General scaling laws of space charge effects in field emission

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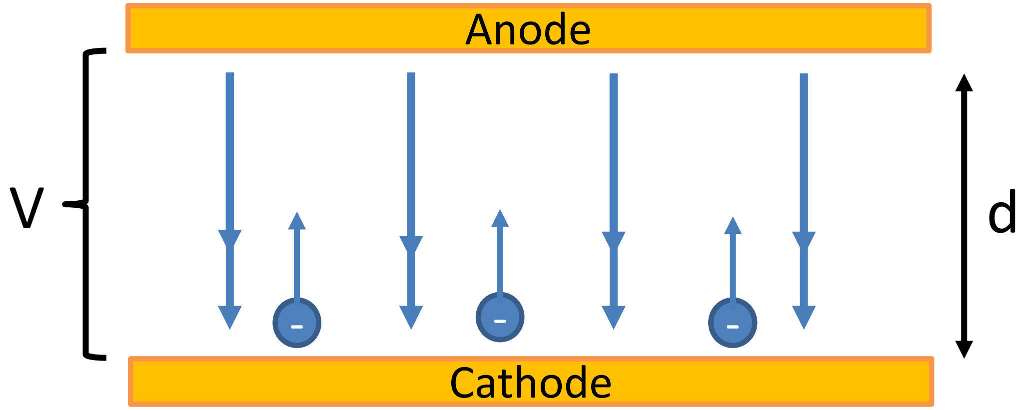
SC-limited field emission

- Child-Langmuir law (1911):

$$J = \frac{I_a}{S} = \frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m_e}}\frac{V_a^{3/2}}{d^2}.$$

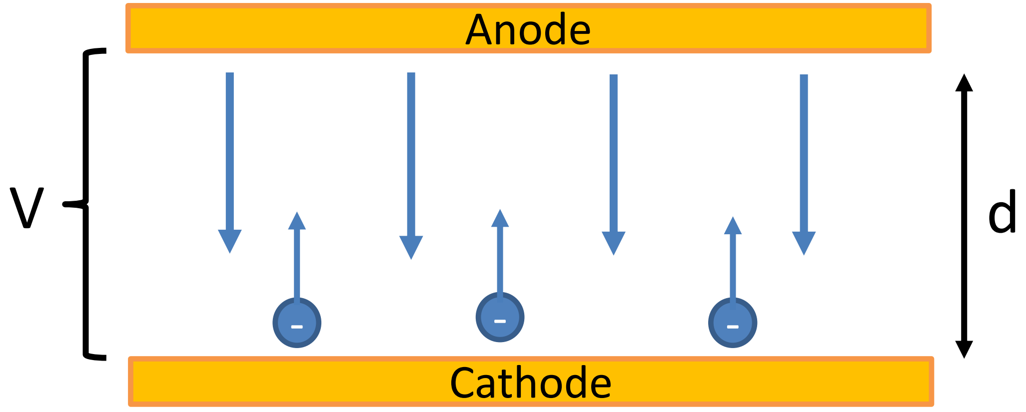


SC-limited field emission

- Poisson & continuity equation:

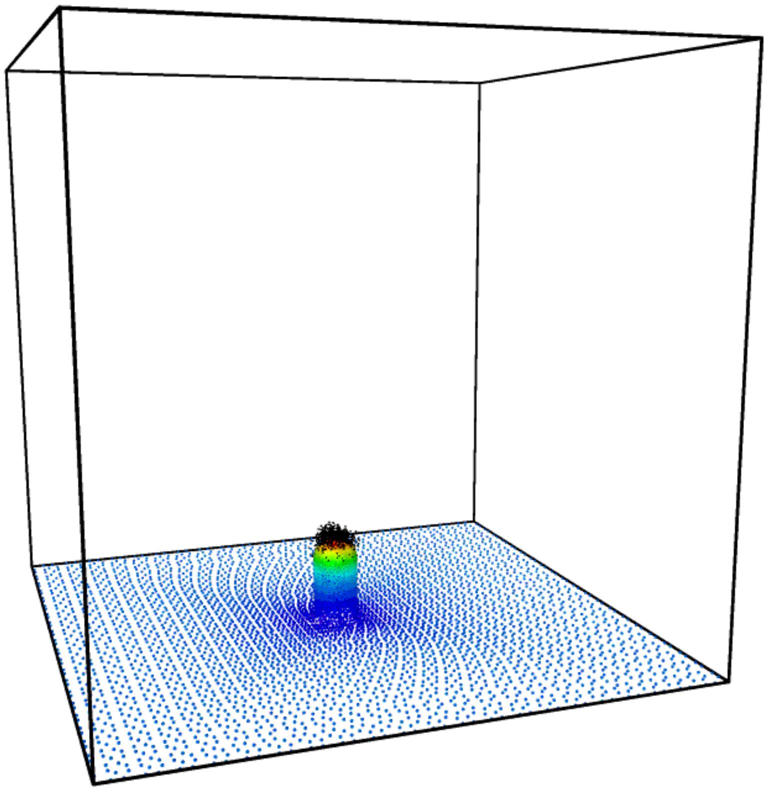
$$\nabla^2 \Phi = kJ(\mathbf{r})\Phi^{-1/2}$$

