

# Status of proton PDFs and rôle of nuclear corrections

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Partly based on work done in collaboration with  
A. D. **M**artin, W. J. **S**tirling and R. S. **T**horne

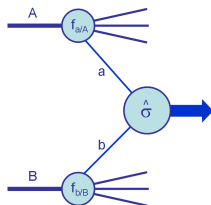
# Introduction

- Protons are not elementary particles: made of **partons**.  
 $\Rightarrow$  **P**arton **D**istribution **F**unctions (**PDFs**) essential to relate theory to experiment at the LHC (and Tevatron, HERA, ...).
- $f_{a/A}(x, Q^2)$  gives *number density* of partons  $a$  in hadron  $A$  with momentum fraction  $x$  at a hard scale  $Q^2 \gg \Lambda_{\text{QCD}}^2$ .

$$\sigma_{AB} = \sum_{a,b=q,g} \int_0^1 dx_a \int_0^1 dx_b f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \hat{\sigma}_{ab}$$

## Outline of talk:

- 1 Introduction to global (proton) PDF analyses.
- 2 Recent developments and current status.
- 3 Benchmark cross sections for the LHC.
- 4 Rôle of nuclear corrections.



# Fixed-order collinear factorisation at hadron colliders

- The “standard” pQCD framework: holds up to formally power-suppressed (“higher-twist”) terms  $\mathcal{O}(\Lambda_{\text{QCD}}^2/Q^2)$ .
- Expand  $\hat{\sigma}_{ab}$ ,  $P_{aa'}$  and  $\beta$  as perturbative series in  $\alpha_S$  ( $\mu_R = \mu_F = Q$ ).

$$\sigma_{AB} = \sum_{a,b=q,g} [\hat{\sigma}_{ab}^{\text{LO}} + \alpha_S(Q^2)\hat{\sigma}_{ab}^{\text{NLO}} + \dots] \otimes f_{a/A}(x_a, Q^2) \otimes f_{b/B}(x_b, Q^2)$$

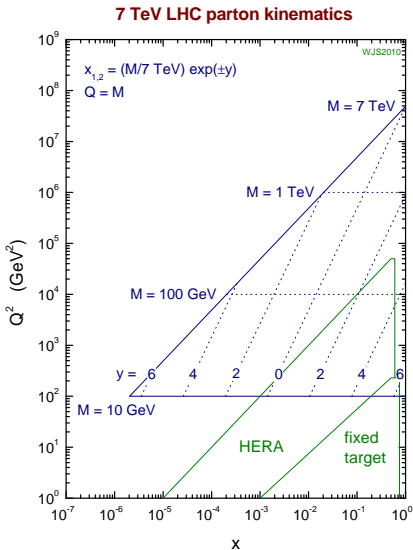
PDF evolution: 
$$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \sum_{a'=q,g} [P_{aa'}^{\text{LO}} + \alpha_S P_{aa'}^{\text{NLO}} + \dots] \otimes f_{a'/A}$$

$\alpha_S$  evolution: 
$$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\text{LO}} \alpha_S^2 - \beta^{\text{NLO}} \alpha_S^3 - \dots$$

- Need to extract input values  $f_{a/A}(x, Q_0^2)$  and  $\alpha_S(M_Z^2)$  from data.
- Structure functions in deep-inelastic scattering (DIS):

$$F_i(x_{\text{Bj}}, Q^2) = \sum_{a=q,g} C_{i,a} \otimes f_{a/A}, \quad C_{i,a} = C_{i,a}^{\text{LO}} + \alpha_S C_{i,a}^{\text{NLO}} + \dots$$

# From HERA *et al.* to the LHC



- PDFs are **universal**.
- Fit existing data from **HERA** and **fixed-target** experiments, together with **Tevatron** data.
- **HERA** *ep* (H1, ZEUS).
- **Fixed-target** experiments:  
*lp, ld*  
 (BCDMS, NMC, E665, SLAC),  
 *$\nu N$*   
 (CCFR, NuTeV, CHORUS),  
*pp, pd* (E866/NuSea).
- **Tevatron** *p $\bar{p}$*  (CDF, DØ).
- DGLAP evolution gives PDFs at higher  $Q^2$  for LHC.

# Paradigm for PDF determination by “global analysis”

- 1 **Parameterise** the  $x$  dependence for each flavour  $a = q, g$  at the input scale  $Q_0^2 \sim 1 \text{ GeV}^2$  in some flexible form, e.g.

$$xf_{a/p}(x, Q_0^2) = A_a x^{\Delta_a} (1-x)^{\eta_a} (1 + \epsilon_a \sqrt{x} + \gamma_a x),$$

subject to number- and momentum-sum rule constraints.

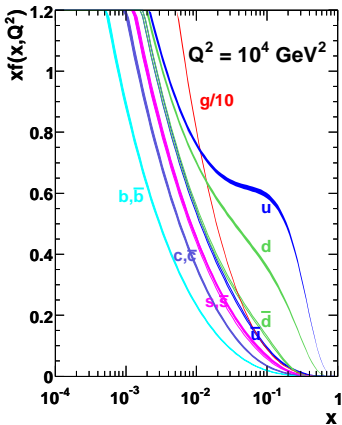
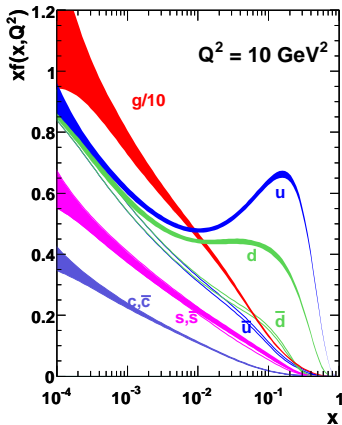
- 2 **Evolve** the PDFs to higher scales  $Q^2 > Q_0^2$  using the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) evolution equations.
- 3 **Convolute** the evolved PDFs with  $C_{i,a}$  and  $\hat{\sigma}_{ab}$  to calculate theory predictions corresponding to a wide variety of data.
- 4 **Vary** the input parameters  $\{A_a, \Delta_a, \eta_a, \epsilon_a, \gamma_a, \dots\}$  to minimise

$$\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left( \frac{\text{Data}_i - \text{Theory}_i}{\text{Error}_i} \right)^2$$

or generalisations to account for *correlated* systematic errors.

# Example of PDFs obtained from global analysis

MSTW 2008 NLO PDFs (68% C.L.)



- **Error bands** shown are obtained from propagation of **experimental** uncertainties on the fitted data points.

# Criteria for choice of tolerance $T = \sqrt{\Delta\chi_{\text{global}}^2}$

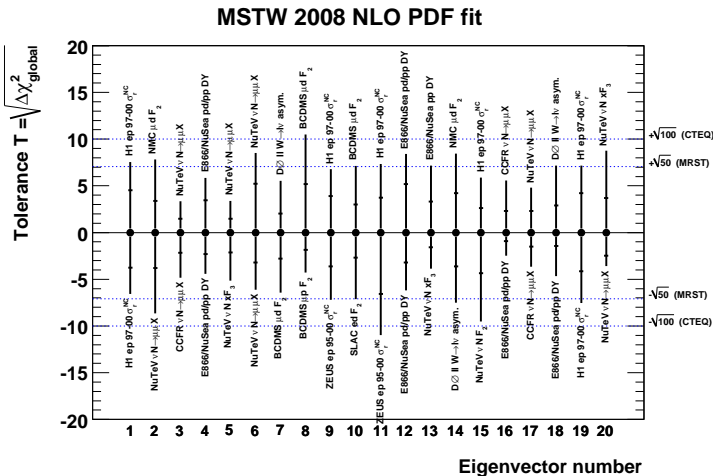
## Parameter-fitting criterion

- $T^2 = 1$  for 68% (1- $\sigma$ ) C.L.,  $T^2 = 2.71$  for 90% C.L.
- **In practice:** minor inconsistencies between fitted data sets, and unknown experimental and theoretical uncertainties, so **not appropriate for global PDF analysis.**

