NLO BFKL with conformal invariance

Grabovsky A. V.

BINP Novosibirsk Russia

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Comparison of BFKL and color dipole approaches in NLO

- Coordinate representation of quark part of the BFKL kernel(NLO):
  V.S. Fadin, R. Fiore and A. Papa, Phys. Lett. **B647** (2007) 179.
- Quark contribution to the kernel of the BK equation (NLO):
  I. Balitsky, Phys. Rev. **D75** (2007).
- Coordinate representation of the gluon part of the BFKL kernel (NLO):
  V.S. Fadin, R. Fiore, A. V. Grabovsky and A. Papa, Nucl. Phys.**B784**, 49 (2007).
- Gluon contribution to the BK kernel (NLO), quasi-conformal kernel:
  I. Balitsky and G. A. Chirilli Phys.Rev.**D77**,014019,(2008);
  Nucl.Phys.**B822**,45, (2009).
- Equivalence of the gluon contributions for forward scattering:
  V.S. Fadin, R. Fiore, A.V. Grabovsky,Nucl.Phys.**B820**, 334,(2009).
- Equivalence of the gluon contributions, quasi-conformal kernel:
  V.S. Fadin, R. Fiore, A.V. Grabovsky Nucl.Phys.**B831**:248-261, (2010).

Applicability area $\sqrt{s} \gg Q \gg \Lambda_{QCD}$

- BFKL:
  - formulated in the momentum space
  - for both colored and colorless particles
  - but linear equation $\rightarrow$ unitarity violation
- BK:
  - formulated in the position space
  - incorporates effects of parton saturation
  - for scattering of colorless particles
- For comparison we need
  - linearized BK kernel
  - BFKL kernel in coordinate space
  - simplification of BFKL kernel for colorless case

Dipole (Möbius) form of BFKL kernel

- For scattering of colorless particles we have:
  - gauge invariance of the incoming particle impact factor $\langle A'\bar{A}|$:
    $\langle A'\bar{A}|\Psi\rangle = 0$ if $\langle q, q'|\Psi\rangle \sim \delta(q), \delta(q')$ or $\langle r, r'|\Psi\rangle$ is independent of r or r',
  - which is saved by the kernel:
    $\langle A'\bar{A}|\hat{\mathcal{K}}^n$ is gauge invariant since $\langle \vec{q}_1, \vec{q}_2|\hat{\mathcal{K}}|\vec{q}_1{}', \vec{q}_2{}'\rangle \to 0|_{\vec{q}_i{}' \to 0}$.
  - and dipole form of the second impact factor $|In\rangle = \frac{1}{\hat{\vec{q}}_1{}^2 \hat{\vec{q}}_2{}^2}|\bar{B}'B\rangle$:
    $\langle \vec{r}_1, \vec{r}_2|In\rangle \to \langle \vec{r}_1, \vec{r}_2|In\rangle - a\langle \vec{r}_1, \vec{r}_1|In\rangle - (1-a)\langle \vec{r}_2, \vec{r}_2|In\rangle$
    $\Rightarrow \langle \vec{r}, \vec{r}|In\rangle = 0$,
- Therefore to find the dipole (Möbius) form of the kernel:
  - we add to $\langle \vec{r}_1, \vec{r}_2|\hat{\mathcal{K}}|\vec{r}_1{}', \vec{r}_2{}'\rangle$ terms without r_1 or r_2 to get
    $\langle \vec{r}_1, \vec{r}_2|\hat{\mathcal{K}}|\vec{r}_1{}', \vec{r}_2{}'\rangle = 0|_{r_1 = r_2}$
  - and drop terms $\sim \delta(\vec{r}_{1'} - \vec{r}_{2'})$.

BFKL kernel in position space

- We do Fourier transform

$$\langle \vec r_1 \vec r_2 | \hat{\mathcal{K}} | \vec r_1{}' \vec r_2{}' \rangle = \int \frac{d^2 q_1}{2\pi} \frac{d^2 q_2}{2\pi} \frac{d^2 q_1'}{2\pi} \frac{d^2 q_2'}{2\pi} \langle \vec q_1, \vec q_2 | \hat{\mathcal{K}} | \vec q_1{}', \vec q_2{}' \rangle e^{i[\vec q_1 \vec r_1 + \vec q_2 \vec r_2 - \vec q_1{}' \vec r_1{}' - \vec q_2{}' \vec r_2{}']} \,,$$

