General scaling laws of space charge effects in field emission

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SC-limited field emission

• Child-Langmuir law (1911):

\[ J = \frac{I_a}{S} = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m_e}} \frac{V_a^{3/2}}{d^2} \]
SC-limited field emission

- Poisson & continuity equation:
  \[ \nabla^2 \Phi = k J(r) \Phi^{-1/2} \]
  \[ \nabla \cdot \mathbf{J} = 0 \]
SC-limited field emission

Particle-in-cell simulation

- Child-Langmuir:
  - 1D, planar, SC limited (F = 0)
- Langmuir-Blodgett:
  - 1D, sphere-cylinder, SC limited
- Stern et al:
  - 1D, planar, general (F > 0)
- Barbour et al:
  - Real geometry -> 1D planar
  - Equivalent Planar Diode
SC-limited field emission

• Equivalent Planar Diode (EPD)
  – Often overestimates SC suppression
  – Underestimates charge density

• Corrected EPD
  – Any 3D emitter is equivalent, regarding SC, to a planar diode of certain characteristics, determined by a single geometry dependent correction factor $\omega$
\[ 3\theta^2 (1 - \theta) = \zeta (4 - 9\zeta) \]

\[ \theta = 1 - \frac{4}{3} \zeta - \frac{5}{9} \zeta^2 - \frac{16}{27} \zeta^4 + \ldots \]

\[ \theta = \frac{F}{F_L} \] - field reduction factor
\[ \zeta = k J \sqrt{V} / F_L^2 \] - space charge strength
\[ F \] - field near cathode
\[ F_L \] - field without space charge (Laplace field)
Corrected EPD

\[ \theta = 1 - \frac{4}{3} \omega \zeta + O(\zeta^2) \]

Ways to obtain correction factor \( \omega \):
- Evaluate a nasty integral
- Use an algebraic equation for sphere & cylinder
- Run particle-in-cell simulation
- Fit from experiment

Comparison with PIC

\[ \gamma = 0.78^\circ \]
\[ r_0 = 315 \text{ nm} \]
\[ R = 6.5 \text{ cm} \]
\[ h = R \]

Collector \((\Phi = V)\)

Vacuum \((\nabla^2 \Phi = -\rho/\varepsilon_0)\)

Emitter \((\Phi = 0)\)
Comparison with PIC

• 2D axisymmetric Femocs*
• Steady-state field calculation
• Adaptive timestep
• Inject single superparticle with a weight that corresponds to the charge it carries

Comparison with PIC

\[ \frac{F}{F_L} = 1 - \frac{4}{3} \omega k \frac{J_s \sqrt{V}}{F_L^2} \]
Comparison with PIC

![Graph showing the comparison between different models: PIC, CEPD (\(\omega = 0.75\)), and EPD (\(\omega = 1\)). The graph plots \(J_s [A/\AA^2]\) against \(F_L [V/\AA]\).]
Comparison with exp.*

*Barbour et al,
Physical Review 92, 45 (1953)
Comparison with exp.

\[ I[A] \]

\[ 10^{-1} \quad 10^{-3} \quad 10^{-5} \quad 10^{-7} \]

\[ 0.05 \quad 0.10 \quad 0.15 \quad 0.20 \quad 0.25 \quad 0.30 \quad 0.35 \quad 0.40 \]

\[ 1/V[1/kV] \]

- \( W = 4.500 \text{ eV} \)
- \( W = 3.283 \text{ eV} \)
- \( W = 2.863 \text{ eV} \)
- \( W = 2.471 \text{ eV} \)

\( \beta \) from Fowler-Nordheim eq.
Comparison with exp.

$W = 4.500 \text{ eV}$

$W = 3.283 \text{ eV}$
Inadequacy of EPD
Conclusions

• By knowing $\omega$ and $\beta$, one can calculate SC suppressed emission current from a 3D emitter
• $\omega$ can be calculated by running PIC (once)
• $\omega$ is scale invariant, therefore it can be tabulated for various geometries
• CEPD model can be used to calculate emission current for various work functions, temperatures etc. without the need to run PIC
• For more details, see arXiv:2008.11984 (currently under review in PRE)
\[ \theta = 1 - \frac{4}{3} \omega \zeta + O(\zeta^2) \]

**General**
\[ \omega \equiv \frac{3}{4} \int_{\Omega} \tilde{\nabla}_r G(\tilde{r}, \tilde{r}') \bigg|_{\tilde{r}_s} \frac{\xi(\tilde{r}')}{\sqrt{\phi_0(\tilde{r}')}} d^3\tilde{r}' \]

**Sphere**
\[ \omega^{(S)} = \frac{3}{4} \frac{(2 - \frac{1}{\tilde{r}}) \log \left( \sqrt{\tilde{r}} + \sqrt{\tilde{r} - 1} \right) - \sqrt{1 - \frac{1}{\tilde{r}}}}{(1 - \frac{1}{\tilde{r}})^{3/2}} \]

**Cylinder**
\[ \omega^{(C)} = \frac{3}{4} \frac{(\tilde{r} + 2\tilde{r} \log(\tilde{r})) D \left( \sqrt{\log(\tilde{r})} \right) - \tilde{r} \sqrt{\log(\tilde{r})}}{[\log(\tilde{r})]^{3/2}} \]

\[ \tilde{r} = \frac{r_{emitter}}{r_{collector}} \]