

# The Relation Good–Walker - Triple-Regge in Diffractive Excitation



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# Content

1. Diffractive excitation often treated by two mechanisms:
  - a) Low mass: Good–Walker
  - b) High mass: Triple Regge

How are these related?

2. High mass diffractive excitation in Good–Walker picture
3. Bare pomeron in Good–Walker picture



# Eikonal approximation

Diffraction and saturation more easily described in impact parameter space

Scattering driven by absorption into inelastic states  $i$ , with weights  $2f_i$

⇒ Elastic amplitude  $T = 1 - e^{-F}$ , with  $F = \sum f_i$

For a structureless projectile we find:

$$\begin{cases} d\sigma_{tot}/d^2b \sim \langle 2T \rangle \\ \sigma_{el}/d^2b \sim \langle T \rangle^2 \\ \sigma_{inel}/d^2b \sim \langle 1 - e^{-\sum 2f_i} \rangle = \sigma_{tot} - \sigma_{el} \end{cases}$$



## Good – Walker

If the projectile has an **internal structure**, the mass eigenstates can differ from the eigenstates of diffraction

Diffractive eigenstates:  $\Phi_n$ ; Eigenvalue:  $T_n$

Mass eigenstates:  $\Psi_k = \sum_n c_{kn} \Phi_n$  ( $\Psi_{in} = \Psi_1$ )

Elastic amplitude:  $\langle \Psi_1 | T | \Psi_1 \rangle = \sum c_{1n}^2 T_n = \langle T \rangle$

$$d\sigma_{el}/d^2b \sim (\sum c_{1n}^2 T_n)^2 = \langle T \rangle^2$$

Amplitude for diffractive transition to mass eigenstate  $\Psi_k$ :

$$\langle \Psi_k | T | \Psi_1 \rangle = \sum_n c_{kn} T_n c_{1n}$$

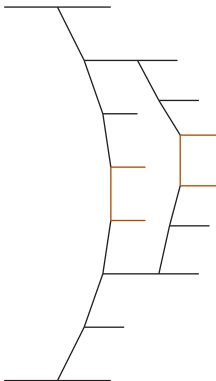
$$d\sigma_{diff}/d^2b = \sum_k \langle \Psi_1 | T | \Psi_k \rangle \langle \Psi_k | T | \Psi_1 \rangle = \langle T^2 \rangle$$

Diffractive excitation determined by the fluctuations:

$$d\sigma_{diff\ ex}/d^2b = d\sigma_{diff} - d\sigma_{el} = \langle T^2 \rangle - \langle T \rangle^2$$



# High energy collisions driven by parton-parton subcollisions (à la PYTHIA)



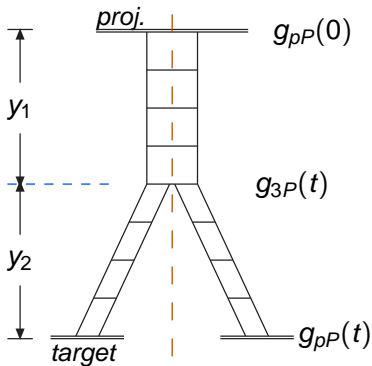
BFKL evolution

Multiple subcollisions  
⇒ saturation



# Diffractive cross sections

## Triple-Regge



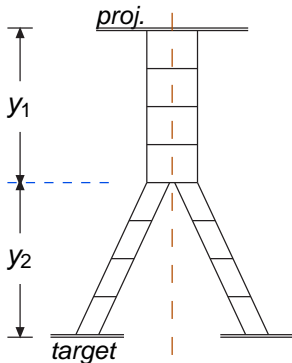
Traditionally fluctuations  
not taken into account

Reggeon parameters and  
couplings fitted to data



# Diffractive cross sections

## Good-Walker



BFKL: Large fluctuations  
 (Mueller–Salam)

Can this reproduce the  
 triple-regge result?