- add & drop some terms to come to the Möbius form,
- which in LO gives the conformally invariant kernel equal to the BK one.
- In NLO for the gluon part we have:

$$\langle \vec r_1 \vec r_2 | \hat{\mathcal{K}}_d^{NLO} | \vec r_1{}' \vec r_2{}' \rangle = \frac{\alpha_s^2(\mu) N_c^2}{4\pi^3} \Big[\delta(\vec r_{11'}) \delta(\vec r_{22'}) \int d\vec\rho \, g^0(\vec r_1, \vec r_2; \vec\rho) + \delta(\vec r_{11'}) g(\vec r_1, \vec r_2; \vec r_2{}') + \delta(\vec r_{22'}) g(\vec r_2, \vec r_1; \vec r_1{}') + \frac{1}{\pi} g(\vec r_1, \vec r_2; \vec r_1{}', \vec r_2{}') \Big]$$

where $\vec r_{i\rho} = \vec r_i - \vec\rho$ and $\vec r_{ij'} = \vec r_i - \vec r_j{}'$.

Explicit Möbius form of the gluon part of the BFKL kernel (NLO)

- Initially obtained g's are not conformally invariant

$$g_0(\vec{r}_1, \vec{r}_2; \vec{\rho}) = 2\pi\zeta(3)\delta\left(\vec{\rho}\right) - g(\vec{r}_1, \vec{r}_2; \vec{\rho}) \ ,$$

$$g_1(\vec{r}_1, \vec{r}_2; \vec{r}_2^{\,\prime}) \ = \frac{11}{6}\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{22'}^{\,2}\vec{r}_{12'}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{r_\mu^2}\right) + \frac{11}{6}\left(\frac{1}{\vec{r}_{22'}^{\,2}} - \frac{1}{\vec{r}_{12'}^{\,2}}\right)\ln\left(\frac{\vec{r}_{22'}^{\,2}}{\vec{r}_{12'}^{\,2}}\right)$$
$$+\frac{1}{2\vec{r}_{22'}^{\,2}}\ln\left(\frac{\vec{r}_{12'}^{\,2}}{\vec{r}_{22'}^{\,2}}\right)\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{12'}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{2\,\vec{r}_{22'}^{\,2}\vec{r}_{12'}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{22'}^{\,2}}\right)\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{12'}^{\,2}}\right),$$

where

$$\ln r_\mu^2 = 2\psi\left(1\right) - \ln\frac{\mu^2}{4} - \frac{3}{11}\left(\frac{67}{9} - 2\zeta(2)\right).$$

  - Terms $\sim 11/6$ are due to renormalization.
  - The other terms are not.

- These g's do not coincide with g's from the linearized BK kernel.

Explicit Möbius form of the gluon part of the BFKL kernel (NLO)

- $g(\vec r_1, \vec r_2; \vec r_1{}', \vec r_2{}')$ is also nonconformal and nonequal to the BK term

$$g(\vec r_1, \vec r_2; \vec r_1{}', \vec r_2{}') = \frac{1}{2\vec r^{\,4}_{1'2'}}\left(\frac{\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}}{d}\ln\left(\frac{\vec r^{\,2}_{12'}\vec r^{\,2}_{21'}}{\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}}\right) - 1\right) + \frac{\vec r^{\,2}_{12}\ln\left(\frac{\vec r^{\,2}_{12}\,\vec r^{\,2}_{1'2'}}{\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}}\right)}{4\vec r^{\,2}_{12'}\vec r^{\,2}_{21'}\,\vec r^{\,2}_{1'2'}}$$

$$+\frac{\vec r^{\,2}_{12}}{d\,\vec r^{\,2}_{1'2'}}\ln\left(\frac{\vec r^{\,2}_{12'}\vec r^{\,2}_{21'}}{\vec r^{\,2}_{11'}\,\vec r^{\,2}_{22'}}\right)\left(\frac{\vec r^{\,2}_{12}\vec r^{\,2}_{1'2'}}{4\vec r^{\,2}_{11'}\,\vec r^{\,2}_{22'}} - 1\right) + \frac{\vec r^{\,2}_{12}\ln\left(\frac{\vec r^{\,2}_{12}\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}\vec r^{\,2}_{1'2'}}{\vec r^{\,4}_{12'}\,\vec r^{\,4}_{21'}}\right)}{4\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}\vec r^{\,2}_{1'2'}}$$

