NEW DEVELOPMENT OF BIRD MODEL

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The possibility that the flow of electrons emitted by a cathode at high dc voltage is essentially due to local emission of a covering dielectric layer is at the basis of the BIRD (Breakdown Induced by Rupture of Dielectric) model. This model assumes that, in presence of sufficient electric field, the electrons trapped in polarization structures of the dielectric layer are extracted by quantum tunnelling effect. As a consequence of the layer electron depletion, the electric field inside the dielectric layer increases and the rupture (breakdown) of the layer itself can occur, provided certain conditions are met. To investigate experimentally the features of this model the High Voltage Short Gap Test Facility (HVSGTF) has recently been built in Padua. The experimental dark current measured at different electrode configurations permits us to test the correctness of the model predictions. In particular, we consider the trend of the current as a function of time and its dependence on the characteristic properties of the dielectric layer. From the theoretical side, we investigate the consistency between a semi-classical model and a simple quantum model.

Content List

1. INTRODUCTION

B.I.R.D. ⟹ **B**REAKDOWN **I**NDUCED by **R**UPTURE of **D**IELECTRIC layer

- Common metals are oxidized

- Even stainless steel is covered by a coating of Chromium Oxide
- Non-metallic impurities are also usually present on metal surfaces

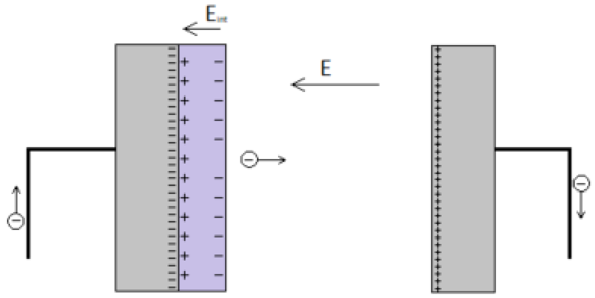
METAL ELECTRODES behaviour in high field is influenced by non-metallic materials

Two different (opposite) APPROACHES

- Fowler-Nordheim - IDEAL conductive metal
- BIRD-model - IDEAL non-conductive dielectric material

2. The BIRD-model

a. IDEAL non-conductive CATHODE layer



b. Tunnelling Effect

From the study of tunnelling effect, it is possible to calculate the decaying probability per unit of time $\lambda(E)$ of a polarization electron and then to write the emitted current density as $J_e = \lambda\sigma$, where σ is the surface density charge.

$$J_e = \varepsilon_0 \lambda (E - E_{int})$$

In an ideal non-conductive cathode E_{int} is not constant in time and so is J_e.

From a semi-classical approach to the problem, it has been found that

$$J_e = \frac{k_{Bird}}{\varepsilon_r}(E - E_{int})\mathrm{E} \cdot exp\left(-\frac{\varepsilon_r E_d}{E}\right)$$

Where ε_r and E_d are the permittivity and dielectric strength of the non-conductive layer, respectively. Furthermore, from the same semi-classical model, we also "guessed" that $k_{Bird} = 37.9\ \frac{\mu A}{kV^2}$.

E. Spada, A. De Lorenzi, N. Pilan and V. Antoni "Theoretical Basis…" *IEEE Trans. on Plasma Science*, vol. 47, no. 5, May 2019, pp. 2759-2764

3. Comparison with F.N. law

Similarity and difference between BIRD-model and F.N. theory

a. Same (approximated) functional shape

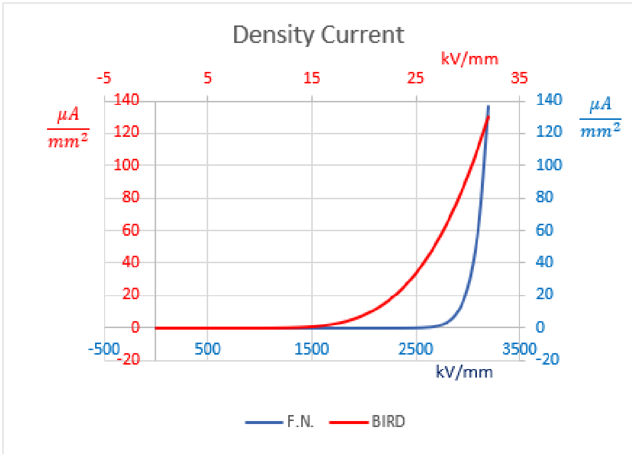
When voltage is applied to electrodes, the starting value of the internal field is given by $E_{int} = \frac{E}{\varepsilon_r}$, where ε_r is the relative permittivity of the layer. If the increasing of E_{int} is negligible, we find then a F.N.-like density current.

