Soft-gluon rescattering in diffractive DIS

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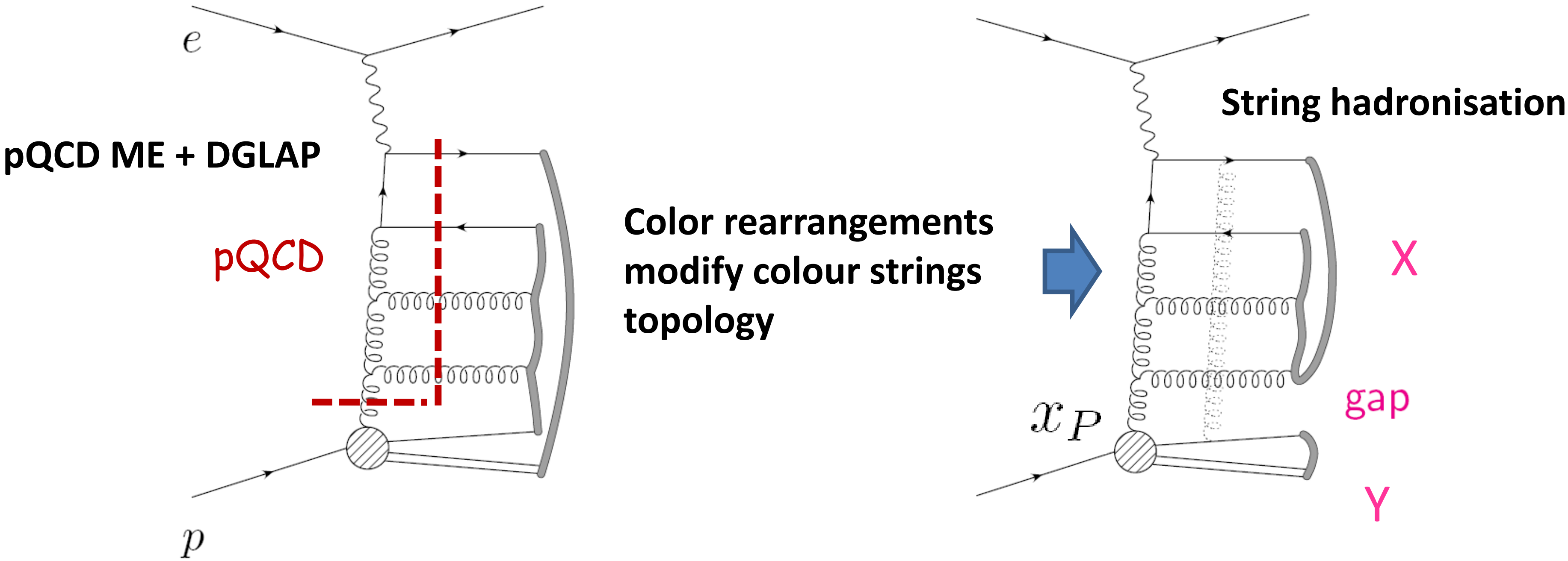
Diffractive Deep Inelastic Scattering: motivation

✓ **The success of Soft Color Interaction (SCI) model (Edin, Ingelman, Rathsman'97)**



- ***Soft interactions among the final state partons and proton remnants (=> proton color field) at small momentum transfers < 1 GeV***

- ***Hard pQCD part (small distances) is not affected by soft interactions (large distances)***

- ***Single parameter - probability for soft colour-anticolour (gluon) exchange***

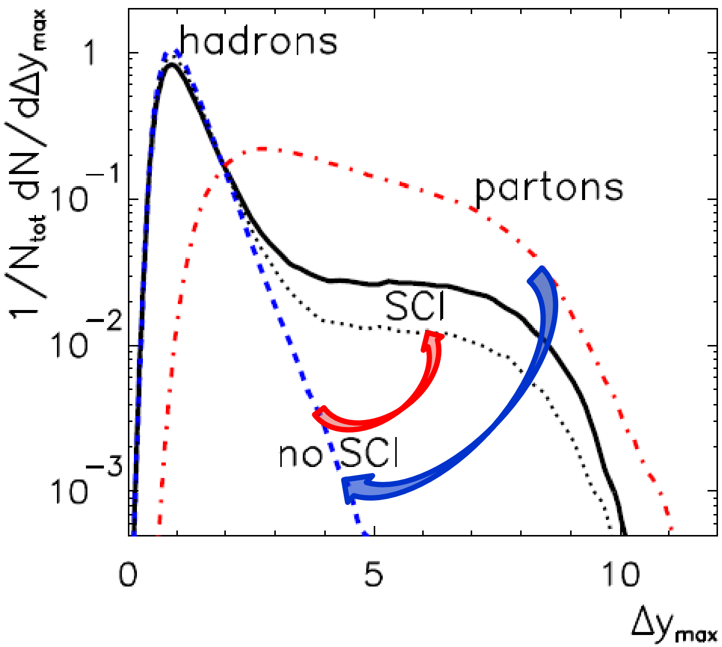
- ***Single model describing all final states: both diffractive and nondiffractive***

Soft Colour Interaction model (SCI)

Add-on to Lund Monte Carlo's LEPTO (ep) and PYTHIA ($p\bar{p}$)

ME + DGLAP PS $> Q_0^2$	$\rightarrow$	SCI model	$\rightarrow$	String hadronisation $\sim \Lambda$
colour ordered parton state		rearranged colour order		modified final state