$$\nabla \cdot \mathbf{J} = 0$$



SC-limited field emission

Particle-in-cell simulation



- Child-Langmuir:
  - 1D, planar, SC limited (F = 0)
- Langmuir-Blodgett:
  - 1D, sphere-cylinder, SC limited
- Stern *et al:*
  - 1D, planar, general (F > 0)
- Barbour *et al:*
  - *Real geometry -> 1D planar*
  - *Equivalent Planar Diode*

SC-limited field emission

- Equivalent Planar Diode (EPD)
  - *Often overestimates SC suppression*
  - *Underestimates charge density*
- *Corrected EPD*
  - *Any 3D emitter is equivalent, regarding SC, to a planar diode of certain characteristics, determined by a single geometry dependent correction factor ω*

EPD

$$3\theta^2(1-\theta) = \zeta(4-9\zeta)$$

$$\approx$$

$$\theta = 1 - \frac{4}{3}\zeta - \frac{5}{9}\zeta^2 - \frac{16}{27}\zeta^4 + \cdots$$

$\theta = F/F_L$ - field reduction factor
$\zeta = kJ\sqrt{V}/F_L^2$ - space charge strength
F - field near cathode
F_L- field without space charge (Laplace field)

Corrected EPD

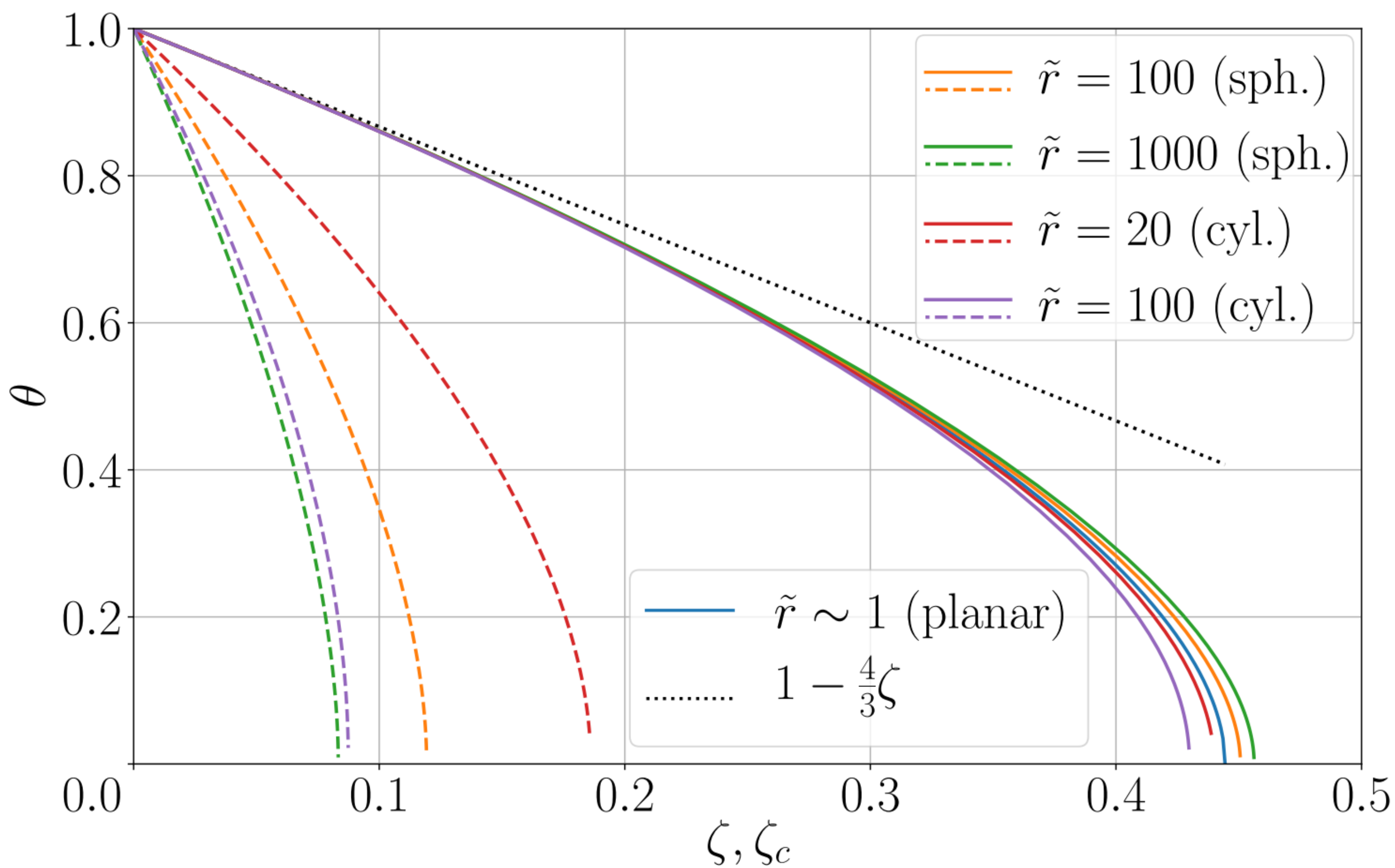
$$\theta = 1 - \frac{4}{3}\omega\zeta + O(\zeta^2)$$