$$J_e = k_1 E^2 e^{-\frac{k_2}{E}}$$

Constants k_1 and k_2 differ considerably from those calculated by F.N., as they depend on the dielectric characteristics rather than those of the metal.

For stainless steel $\Phi \approx 5.0\ eV$.   For Cr_2O_3 $\varepsilon_r = 12.5$ and $E_d = 7.9\ \frac{kV}{mm}$

	k_1		k_2	
F.N.	$\frac{e^3}{8\pi h\Phi}$	$3.1 \cdot 10^5\ \frac{\mu A}{kV^2}$	$\frac{8\pi(2m\Phi^3)^{\frac{1}{2}}}{3eh}$	$76 \cdot 10^3\ \frac{kV}{mm}$
BIRD	$\frac{k_{Bird}}{\varepsilon_r}\left(1 - \frac{1}{\varepsilon_r}\right)$	$2.8\ \frac{\mu A}{kV^2}$	$\varepsilon_r E_d$	$99\ \frac{kV}{mm}$



It turns out that despite of the much greater k_1 factor, that appears in the F.N. theory, a much greater current results in the BIRD model. The exponential factor is clearly prevalent.

b. Long-time evolution of the electric current density

Looking at the complete density current,

$$J_e = \frac{k_{Bird}}{\varepsilon_r}(E - E_{int})\mathrm{E} \cdot exp\left(-\frac{\varepsilon_r E_d}{E}\right)$$

we see that it explicitly depends on E_{int}. Since the internal field increases from $E_{int} = \frac{E}{\varepsilon_r}$ up to the value $E_{int} = E$, we expect that the emitted current decreases in time.

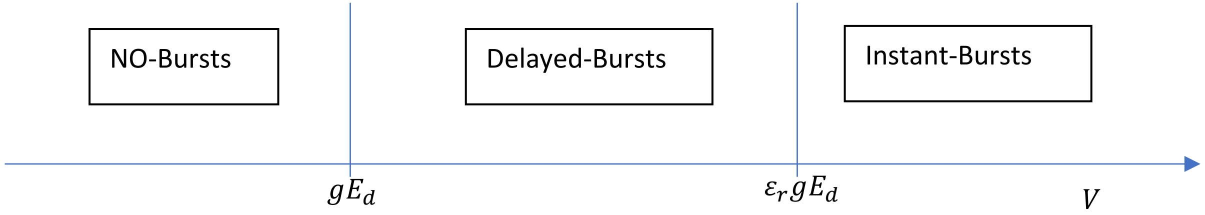
Two BIRD model current density

- "Complete law" $J_e = \frac{k_{Bird}}{\varepsilon_r}(E - E_{int})\mathrm{E} \cdot exp\left(-\frac{\varepsilon_r E_d}{E}\right)$ Long-time evolution
- "Approximated F.N.-like law" $J_e = \frac{k_{Bird}}{\varepsilon_r}\left(1 - \frac{1}{\varepsilon_r}\right) E^2 e^{-\frac{\varepsilon_r E_d}{E}}$ Short-time evolution

c. BURSTS

When E_{int} exceeds the dielectric strength E_d a local disruption occurs.