Size Δy_{max} of largest gap in DIS events



SCI $\Rightarrow$ plateau in Δy_{max}
characteristic for diffraction

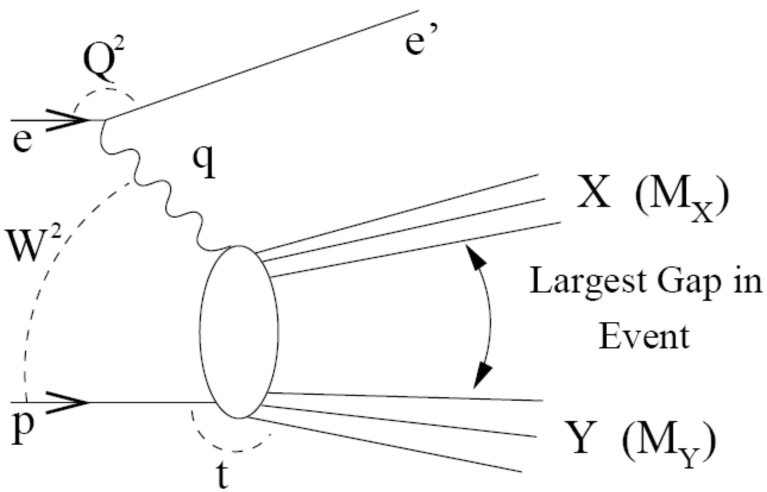
Small parameter sensitivity
— $P = 0.5$
$\cdots$ $P = 0.1$

Gap-size is infrared sensitive observable !

Large gaps at parton level
normally string across $\rightarrow$ hadrons fill up
SCI $\rightarrow$ new string topologies, some with gaps

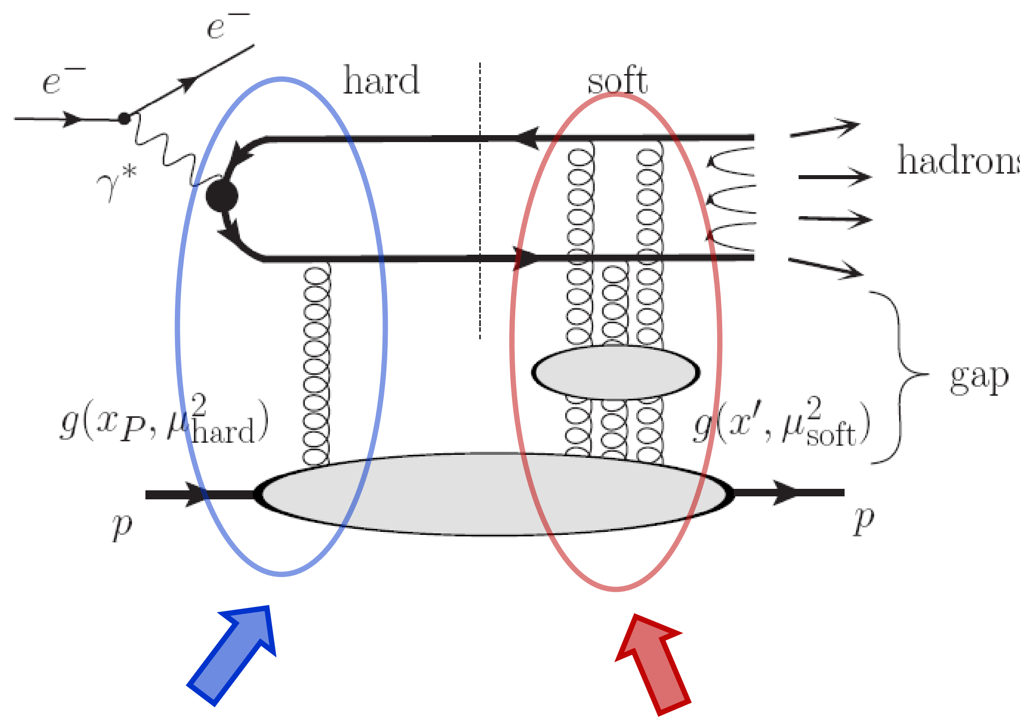
QCD rescattering theory

Diffractive DIS at HERA



Soft gluons cannot resolve quarks dynamically → but they always couple to quark current!

QCD rescattering model



Hard part
conventional
(small distance)

Soft part:
color-screening (octet)
multigluon exchange
(large distance)

Diffractive DIS at HERA: Final State Interactions in the dipole picture

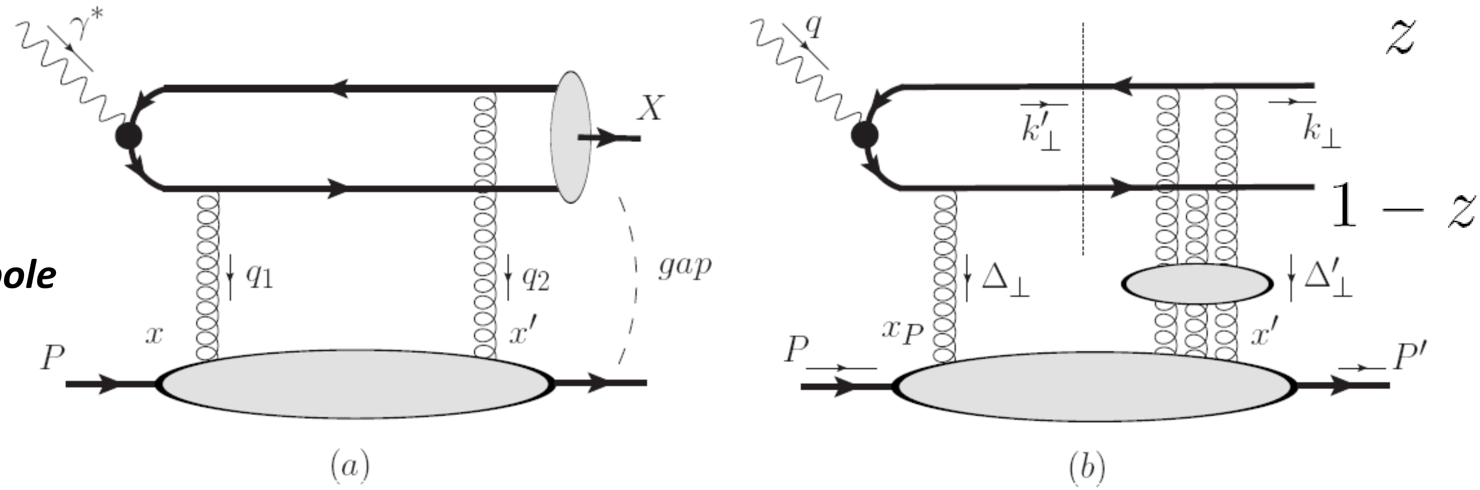
Basic variables: $$x \equiv \frac{Q^2}{2Pq} = \frac{Q^2}{Q^2+W^2}, \quad \beta = \frac{Q^2}{Q^2+M_X^2}, \quad x_P = \frac{x}{\beta}, \quad t = (P'-P)^2$$