Ways to obtain correction factor ω:

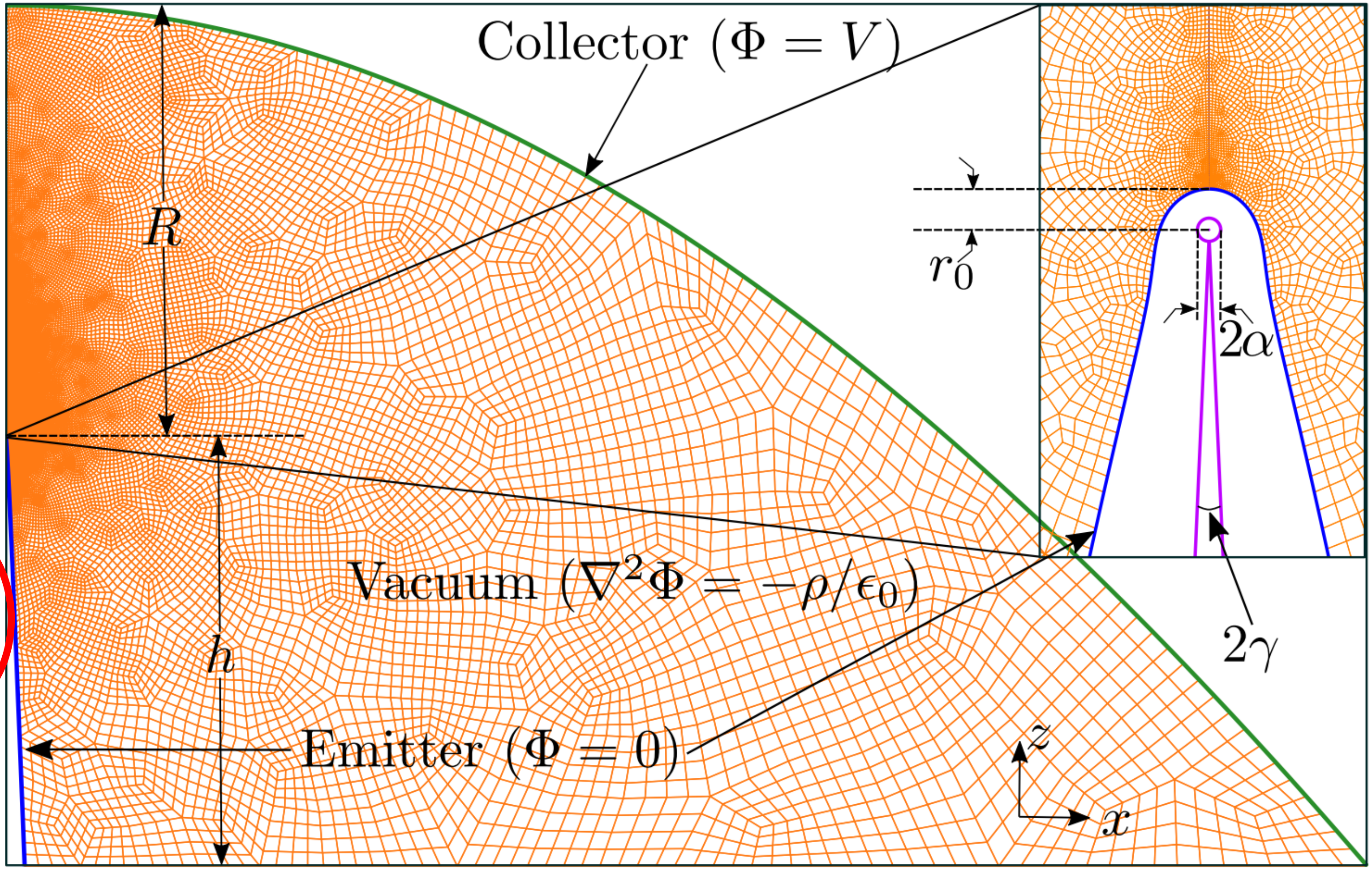
- Evaluate a nasty integral
- Use an algebraic equation for sphere & cylinder
- Run particle-in-cell simulation
- Fit from experiment