So, putting $E = \frac{V}{g}$ we find three intervals for the gap voltage V:



For example, for stainless steel electrodes (covered with a Cr_2O_3 layer) and gap $g = 0.5\ mm$, we have

$gE_d \approx 4.0\ kV$ and $\varepsilon_r gE_d \approx 49.3\ kV$

Note that

- If a geometric amplification β factor is present the two limits are lowered to the $\frac{gE_d}{\beta}$ and $\frac{\varepsilon_r gE_d}{\beta}$ values, respectively. (both limits shrink)
- If the dielectric layer is not ideal i.e., a small conductivity is allowed, the upper limit $\varepsilon_r gE_d$ remains unchanged, while the lower limit raises. (the Delayed-Bursts range narrows)

4. Theory

From a Semi-Classical BIRD-model to a Quantum BIRD-model (a road still steep uphill)

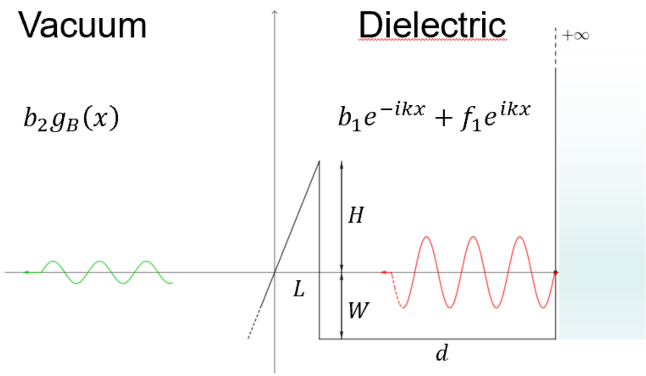
The semiclassical approach led us to the electron current density

$$J_e \approx k(E - E_{int})(E - \varepsilon_r E_d) \approx k(E - E_{int})Ee^{-\frac{\varepsilon_r E_d}{E}}$$

Now we need a more rigorous calculation. A Quantum BIRD model.

The Transmission coefficient

The polarization electron is confined in a small region near the dielectric surface and the simplest confining model for it is a well with a Triangular Barrier on one side and an infinite barrier at the other.



The transmission coefficient through the triangular barrier has long been known

$$T = \frac{4k}{k_A\sqrt{x_L}} exp\left(-\frac{4}{3}x_L^{\frac{3}{2}}\right) \qquad x_L = k_A L = \left(\frac{2m}{h^2}\right)^{\frac{1}{3}} \frac{H}{(eE)^{\frac{2}{3}}} \stackrel{\text{def}}{=} \left(\frac{3}{4}\frac{\varepsilon_r E_d}{E}\right)^{\frac{2}{3}}$$

(Here we adopt Forbes notation, in the deep tunnel limit.)

«G. Forbes and J. Deane» Proc. R. Soc. A (2011) 467, 2927-2947

We can calculate the current density as $J_e = \lambda\sigma$

The decaying probability per unit of time λ and the transmission coefficient T (that is the probability that an incoming electron pass through the barrier) are closely related to each other.

They are <u>approximatively</u> proportional quantities: $\lambda = fT$ where f is the frequency $f = \frac{\hbar k}{2md}$ of a bouncing wave in a well of dimension d:

$$\lambda = \frac{\hbar k}{2md} T$$

Gamow pre-factor

In his seminal paper (1928) Gamow calculated the radioactive α-decay constant of heavy atoms using

$$\lambda = \frac{4hk\sin^2\theta}{\pi m\left[1+\left(\frac{k'}{k^0}\right)^2\right]2(l+q_0)_k} \cdot e^{-\frac{4\pi l\sqrt{2m}}{h}\sqrt{U_0-E}}$$

G. Gamow, ZP, **51,** 204

That in our notation can be written as

$$\lambda = \frac{\hbar k}{2m(d+L)} T$$

Using Gamow's result, it is possible to write, in the limit $\frac{d}{L} \ll 1$,

$$\lambda = \frac{\hbar k}{2m(L+d)} T \approx \frac{\hbar k}{2mL} T$$

Thus,

$$\lambda \approx \frac{\hbar k}{2m} \frac{1}{(k_A L)} \frac{4k}{\sqrt{x_L}} exp\left(-\frac{4}{3} x_L^{\frac{3}{2}}\right) = \frac{2\hbar k^2}{m} x_L^{-\frac{3}{2}} exp\left(-\frac{4}{3} x_L^{\frac{3}{2}}\right) = \frac{8\hbar k^2}{3m\varepsilon_r E_d} \mathrm{E} \cdot exp\left(-\frac{\varepsilon_r E_d}{\mathrm{E}}\right)$$

In the end…

$$J_e = \varepsilon_0 \lambda (E - E_{int}) = k(E - E_{int}) \mathrm{E} \cdot exp\left(-\frac{\varepsilon_r E_d}{E}\right)$$

The same law as in the semi-classical model Good!