$$+\frac{\ln\left(\frac{\vec r^{\,2}_{12}\vec r^{\,2}_{1'2'}}{\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}}\right)}{4\vec r^{\,2}_{12'}\,\vec r^{\,2}_{21'}} + \frac{\ln\left(\frac{\vec r^{\,2}_{12}}{\vec r^{\,2}_{1'2'}}\right)}{4\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}} + \frac{\ln\left(\frac{\vec r^{\,2}_{22'}}{\vec r^{\,2}_{12}}\right)}{2\vec r^{\,2}_{11'}\,\vec r^{\,2}_{12'}} + \frac{\ln\left(\frac{\vec r^{\,2}_{12}\vec r^{\,2}_{1'2'}}{\vec r^{\,2}_{12'}\vec r^{\,2}_{22'}}\right)}{2\vec r^{\,2}_{11'}\,\vec r^{\,2}_{1'2'}} + \frac{\ln\left(\frac{\vec r^{\,2}_{12}\vec r^{\,2}_{11'}}{\vec r^{\,2}_{22'}\,\vec r^{\,2}_{1'2'}}\right)}{2\vec r^{\,2}_{12'}\vec r^{\,2}_{1'2'}}$$

$$+\frac{\vec r^{\,2}_{22'}\ln\left(\frac{\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}}{\vec r^{\,2}_{12}\,\vec r^{\,2}_{1'2'}}\right)}{2\vec r^{\,2}_{12'}\vec r^{\,2}_{21'}\vec r^{\,2}_{1'2'}} + \frac{\vec r^{\,2}_{12}\ln\left(\frac{\vec r^{\,2}_{11'}}{\vec r^{\,2}_{1'2'}}\right)}{2\vec r^{\,2}_{11'}\vec r^{\,2}_{12'}\,\vec r^{\,2}_{22'}} + \frac{\vec r^{\,2}_{21'}\ln\left(\frac{\vec r^{\,2}_{21'}\vec r^{\,2}_{1'2'}}{\vec r^{\,2}_{12}\,\vec r^{\,2}_{11'}}\right)}{2\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}\vec r^{\,2}_{1'2'}} + (1 \leftrightarrow 2),$$

$$d = \vec r^{\,2}_{12'}\vec r^{\,2}_{21'} - \vec r^{\,2}_{11'}\vec r^{\,2}_{22'}.$$

Freedom in the definition of the kernel

- We need the discontinuity of the amplitude for $A+B \to A'+B'$

$$-4i(2\pi)^{D-2}\delta(\vec{q}_A - \vec{q}_B)\text{disc}_s\mathcal{A}_{AB}^{A'B'} = \langle A'\bar{A}|e^{\ln(s/s_0)\hat{\mathcal{K}}}\frac{1}{\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}}|\bar{B}'B\rangle\ ,$$

- hence, we can change the kernel and the impact factors via an arbitrary operator:

$$\hat{\mathcal{K}} \to \hat{\mathcal{O}}^{-1}\hat{\mathcal{K}}\hat{\mathcal{O}}\ ,\quad \langle A'\bar{A}| \to \langle A'\bar{A}|\hat{\mathcal{O}}\ ,\quad \frac{1}{\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}}|\bar{B}'B\rangle \to \hat{\mathcal{O}}^{-1}\frac{1}{\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}}|\bar{B}'B\rangle\ .$$

- However, the LO kernel is fixed by the LO BK kernel. Therefore, we have the following transformations with $\hat{\mathcal{O}} = 1 - \hat{O},\quad \hat{O} \sim g^2$:

$$\hat{\mathcal{K}} \to \hat{\mathcal{K}} - [\hat{\mathcal{K}}^B, \hat{O}]\ ,\quad \hat{\mathcal{K}}^B = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r^B\ .$$

Structure of the gluon part in NLO

- We have the convenient decomposition:

$$\hat{\mathcal{K}} = \hat{\mathcal{K}}_p + \hat{\mathcal{K}}_{s1} + \hat{\mathcal{K}}_{s2} \ ,$$

  - $\hat{\mathcal{K}}_p$ — planar part (contribution of trajectories and octet kernel),
  - $\hat{\mathcal{K}}_{s1}$ — symmetric part of 2-gluon contribution,
  - $\hat{\mathcal{K}}_{s2}$ — subtraction term from the separation of KMRK and MRK.