## Hypothesis-testing criterion (proposed by CTEQ)

- Much weaker: treat PDF sets obtained from eigenvectors of covariance matrix as **alternative hypotheses.**
- Determine  $T^2$  from the criterion that **each data set should be described within its 90% C.L. limit.** Very roughly, a “good” fit has  $\chi^2 \simeq N_{\text{pts.}} \pm \sqrt{2N_{\text{pts.}}}$  for each data set.
- **CTEQ:**  $T^2 = 100$  for 90% C.L. limit, **MRST:**  $T^2 = 50$ .
- Large  $T^2$  also for *nuclear* PDFs, e.g.  $T^2 = 50$  for **EPS09.**

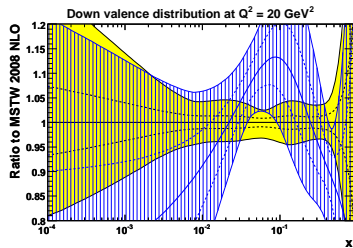
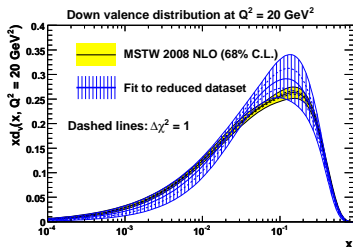
# Dynamic tolerance: different for each eigenvector



- Outer (inner) error bars give tolerance for 90% (68%) C.L.



# Test of dynamic tolerance: fit to reduced dataset



- Fit to **reduced dataset** comprising **589** DIS data points, cf. **2699** data points in **global** fit.
- Errors given by  $T^2 = 1$  don't overlap  $\Rightarrow$  inconsistent data sets included in global fit.
- **Dynamic tolerance**  $T^2 > 1$  **accommodates** mildly inconsistent data sets.
- **Issues:**
  - ①  $T^2 > 1$  not rigorous?
  - ② Dependence on input parameterisation?

# Alternative approach: NNPDF Collaboration

## MSTW approach [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)] (CTEQ similar)

Parameterisation	$xf_{a/p} \sim A_a x^{\Delta_a} (1-x)^{\eta_a} (1 + \epsilon_a \sqrt{x} + \gamma_a x)$
Minimisation	Non-linear least-squares (Marquardt method)
Error propagation	Hessian method with dynamical tolerance
Application	Use best-fit and 40 eigenvector PDF sets

## NNPDF approach [[arXiv:1002.4407](https://arxiv.org/abs/1002.4407), and references therein]

Parameterisation	Neural network (37 free parameters per PDF)
Minimisation	Genetic algorithm (stop before overlearning)
Error propagation	Generate $N_{\text{rep}} \sim \mathcal{O}(1000)$ MC data replicas
Application	Calculate average and s.d. over $N_{\text{rep}}$ PDF sets

- It would be interesting to apply the NNPDF approach to the determination of *nuclear PDFs*, where parameterisation bias is more relevant than for *proton PDFs* due to less available data.

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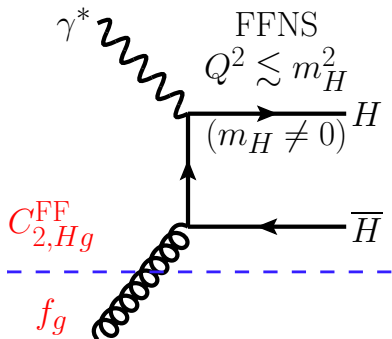
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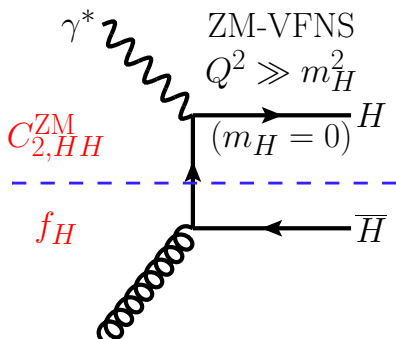
- It would be interesting to apply the NNPDF approach to the determination of *nuclear PDFs*, where parameterisation bias is more relevant than for *proton* PDFs due to less available data.

# Heavy quark contribution to DIS structure function $F_2$



Fixed flavour number scheme

- No heavy quark PDF.
- Includes  $\mathcal{O}(m_H^2/Q^2)$  terms.
- No resummation of  $\alpha_S \ln(Q^2/m_H^2)$  terms.



Zero-mass variable flavour number scheme

- Use heavy quark PDF.
- Mass dependence neglected.
- Resums  $\alpha_S \ln(Q^2/m_H^2)$  terms similar to light quarks.

# General-mass variable flavour number scheme (GM-VFNS)

- Interpolate between two well-defined regions.
- FFNS for  $Q^2 \leq m_H^2$ , ZM-VFNS for  $Q^2 \gg m_H^2$ .
- Details of interpolation are ambiguous ( $\Rightarrow$  **uncertainty**).

## CTEQ6.1 (ZM-VFNS) $\rightarrow$ CTEQ6.5 (GM-VFNS)

- **8% increase** in  $W$  and  $Z$  cross sections at LHC (14 TeV).

## MRST 1998–2006 and MSTW 2008

- MRST/MSTW group have used a GM-VFNS since 1998.

## NNPDF 1.0–2.0

- Fits still use **ZM-VFNS** ( $\rightarrow$  GM-VFNS [[arXiv:1002.4407](https://arxiv.org/abs/1002.4407)]).

Recent comparison between GM-VFNS used by different groups:

**J. Rojo, S. Forte, J. Huston, P. Nadolsky, P. Nason, F. Olness, R. Thorne and G. W.**

*“The Les Houches benchmarks for GM-VFN heavy quark schemes in DIS”* [[arXiv:1003.1241](https://arxiv.org/abs/1003.1241)]



# Most recent public NLO PDF sets from fitting groups

MSTW08 [arXiv:0901.0002]

GM-VFNS **global** fit to DIS, DY and jet data.

CTEQ6.6 [arXiv:0802.0007]

GM-VFNS **global** fit to DIS, DY and jet data.

NNPDF2.0 [arXiv:1002.4407]

**ZM-VFNS global** fit to DIS, DY and jet data.

HERAPDF1.0 [arXiv:0911.0884]

GM-VFNS fit only to inclusive HERA data.

ABKM09 [arXiv:0908.2766, with NLO update in January 2010]

FFNS fit to DIS and DY data.

GJR08 [arXiv:0709.0614]

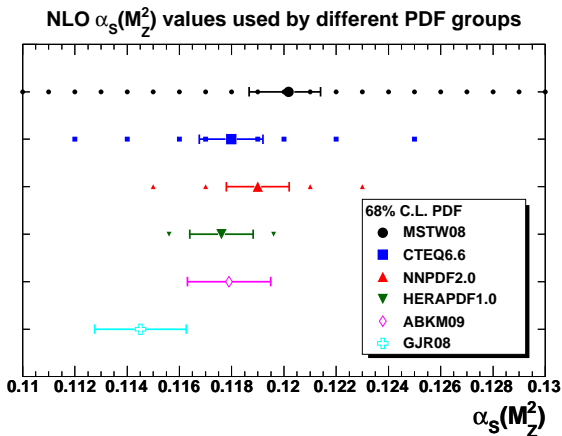
FFNS 'dynamical' fit to DIS, DY and jet data.

- Only MSTW08, ABKM09 and JR09 have **NNLO** PDFs.