But …. not too much…

… Gamow pre-factor seems to be wrong!

Can BIRD density current correctly fit experimental data?

5. Amplification factor

Previous study showed that total emitted current, at different gap distance, cannot be fitted by a law that depends only on the electric field. This is the same problem found in the so - called Total-Voltage-Effect.

Instead, we found that density current also depends on an amplification factor that depends in turn only on gap voltage.

This amplification current has been experimentally found to be of the type

$$A(V) = 1 + k_0 e^{-\frac{k_3}{V}}$$

SUMMING-UP

The BIRD model, with the amplification factor, can be summarized in the laws:

- $I_e = S\left(1 + k_0 e^{-\frac{k_3}{V}}\right)\frac{k_{Bird}}{\varepsilon_r}(\beta E - E_{int})\beta E \cdot exp\left(-\frac{\varepsilon_r E_d}{\beta E}\right)$ for “long time”
- $I_e = S\left(1 + k_0 e^{-\frac{k_3}{V}}\right)\frac{k_{Bird}}{\varepsilon_r}\left(1 - \frac{1}{\varepsilon_r}\right)(\beta E)^2 \cdot exp\left(-\frac{\varepsilon_r E_d}{\beta E}\right)$ for “short time” (F.N.-like law)

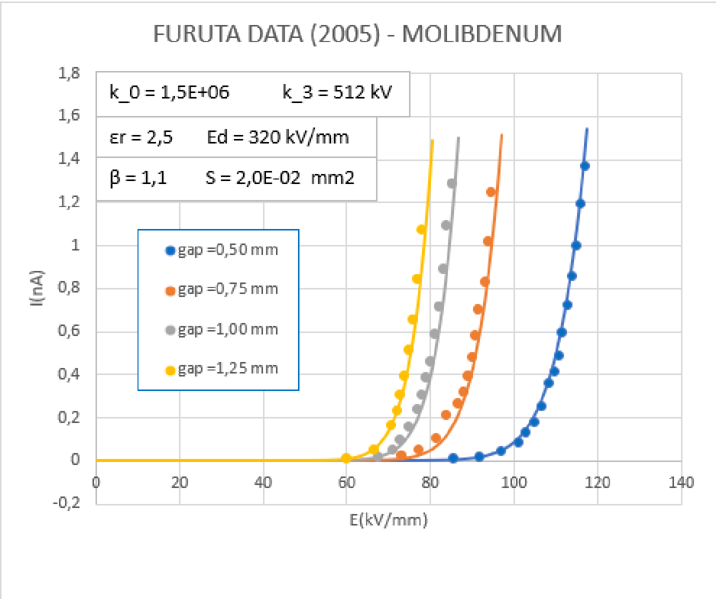
(Here we introduced also the local geometric amplification factor β)

6. Experiments

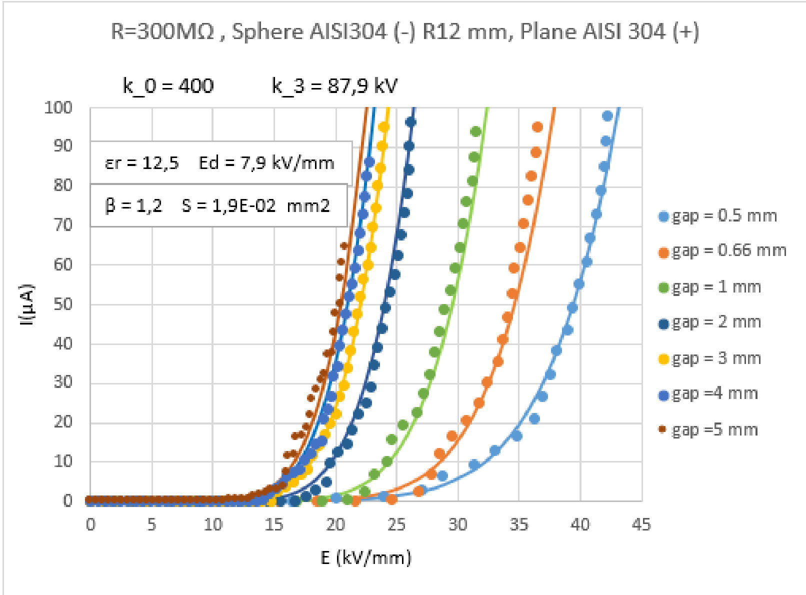
ANALISYS OF DATA WITH “short-time” F.N.-like law

