Next-to-leading order Photon Impact Factor in Deep Inelastic Scattering at small-x

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Low x in Kavala 23 June 2010

Outline

- High-energy scattering and Wilson lines in quantum mechanics, QED and QCD.
- Light-cone OPE versus OPE in color dipoles.
- The LO evolution of color dipoles: BK equation.
- NLO amplitudes and NLO BK equation in $\mathcal{N}$=4 SYM.
- NLO amplitudes for deep inelastic scattering in QCD.
- Conclusions.

Conformal invariance of the BK equation

Formally, a light-like Wilson line

$$[\infty p_1 + x_\perp, -\infty p_1 + x_\perp] \;=\; \mathrm{Pexp}\left\{ ig\int_{-\infty}^{\infty} dx^+ \, A_+(x^+, x_\perp) \right\}$$

is invariant under inversion (with respect to the point with $x^- = 0$).

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Indeed,
$(x^+, x_\perp)^2 = -x_\perp^2 \Rightarrow$ after the inversion $x_\perp \to x_\perp / x_\perp^2$ and $x^+ \to x^+ / x_\perp^2$

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$$[\infty p_1 + x_\perp, -\infty p_1 + x_\perp] \;\to\; \mathrm{Pexp}\left\{ig\int_{-\infty}^{\infty} d\frac{x^+}{x_\perp^2} \, A_+\Big(\frac{x^+}{x_\perp^2}, \frac{x_\perp}{x_\perp^2}\Big)\right\} \;=\; [\infty p_1 + \frac{x_\perp}{x_\perp^2}, -\infty p_1 + \frac{x_\perp}{x_\perp^2}]$$

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$$[\infty p_1 + x_\perp, -\infty p_1 + x_\perp] \to \text{Pexp}\left\{ig\int_{-\infty}^{\infty} d\frac{x^+}{x_\perp^2} A_+\left(\frac{x^+}{x_\perp^2}, \frac{x_\perp}{x_\perp^2}\right)\right\} = [\infty p_1 + \frac{x_\perp}{x_\perp^2}, -\infty p_1 + \frac{x_\perp}{x_\perp^2}]$$

$\Rightarrow$The dipole kernel is invariant under the inversion $V(x_\perp) = U(x_\perp/x_\perp^2)$

$$\frac{d}{d\eta}\text{Tr}\{V_x V_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int \frac{d^2z}{\not{z}^4} \frac{(x-y)^2 \, \not{z}^4}{(x-z)^2(z-y)^2}[\text{Tr}\{V_x V_z^\dagger\}\text{Tr}\{V_z V_y^\dagger\} - N_c \text{Tr}\{V_x V_y^\dagger\}]$$

Conformal invariance of the BK equation

SL(2,C) for Wilson lines

$$\hat{S}_- \equiv \frac{i}{2}(K^1 + iK^2), \quad \hat{S}_0 \equiv \frac{i}{2}(D + iM^{12}), \quad \hat{S}_+ \equiv \frac{i}{2}(P^1 - iP^2)$$

$$[\hat{S}_0, \hat{S}_\pm] = \pm\hat{S}_\pm, \quad \frac{1}{2}[\hat{S}_+, \hat{S}_-] = \hat{S}_0,$$

$$[\hat{S}_-, \hat{U}(z,\bar{z})] = z^2\partial_z\hat{U}(z,\bar{z}), \quad [\hat{S}_0, \hat{U}(z,\bar{z})] = z\partial_z\hat{U}(z,\bar{z}), \quad [\hat{S}_+, \hat{U}(z,\bar{z})] = -\partial_z\hat{U}(z,\bar{z})$$

$z \equiv z^1 + iz^2, \bar{z} \equiv z^1 + iz^2, \quad U(z_\perp) = U(z,\bar{z})$

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Conformal invariance of the evolution kernel

$$\frac{d}{d\eta}[\hat{S}_-, \mathrm{Tr}\{U_x U_y^\dagger\}] = \frac{\alpha_s N_c}{2\pi^2}\int dz\, K(x,y,z)[\hat{S}_-, \mathrm{Tr}\{U_x U_y^\dagger\}\mathrm{Tr}\{U_x U_y^\dagger\}]$$

$$\Rightarrow \left[x^2\frac{\partial}{\partial x} + y^2\frac{\partial}{\partial y} + z^2\frac{\partial}{\partial z}\right]K(x,y,z) = 0$$

