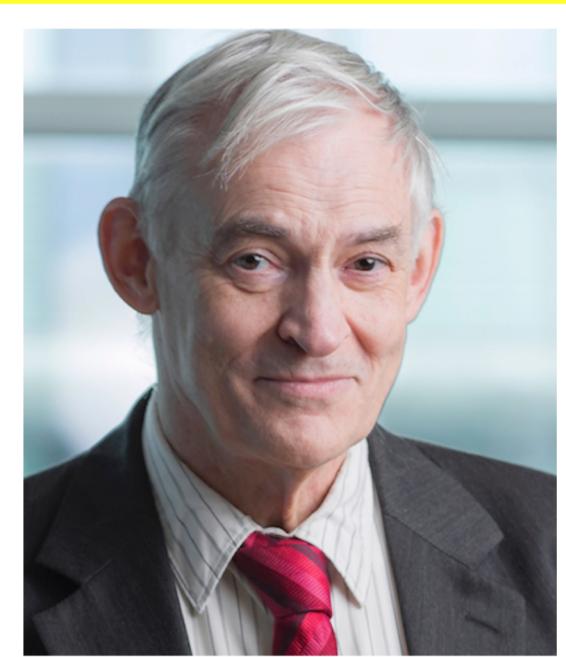


A TUTORIAL COMMENTARY ON THE SCHOTTKY CONSTANT

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SUMMARY

The Schottky constant plays a central role in modern theories of field electron emission, thermal electron emission, and ionic field evaporation, particularly since the 1970s reforms in the international system of measurement, but it is not widely recognised as a useful universal constant. This Poster provides a brief "tutorial" introduction to the Schottky constant, primarily for those not familiar with it. It provides a definition, and proof of relevant equations. It indicates the main contexts in which the Schottky constant is used, and demonstrates that the Schottky constant is a "property of the world" that is represented by technically different physical quantities in different equation systems.

1. INTRODUCTION

Electrons are held into materials by a surface barrier, attributed in simple models to image attraction between the electron and the surface. Applying a classical electrostatic field of appropriate polarity lowers this barrier, and can reduce it to zero if the field magnitude F is large enough.

Image effects in conducting spheres, and the resulting existence of a field-lowered barrier, were discussed by Kelvin in 1849 and by Maxwell in 1891. In 1903, when discussing [1, p. 386] observed spark discharges [2] between closely spaced planar surfaces, J.J. Thomson suggested the cause of sparking might be electron emission due to barrier lowering. However, relevant planar-geometry equations were first clearly formulated by Schottky in 1914 [3]. Hence the barrier-lowering effect is known as the **Schottky effect**.

Schottky's original treatment used "electric potentials", and the Gaussian equation system. Modern treatments discuss the component of electron total energy in the direction (z) normal to the material surface, and use the modern **International System of Quantities (ISQ)** [4], which puts the **vacuum electric permittivity** ϵ_0 in Coulomb's Law. For a good conductor, such as a metal, the energy barrier is described by an energy-like quantity (the so-called **electron motive energy**) $M^{SN}(z)$ given (for a classical planar surface) by

$$M^{SN}(z) = H - eFz - e^2/16\pi\epsilon_0 z. \quad (1)$$

where H is the barrier's **zero-field height**, and e is the **elementary positive charge**. This barrier is often called the **Schottky-Nordheim (SN) barrier**.

2. SCHOTTKY REDUCTION AND THE SCHOTTKY CONSTANT

In the presence of a barrier-reducing field, the maximum height M_{max} of a SN barrier is less than its zero-field height H by an energy Δ_s derived as follows. The condition for the barrier maximum is

$$dM^{SN}/dz = -eF + e^2/16\pi\epsilon_0 z^2 = 0, \quad (2)$$

or

$$eFz = e^2/16\pi\epsilon_0 z, \quad (3)$$

or

$$z = (e/16\pi\epsilon_0)^{1/2} F^{-1/2}. \quad (4)$$

Hence, from (1) & (3):

$$M_{max} = H - 2eFz. \quad (5)$$

Hence:

$$\Delta_s = H - M_{max} = 2eFz. \quad (6)$$

Hence, using (4):

$$\Delta_s = c F^{1/2}, \quad (7)$$

where

$$c = (e^3/4\pi\epsilon_0)^{1/2}. \quad (8)$$

The energy Δ_s is called the **Schottky reduction (or Schottky lowering)**. For SN barriers, Δ_s depends only on the applied field and is proportional to $F^{1/2}$. The coefficient of proportionality c (sometimes written c_s) is an universal constant defined as above and now often called the **Schottky constant**. In the "customary units" often used in field emission, c has the value

$$c \approx 1.199985 \text{ eV (V/nm)}^{-1/2} \approx 3.794686 \times 10^{-5} \text{ eV (V/m)}^{-1/2}. \quad (9)$$

For those unfamiliar with it, this derivation can seem unexpectedly "tricky".

The numerical value of c was first given (but in cm-based units) in eq. (6) of Schottky's 1914 paper [3], but the name "Schottky constant" is much more recent, and is still not widely recognised.

