_I}{\sigma_{E_0}} = \frac{\beta}{I} \frac{\partial I}{\partial E}$$

- The estimated  $\beta$  is therefore

$$\beta = \frac{E(\eta)}{E_0}$$

- The code needed to implement this method is freely available at <https://github.com/SagyLachmann/Statistical-analysis-of-dark-currents>



# How to use the code

The screenshot shows a GitHub repository page for 'Statistical-analysis-of-dark-currents' by SagyLachmann. The repository is on the 'main' branch and has 1 branch and 0 tags. It contains 8 files: README.md, calculateCurrentPDF.m, createVarEref.m, mats.mat, mean and std relative error.mat, supply.m, transparency.m, and varEestimation.m. The README.md file is selected, showing its content: 'Statistical-analysis-of-dark-currents' and 'OS code for the paper "Statistical analysis of dark currents"'. The right sidebar shows the repository's metadata: 1 Watch, 0 Stars, and 0 Forks. The 'About' section describes the repository as 'OS code for the paper "Statistical analysis of dark currents"'. The 'Releases' and 'Packages' sections indicate that no releases or packages have been published. The 'Languages' section shows that the repository is 100.0% MATLAB.

GitHub - SagyLachmann/Statistical-analysis-of-dark-currents

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main 1 branch 0 tags Go to file Code

SagyLachmann Add files via upload ca79d8b 25 days ago 3 commits

README.md	Initial commit	2 months ago
calculateCurrentPDF.m	Add files via upload	25 days ago
createVarEref.m	Add files via upload	25 days ago
mats.mat	Add files via upload	2 months ago
mean and std relative error.mat	Add files via upload	2 months ago
supply.m	Add files via upload	2 months ago
transparency.m	Add files via upload	2 months ago
varEestimation.m	Add files via upload	25 days ago

README.md

## Statistical-analysis-of-dark-currents

OS code for the paper "Statistical analysis of dark currents"

About

OS code for the paper "Statistical analysis of dark currents"

Readme

Releases

No releases published

Packages

No packages published

Languages

- MATLAB 100.0%



MATLAB R2019b - academic use

The image shows the MATLAB R2019b interface with a file explorer window open. The window title is "MATLAB R2019b - academic use". The interface includes a ribbon with tabs for HOME, PLOTS, and APPS. The ribbon contains various icons for file operations (New Script, New Live Script, New, Open, Compare, Find Files), data import, saving, and workspace management (New Variable, Open Variable, Clear Workspace). The file explorer window shows the current folder path: C:\Users\googo\Downloads\Statistical-analysis-of-dark-currents-main. The file list includes:

- calculateCurrentPDF.m
- createVarEref.m
- mats.mat
- mean and std relative error.mat
- README.md
- supply.m
- transparency.m
- varEstimation.m