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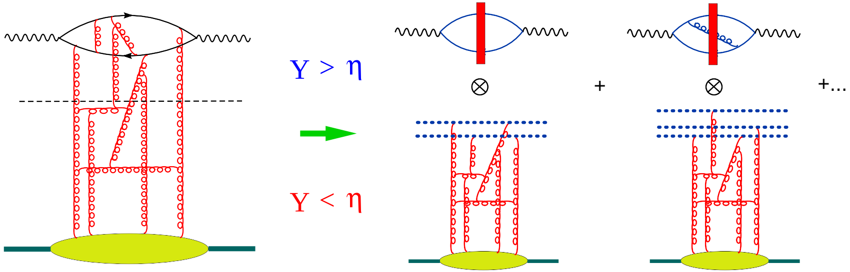
In the leading order - OK. In the NLO - ?



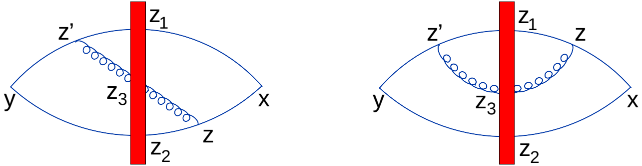
The high-energy operator expansion is

$$T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}(z_1,z_2)\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

$$+ \int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}(z_1,z_2,z_3)[\frac{1}{N_c}\mathrm{Tr}\{T^n\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}T^n\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - \mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

In the leading order - conf. invariant impact factor

$$I_{\rm LO} = \frac{x_+^{-2}y_+^{-2}}{\pi^2\mathcal{Z}_1^2\mathcal{Z}_2^2}, \qquad \mathcal{Z}_i \equiv \frac{(x-z_i)_\perp^2}{x_+} - \frac{(y-z_i)_\perp^2}{y_+} \qquad CCP, 2007$$



$$I^{\rm NLO}(x,y;z_1,z_2,z_3;\eta) \;=\; -\,I^{\rm LO}\times\frac{\lambda}{\pi^2}\frac{z_{13}^2}{z_{12}^2z_{23}^2}\Big[\ln\frac{\sigma s}{4}\mathcal{Z}_3-\frac{i\pi}{2}+C\Big]$$

The NLO impact factor is not Möbius invariant $\Rightarrow$ the color dipole with the cutoff η is not invariant

However, if we define a composite operator (a - analog of μ^{-2} for usual OPE)

$$\big[{\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]^{\rm conf} \;=\; {\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}$$
$$+\,\frac{\lambda}{2\pi^2}\int d^2z_3\;\frac{z_{12}^2}{z_{13}^2z_{23}^2}\big[{\rm Tr}\{T^n\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^n\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\}-N_c{\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]\ln\frac{az_{12}^2}{z_{13}^2z_{23}^2}\;+\;O(\lambda^2)$$

the impact factor becomes conformal in the NLO.

Non-linear evolution equation at NLO

$$\frac{d}{d\eta}Tr\{U_xU^\dagger_y\} =$$
$$\int \frac{d^2z}{2\pi^2}\left(\alpha_s\frac{(x-y)^2}{(x-z)^2(z-y)^2} + \alpha_s^2 K_{NLO}(x,y,z)\right)[Tr\{U_xU^\dagger_z\}Tr\{U_zU^\dagger_y\} - N_c Tr\{U_zU^\dagger_y\}] +$$
$$\alpha_s^2\int d^2z d^2z'\left(K_4(x,y,z,z')\{U_x, U^\dagger_{z'}, U_z, U^\dagger_y\} + K_6(x,y,z,z')\{U_x, U^\dagger_{z'}, U_{z'}, U_z, U^\dagger_z, U^\dagger_y\}\right)$$

K_{NLO} is the next-to-leading order correction to the dipole kernel and K_4 and K_6 are the coefficients in front of the (tree) four- and six-Wilson line operators with arbitrary white arrangements of color indices.