a. Old data

FURUTA et al. (2005)



HVSGTF Padova - June 2019



Temptative results:

- F.N.-like law correctly fit experimental data.
- The amplification factor $A(V) = \left(1 + k_0 e^{-\frac{k_3}{V}}\right)$ depends on the type of the electrodes surface dielectric layer.
- Assuming $k_2 = \varepsilon_r E_d$ provides reliable β values. ($\beta \approx 1.1 \div 1.2$)
- Assuming $k_{Bird} = 37.9 \frac{\mu A}{kV^2}$ provides reliable S values. ($S \approx 10^{-2} mm^2$)

In 2020, COVID permitting, some other experiments were performed. Our main goals were:

- To confirm old data analysis, using “short-time” F.N.-like law.
- To better understand the role of the bursts in the conditioning phase.
- To gather new information about “long-time” BIRD law.

b. New HVSGTF data

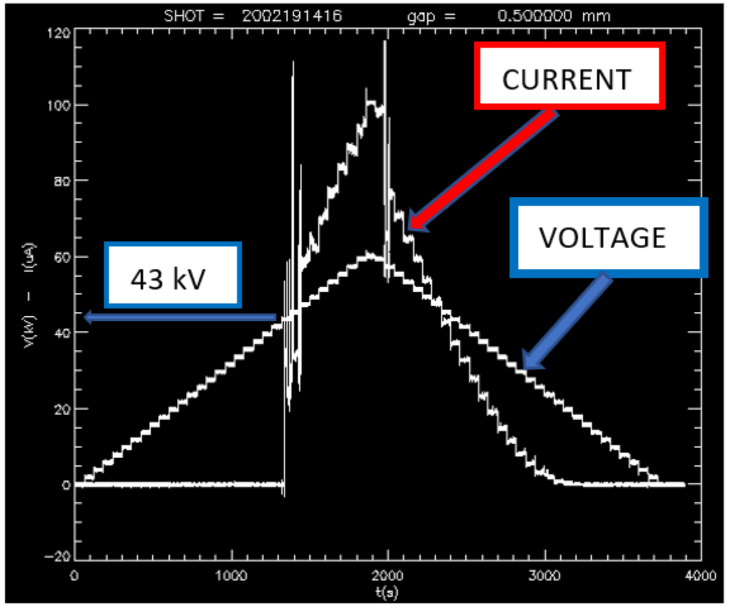
One of these campaigns of measures was started in February 2020. Stainless steel electrodes have been **carefully prepared with fine processing**. We changed the electrode gap distance from a 0.5 mm to 5.0 mm and then back again to 0.5 mm. For each measure, voltage was increased by 2 kV, step by step, from zero to a maximum voltage and back again to zero. Each step lasts for about 60 s.