# Benchmark cross sections: where do we stand?

- **PDF4LHC** exercise: results presented by G.W. on 26th March.
- Use most recent public NLO PDFs from **all** fitting groups to calculate LHC benchmark processes:  $W^\pm$ ,  $Z^0$ ,  $t\bar{t}$ ,  $gg \rightarrow H$ .
- Comparisons at **NLO** since only three groups have NNLO PDFs.
- **Complication**: all groups use different values of  $\alpha_S(M_Z^2)$ . However:
  - All groups now provide a prescription for “PDF+ $\alpha_S$ ” uncertainties.
  - Most provide alternative PDF sets for different  $\alpha_S(M_Z^2)$  values.
- **Aims of exercise**:
  - ① Establish degree of compatibility and identify **outliers**.
  - ② Compare cross sections at **same**  $\alpha_S$  values.
  - ③ To what extent are differences in predictions due to **different  $\alpha_S$  values** used by each group, rather than differences in PDFs?
- Plots:  
<http://projects.hepforge.org/mstwpdf/pdf4lhc/>

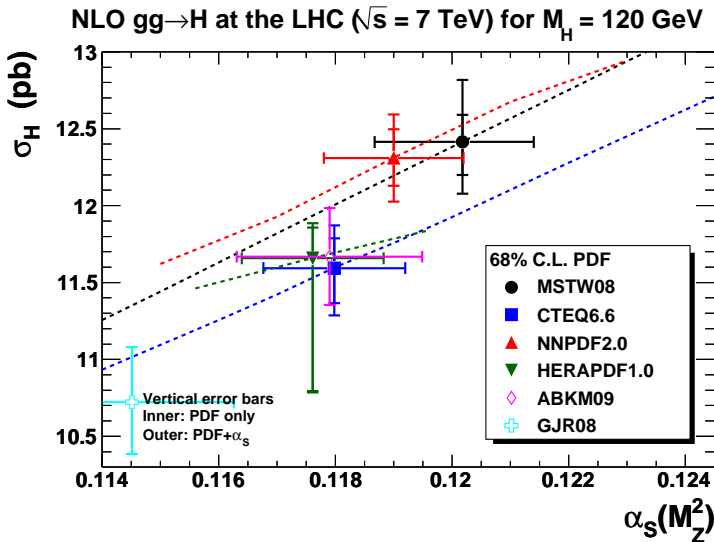
# Values of $\alpha_S(M_Z^2)$ used by different fitting groups



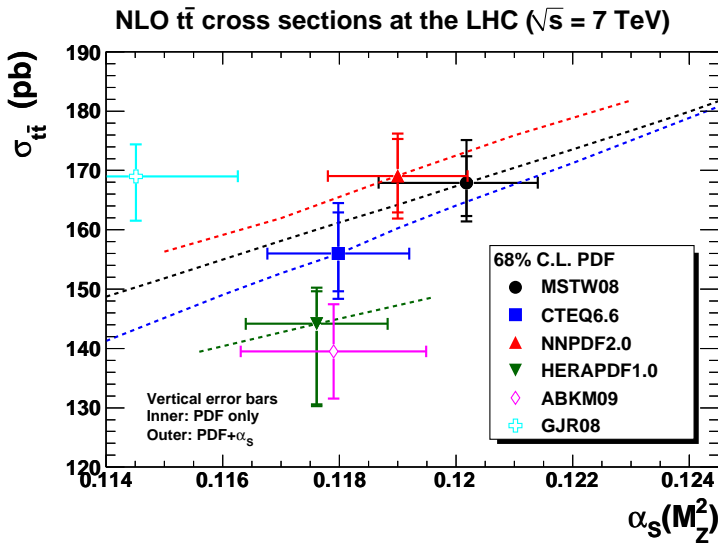
- $\alpha_S(M_Z^2)$  for MSTW08, ABKM09 and GJR08 obtained from fit.
- $\alpha_S(M_Z^2)$  for other groups applied as an external constraint.
- Smaller symbols indicate alternative  $\alpha_S(M_Z^2)$  values provided.

(NNPDF and HERAPDF now provide more values in the latest LHAPDF V5.8.3 released 18th May.)

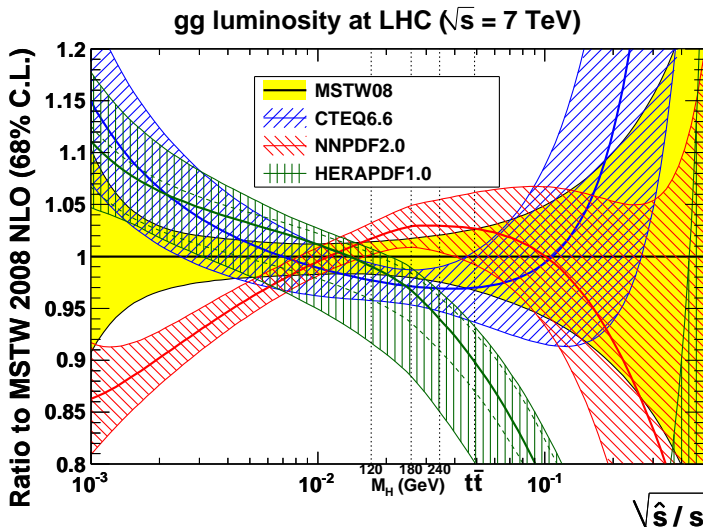
# Higgs ( $M_H = 120$ GeV) total cross section vs. $\alpha_S(M_Z^2)$



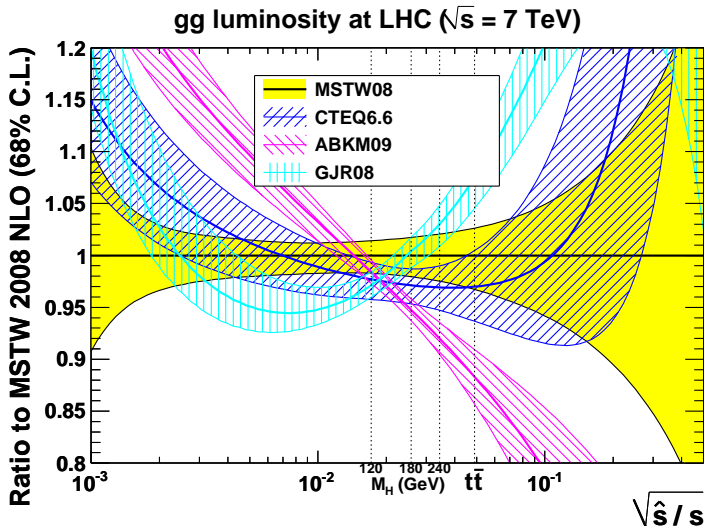
# $t\bar{t}$ total cross section vs. $\alpha_s(M_Z^2)$



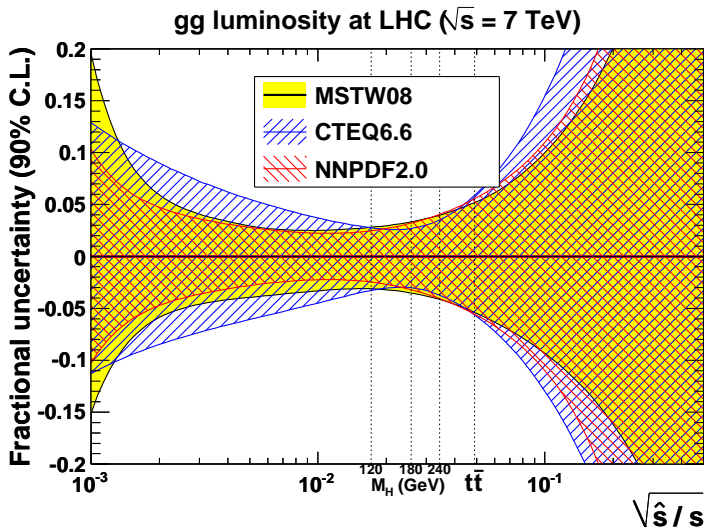
# Ratio of gluon-gluon luminosity functions (1)