Definition of the NLO kernel

In general

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} = \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + \alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} + O(\alpha_s^3)$$

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$$\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} = \frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} - \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + O(\alpha_s^3)$$

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We calculate the “matrix element” of the r.h.s. in the shock-wave background

$$\langle\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle = \frac{d}{d\eta}\langle\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle - \langle\alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle + O(\alpha_s^3)$$

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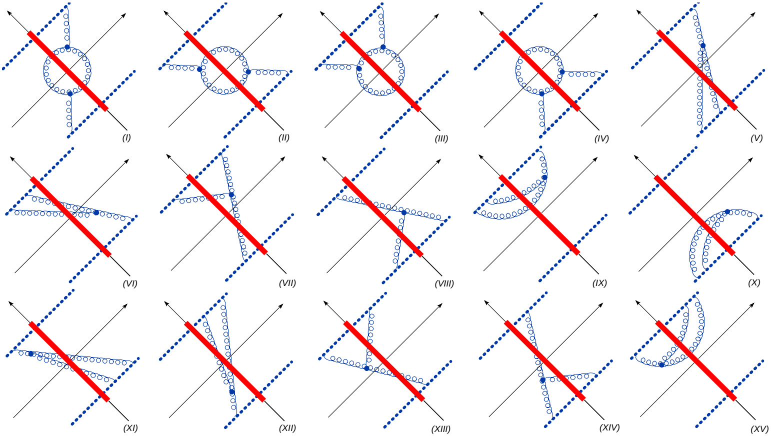
Subtraction of the (LO) contribution (with the rigid rapidity cutoff)
$\Rightarrow$ $\left[\frac{1}{v}\right]_+$ prescription in the integrals over Feynman parameter v

Typical integral

$$\int_0^1 dv\,\frac{1}{(k-p)_\perp^2 v + p_\perp^2(1-v)}\left[\frac{1}{v}\right]_+ = \frac{1}{p_\perp^2}\ln\frac{(k-p)_\perp^2}{p_\perp^2}$$

Diagrams with 2 gluons interaction



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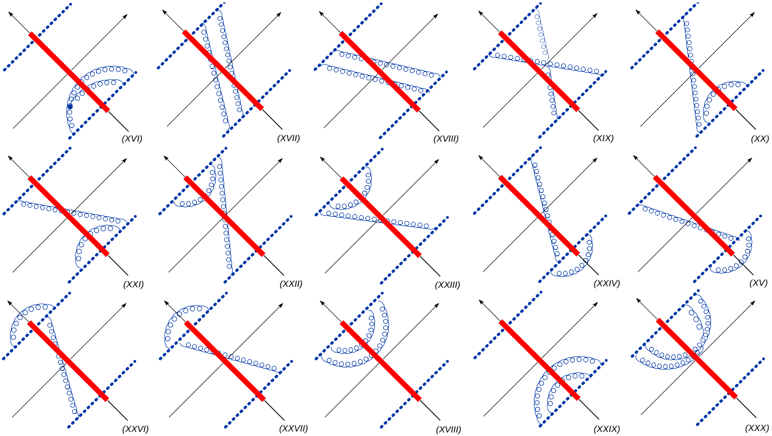

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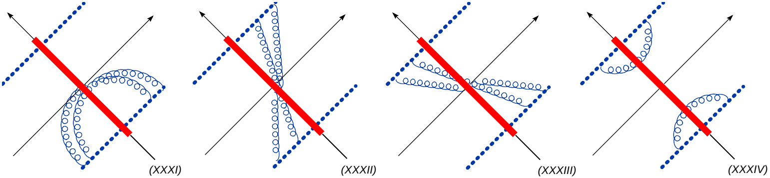
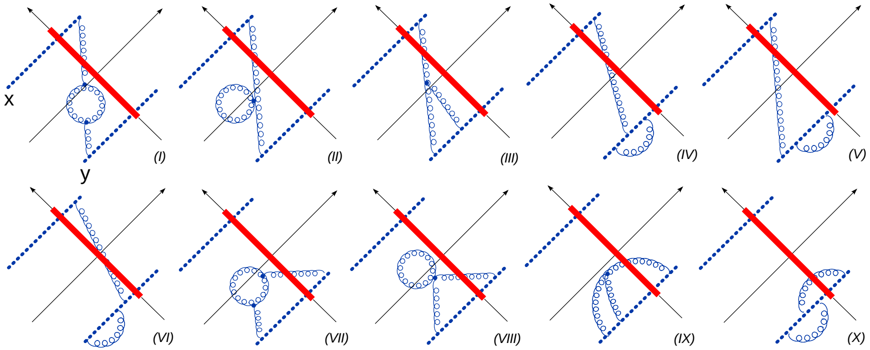
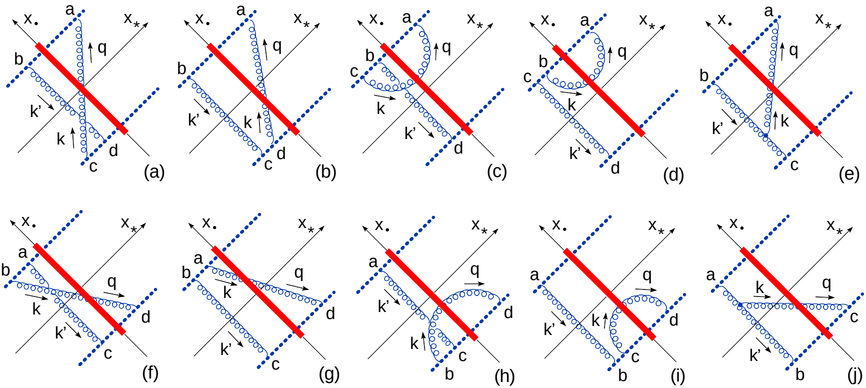