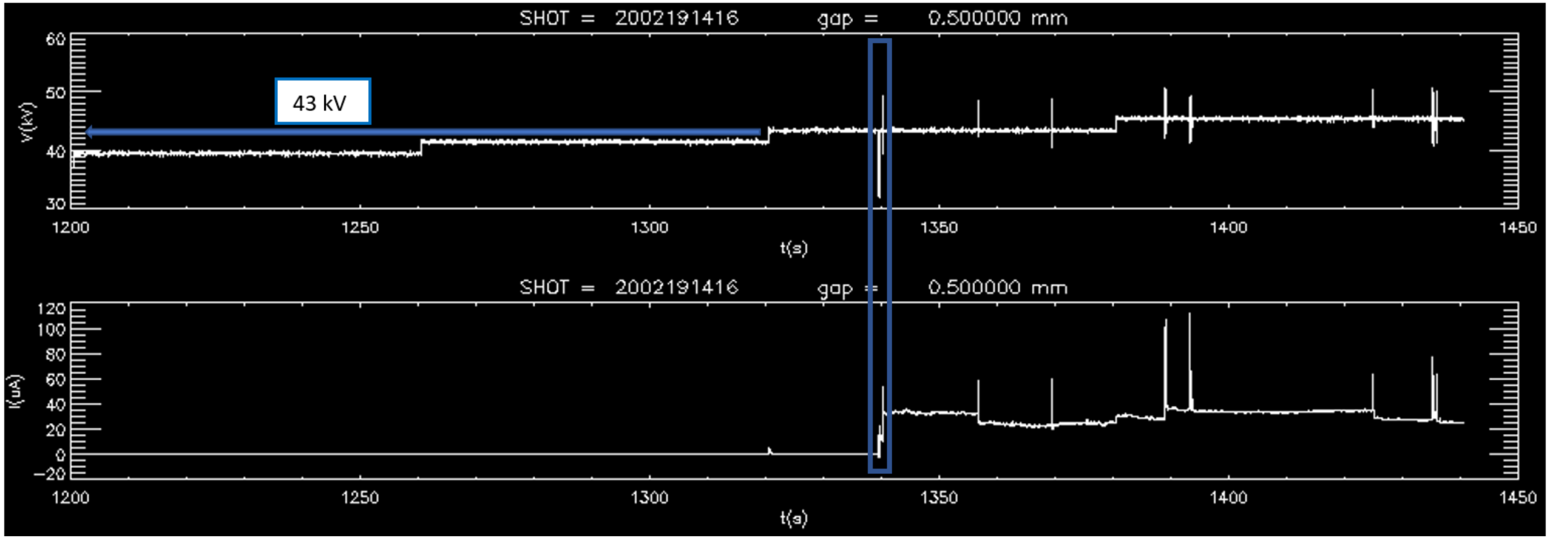
Date	Negative electrode	Positive electrode	gap length [mm]	Maximum voltage in high vacuum [kV]	Sessions #
19/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	0,5	43	2002191416
19/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	1	60	2002191529
20/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	2	83	2002200932
20/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	3	84	2002201138
20/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	4	85	2002201345
20/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	5	87	2002201537
21/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	4	85	2002210901
21/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	3	86	2002211058
21/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	2	82	2002211321
21/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	1	55	2002211528
24/02/2020	SS sphere , ϕ = 24mm	SS plane , ϕ = 108mm	0,5	29	2002240907

Note that for $g = 0.5\ mm$ we expect (BIRD-model) an upper limit voltage (Instant-Burst) for $\beta \approx 1$, at $V = 49.3\ kV$.

From the start of this campaign, no current was measured below $43\ kV$. Once this voltage has been reached, we had a First Burst.

- It was not an Instant-Burst, it takes few tenth of seconds at $V = 43\ kV$ to happen.
- From then on others bursts appear and a more or less regular current was measured.
- Sometime bursts increase the electron emission, sometime they decrease it (usual conditioning process). In other cases, they don't change electron emission at all.