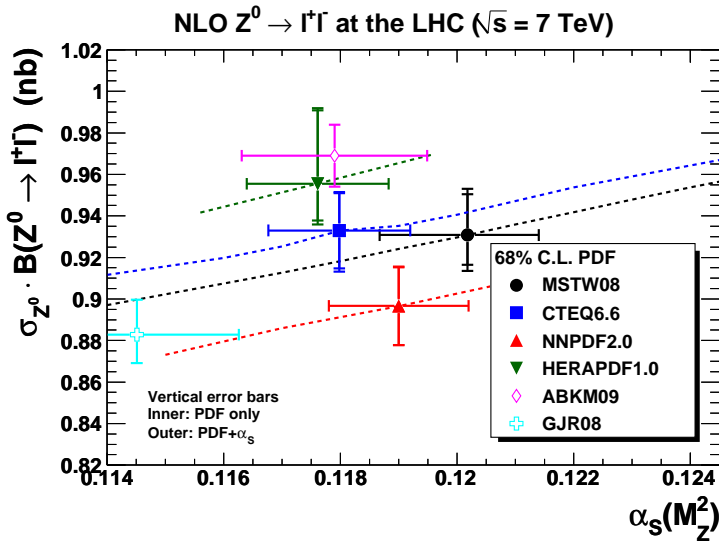
## Ratio of gluon-gluon luminosity functions (2)



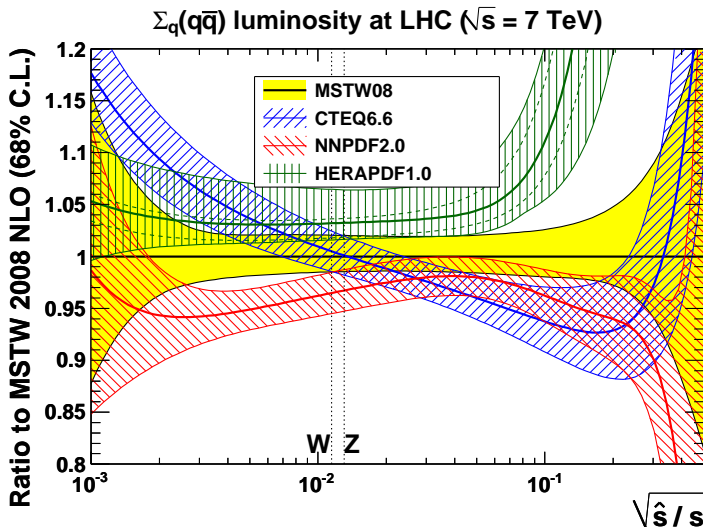
# Fractional uncertainty in gluon-gluon luminosity



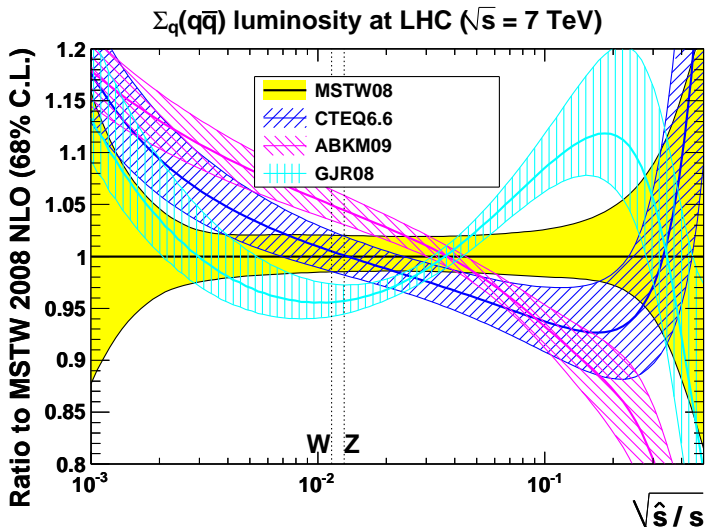


$Z^0$  total cross section vs.  $\alpha_S(M_Z^2)$ 

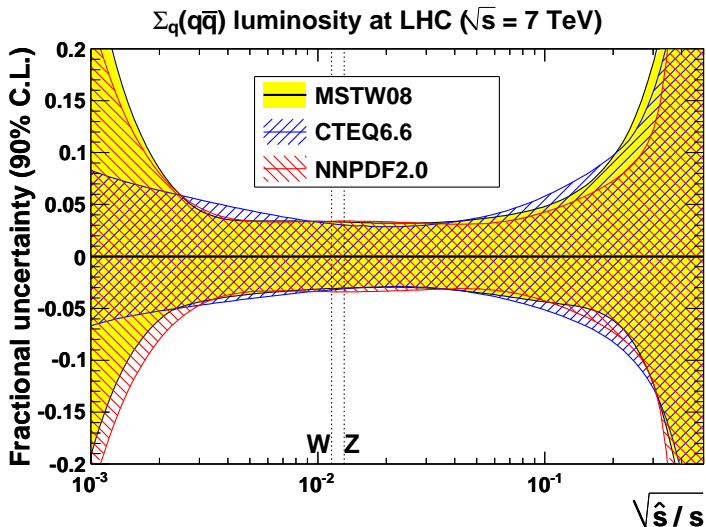
# Ratio of quark–antiquark luminosity functions (1)



# Ratio of quark-antiquark luminosity functions (2)



# Fractional uncertainty in quark-antiquark luminosity



# Summary: current status of proton PDFs

- Highlight major differences in data and theory between groups.

	MSTW08	CTEQ6.6	NNPDF2.0	HERAPDF1.0	ABKM09	GJR08
HERA DIS	✓	✓	✓	✓	✓	✓
Fixed-target DIS	✓	✓	✓	✗	✓	✓
Fixed-target DY	✓	✓	✓	✗	✓	✓
Tevatron $W,Z$	✓	✓	✓	✗	✗	✗
Tevatron jets	✓	✓	✓	✗	✗	✓
GM-VFNS	✓	✓	✗	✓	✗	✗
NNLO	✓	✗	✗	✗	✓	✓

- *Three NLO* global fits, but only *one NNLO* global fit.
- **CTEQ6.6** only uses Tevatron Run I data, not Run II.
- Only **NNPDF2.0** and **HERAPDF1.0** use *combined* HERA data.

# Which processes constrain different PDFs?

- Processes included in MSTW 2008 analysis [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)]:

Process	Subprocess	Partons	x range
$l^\pm \{p, n\} \rightarrow l^\pm X$	$\gamma^* q \rightarrow q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$l^\pm n/p \rightarrow l^\pm X$	$\gamma^* d/u \rightarrow d/u$	$d/u$	$x \gtrsim 0.01$
$pp \rightarrow \mu^+ \mu^- X$	$u\bar{u}, d\bar{d} \rightarrow \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$p n/pp \rightarrow \mu^+ \mu^- X$	$(u\bar{d})/(u\bar{u}) \rightarrow \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \rightarrow \mu^-(\mu^+) X$	$W^* q \rightarrow q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \rightarrow \mu^- \mu^+ X$	$W^* s \rightarrow c$	$s$	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \rightarrow \mu^+ \mu^- X$	$W^* \bar{s} \rightarrow \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \rightarrow e^\pm X$	$\gamma^* q \rightarrow q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \rightarrow \bar{\nu} X$	$W^+ \{d, s\} \rightarrow \{u, c\}$	$d, s$	$x \gtrsim 0.01$
$e^\pm p \rightarrow e^\pm c\bar{c} X$	$\gamma^* c \rightarrow c, \gamma^* g \rightarrow c\bar{c}$	$c, g$	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \rightarrow \text{jet} + X$	$\gamma^* g \rightarrow q\bar{q}$	$g$	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \rightarrow \text{jet} + X$	$gg, qg, q\bar{q} \rightarrow 2j$	$g, q$	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \rightarrow (W^\pm \rightarrow l^\pm \nu) X$	$ud \rightarrow W, \bar{u}\bar{d} \rightarrow W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \rightarrow (Z \rightarrow l^+ l^-) X$	$uu, dd \rightarrow Z$	$d$	$x \gtrsim 0.05$

- Nuclear corrections needed for  $n$  (deuteron) and  $N$  ( $Fe$  or  $Pb$ ).