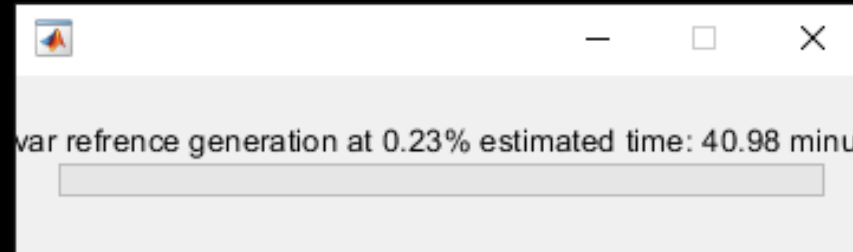




Command Window

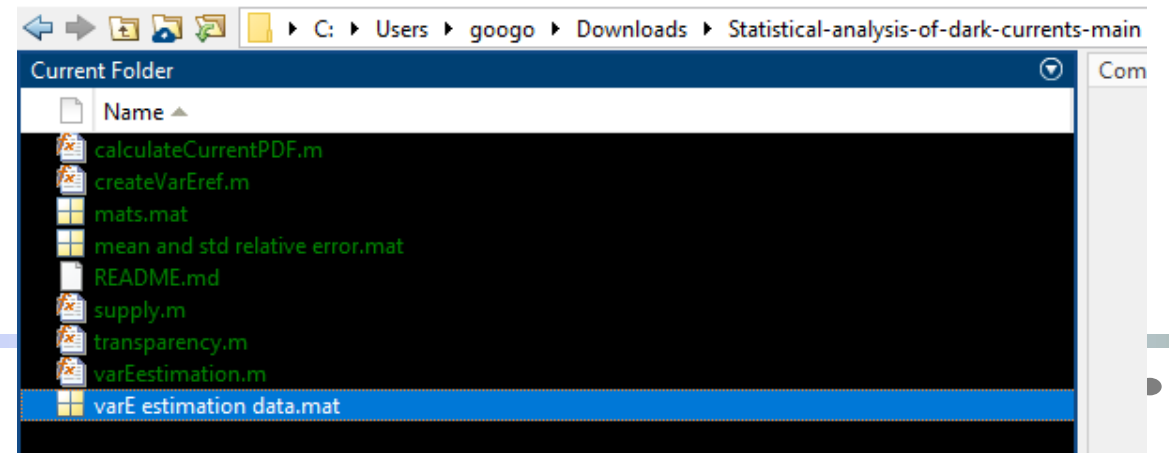
```
>> data=createVarEref('gauss', [5000 10 1], [500 1000], [19 30]*1e6);
```

fx

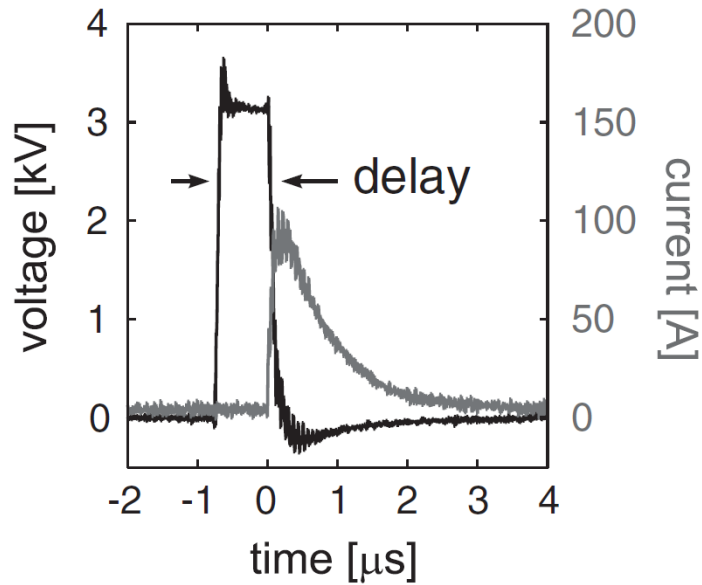


- type=gauss/saw/sine
- params=[ $\sigma_{E_0}$  amplitude, # of points, integration time] (Volts/m, #, seconds)
- beta\_range=range of expected  $\beta$  values
- E0\_range=range of applied fields that will be used (Volts/m)