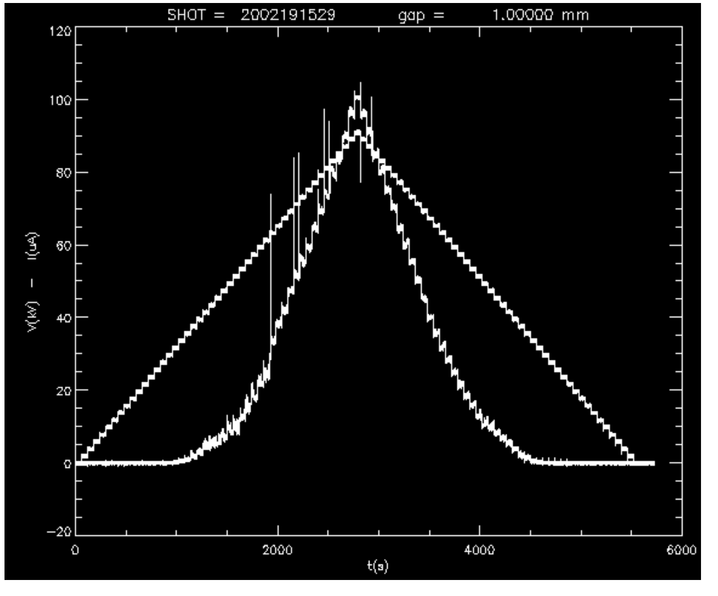




In the light of the BIRD model, we can guess that

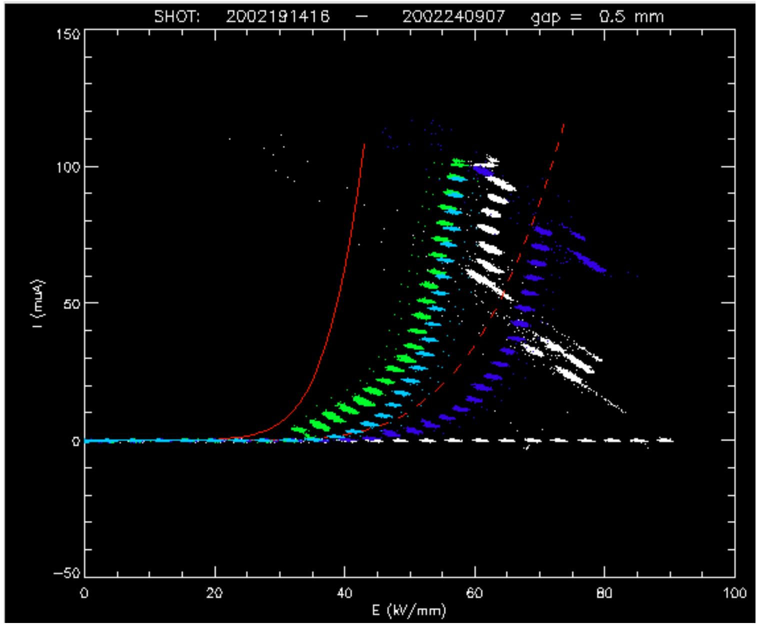
- Before the First Burst, the β factor was pretty close to unity ($\beta \approx 1$)
- The First Burst deteriorate the surface and new regions S_β of higher β appear.
- From then on, current is emitted from higher β-regions.

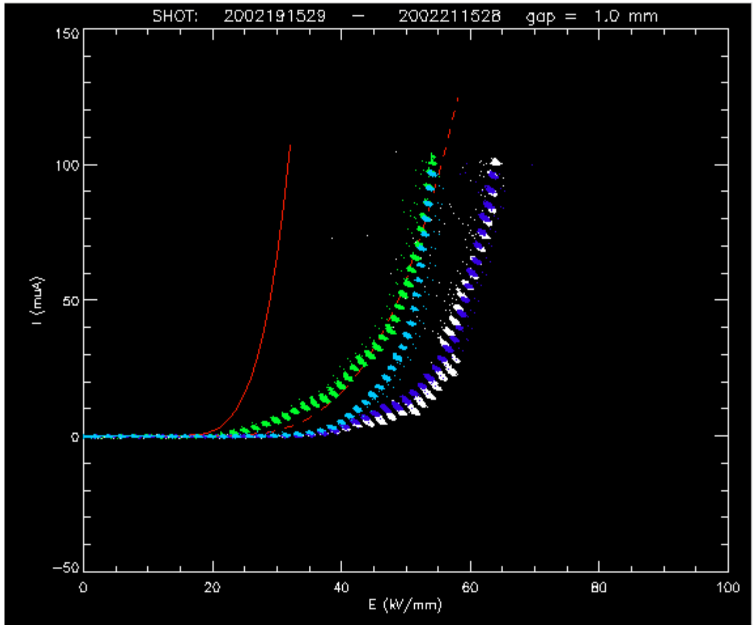
We show here the successive signal, at $g = 1.0\ mm$



We see a more regular increase of current with few bursts added.