<u>Hard scale</u>

$$Q^2 = -q^2$$

Leading contribution ***provided by quark dipole***

$$\beta = x/x_P \to 1$$



Invariant mass ***of X system and*** *c.m.s energy*

$$M_X^2 = \frac{1-\beta}{\beta}Q^2, \quad W^2 \simeq \frac{Q^2}{x_P\beta},$$

Working *domain of interest:*

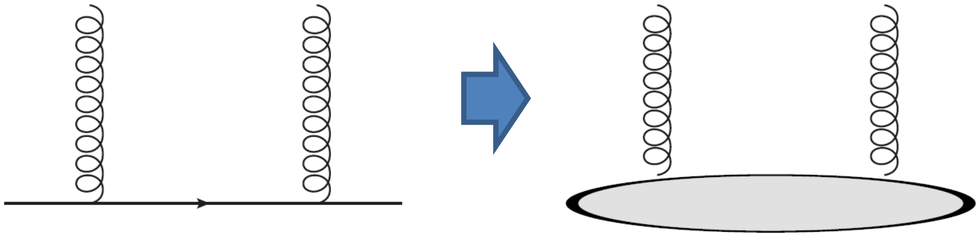
$$x_P \ll 1, \quad M_X \ll W \quad |t| \ll Q^2, M_X^2 \quad x' \ll x_P$$

$$\varepsilon^2 = z(1-z)Q^2 + m_q^2, \quad k_\perp^2 = z(1-z)M_X^2 - m_q^2$$

The hard QCD factorization scale ***= quark virtuality!***

$$\mu_F^2 = k_\perp^2 + \varepsilon^2 = z(1-z)\frac{Q^2}{\beta}$$

Unintegrated gluon density



$C_F \alpha_s / \pi$ $\qquad f_g^{\rm off}(x_P, x', \Delta_\perp^2, \Delta_\perp'^2, \mu_F^2)$

Off-diagonal UGDF

$$\mathcal{F}_g^{\rm off} \simeq \sqrt{\mathcal{F}_g(x_P, \Delta_\perp^2, \mu_F^2)\mathcal{F}_g(x', {\Delta'_\perp}^2, \mu_{\rm soft}^2)}\,,$$

$$\frac{f_g(x, \Delta_\perp^2)}{\Delta_\perp^2} \equiv \mathcal{F}(x, \Delta_\perp^2) \to {\rm const}\,, \quad \Delta_\perp^2 \to 0$$

In the impact parameter space:

$$\mathcal{V}(\mathbf{b}, \mathbf{r}) = \frac{1}{\alpha_s(\mu_{\rm soft}^2)} \int \frac{d^2\Delta_\perp}{(2\pi)^2} \sqrt{x_P}\,\mathcal{F}_g^{\rm off} \times \left\{e^{-i\mathbf{r}\boldsymbol{\Delta}_\perp} - e^{i\mathbf{r}\boldsymbol{\Delta}_\perp}\right\} e^{i\mathbf{b}\boldsymbol{\Delta}_\perp}.$$

Analytic coupling at the soft scale

$$\alpha_s^{\rm soft} = \mathcal{A}_1(\Lambda_{\rm QCD}) \simeq 0.7$$

Gaussian Ansatz

$$\sqrt{x_P}\mathcal{F}_g^{\rm off} \simeq \sqrt{x_P g(x_P, \mu_F^2)\, x' g(x', \mu_{\rm soft}^2)}\, f_G(\Delta_\perp^2),$$

$$f_G(\Delta_\perp^2) = 1/(2\pi\rho_0^2)\, \exp\left(-\Delta_\perp^2/2\rho_0^2\right),$$

***Soft hardronic scale* – transverse proton radius** $\qquad r_p \sim 1/\rho_0$

Diffractive slope

$$\sim \exp(B_D t) \qquad B_D = 1/\rho_0^2 \simeq 6.9 \pm 0.2 \text{ GeV}^2 \qquad \rho_0 \simeq 380 \text{ MeV}$$

Hard-soft factorization scheme

✓ The total **amplitude**

loop integration + cutting rules

$$M(\delta) \sim \int \frac{d^2\Delta_\perp}{(2\pi)^2} \cdot M^{hard}(\mathbf{\Delta}_\perp) \cdot M^{soft}(\boldsymbol{\delta} - \mathbf{\Delta}_\perp) \qquad \delta \equiv \sqrt{-t} = |\mathbf{\Delta}_\perp + \mathbf{\Delta}'_\perp|$$

to the **impact parameter representation** →

$$M(\delta) \sim \int d^2 b e^{-i\boldsymbol{\delta}\mathbf{b}} \hat{M}^{hard}(\mathbf{b}) \cdot \hat{M}^{soft}(\mathbf{b})$$