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$pn/pp \rightarrow \mu^+ \mu^- X$	$(u\bar{d})/(u\bar{u}) \rightarrow \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \rightarrow \mu^-(\mu^+) X$	$W^* q \rightarrow q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
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$p\bar{p} \rightarrow (Z \rightarrow l^+ l^-) X$	$uu, dd \rightarrow Z$	$d$	$x \gtrsim 0.05$

- Nuclear corrections needed for  $n$  (deuteron) and  $N$  ( $Fe$  or  $Pb$ ).

Data sets fitted in MSTW 2008 NLO analysis [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)]

Data set	$\chi^2 / N_{\text{pts.}}$	Data set	$\chi^2 / N_{\text{pts.}}$
H1 MB 99 $e^+p$ NC	9 / 8	BCDMS $\mu p F_2$	182 / 163
H1 MB 97 $e^+p$ NC	42 / 64	BCDMS $\mu d F_2$	190 / 151
H1 low $Q^2$ 96–97 $e^+p$ NC	44 / 80	NMC $\mu p F_2$	121 / 123
H1 high $Q^2$ 98–99 $e^-p$ NC	122 / 126	NMC $\mu d F_2$	102 / 123
H1 high $Q^2$ 99–00 $e^+p$ NC	131 / 147	NMC $\mu n / \mu p$	130 / 148
ZEUS SVX 95 $e^+p$ NC	35 / 30	E665 $\mu p F_2$	57 / 53
ZEUS 96–97 $e^+p$ NC	86 / 144	E665 $\mu d F_2$	53 / 53
ZEUS 98–99 $e^-p$ NC	54 / 92	SLAC $ep F_2$	30 / 37
ZEUS 99–00 $e^+p$ NC	63 / 90	SLAC $ed F_2$	30 / 38
H1 99–00 $e^+p$ CC	29 / 28	NMC/BCDMS/SLAC $F_L$	38 / 31
ZEUS 99–00 $e^+p$ CC	38 / 30	E866/NuSea $pp$ DY	228 / 184
<b>H1/ZEUS <math>e^\pm p F_2^{\text{charm}}</math></b>	107 / 83	E866/NuSea $pd/pp$ DY	14 / 15
<b>H1 99–00 <math>e^+p</math> incl. jets</b>	19 / 24	<b>NuTeV <math>\nu N F_2</math></b>	49 / 53
<b>ZEUS 96–97 <math>e^+p</math> incl. jets</b>	30 / 30	<b>CHORUS <math>\nu N F_2</math></b>	26 / 42
<b>ZEUS 98–00 <math>e^\pm p</math> incl. jets</b>	17 / 30	<b>NuTeV <math>\nu N xF_3</math></b>	40 / 45
<b>DØ II <math>p\bar{p}</math> incl. jets</b>	114 / 110	<b>CHORUS <math>\nu N xF_3</math></b>	31 / 33
<b>CDF II <math>p\bar{p}</math> incl. jets</b>	56 / 76	<b>CCFR <math>\nu N \rightarrow \mu\mu X</math></b>	66 / 86
<b>CDF II <math>W \rightarrow l\nu</math> asym.</b>	29 / 22	<b>NuTeV <math>\nu N \rightarrow \mu\mu X</math></b>	39 / 40
<b>DØ II <math>W \rightarrow l\nu</math> asym.</b>	25 / 10	<b>All data sets</b>	<b>2543 / 2699</b>
<b>DØ II <math>Z</math> rap.</b>	19 / 28		
<b>CDF II <math>Z</math> rap.</b>	49 / 29		

- **Bold** = New w.r.t. MRST 2006 fit.

- **Red** = Needs nuclear corrections.



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H1 high $Q^2$ 99–00 $e^+p$ NC	131 / 147	<b>NMC <math>\mu n / \mu p</math></b>	130 / 148
ZEUS SVX 95 $e^+p$ NC	35 / 30	E665 $\mu p F_2$	57 / 53
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ZEUS 98–99 $e^-p$ NC	54 / 92	SLAC $ep F_2$	30 / 37
ZEUS 99–00 $e^+p$ NC	63 / 90	<b>SLAC <math>ed F_2</math></b>	30 / 38
H1 99–00 $e^+p$ CC	29 / 28	NMC/BCDMS/SLAC $F_L$	38 / 31
ZEUS 99–00 $e^+p$ CC	38 / 30	E866/NuSea $pp$ DY	228 / 184
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<b>DØ II <math>Z</math> rap.</b>	19 / 28		
<b>CDF II <math>Z</math> rap.</b>	49 / 29		

- **Bold** = New w.r.t. MRST 2006 fit.
- **Red** = Needs nuclear corrections.

## Why do we use data taken on nuclear targets?

- Assume isospin symmetry ( $u \equiv u^p = d^n$ ,  $d \equiv d^p = u^n$ ) and isoscalar target ( $N = (p + n)/2$ ), then structure functions are:

### Neutral-current DIS in charged-lepton–nucleon scattering

$$F_2^{\ell^\pm p} = x \left[ \frac{4}{9} (u + \bar{u} + \dots) + \frac{1}{9} (d + \bar{d} + \dots) \right]$$

$$F_2^{\ell^\pm d} = (F_2^{\ell^\pm p} + F_2^{\ell^\pm n})/2 = \frac{5}{18} x (u + \bar{u} + d + \bar{d}) + \dots$$

### Charged-current DIS in neutrino–nucleus scattering

$$F_2 \equiv (F_2^{\nu N} + F_2^{\bar{\nu} N})/2 = x (u + \bar{u} + d + \bar{d} + \dots)$$

$$xF_3 \equiv (xF_3^{\nu N} + xF_3^{\bar{\nu} N})/2 = x (u - \bar{u} + d - \bar{d} + \dots)$$

- $\nu N \rightarrow \mu\mu X$  constrains  $s$ ;  $\bar{\nu} N \rightarrow \mu\mu X$  constrains  $\bar{s}$ .

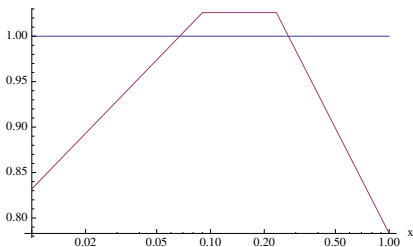
Deuterium and neutrino DIS data provide **flavour separation!**

# Treatment of nuclear corrections by MRST (1998–2006)

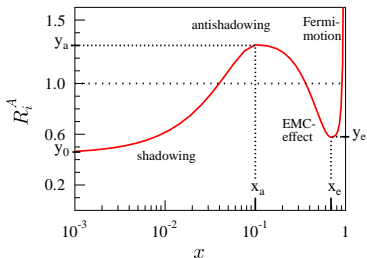
- Fit  $\nu N$  data on  $F_2$  and  $xF_3$  from CCFR (iron target).
- Empirical nuclear correction “RATFE” derived from  $\mu$ - $Fe$  data.
- Correction assumed to be  $Q^2$ -/process-/flavour- independent.