- It enjoys the properties:
  - $g_{s2}(\vec{r}_1, \vec{r}_2; \vec{r}_1{}', \vec{r}_2{}') = g_p(\vec{r}_1, \vec{r}_2; \vec{r}_1{}', \vec{r}_2{}') - \left[\frac{(\vec{r}_{22'}\ \vec{r}_{12})}{\vec{r}^{\,2}_{11'} \vec{r}^{\,2}_{22'} \vec{r}^{\,2}_{1'2'}} \ln\left(\frac{\vec{r}^{\,2}_{12'}}{\vec{r}^{\,2}_{1'2'}} \right) + (1 \leftrightarrow 2) \right]$
  - $\hat{\mathcal{K}}_{s2} = -\frac{1}{4} \left[\hat{\mathcal{K}}^B_{\mathbf{r}}, \hat{\mathcal{K}}^B_r \ln \hat{\vec{q}}^{\,2}_{11'} \right]$ is very similar to the necessary $[\hat{\mathcal{K}}^B, \hat{O}]$,
  - but $\left[\hat{\mathcal{K}}^B, \hat{\mathcal{K}}^B_r \ln \hat{\vec{q}}^{\,2}_{11'} \right]$ is divergent.

Construction of the operator $\hat{O}$

- The operator eliminating the difference between the gluon parts of BFKL and BK kernels is

$$\langle \vec{q}_1, \vec{q}_2 | \hat{O} | \vec{q}_1{}', \vec{q}_2{}' \rangle = -\frac{1}{2} \langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}}_r^B \ln \hat{\vec{q}}_{11'}^{\,2} | \vec{q}_1{}', \vec{q}_2{}' \rangle \;-\; \frac{1}{2} \times$$

$$\times \underbrace{\frac{-\alpha_s N_c}{2\pi^2} \delta(\vec{q}_{22'}) \delta\left(\vec{q}_{11'}\right) \int d^{2+2\epsilon} k \left(\frac{2}{\vec{k}^{\,2}} - \frac{\vec{k}(\vec{k}-\vec{q}_1)}{\vec{k}^{\,2}(\vec{k}-\vec{q}_1)^2} - \frac{\vec{k}(\vec{k}-\vec{q}_2)}{\vec{k}^{\,2}(\vec{k}-\vec{q}_2)^2} \right)} \ln \vec{k}^{\,2}.$$

$$\delta(\vec{q}_{22'}) \delta\left(\vec{q}_{11'}\right) \left(\omega\left(\vec{q}_1^{\,2}\right) + \omega(\vec{q}_2^{\,2}) \right)$$

- Next, all nonconformal terms without 11/6 cancel via $\hat{O}_1$ from I. Balitsky and G. A. Chirilli Nucl.Phys.B822,45, (2009) $\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{O}}_1 | \vec{r}_1{}' \vec{r}_2{}' \rangle =$

$$= \frac{\alpha_s(\mu) N_c}{4\pi^2} \int d\vec{\rho} \frac{\vec{r}_{12}^{\,2} \ln\left(\frac{\vec{r}_{12}{}^2}{\vec{r}_{1\rho}^{\,2} \vec{r}_{2\rho}^{\,2}}\right)}{\vec{r}_{1\rho}^{\,2} \vec{r}_{2\rho}^{\,2}} \left[\delta(\vec{r}_{11'}) \delta(\vec{r}_{2'\rho}) + \delta(\vec{r}_{1'\rho}) \delta(\vec{r}_{22'}) - \delta(\vec{r}_{11'}) \delta(r_{22'}) \right] .$$

Quasi-conformal form for the gluon part of the BFKL kernel

- For $\hat{\mathcal{K}} - [\hat{\mathcal{K}}^B, \hat{O} + \hat{O}_1]$ we have

$$g^0(\vec{r}_1, \vec{r}_2; \vec{\rho}) = 6\pi\zeta(3)\,\delta(\vec{\rho}) - g(\vec{r}_1, \vec{r}_2; \vec{\rho})\ ,$$