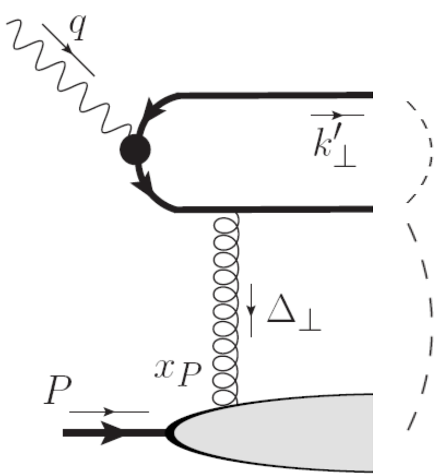
factorisation of the b-dependence

Factorization condition in the impact parameter space

$$\Delta_\perp \sim \Delta'_\perp \ll k'_\perp \sim k_\perp \longrightarrow r \ll b$$

✓ **Hard part**

$$M^{hard}_{L,T}(\Delta_\perp, k'_\perp) =$$



$r \sim 1/k'_\perp$

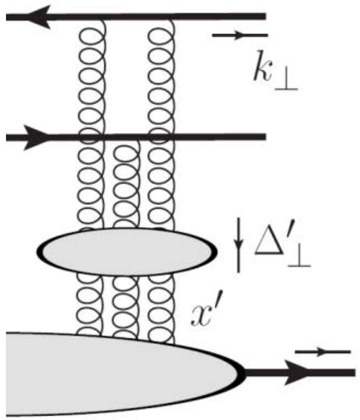
$b \sim 1/\Delta_\perp$

$$= \int d^2\mathbf{r} d^2\mathbf{b}\, \hat{M}^{hard}_{L,T}(\mathbf{b}, \mathbf{r}) e^{-i\mathbf{r}\mathbf{k}'_\perp} e^{-i\mathbf{b}\mathbf{\Delta}_\perp}$$

Soft gluon "exponentiation"

✓ **Soft gluon exchanges generate only the phase shifts – to be resummed to all orders!**

$$M^{soft}(\Delta'_\perp, k_\perp) =$$



the large N_c limit – planar diagrams only!

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} M_1^{soft} = \mathcal{A}\, e^{-i\mathbf{r}\mathbf{k}_\perp} \frac{1}{\Delta'^2_\perp} \left[e^{-i\mathbf{r}\boldsymbol{\Delta}'_\perp} - 1 \right],$$

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} M_2^{soft} = \frac{\mathcal{A}^2}{2!} e^{-i\mathbf{r}\mathbf{k}_\perp} \times$$
$$\int \frac{d^2\Delta'_{2\perp}}{(2\pi)^2} \frac{1}{\Delta'^2_{1\perp}\Delta'^2_{2\perp}} \left[e^{-i\mathbf{r}\boldsymbol{\Delta}'_\perp} - e^{-i\mathbf{r}\boldsymbol{\Delta}'_{2\perp}} - e^{-i\mathbf{r}\boldsymbol{\Delta}'_{1\perp}} + 1 \right]$$

etc ...

Fourier transform →

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} \hat{M}_1^{soft} = e^{-i\mathbf{r}\mathbf{k}_\perp} \mathcal{A} \cdot \mathcal{W}(\mathbf{b},\mathbf{r}),$$

$$e^{-i\mathbf{r}\mathbf{k}'_\perp} \hat{M}_2^{soft} = e^{-i\mathbf{r}\mathbf{k}_\perp} \frac{\mathcal{A}^2 \cdot \mathcal{W}(\mathbf{b},\mathbf{r})^2}{2!}, \quad \dots$$

Soft gluon rescattering amplitude →

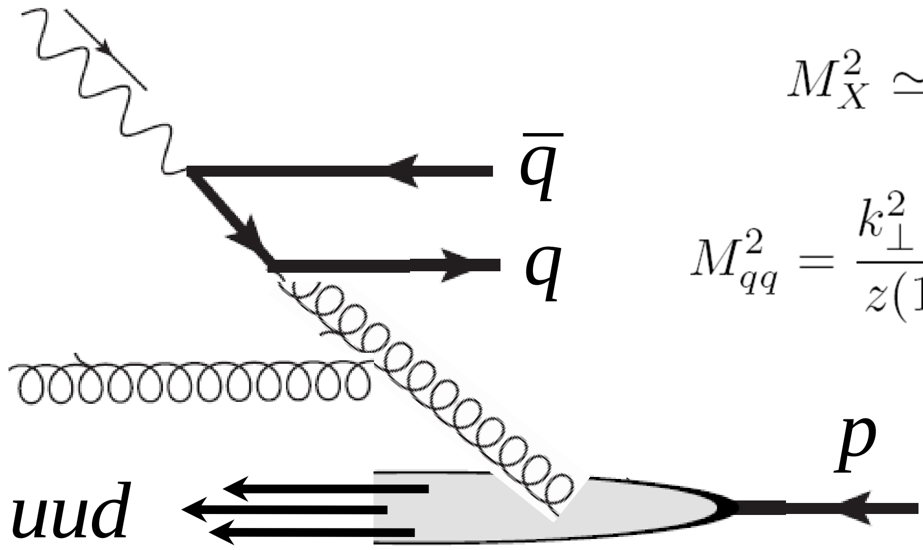
$$e^{-i\mathbf{r}\mathbf{k}'_\perp} \hat{M}^{soft}(\mathbf{b},\mathbf{r}) = -e^{-i\mathbf{r}\mathbf{k}_\perp} \left(1 - e^{\mathcal{A}\cdot\mathcal{W}(\mathbf{b},\mathbf{r})}\right)$$

$$\mathcal{A} = i g_s^2 C_F/2 \qquad \mathcal{W}(\mathbf{b},\mathbf{r}) = \frac{1}{2\pi} \ln \frac{|\mathbf{b}-\mathbf{r}|}{|\mathbf{b}|}$$

Inspired by Brodsky et al, PRD65, 114025 (2002)