Correction used by MRST:

RATFE(x)



Compare to generic correction:



[EPS09, arXiv:0902.4154]

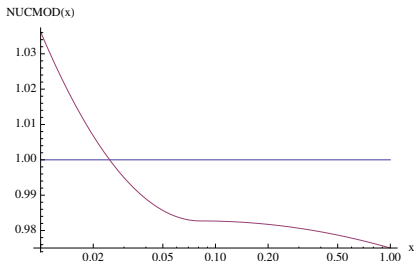
- Similar treatment in standard CTEQ fits ( $\rightarrow$  I. Schienbein).
- NNPDF use  $\nu N$  data ( $Fe$  and  $Pb$ ) but **without** nuclear effects.

# Improvements in MRST 2006 → MSTW 2008

- CCFR  $F_2$  and  $xF_3$  data → **NuTeV** ( $Fe$ ) and **CHORUS** ( $Pb$ ).
- Apply  $x$ -,  $Q^2$ -,  $A$ - and flavour- **dependent** nuclear correction:

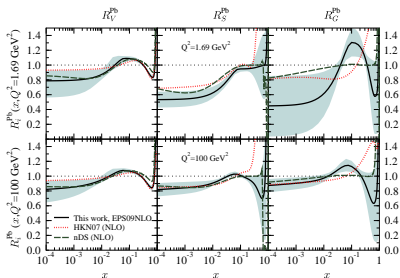
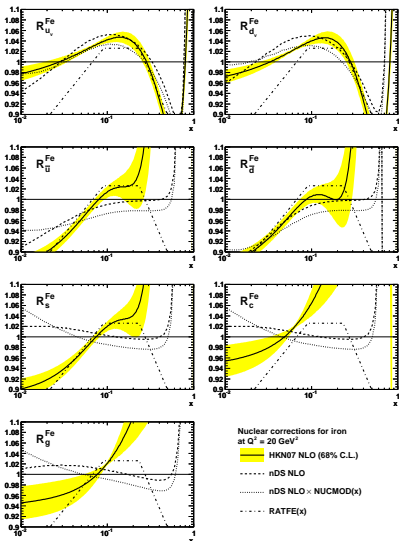
$$R_f^A(x, Q^2) = f^A(x, Q^2)/f(x, Q^2).$$

- Only NLO fit available in 2006 was **nDS** [de Florian and Sassot, '03].
- Determination of  $R_f^A$  used free proton PDFs as input (**GRV98**).  
In principle, some degree of **circularity** (→ I. Schienbein).

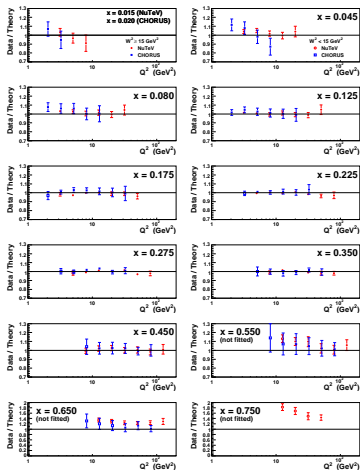
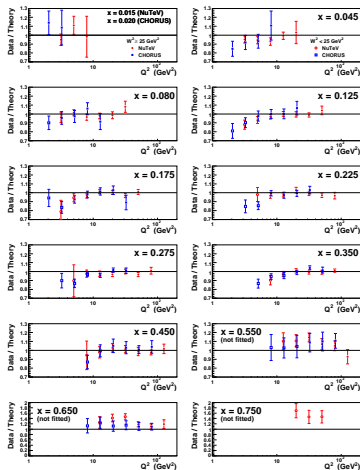


- Allow for **uncertainty** with extra modification function ( $Q^2$ -,  $A$ -,  $f$ - independent).
- Three parameters allowed to go free in global fits, which enter covariance matrix **used for final error propagation**.

# Comparison of nuclear corrections for $Fe$ and $Pb$



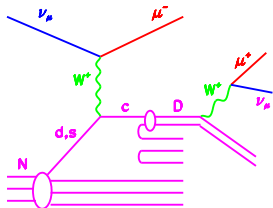
- Two later NLO fits with uncertainties (HKN07, EPS09).
- Reasonable agreement for most relevant  $R_u^A$  and  $R_d^A$ .
- Larger uncertainties present for  $R_s^A$  and (less relevant)  $R_g^A$ .

Description of NuTeV and CHORUS data on  $F_2$  and  $xF_3$ Ratio of NuTeV and CHORUS  $F_2$  data to MSTW 2008 NNLO PDF fitRatio of NuTeV and CHORUS  $xF_3$  data to MSTW 2008 NNLO PDF fit

$\chi^2$ for $F_2$ data	LO	NLO	NNLO
NuTeV (49 pts.)	49	49	46
CHORUS (37 pts.)	21	26	29

$\chi^2$ for $xF_3$ data	LO	NLO	NNLO
NuTeV (45 pts.)	62	40	34
CHORUS (33 pts.)	44	31	26

# NuTeV/CCFR dimuon cross sections and strangeness



$$\frac{d\sigma}{dx dy}(\nu_\mu N \rightarrow \mu^+ \mu^- X) = B_c \mathcal{A} \frac{d\sigma}{dx dy}(\nu_\mu N \rightarrow \mu^- c X)$$

$$\propto |V_{cs}|^2 \xi s^A(\xi, Q^2) + |V_{cd}|^2 \dots$$

- $\nu_\mu$  and  $\bar{\nu}_\mu$  cross sections constrain  $s^A$  and  $\bar{s}^A$ .
- Again, use  $R_f^A$  (nDS) to relate to **proton** PDFs.
- Can **relax assumption** made in previous fits that

$$s(x, Q_0^2) = \bar{s}(x, Q_0^2) = \frac{\kappa}{2} [\bar{u}(x, Q_0^2) + \bar{d}(x, Q_0^2)], \text{ with } \kappa \approx 0.5.$$

- MSTW **parameterise** at input scale of  $Q_0^2 = 1 \text{ GeV}^2$  in the form:

$$x s(x, Q_0^2) + x \bar{s}(x, Q_0^2) = A_+ (1-x)^{\eta_+} x S(x, Q_0^2),$$

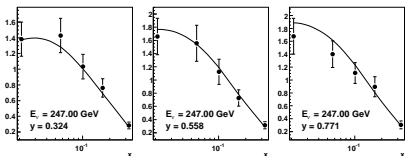
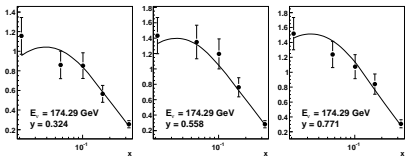
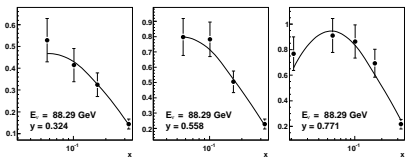
$$x s(x, Q_0^2) - x \bar{s}(x, Q_0^2) = A_- x^{0.2} (1-x)^{\eta_-} (1-x/x_0).$$

- $x_0$  fixed by zero strangeness:  $\int_0^1 dx [s(x, Q_0^2) - \bar{s}(x, Q_0^2)] = 0$ .