The so-called **zero-barrier field (for the SN barrier)** $F_0^{SN}(H)$, which is the field needed to reduce a SN barrier of zero-field height H to zero (i.e., to reduce M_{max} to zero), is found by setting $\Delta_s = 0$ in eq. (7). This yields

$$F_0^{SN}(H) = c^2 H^2 \approx (0.6944615 \text{ V/nm}) (H/\text{eV})^2. \quad (10)$$

The numerical value of c^{-2} (for fields in V/cm) was first given by Schottky in his 1923 paper [5] [see Table 1, on p. 83 – the value given in eq. (5) is incorrect], where he unsuccessfully tried to explain **field electron emission (FE)** as an effect occurring at field $F_0^{SN}(\phi)$, where ϕ is the relevant local work function.

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3. GENERALIZATION TO OTHER BARRIER FORMS

The algebraic results just discussed refer specifically to the SN barrier. However, physically analogous effects occur with any barrier of the mathematical form:

$$M^{SN}(z) = H - CFz - B/z, \quad (11)$$

where B and C are physical constants, and with any barrier of geometrically similar form. It seems reasonable to let the terms "Schottky reduction" and "the Schottky effect" also apply to these physically analogous effects, but to restrict the name "Schottky constant" to the parameter c as just defined for the SN barrier. However, for barriers of form (11) there will be, in each case, a formula similar to (7), with a specific barrier reduction constant equal to a constant times c . An example is eq. (15): here the multiplying constant is $n^{3/2}$.

4. SOME COMMON APPLICATIONS OF THE SCHOTTKY CONSTANT

(a) **Classical Schottky emission.** The classical Richardson-Schottky equation for emission current density J is conveniently written

$$J = A_0 T^2 \exp[cF^{1/2}/k_B T] \exp[-F^{1/2}/k_B T], \quad (12)$$

where T is thermodynamic temperature, k_B is the Boltzmann constant, and A_0 is the universal theoretical Richardson constant.

(b) **Murphy-Good (MG) FE theory.** MG's 1956 FE theory corrected errors in the original 1928 treatment of Fowler and Nordheim. A modern version [6] of MG theory uses a parameter f , the so-called **scaled field for a SN barrier of zero-field height** ϕ , defined by

$$f = F/F_0^{SN}(\phi) = c^2 \phi^{-2} F. \quad (13)$$

Older versions of MG FE theory used the **Nordheim parameter** y , given by

$$y = +\sqrt{f} = c\phi^{-1} F^{1/2}. \quad (14)$$

Nowadays, use of f , rather than y , is normally to be preferred [7], particularly when discussing current-voltage characteristics. (Thus, in MG theory, it is best to express the correction factor " v_F " in the exponent as a function of f .)

(c) **Positive-ion field-evaporation theory,** as used in field ion microscopy and atom probe microscopy [7]. A parameter of some interest is the so-called **zero-barrier escape field** F_n^e at which the activation energy for an ion of charge ne to escape from the surface becomes zero. The so-called **basic thermodynamic formula** [8] (previously called the "image-hump formula") is an approximation, analogous to (9), in which the escape field is given by

$$F_n^e \approx n^{-3} c^{-2} K_n^2, \quad (15)$$

where K_n is the activation energy needed for ion escape when the field is zero. K_n is a parameter analogous to the zero-field barrier height H .

In existing literature, often c is replaced by its definition (8), or a 1960s-style (now dimensionally inconsistent) formula is used in which c is replaced by a pure number. Formulae are neater (and in the latter case are more correctly formulated) when the universal constant c is used, with the Schottky constant c defined separately or with an appropriate citation given to its definition.

5. THE SCHOTTKY CONSTANT IN OTHER EQUATION SYSTEMS

Equation (8) is an ISQ-system equation, and the *mathematical* quantities it contains are defined within this system. It can be generalised to become a statement about **physical properties** of the world, written:

$$\text{Schottky-reduction} = \text{Schottky-constant} \times (\text{Field-quantity})^{1/2} \quad (16)$$

Table 1 compares the forms this relationship takes in the ISQ, Gaussian and Hartree equation systems, all of which have been used in FE. The subscripts "ISQ", "s", and "H" label quantities belonging to these systems, respectively.

Equation system	Schottky-constant	Reduction expression
ISQ	$(e_{ISQ}^{-3}/4\pi\epsilon_0)^{1/2}$	$(e_{ISQ}^{-3}/4\pi\epsilon_0)^{1/2} F_{ISQ}$
Gaussian	$e_s^{3/2}$	$e_s^{3/2} F_s^{1/2}$
Hartree	1	$F_H^{1/2}$

An intention of the 1970s reforms in the international system of measurement, made by international and national standards authorities ultimately on behalf of Governments, was that the ISQ should become the primary equation system for scientific communication. In FE literature, this 50-year old imperative is mostly implemented, but not yet completely (partly due to the re-printing of pre-reform textbooks without formula updating, and to slow adoption of the ISQ in Russian-language FE literature).