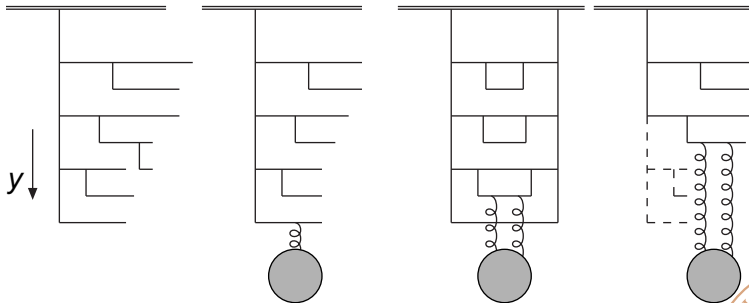
$\langle \langle T \rangle_{\text{target}}^2 \rangle_{\text{proj}}$  gives diffractive scattering with  $M_X^2 < \exp(y_1)$

Vary  $y_1$  gives  $d\sigma/dM_X^2$



# Diffractive eigenstates

Parton cascades, which can come on shell through interaction with the target



*virtual cascade*

*inelastic int.*

*elastic scatt.*

*diffractive exc.*

Cf. Miettinen–Pumplin (1978), Hatta *et al.* (2006)<sup>1</sup>

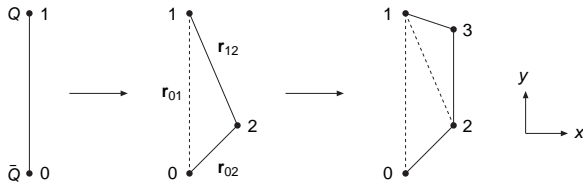
<sup>1</sup>Hatta-Iancu-Marquet-Soyez-Triantafyllopoulos





## Dipole cascade models

Mueller Dipole Model: Formulation of LL BFKL in transverse coordinate space



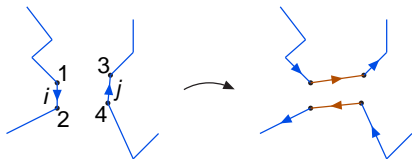
Emission probability:  $\frac{dP}{dy} = \frac{\bar{\alpha}}{2\pi} d^2\mathbf{r}_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2}$

Color screening: Suppression of large dipoles  
 $\sim$  suppression of small  $k_{\perp}$  in BFKL



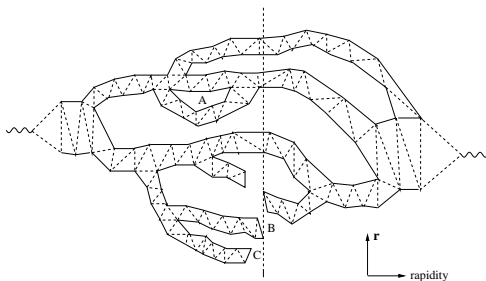
## Dipole-dipole scattering

Single gluon exchange  $\Rightarrow$  Color reconnection



Born amplitude:

$$f_{ij} = \frac{\alpha_s^2}{2} \ln^2 \left( \frac{r_{13}r_{24}}{r_{14}r_{23}} \right)$$



# Lund Dipole Cascade model<sup>2</sup>

The Lund model is a generalization of Mueller's dipole model, with the following improvements:

- ▶ Include NLL BFKL effects
- ▶ Include Nonlinear effects in evolution
- ▶ Include Confinement effects

MC: DIPSY

Initial state wavefunctions:

$\gamma^*$ : Given by perturbative QCD.  $\Psi_{T,L}(r, z; Q^2)$

**proton**: Dipole triangle

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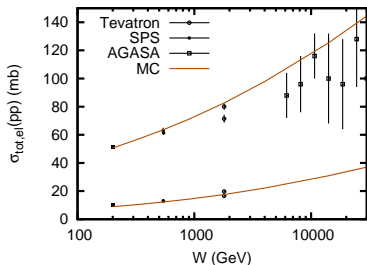
<sup>2</sup>Avsar-Flensburg-GG-Lönblad



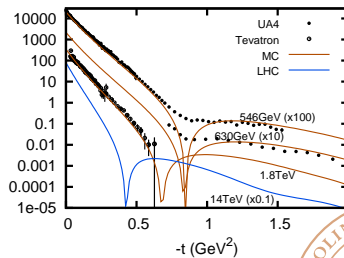
# Total and elastic cross sections

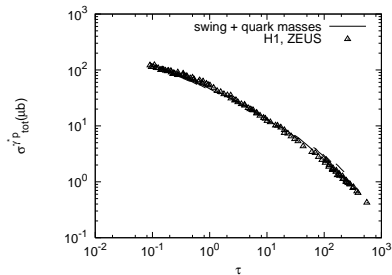
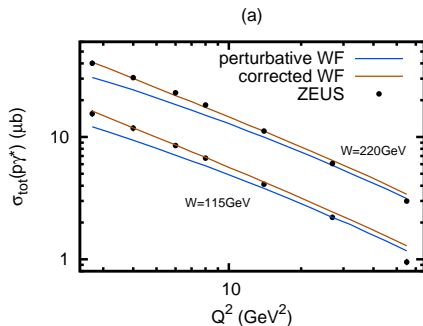
*pp*