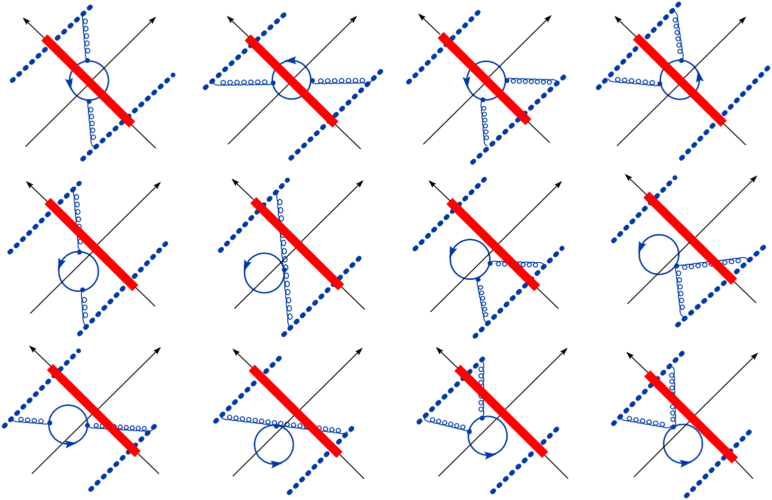
"Running coupling" diagrams



$1 \to 2$ dipole transition diagrams



$\mathcal{N} = 4$ SYM diagrams (scalar and gluino loops)



Evolution equation for color dipole in $\mathcal{N}=4$ (I.B. and G. A. C.)

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}$$
$$= \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big\{1-\frac{\alpha_sN_c}{4\pi}\Big[\frac{\pi^2}{3}+2\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}\Big]\Big\}$$
$$\times\,[\mathrm{Tr}\{T^a\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^a\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\}-N_c\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]$$
$$-\frac{\alpha_s^2}{4\pi^4}\int\frac{d^2z_3d^2z_4}{z_{34}^4}\,\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\Big[1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{23}^2z_{14}^2}\Big]\ln\frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}$$
$$\times\,\mathrm{Tr}\{[T^a,T^b]\hat U^\eta_{z_1}T^{a'}T^{b'}\hat U^{\dagger\eta}_{z_2}+T^bT^a\hat U^\eta_{z_1}[T^{b'},T^{a'}]\hat U^{\dagger\eta}_{z_2}\}(\hat U^\eta_{z_3})^{aa'}(\hat U^\eta_{z_4}-\hat U^\eta_{z_3})^{bb'}$$

NLO kernel = Non-conformal term + Conformal term.

Non-conformal term is due to the non-invariant cutoff $\alpha<\sigma=e^{2\eta}$ in the rapidity of Wilson lines.

$$
\begin{aligned}
&\frac{d}{d\eta}\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\\
&= \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big\{1-\frac{\alpha_sN_c}{4\pi}\Big[\frac{\pi^2}{3}+2\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}\Big]\Big\}\\
&\times\big[\mathrm{Tr}\{T^a\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^a\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\}-N_c\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]\\
&-\frac{\alpha_s^2}{4\pi^4}\int\frac{d^2z_3d^2z_4}{z_{34}^4}\,\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\Big[1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{23}^2z_{14}^2}\Big]\ln\frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}\\
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\end{aligned}
$$

NLO kernel = Non-conformal term + Conformal term.