**R. G. Forbes, Journal of Applied Physics 104, 084303 (2008)*

Behavior of the model



Comparison with PIC



$\gamma = 0.78°$
$r_0 = 315\ nm,$
$R = 6.5\ cm,$
$h = R$

Comparison with PIC

- 2D axisymmetric Femocs*
- Steady-state field calculation
- Adaptive timestep
- Inject single superparticle with a weight that corresponds to the charge it carries

Phys. Rev. E* ***101*, 053307 (2020)*

Comparison with PIC

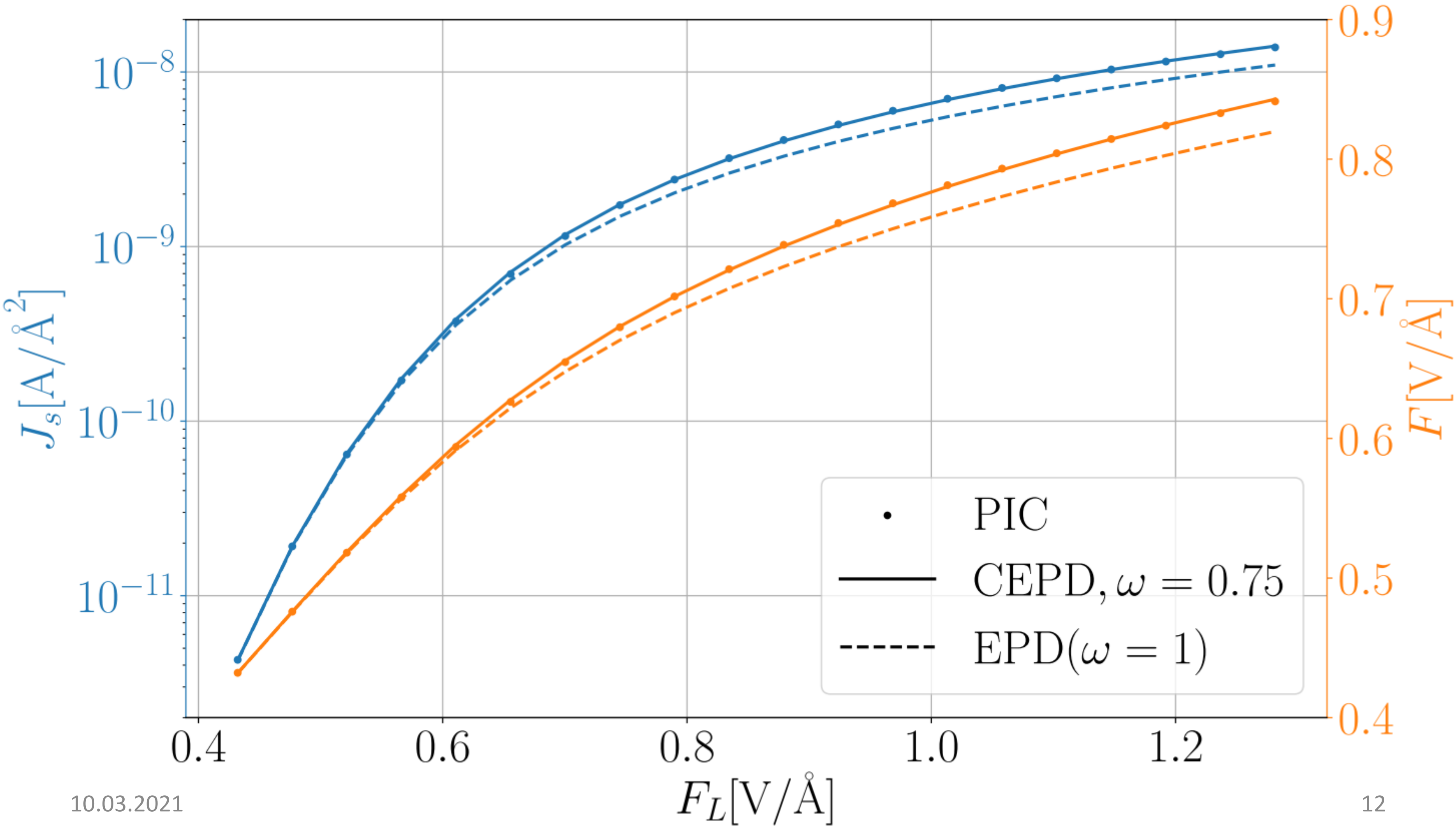
fit

ramp

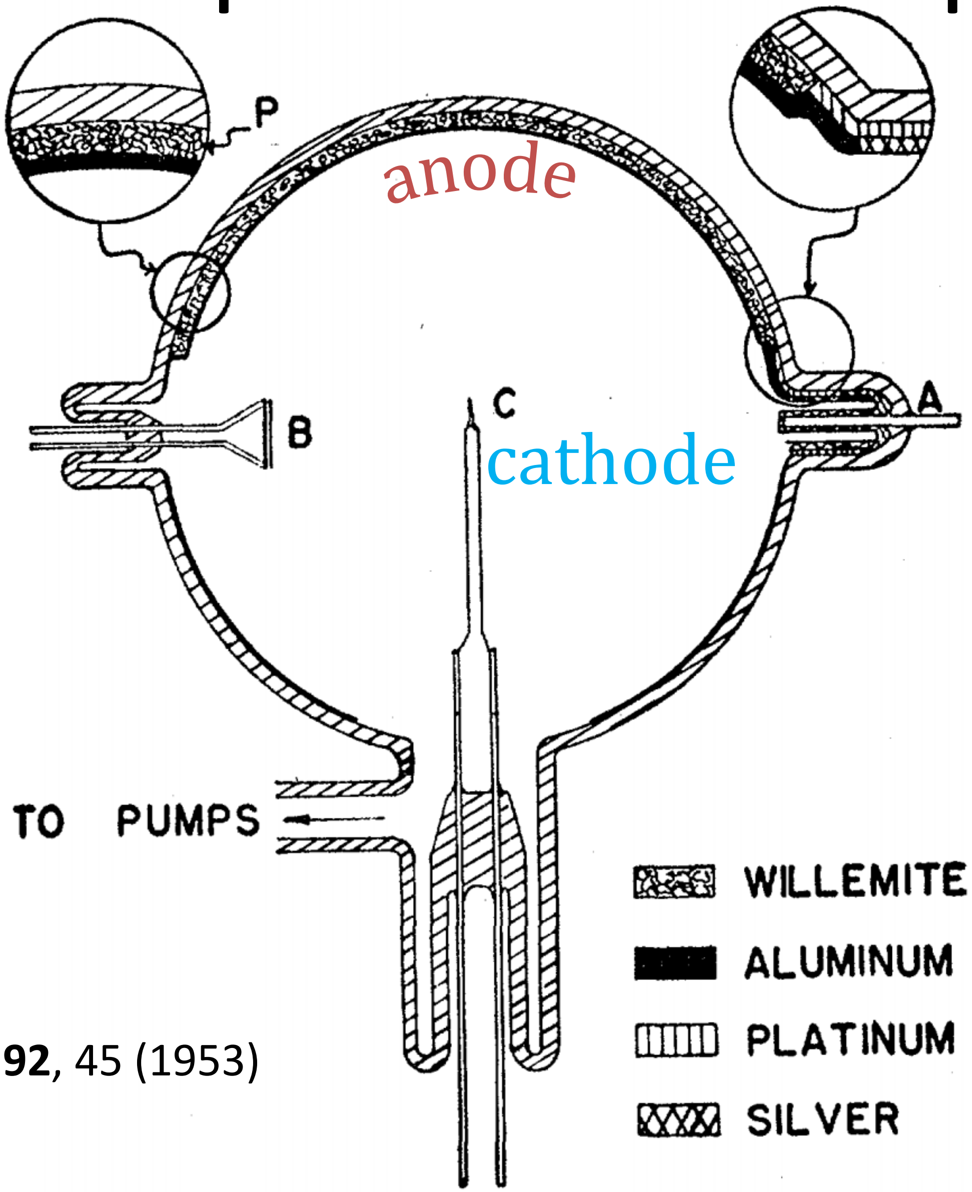
$$\frac{F}{F_L} = 1 - \frac{4}{3}\omega k \frac{J_s\sqrt{V}}{F_L^2}$$