Gluonic contribution @ large M_X

Gluon radiated from "hard" gluon is far away in *p*-space from $q\bar{q}$
$\rightarrow$ leading contribution to large M_X



$$M_X^2 \simeq M_{qq}^2 + \frac{p_\perp^2}{z'} \rightarrow \infty,$$

$$M_{qq}^2 = \frac{k_\perp^2 + m_q^2}{z(1-z)}, \quad z_g = \frac{z'}{x_P} + \beta\left(1 + \frac{M_{qq}^2}{Q^2}\right)$$

$$z' \ll z \qquad p_\perp \sim k_{1,\perp} \gg k_{2,\perp}$$

$\rightarrow$ Altarelli-Parisi splitting $\otimes$ $q\bar{q}$-dipole $\otimes$ multiple gluon exchange

$$x_P F_{q\bar{q}g}^{D(4)} \simeq \frac{1}{N_c^2} \int \frac{dt_g dz_g}{t_g + m_g^2} P_{gg}(z_g) \frac{\alpha_s(t_g)}{2\pi} x_P F_{q\bar{q}}^{D(4)}$$

Diffractive structure function: results

Data are given in terms of the reduced cross section

$$\sqrt{s} = 318 \ \mathrm{GeV}$$

$$x_P \sigma_r^{D(3)} = x_P F_{q\bar{q},T}^{D(3)} + \frac{2-2y}{2-2y+y^2} x_P F_{q\bar{q},L}^{D(3)} + x_P F_{q\bar{q}g}^{D(3)} \qquad y = Q^2/(s x_B) \le 1$$

Quark dipole contribution:

$$x_P F_L^{D(4)} = \mathcal{S}\, Q^4 M_X^2 \int_{z_{min}}^{\frac{1}{2}} dz (1-2z)\, z^2 (1-z)^2 |J_L|^2$$

$$x_P F_T^{D(4)} = 2\mathcal{S}\, Q^4 \int_{z_{min}}^{\frac{1}{2}} dz (1-2z) \left\{(1-z)^2 + z^2\right\} |J_T|^2$$

$$\mathcal{S} = \sum_q e_q^2/(2\pi^2 N_c^3)$$

Amplitudes:

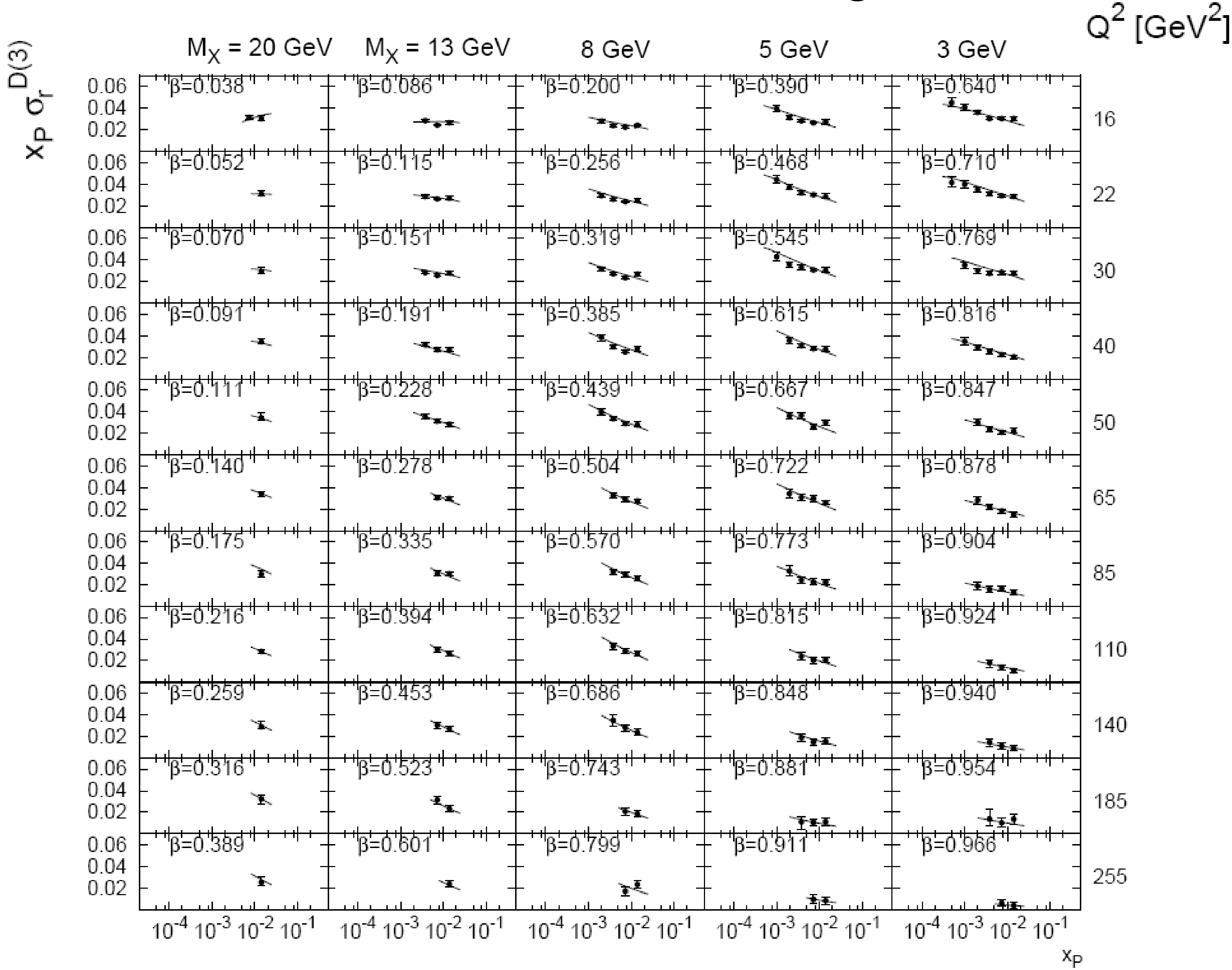
$$J_L = i\alpha_s(\mu_F^2) \int d^2\mathbf{r} d^2\mathbf{b}\, e^{-i\boldsymbol{\delta}\mathbf{b}} e^{-i\mathbf{r}\mathbf{k}_\perp} K_0(\varepsilon r) \times \mathcal{V}(\mathbf{b},\mathbf{r}) \left[1 - e^{\mathcal{A}\mathcal{W}}\right],$$