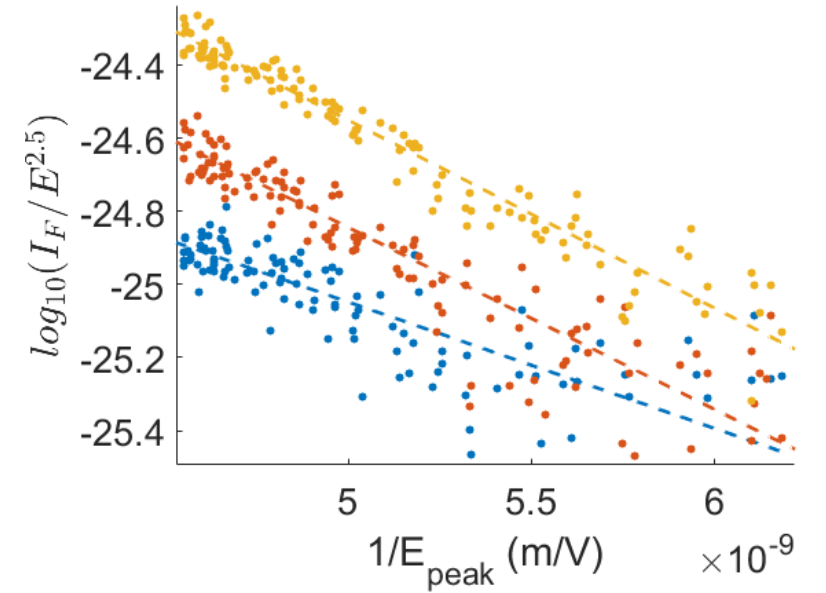
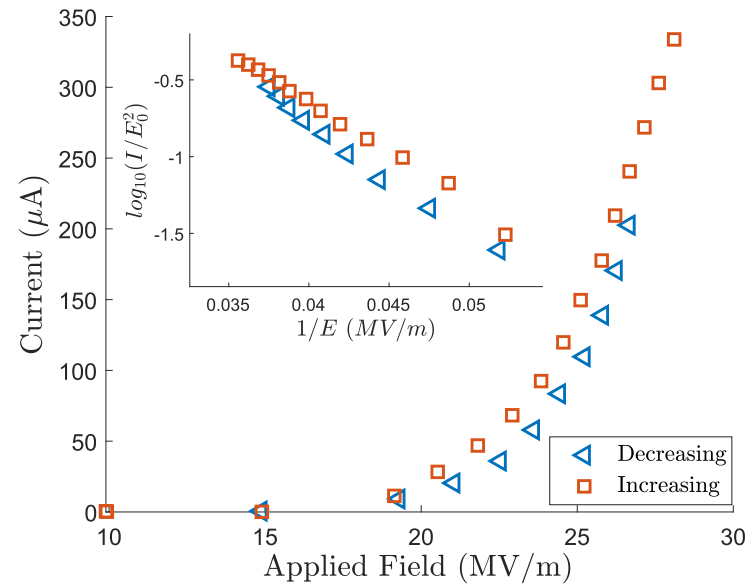
- Save the generated structure 'data'



# MEASURE!



A. Descoeur et. al, Investigation of the dc vacuum breakdown mechanism, 2009



J. Paszkiewicz, Temporal and Spatial Measurement of Field Emission, Mini MeV Arc 2018



# MEASURE!



```
>> m=10;
E=(randn(1,m)*50+1.95e5)*100;
for j=1:m
temp=calculateCurrentPDF('Cu', E(j)/100*540, 293, 1,1);%returns 2.5e4 points
temp_I=sum(temp,2);
I(j)=mean(temp_I);
end
```