measure

Comparison with PIC

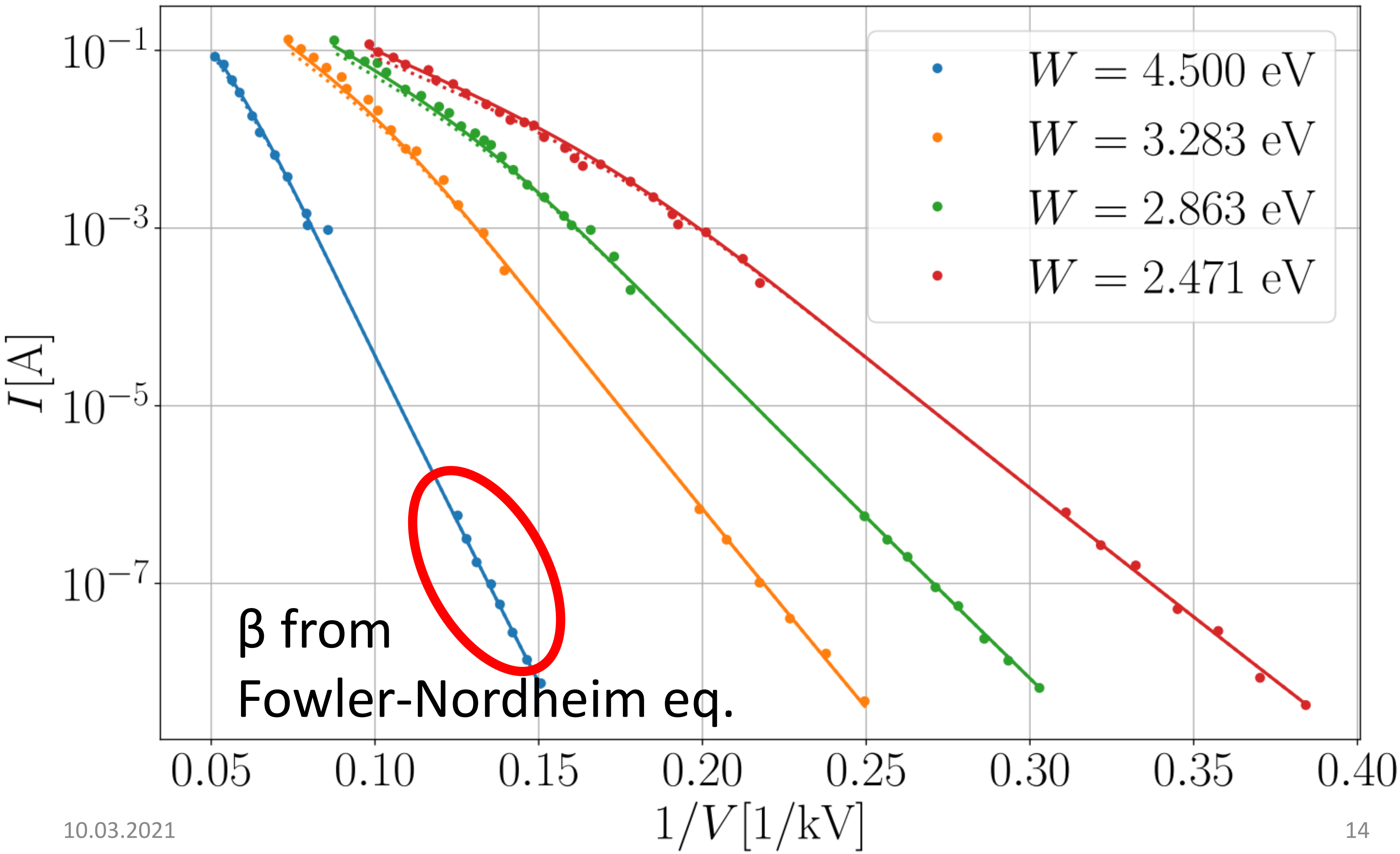


Comparison with exp.*