$$J_T = i\alpha_s(\mu_F^2) \int d^2\mathbf{r} d^2\mathbf{b}\, e^{-i\boldsymbol{\delta}\mathbf{b}} e^{-i\mathbf{r}\mathbf{k}_\perp} \varepsilon K_1(\varepsilon r) \times \frac{r_x \pm i r_y}{r} \mathcal{V}(\mathbf{b},\mathbf{r}) \left[1 - e^{\mathcal{A}\mathcal{W}}\right].$$

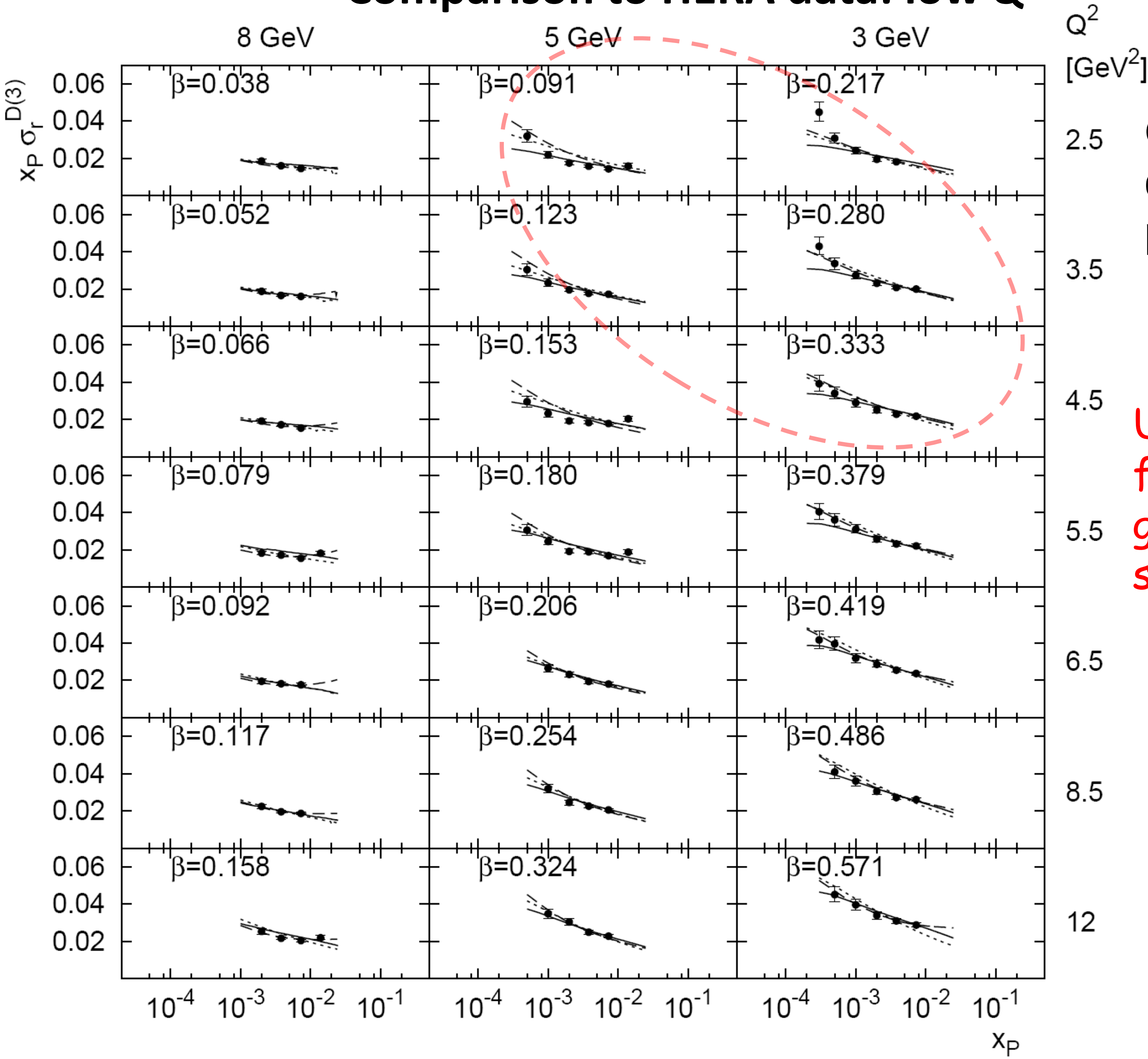
Gluonic dipole contribution:

$$x_P F_{q\bar{q}g}^{D(4)} \simeq \frac{1}{N_c^2} \int \frac{dt_g dz_g}{t_g + m_g^2} P_{gg}(z_g) \frac{\alpha_s(t_g)}{2\pi} x_P F_{q\bar{q}}^{D(4)}$$