## Description of NuTeV dimuon cross sections

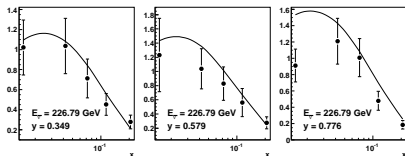
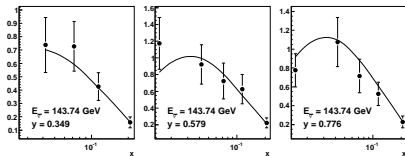
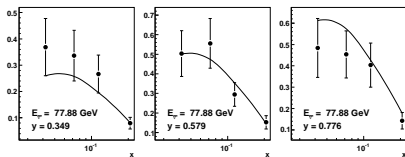
$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}$$

MSTW 2008 NNLO PDF fit,  $\chi^2 = 13$  for 21 DOF



$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}$$

MSTW 2008 NNLO PDF fit,  $\chi^2 = 32$  for 19 DOF

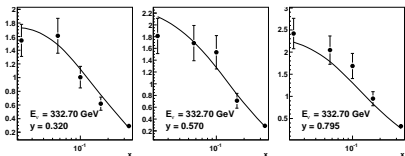
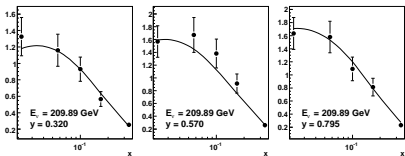
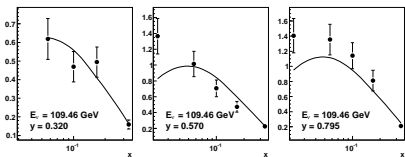




## Description of CCFR dimuon cross sections

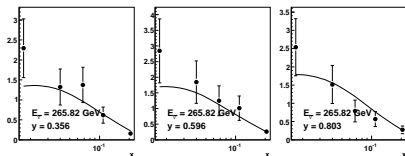
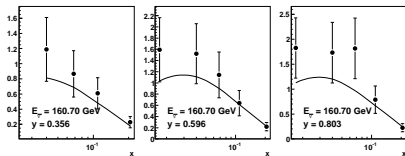
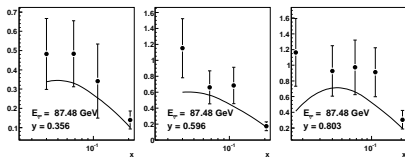
$$\text{CCFR } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\nu_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^2$$

MSTW 2008 NNLO PDF fit,  $\chi^2 = 38$  for 44 pts.



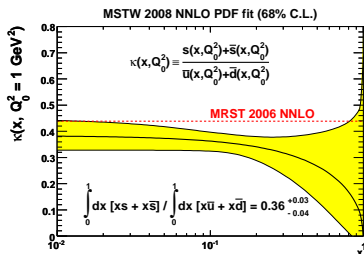
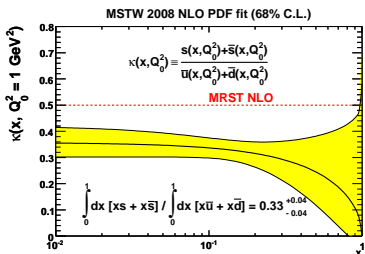
$$\text{CCFR } \frac{100\pi}{G_F^2 M_N E_{\bar{\nu}}} \frac{d\sigma}{dx dy} (\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^2$$

MSTW 2008 NNLO PDF fit,  $\chi^2 = 31$  for 42 pts.

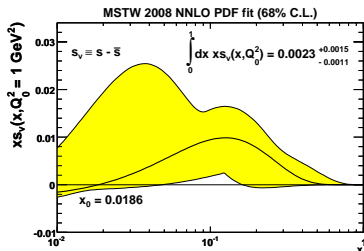
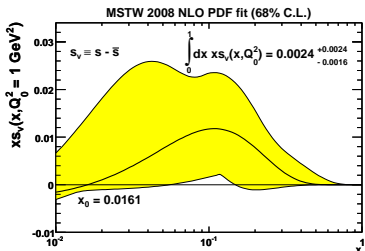


# Strange quark and antiquark distributions

- Ratio of strange fit sea to non-strange sea:  $(s + \bar{s}) / (\bar{u} + \bar{d})$ .



- Strange sea asymmetry:  $x\bar{s} - x\bar{u}$ .



# Resolutions to NuTeV $\sin^2 \theta_W$ “anomaly”

- NuTeV extraction [[hep-ex/0110059](#)] of  $\sin^2 \theta_W$  from

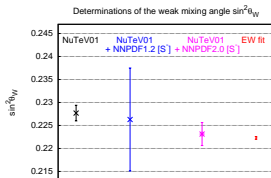
$$R^- \equiv \frac{\sigma(\nu_\mu N \rightarrow \nu_\mu X) - \sigma(\bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \rightarrow \mu^- X) - \sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)} \approx \frac{1}{2} - \sin^2 \theta_W$$

was  $\sim 3\sigma$  above the Standard Model prediction.

- New Physics? Or neglected PDF-related uncertainties, e.g.

## 1 Strange sea asymmetry? ( $s \neq \bar{s}$ )

- MSTW  $s - \bar{s}$  removes  $1-2\sigma$  of NuTeV  $\sin^2 \theta_W$  “anomaly”.
- Unbiased parameterisation (NNPDF) removes **all** of it.



## 2 Isospin violation? ( $u^P \neq d^P, d^P \neq u^P$ )

- Generated by QED corrections to parton evolution.
- Found by MRST [[hep-ph/0411040](#)] to remove a little more than  $1\sigma$  of the total discrepancy in the NuTeV  $\sin^2 \theta_W$ .

## 3 Nuclear corrections? ( $R_{u\nu}^A \neq R_{d\nu}^A$ ) [[Eskola and Paukkunen, '06](#)]

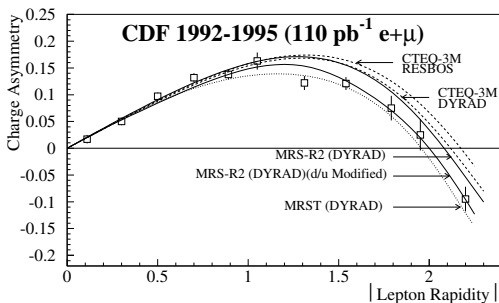
# Nuclear modifications in the deuteron

- MSTW08 fit used  $F_2^d$  data from BCDMS, NMC, E665, SLAC.
- Deuteron corrections expected to be **small** (few % at most).
- **Neglected** by most proton PDF groups (e.g. CTEQ, NNPDF) and in most fits of nuclear PDFs (exception: HKN07).
- $F_2^d$  data used in all MRST/MSTW fits were **corrected** for nuclear shadowing using a model [Badelek and Kwiecinski, '94]:
  - Calculated using a vector-meson-dominance mechanism.
  - Correction of 1% at  $x = 0.01$  and  $Q^2 = 4 \text{ GeV}^2$ .
  - **No** nuclear modification at **medium/large**  $x$ .
- Recently we re-examined deuteron corrections, prompted by apparent discrepancies with new precise Tevatron data on the  $W \rightarrow \ell\nu$  **charge asymmetry**. **Preliminary** studies.

# $W \rightarrow \ell\nu$ charge asymmetry from Tevatron Run I

$$A_W(y_W) = \frac{d\sigma(W^+)/dy_W - d\sigma(W^-)/dy_W}{d\sigma(W^+)/dy_W + d\sigma(W^-)/dy_W} \approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}$$

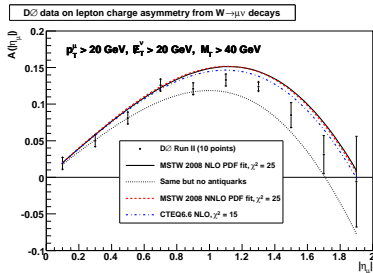
But measure  $A_\ell(\eta_\ell) = \frac{d\sigma(\ell^+)/d\eta_\ell - d\sigma(\ell^-)/d\eta_\ell}{d\sigma(\ell^+)/d\eta_\ell + d\sigma(\ell^-)/d\eta_\ell}$ , constrains  $d/u$  ratio.