$\sigma_{tot}$  and  $\sigma_{el}$



$d\sigma/dt$



$\gamma^* p$ 

Satisfies geometric scaling

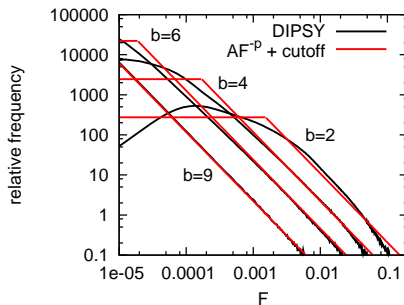


# How large are the fluctuations?

$$\gamma^* p: \quad \frac{dP}{dF} \approx A F^{-p} \quad (\text{with cutoff for small } F\text{-values})$$

$$\Rightarrow d\sigma_{\text{diff.ex.}}/d\sigma_{\text{tot}} \approx (1 - 1/2^{2-p})$$

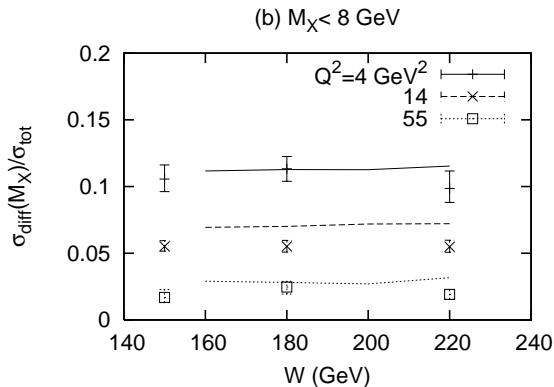
$$W = 220 \quad Q^2 = 14$$



The power  $p$  is independent of  $b$  (but grows slowly with  $Q^2$ )

$\gamma^* p$ 

Example  $M_X < 8 \text{ GeV}$ ,  $Q^2 = 4, 14, 55 \text{ GeV}^2$ .

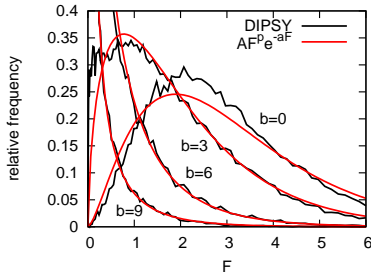


$pp$ : Born approximation: large fluctuations

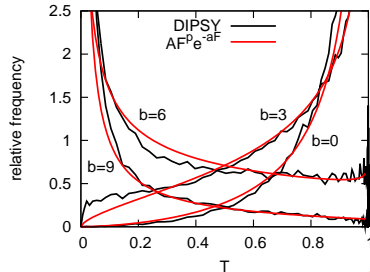
$$\frac{dP}{dF} \approx A F^p e^{-aF}$$

Born ampl.

$W = 2 \text{ TeV}$



Unitarized ampl.



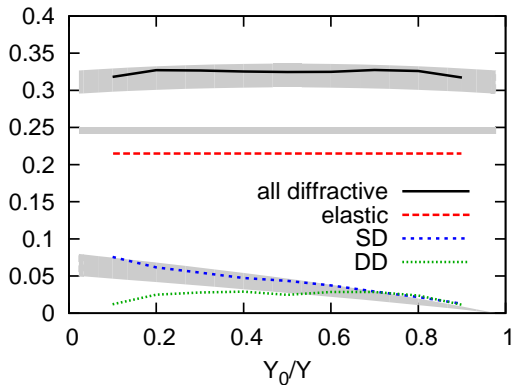
$a$  is independent of  $b$ .  $d\sigma_{d.ex.}/d\sigma_{tot}|_{Born} = 1/2a \sim 1/3$

$\langle F \rangle$  is large  $\Rightarrow$  Unitarity effects important

$\sim$  enhanced diagrams in triple-regge formalism





$pp$  1.8 TeV

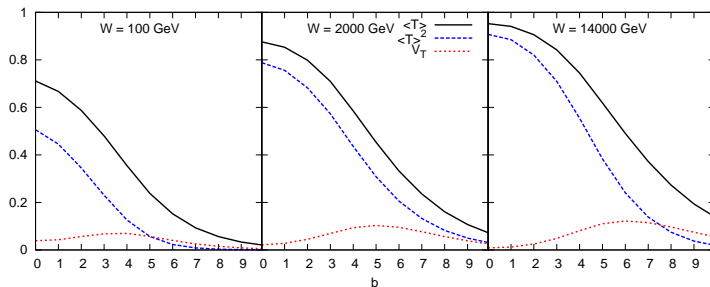
Saturation  $\Rightarrow$  Factorization breaking in diffractive excitation