Diffractive structure function: large Q^2



Comparison to HERA data: low Q^2

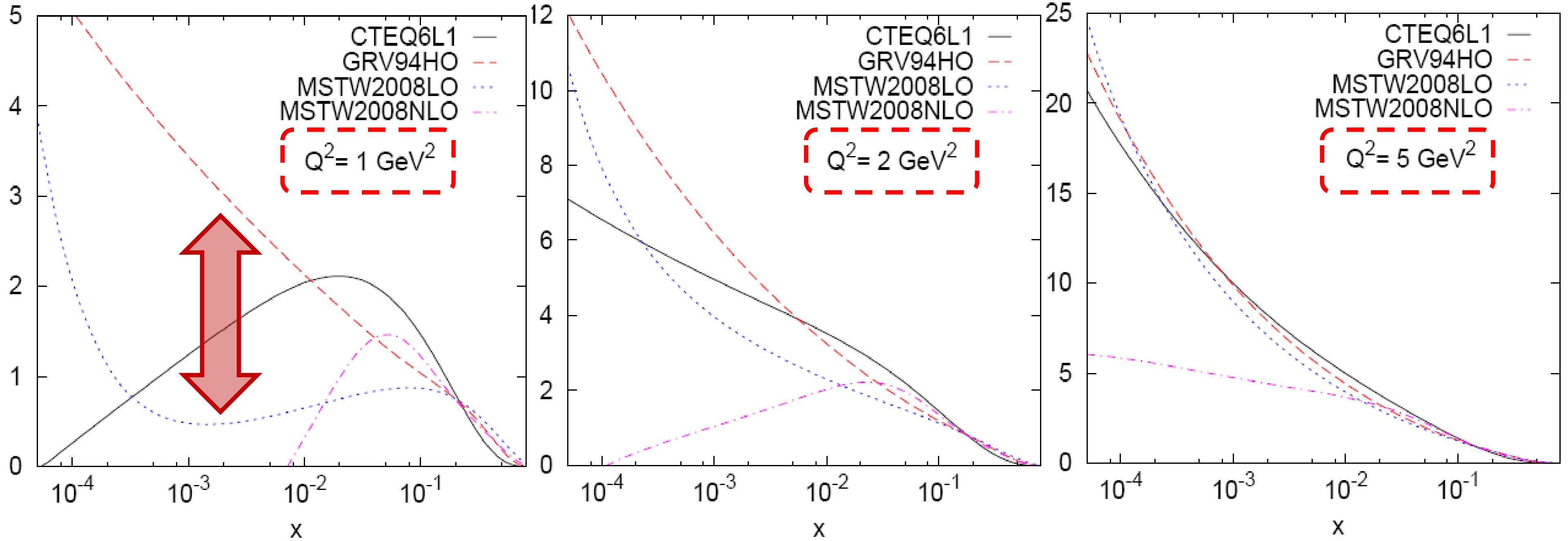


Curves for different $xg(x,\mu)$ parametrizations

Uncertainty from unknown gluon density at small x & scale

Possibility to extract $g(x,\mu_F)$ at $x \sim 10^{-4}$ $\mu_F < \sim 1$ GeV

Gluon density parametrizations at low-*x* and low Q^2

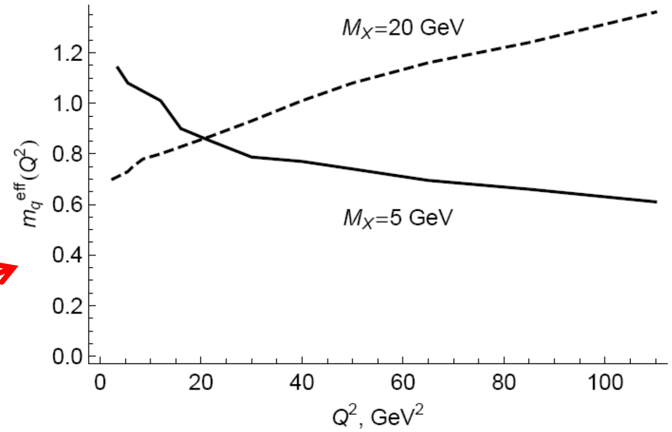


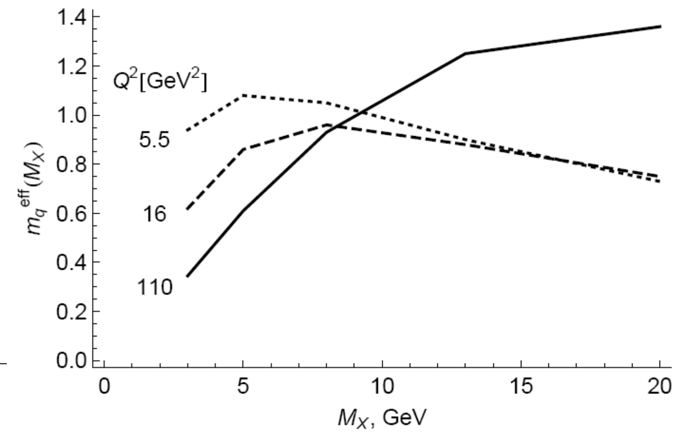
Large differences at $x<\sim10^{-2}$ and $Q^2<\sim2\ GeV^2$!!

→ Unknown gluon density in this region !!!

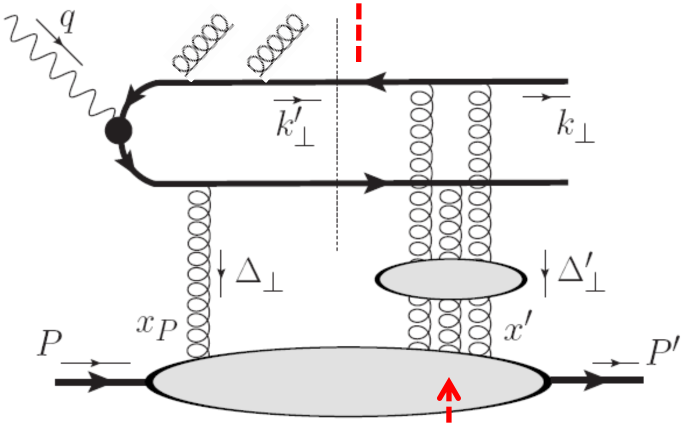
Model parameters

Effective mass m_{eff}
of 0.3—1.2 GeV
of quark in dipole
from dressing-up
with gluon radiation