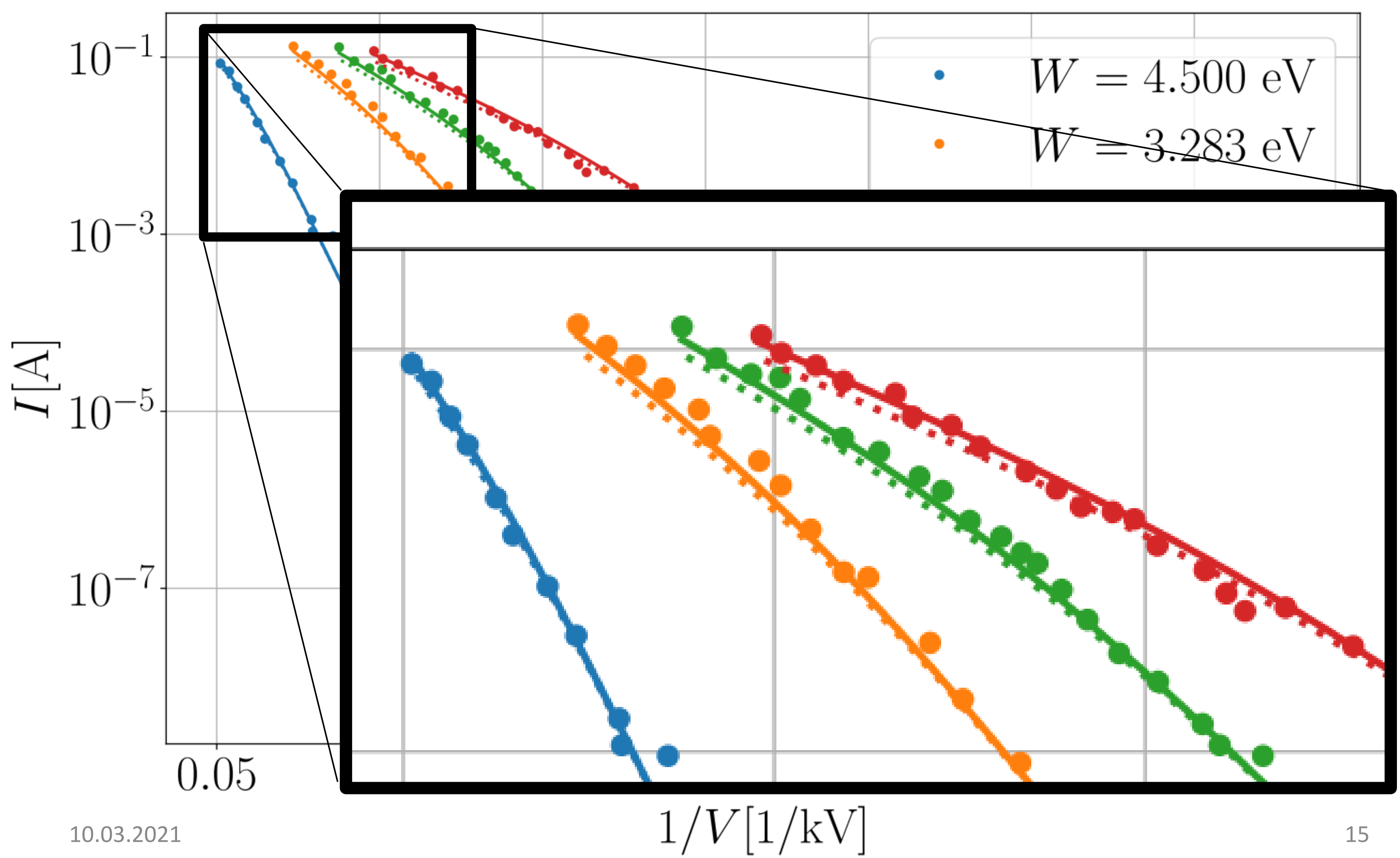


*Barbour *et al*,
Physical Review **92**, 45 (1953)