## Impact parameter profile

Central collisions:  $\langle T \rangle$  large  $\Rightarrow$  Fluctuations small

Peripheral collisions:  $\langle T \rangle$  small  $\Rightarrow$  Fluctuations small



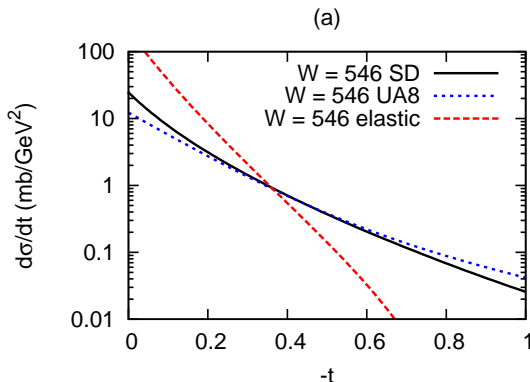
Largest fluctuations when  $\langle F \rangle \sim 1$  and  $\langle T \rangle \sim 0.5$

Circular ring expanding to larger radius at higher energy



# $t$ -dependence

## Single diffractive and elastic cross sections

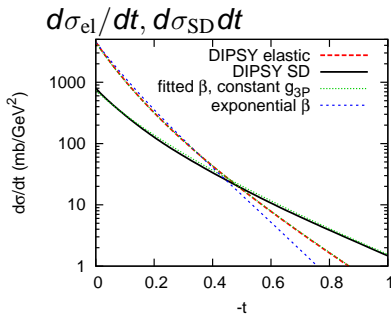
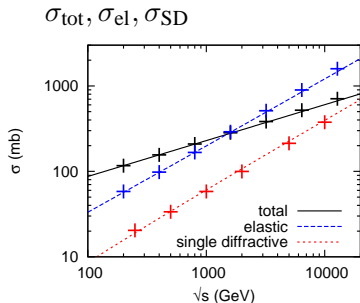


Agrees with fit to UA8 data



# Triple-Regge parameters

**BARE** pomeron without saturation effects



Triple-Regge fit with a single pomeron pole

$$\alpha(0) = 1.21, \quad \alpha' = 0.2 \text{ GeV}^{-2}$$

$$g_{pP}(t) = (5.6 \text{ GeV}^{-1}) e^{1.9t}, \quad g_{3P}(t) = 0.31 \text{ GeV}^{-1}$$



## Compare with triple-regge fits:

$$\alpha(0) = 1.21, \quad \alpha' = 0.2 \text{ GeV}^{-2}$$

$$g_{pP}(t) = (5.6 \text{ GeV}^{-1}) e^{1.9t}, \quad g_{3P}(t) = 0.31 \text{ GeV}^{-1}$$

Ryskin *et al.*:  $\alpha(0) = 1.3, \quad \alpha' \leq 0.05 \text{ GeV}^{-2}$

Kaidalov *et al.*:  $\alpha(0) = 1.12, \quad \alpha' = 0.22 \text{ GeV}^{-2}$

Goulianos:  $\alpha(0) = 1.21, \quad \alpha' = 0.2 \text{ GeV}^{-2}$

### Note:

Fit  $\sim$  single pomeron pole (not a cut)

$g_{3P}$ : magnitude  $\sim$  pert. QCD estimate by Bartels-Ryskin-Vacca  
( $\pi g_{3P} \sim 0.2 - 1.7 \text{ GeV}^{-1}$ )



## Summary

- ▶ In the eikonal approximation diffractive excitation is directly determined by the **fluctuations** in the scattering process
- ▶ The fluctuations in BFKL evolution are **large**
- ▶ Implemented in the Lund Dipole Cascade Model they reproduce the triple-pomeron formula for diffractive excitation to small and large masses in  $\gamma^*p$  and  $pp$  collisions, **with no extra parameters**
- ▶ In  $pp$  the fluctuations are large for the Born amplitudes, but strongly **suppressed by unitarity** above  $\sim 20$  GeV  
⇒ Factorization breaking
- ▶ The model result gives a **bare pomeron**, which is a **simple pole** with  $\alpha(0) = 1.21$  and  $\alpha' = 0.2 \text{ GeV}^{-1}$   
and with  $g_{3P}(t)$  approximately constant =  $0.31 \text{ GeV}^{-1}$   
(With saturation the cross section grows  $\sim s^{0.1}$  up to  $\sim 50 \text{ TeV}$ )