$$\beta_{actual} = 540, \beta_{estimated} = 540.5 \pm 106.7$$

error  $\cong 0.09\%$

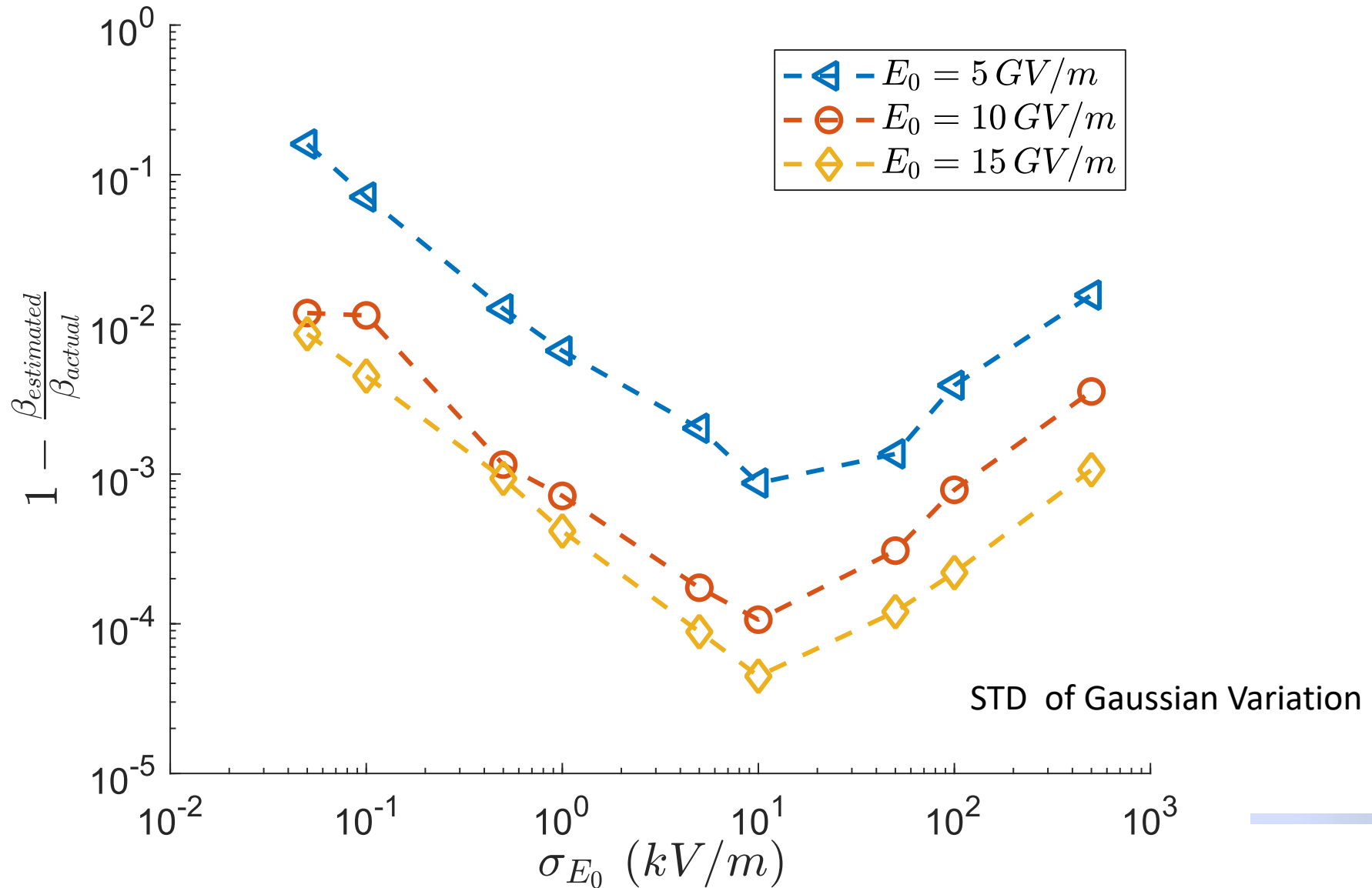
```
>> m=10;
E=(randn(1,m)*50+2.44e5)*100;
for j=1:m
temp=calculateCurrentPDF('Cu', E(j)/100*797, 293, 1,1);%returns 2.5e4 points
temp_I=sum(temp,2);
I(j)=mean(temp_I);
end
```

$$\beta_{actual} = 797, \beta_{estimated} = 795.8 \pm 167.9$$

error  $\cong 0.15\%$



# Choose Your Variation Carefully!



# Shot noise estimation

$$N(W) = \frac{4\pi m k_B T}{h^3} L\left(\frac{W - \mu}{kT}\right); \quad L(x) = \ln(1 + e^{-x})$$

$$D(W) = \left\{ 1 + \exp\left[\frac{4\sqrt{2}}{3} \left(\frac{E\hbar^4}{m^2 e^5}\right)^{-1/4} y^{-3/2} v(y)\right]\right\}^{-1} =$$