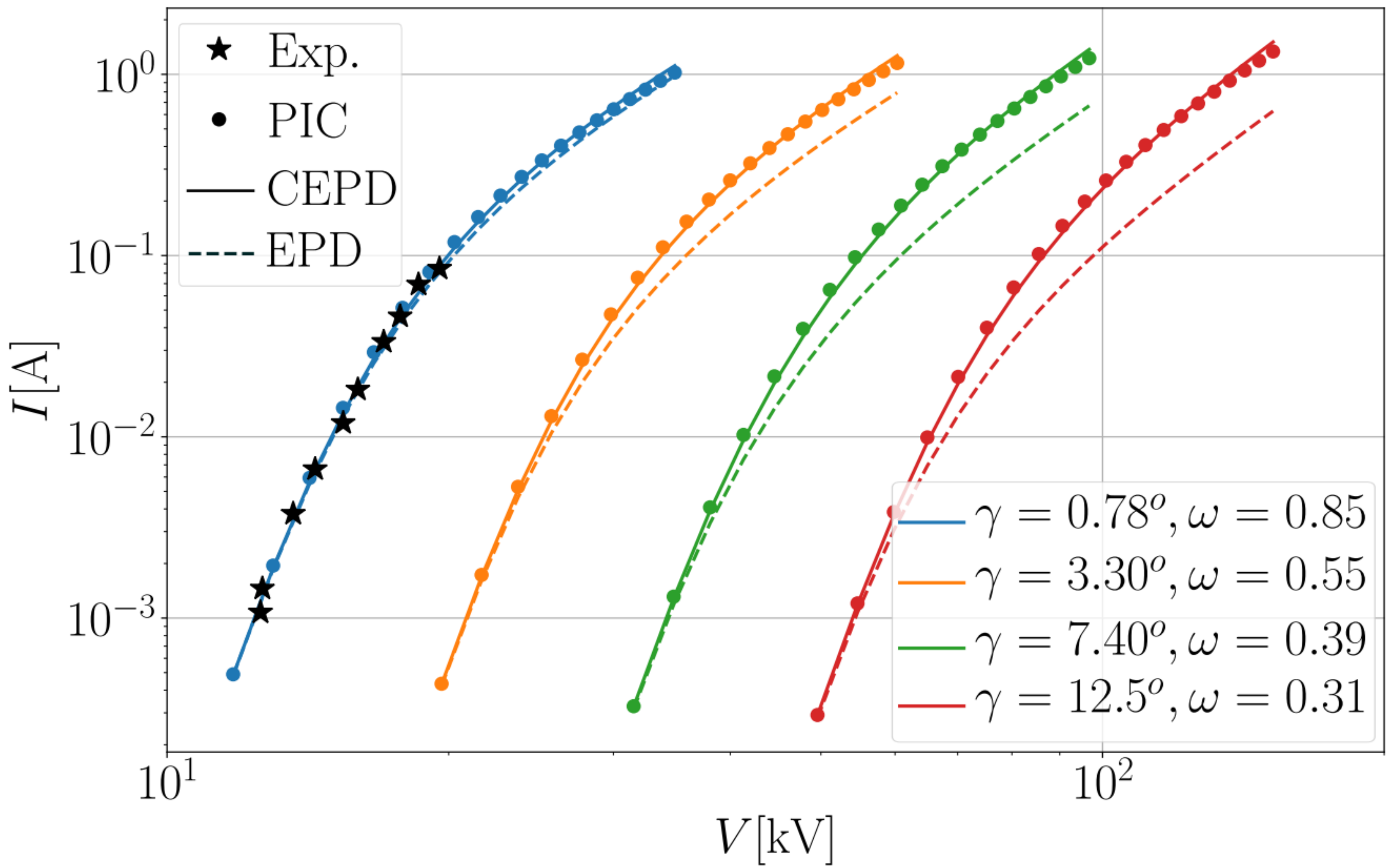
Comparison with exp.



Comparison with exp.



Inadequacy of EPD



Conclusions

- By knowing ω and β, one can calculate SC suppressed emission current from a 3D emitter
- ω can be calculated by running PIC (once)
- ω is scale invariant, therefore it can be tabulated for various geometries
- CEPD model can be used to calculate emission current for various work functions, temperatures etc. without the need to run PIC
- For more details, see *arXiv:2008.11984* (currently under review in PRE)

Corrected EPD

$$\theta = 1 - \frac{4}{3}\omega\zeta + O(\zeta^2)$$

General → $$\omega \equiv \frac{3}{4}\int_\Omega \tilde{\nabla}_{\mathbf{r}} G(\tilde{\mathbf{r}}, \tilde{\mathbf{r}}')\Big|_{\tilde{\mathbf{r}}_s} \frac{\xi(\tilde{\mathbf{r}}')}{\sqrt{\phi_0(\tilde{\mathbf{r}}')}} d^3\tilde{\mathbf{r}}'$$

Sphere → $$\omega^{(S)} = \frac{3}{4}\frac{\left(2 - \frac{1}{\tilde{r}}\right)\log\left(\sqrt{\tilde{r}} + \sqrt{\tilde{r} - 1}\right) - \sqrt{1 - \frac{1}{\tilde{r}}}}{\left(1 - \frac{1}{\tilde{r}}\right)^{3/2}},$$

Cylinder → $$\omega^{(C)} = \frac{3}{4}\frac{(\tilde{r} + 2\tilde{r}\log(\tilde{r}))D\left(\sqrt{\log(\tilde{r})}\right) - \tilde{r}\sqrt{\log(\tilde{r})}}{[\log(\tilde{r})]^{3/2}},$$

$$\tilde{r} = \frac{r_{emitter}}{r_{collector}}$$