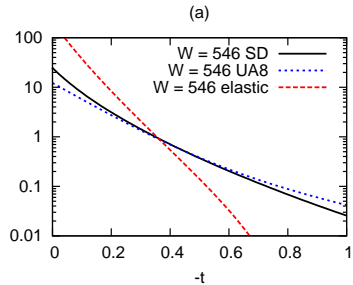
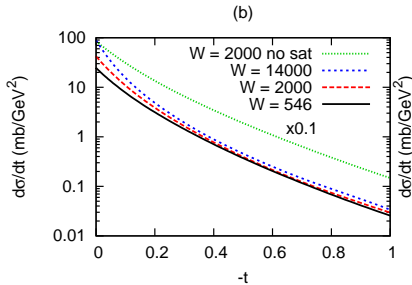
## Extra slides

Triple-pomeron formulae:

$$\begin{aligned} \sigma_{\text{tot}} &= \beta^2(0) s^{\alpha(0)-1}, \\ \frac{d\sigma_{\text{el}}}{dt} &= \frac{1}{16\pi} \beta^4(t) s^{2(\alpha(t)-1)}, \\ M_X^2 \frac{d\sigma_{\text{SD}}}{dt d(M_X^2)} &= \frac{1}{16\pi} \beta^2(t) \beta(0) g_{3P}(t) \left( \frac{s}{M_X^2} \right)^{2(\alpha(t)-1)} \left( M_X^2 \right)^{\alpha(0)-1}, \\ \beta(t) &\equiv g_{pP}(t) \end{aligned}$$



## Energy dependence and effect of saturation on $d\sigma/dt$



Energy dependence, and result without saturation at 2 TeV

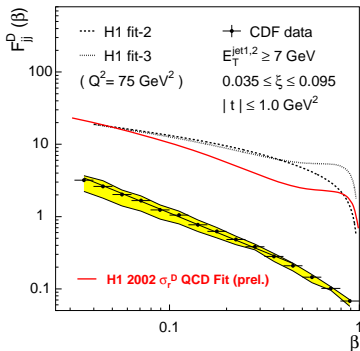
546 GeV compared with a fit to UA8 data, and with elastic scattering





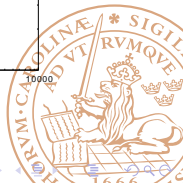
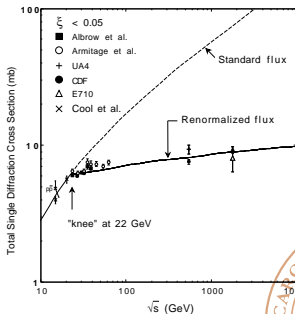
## Factorization breaking

Difference between  
 $pp$  and  $\gamma^*p$



Cf. Goulianos' saturation of  
 pomeron flux

$pp$  scattering



## $pp$ scattering

$\langle T \rangle$  and  $V_T$  for the simple parametrization  $\frac{dP}{dF} \approx AF^p e^{-aF}$

$$\langle T \rangle = 1 - \left(\frac{a}{a+1}\right)^{p+1} = 1 - \left(\frac{a}{a+1}\right)^{a\langle F \rangle} \rightarrow 1 \text{ when } \langle F \rangle \rightarrow \infty$$

$$V_T = \left(\frac{a}{a+2}\right)^{p+1} - \left(\frac{a}{a+1}\right)^{2p+2} \rightarrow 0 \text{ when } \langle F \rangle \rightarrow \infty$$



## Diffractive final states

Coherence effects important for subtracting el. scatt.

$$d\sigma_n = c_n^2 \left( \sum_m d_m^2 t_{nm} - \langle t \rangle \right)^2$$

$$\langle t \rangle = \sum_n \sum_m c_n^2 d_m^2 t_{nm}$$



## Toy model

(Abelian emissions; no saturation)

$$\Psi_{in} = \prod_i (\alpha_i + \beta_i) |0\rangle$$

parton  $i$  produced with prob.  $|\beta_i|^2$ , interacts with weight  $f_i$

Diff. exc. states:

$$\Psi_j = (-\beta_j + \alpha_j) \prod_{i \neq j} (\alpha_i + \beta_i) |0\rangle$$

$$d\sigma_{el} \sim (\sum_i \beta_i^2 f_i)^2$$

$$d\sigma_j \sim \alpha_j^2 \beta_j^2 f_j^2$$