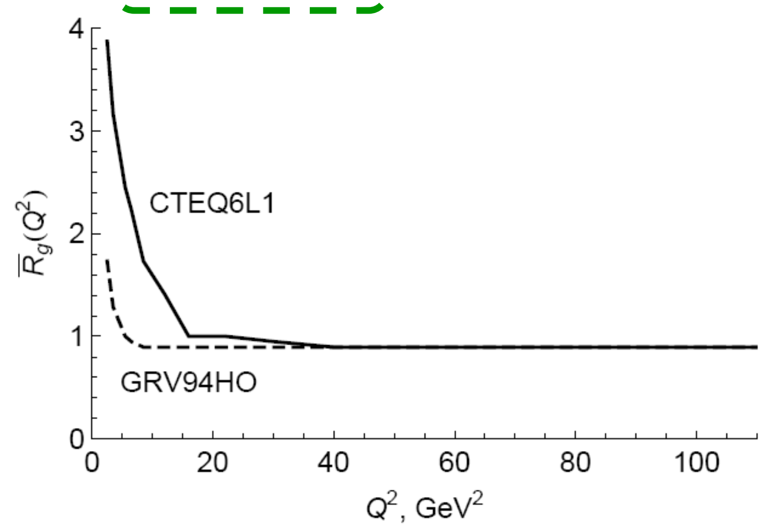




$$m_q \rightarrow m_{eff}$$

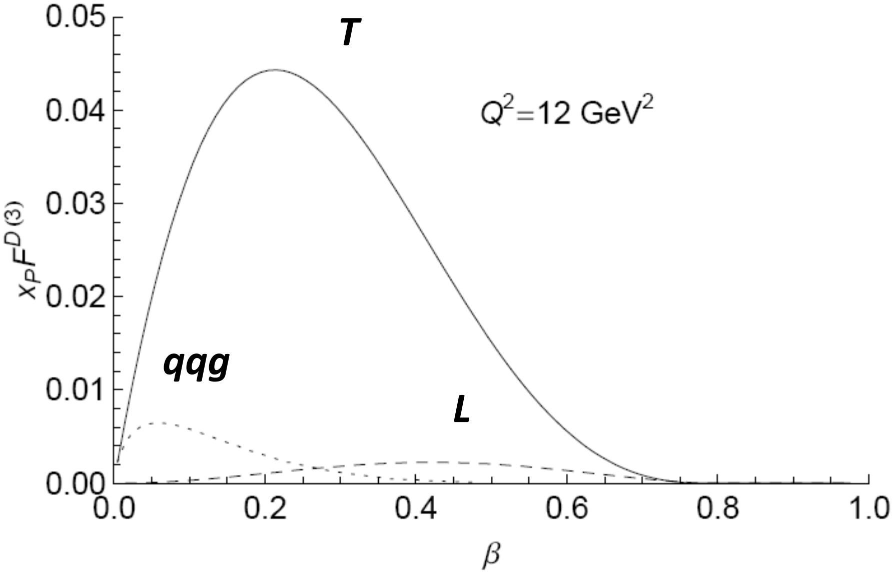


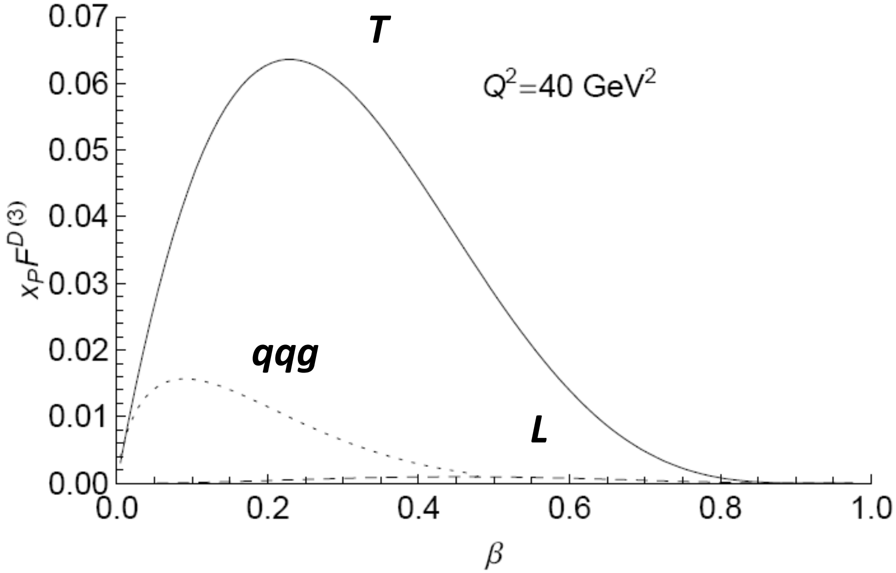
$$\sqrt{x_P}\mathcal{F}_g^{\rm off} \simeq \bar{R}_g(x', \mu^2_{\rm soft})\sqrt{x_P g(x_P, \mu_F^2)}\, f_G(\Delta_\perp^2)$$



Soft gluon density function R_g
= constant ≈ 1, except at small Q^2

Photon polarization contributions and mass spectrum





Gluonic contribution increases at high M_X and Q^2 !!!

Summary

✓ We constructed the QCD based model for the soft gluon rescattering.

✓ The model works basically well and leads to a good description of the HERA data on the diffractive structure function in almost all bins in photon virtuality and invariant mass of the final hadronic system.

✓ At lower xP and Q^2 the uncertainties in conventional parton densities become significant, and some improvement of the data description is still needed.