$$= \left[1 + \exp\left(H(U - W)^{3/2} v(y) E^{-1}\right)\right]^{-1}$$

Numerical  
Gaussian  
Distribution



$$\mu_W = SD(W)N(W)$$

$$\sigma_W = Sdt^{-1/2} \sqrt{(1 - D(W))D(W)N(W)}$$

Integration  
by energy

$$\mu = S \int D(W)N(W)dW$$

$$\sigma = Sdt^{-1/2} \sqrt{\int (1 - D(W))D(W)N(W)dW}$$

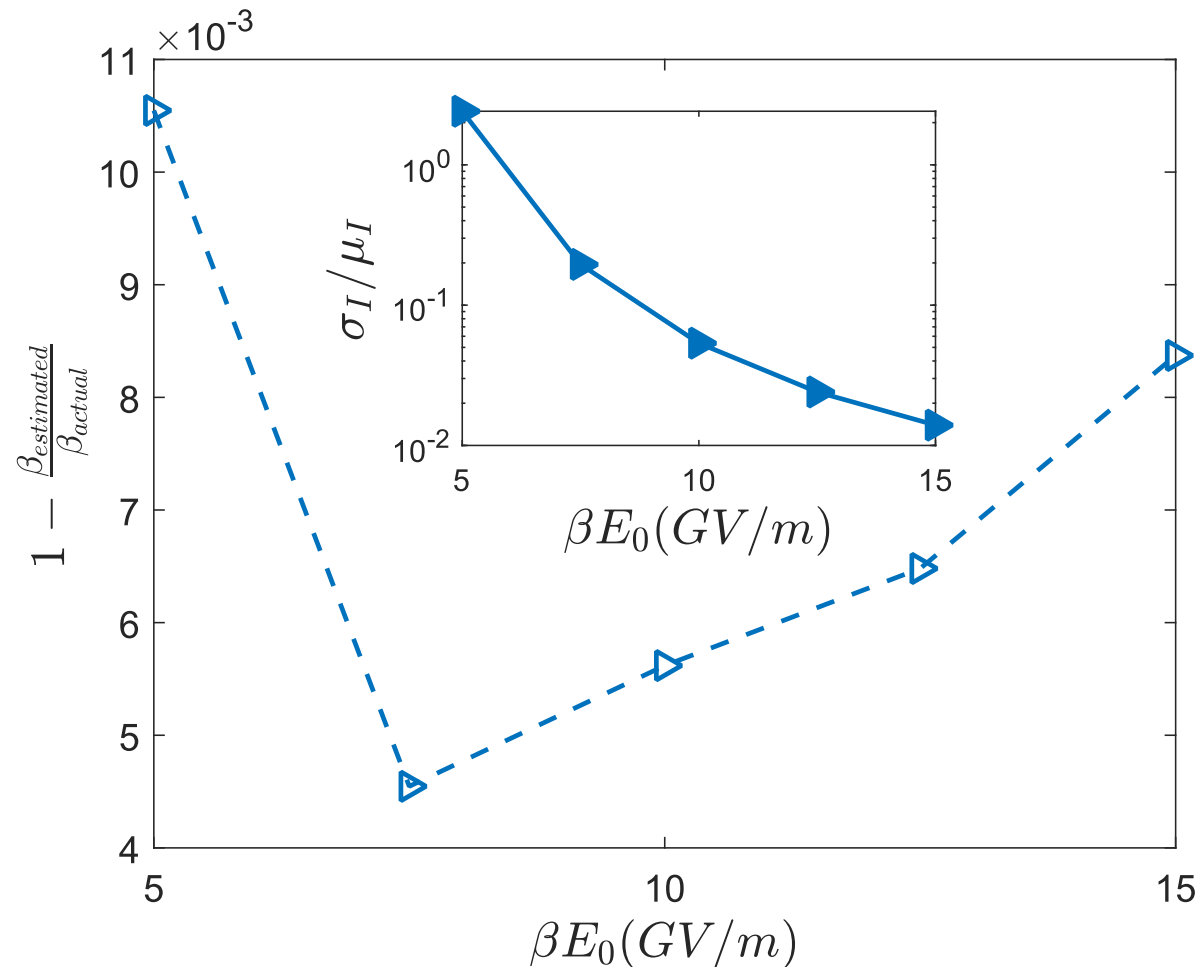


# Shot noise estimation

$$\mu = S \int D(W, E) N(W) dW$$

$$\sigma = S dt^{-1/2} \sqrt{\int (1 - D(W, E)) D(W, E) N(W) dW}$$

$$\tilde{D}(E) = 1 - \frac{\sigma}{\mu}$$



# Conclusions

- The current method of  $\beta$  estimation has several limitations.
- The methods presented offer new possibilities for the monitoring and characterization of high field systems.
- The field variation method is ready for implementation with the online code <https://github.com/SagyLachmann/Statistical-analysis-of-dark-currents>
- For shot noise implementation you are welcome to contact me at [sagy.lachmann@mail.huji.ac.il](mailto:sagy.lachmann@mail.huji.ac.il) .

# THANK YOU!