$$g(\vec{r}_1, \vec{r}_2; \vec{r}_2^{\,\prime}) = \frac{11}{6}\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{22'}^{\,2}\vec{r}_{12'}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{r_\mu^2}\right) + \frac{11}{6}\left(\frac{1}{\vec{r}_{22'}^{\;2}} - \frac{1}{\vec{r}_{12'}^{\;2}}\right)\ln\left(\frac{\vec{r}_{22'}^{\,2}}{\vec{r}_{12'}^{\,2}}\right),$$

and

$$g(\vec{r}_1, \vec{r}_2; \vec{r}_1^{\,\prime}, \vec{r}_2^{\,\prime}) = \frac{1}{\vec{r}_{1'2'}^{\,4}}\left(\frac{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2} - 2\vec{r}_{12}^{\,2}\,\vec{r}_{1'2'}^{\,2}}{d}\ln\left(\frac{\vec{r}_{12'}^{\,2}\,\vec{r}_{21'}^{\,2}}{\vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}}\right) - 1\right)$$
$$+ \frac{1}{2\vec{r}_{11'}^{\;2}\,\vec{r}_{22'}^{\;2}}\ln\left(\frac{\vec{r}_{12'}^{\,2}\,\vec{r}_{21'}^{\,2}}{\vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}}\right)\left(\frac{\vec{r}_{12}^{\;4}}{d} - \frac{\vec{r}_{12}^{\;2}}{\vec{r}_{1'2'}^{\;2}}\right) + \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{11'}^{\;2}\vec{r}_{22'}^{\;2}\vec{r}_{1'2'}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}\vec{r}_{1'2'}^{\,2}}{\vec{r}_{12'}^{\,2}\,\vec{r}_{21'}^{\,2}}\right).$$

These functions

- have nonconformal terms of renormalization origin only,
- coincide with the BK ones.

Conclusion

- We have the quasi-conformal gluon part of the NLO BFKL kernel identical to the linearized BK one.
- As a result, since the scalar and the fermion parts are already quasi-conformal and identical, we have
  - the quasi-conformal NLO BFKL kernel identical to the linearized BK one in QCD;
  - the conformally invariant NLO BFKL kernel identical to the linearized BK one in SUSY N=4.

THANK YOU FOR YOUR ATTENTION!

Bonus. Conformally invariant NLO BFKL kernel for SUSY N=4 with $\beta_0 = 0$:

$$\langle\vec{r}_1\vec{r}_2|\hat{\mathcal{K}}_M|\vec{r}_1{}'\vec{r}_2{}'\rangle = \langle\vec{r}_1\vec{r}_2|\hat{\mathcal{K}}^B_M|\vec{r}_1{}'\vec{r}_2{}'\rangle\left(1 - \frac{\alpha_s N_c \zeta(2)}{2\pi}\right.$$

$$+\frac{\alpha_s^2 N_c^2}{4\pi^4}\left[\frac{\ln\left(\frac{\vec{r}^{\,2}_{12'}\,\vec{r}^{\,2}_{21'}}{\vec{r}^{\,2}_{11'}\vec{r}^{\,2}_{22'}}\right)}{2\vec{r}^{\,2}_{11'}\,\vec{r}^{\,2}_{22'}}\left(\frac{\vec{r}^{\,4}_{12}}{\vec{r}^{\,2}_{12'}\vec{r}^{\,2}_{21'} - \vec{r}^{\,2}_{11'}\vec{r}^{\,2}_{22'}} - \frac{\vec{r}^{\,2}_{12}}{\vec{r}^{\,2}_{1'2'}}\right)\right.$$

$$\left.\left.+\frac{\vec{r}^{\,2}_{12}\ln\left(\frac{\vec{r}^{\,2}_{12}\vec{r}^{\,2}_{1'2'}}{\vec{r}^{\,2}_{12'}\,\vec{r}^{\,2}_{21'}}\right)}{\vec{r}^{\,2}_{11'}\vec{r}^{\,2}_{22'}\vec{r}^{\,2}_{1'2'}} + 6\pi^2\zeta\left(3\right)\delta(\vec{r}_{11'})\delta(r_{22'})\right]\right) .$$

Bonus. Freedom in the definition of the energy scale s_0

$$\left(\frac{s}{\hat{f}_L\hat{f}_R}\right)^{\hat{\mathcal{K}}}_{LR} = \left(1 - \ln \hat{f}_L \hat{\mathcal{K}}_B\right) s^{\hat{\mathcal{K}}} \left(1 - \hat{\mathcal{K}}_B \ln \hat{f}_R\right) = \left(\frac{1}{\hat{f}_L}\right)^{\hat{\mathcal{K}}}_L s^{\hat{\mathcal{K}}} \left(\frac{1}{\hat{f}_R}\right)^{\hat{\mathcal{K}}}_R .$$