“Short-time” F.N.-like law analysis





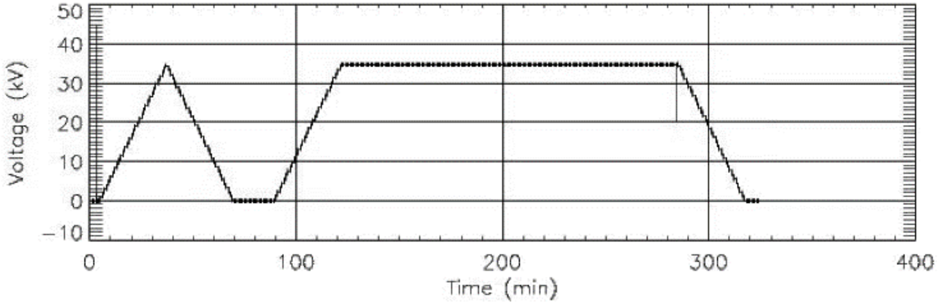
□ White	FIRST – INCREASING - VOLTAGE
■ Blue	FIRST – DECREASING - VOLTAGE
■ Green	SECOND – INCREASING - VOLTAGE
■ Azure	SECOND – DECREASING - VOLTAGE
Solid-Red-line	“Old Data” – Interpolation
Dashed-Red-line	Solid-Red-Line divided by 30

NO DEFINTE CONCLUSION CAN BE INFERRED about reliability of the “Short-time” F.N.-like law

(the system is not well conditioned)

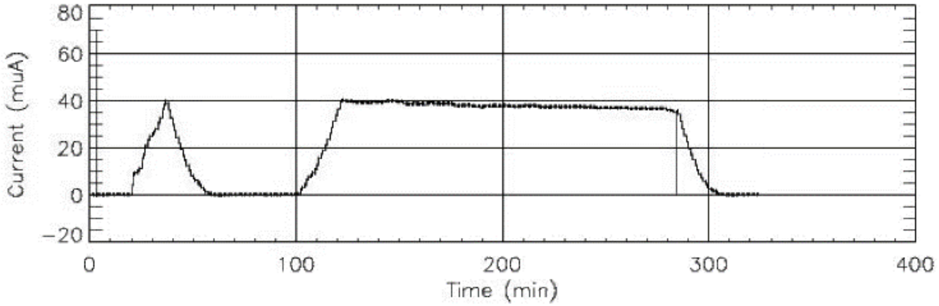
Decreasing current

$$I_e = S\left(1 + k_0 e^{-\frac{k_3}{V}}\right)\frac{k_{Bird}}{\varepsilon_r}(\beta E - E_{int})\beta E \cdot exp\left(-\frac{\varepsilon_r E_d}{\beta E}\right)$$



Voltage and Current time evolution for a gap distance $g = 1.0\, mm$.

In about three hours of constant voltage (35 kV) the current slowly decreases few micro-Amps.



Long time $\geqslant 10^2\ min$

Short time $\leqslant 10^2\ s$

EXPERIMENT CONFIRM THE DECREASING TIME- BEHAVIOR OF CURRENT

7. SUMMARY

BIRD–model + Amplification factor

UP-SIDE

a. It takes DIELECTRIC LAYER into account
b. Fowler-Nordheim – like law (with more realistic coefficients).
c. Correct long-time prediction (decreasing of current).
d. Good fit for well-conditioned systems (to be confirmed).
e. Plausible values for β and S.
f. Explain the nature of BURSTS.
g. Some theoretical indication from Triangular Barrier + Gamow's α-decay theory.

DOWN-SIDE

a. Lack of a solid quantum BIRD model (Gamow's pre-factor seems to be wrong).
b. Amplification factor not yet theoretically understood.

8. FUTURE GOALS

EXPERIMENTs

a. Low-Voltage - Low-current measures (Stainless steel $V \preccurlyeq 8\ kV, \quad I =.....$)

b. Different dielectric coatings (to test different ε_r and E_d)

THEORY

a. Gamow pre-factor investigation

b. Quantum BIRD (ideal) theory

c. Addition of “small” current from the metal to the outer dielectric surface

Thanks for ATTENTION

.... and PATIENCE