[hep-ex/9809001]

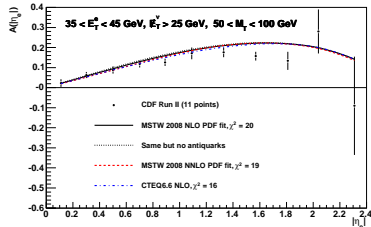
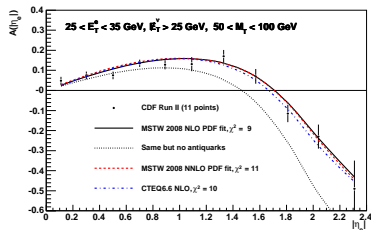
# $W \rightarrow \ell\nu$ charge asymmetry from Tevatron Run II

- Run II data in **MSTW 2008** fit.
- Mainly constraint on **down** quark.
- Antiquarks important at low  $p_T^\ell$ .

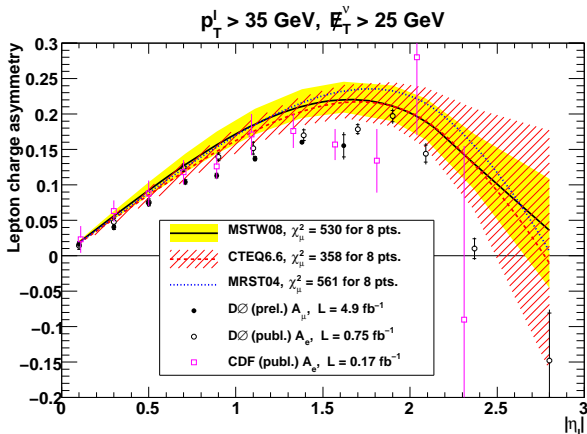


[Data: [arXiv:0709.4254](https://arxiv.org/abs/0709.4254)]

CDF data on lepton charge asymmetry from  $W \rightarrow e\nu$  decays



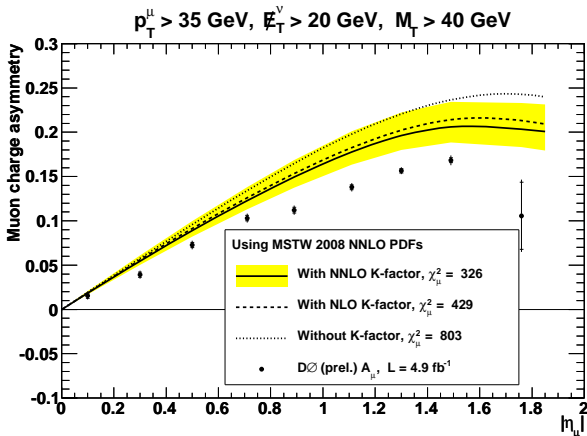
[Data: [hep-ex/0501023](https://hep-ex.org/2005/10/23/)]

Latest DØ data on  $W \rightarrow \ell\nu$  charge asymmetryNew DØ data:  $A_e$  [arXiv:0807.3367] and  $A_\mu$  [DØ Note 5976-CONF]

- Problems describing new data at NLO, especially for  $p_T^\ell > 35 \text{ GeV}$ .

# Latest $D\bar{D}$ data on $W \rightarrow \ell\nu$ charge asymmetry

- Effect of NNLO (or  $p_T^W$ -resummation, RESBOS) is small.



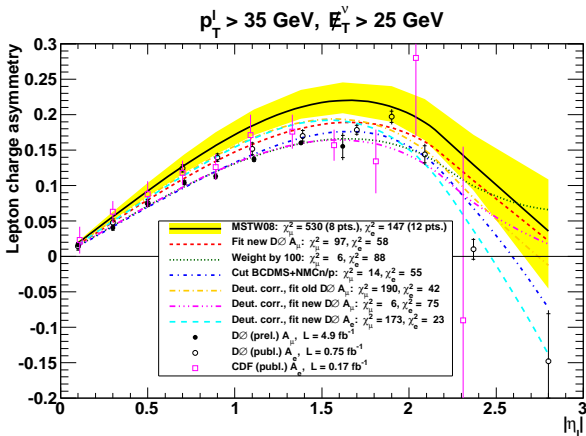
NNLO: [Catani, Cieri, Ferrera, de Florian, Grazzini, arXiv:0903.2120](#)

(Previous calculation: [Melnikov, Petriello, hep-ph/0609070, FEWZ](#))



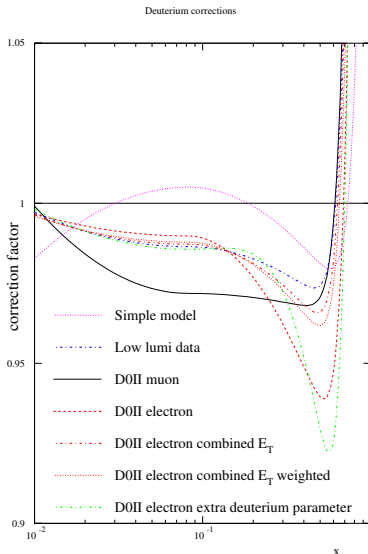
# Latest $D\bar{D}$ data on $W \rightarrow \ell\nu$ charge asymmetry

- Can the PDFs be refitted to describe the new data?



- Allowing **free deuteron corrections** aids description of  $D\bar{D}$  data.
- But not possible to describe both  $A_\mu$  and  $A_e$  simultaneously.

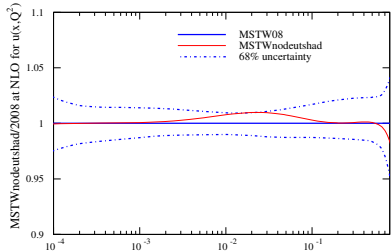
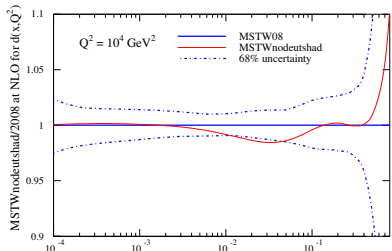
# Deuteron corrections determined from global fits



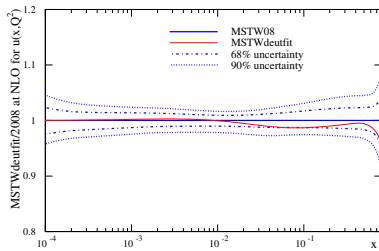
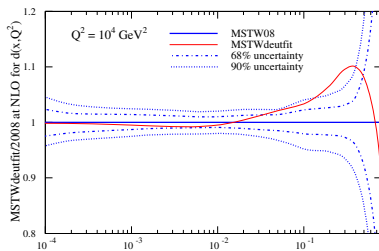
- **Parameterise** deuteron corrections as a smooth  $Q^2$ -independent function with 4 free parameters.
- **Helps resolve tension** between  $F_2^d$  data and Tevatron  $W \rightarrow \ell\nu$  data.
- But corrections are **larger** and a different shape than theoretical expectations (“**Simple model**”).
- Particularly large corrections needed for DØ  $W \rightarrow \mu\nu$  charge asymmetry data.

# Effect of deuteron corrections on $d$ - and $u$ -quark PDFs

No deuteron shadowing:



Deuteron corrections fitted:



# Summary

## Status of proton PDFs (**global** fits in GM-VFNS)

- MSTW08/CTEQ6.6 at NLO, but only MSTW08 at NNLO.
- NNPDF a viable alternative **after** GM-VFNS implemented.

## Rôle of nuclear corrections in global fits of proton PDFs

- Neutrino DIS data and deuteron structure functions are important to provide **flavour separation** of proton PDFs.
- Plenty of room for improvement in treatment of **nuclear corrections** (→ next talks by H. Paukkunen and I. Schienbein).
- Need reliable estimate of nuclear corrections (including deuteron), and propagation of uncertainties through to proton PDFs, to be used for precision LHC physics.