Shift of s_0 leads to the folloing shift of both impact factors(NLO)

$$\langle A'\bar{A}| \left(\frac{s}{s_0}\right)^{\hat{\mathcal{K}}} \frac{1}{\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle = \langle \widetilde{A'\bar{A}}| \left(\frac{s}{\hat{f}_L\hat{f}_R}\right)^{\hat{\mathcal{K}}}_{LR} |\widetilde{\bar{B}'B}\rangle,$$

$$\langle \widetilde{A'\bar{A}}| = \langle A'\bar{A}| \left(1 + \ln \frac{\hat{f}_L}{\sqrt{s_0}} \hat{\mathcal{K}}_B\right), \quad |\widetilde{\bar{B}'B}\rangle = \left(1 + \hat{\mathcal{K}}_B \ln \frac{\hat{f}_R}{\sqrt{s_0}}\right) \frac{1}{\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle.$$

Bonus. Freedom in the definition of the energy scale s_0

Shift of s_0 leads to the changes in one impact factor and the kernel (NLO)

$$\left(1+\ln \hat{f}_L \hat{\mathcal{K}}_B\right) \hat{\mathcal{K}} \left(1+\ln \hat{f}_L \hat{\mathcal{K}}_B\right)^{-1} = \hat{\mathcal{K}} - \left[\hat{\mathcal{K}}_B, \ln \hat{f}_L \hat{\mathcal{K}}_B\right] = \hat{\mathcal{K}}',$$

$$\left(1+\ln \hat{f}_L \hat{\mathcal{K}}_B\right) \left(\frac{s}{\hat{f}_L \hat{f}_R}\right)^{\hat{\mathcal{K}}}_{LR} \left(1+\ln \hat{f}_L \hat{\mathcal{K}}_B\right)^{-1} = \left(\frac{s}{\hat{f}_L \hat{f}_R}\right)^{\hat{\mathcal{K}}'}_{LR}.$$

$$\langle A'\bar{A}| \left(\frac{s}{s_0}\right)^{\hat{\mathcal{K}}} \frac{1}{\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle = \langle A'\bar{A}| \left(\frac{s}{\hat{f}_L \hat{f}_R}\right)^{\hat{\mathcal{K}}'}_{LR} |\overline{\bar{B}'B}\rangle\ ,$$

$$|\overline{\bar{B}'B}\rangle = \left(1 + \hat{\mathcal{K}}_B \ln \frac{\hat{f}_R}{\sqrt{s_0}} + \ln \frac{\hat{f}_L}{\sqrt{s_0}} \hat{\mathcal{K}}_B\right) \frac{1}{\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle.$$

Bonus. Difference of forward BFKL and BK kernels.

$$\langle\vec{x}|\hat{\mathcal{K}}_m - \hat{\mathcal{K}}_{BC}|\vec{z}\rangle = \frac{\alpha_s^2(\mu)N_c^2}{4\pi^3}\left\{ 2\pi\zeta(3)\,\delta(\vec{x}-\vec{z}) - \frac{11}{3}\frac{x^2}{(x-z)^2z^2}(\ln 2 - C)\right.$$

$$\left. + \frac{x^2}{(x-z)^2z^2}\ln\left(\frac{x^2}{z^2}\right)\ln\left(\frac{x^2\,z^2}{(x-z)^4}\right)\right\}.$$

with $x = \vec{r}_{12},\ \ z = \vec{r}_{1'2'}$

$$\langle\vec{x}|\left[\hat{\mathcal{K}}^B\,,\hat{\mathcal{K}}^B\ln\left(\hat{q}_1^2\right)\right]_d|\vec{z}\rangle = -\frac{\alpha_s^2(\mu^2)N_c^2}{2\pi^3}\frac{x^2}{(x-z)^2z^2}\ln\left(\frac{x^2}{z^2}\right)\ln\left(\frac{x^2z^2}{(x-z)^{\,4}}\right).$$

The following operator kills this term

$$\hat{O} = \frac{1}{2}\hat{\mathcal{K}}^B\ln\left(\hat{\vec{q}}_1^{\,2}\right).$$