Non-conformal term is due to the non-invariant cutoff $\alpha<\sigma=e^{2\eta}$ in the rapidity of Wilson lines.

For the conformal composite dipole the result is Möbius invariant

Evolution equation for composite conformal dipoles in $\mathcal{N} = 4$

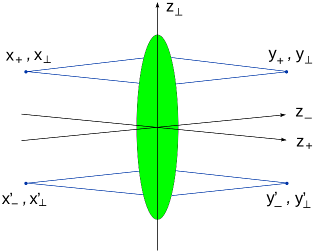
(I.B. and G. A. C.)

$$
\begin{aligned}
&\frac{d}{d\eta}\big[\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]^{\rm conf} \\
&= \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[1-\frac{\alpha_sN_c}{4\pi}\frac{\pi^2}{3}\Big]\big[\mathrm{Tr}\{T^a\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^a\hat U_{z_3}\hat U^{\dagger\eta}_{z_2}\}-N_c\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]^{\rm conf} \\
&\quad-\frac{\alpha_s^2}{4\pi^4}\int d^2z_3d^2z_4\frac{z_{12}^2}{z_{13}^2z_{24}^2z_{34}^2}\Big\{2\ln\frac{z_{12}^2z_{34}^2}{z_{14}^2z_{23}^2}+\Big[1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{14}^2z_{23}^2}\Big]\ln\frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}\Big\} \\
&\quad\times\mathrm{Tr}\{[T^a,T^b]\hat U^\eta_{z_1}T^{a'}T^{b'}\hat U^{\dagger\eta}_{z_2}+T^bT^a\hat U^\eta_{z_1}[T^{b'},T^{a'}]\hat U^{\dagger\eta}_{z_2}\}[(\hat U^\eta_{z_3})^{aa'}(\hat U^\eta_{z_4})^{bb'}-(z_4\to z_3)]
\end{aligned}
$$

Now Möbius invariant!

$$(x-y)^4(x'-y')^4\langle\mathcal{O}(x)\mathcal{O}^\dagger(y)\mathcal{O}(x')\mathcal{O}^\dagger(y')\rangle$$

Regge limit: $x_+ \to \rho x_+,\ x'_+ \to \rho x'_+, y_- \to \rho' y_-,\ y'_- \to \rho' y-_-$ $\rho, \rho' \to \infty$

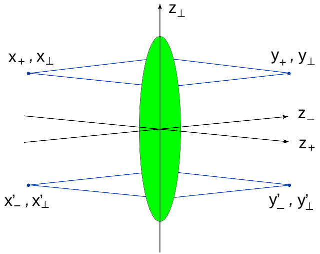


Regge limit symmetry in a conformal theory: 2-dim conformal Möbius group $SL(2, C)$.

Small-x (Regge) limit in the coordinate space

$$(x-y)^4(x'-y')^4\langle\mathcal{O}(x)\mathcal{O}^\dagger(y)\mathcal{O}(x')\mathcal{O}^\dagger(y')\rangle$$

Regge limit: $x_+ \to \rho x_+,\ x'_+ \to \rho x'_+,\ y_- \to \rho' y_-,\ y'_- \to \rho' y-_-$ $\quad \rho, \rho' \to \infty$



LLA: $\alpha_s \ll 1,\ \alpha_s \ln\rho \sim 1,\ \Rightarrow \sum(\alpha_s \ln\rho)^n \equiv$ BFKL pomeron.
LLA $\Leftrightarrow$ tree diagrams $\Rightarrow$ the BFKL pomeron is Möbius invariant .

NLO LLA: extra α_s: $\sum \alpha_s(\alpha_s \ln\rho)^n \equiv$ NLO BFKL

In conformal theory ($\mathcal{N}=4$ SYM) the NLO BFKL for composite conformal dipole operator is Möbius invariant.

In a conformal theory the amplitude $A(x,y;x',y')$ depends on two conformal ratios which can be chosen as

$$R = \frac{(x-x')(y-y')^2}{(x-y)^2(x'-y')^2},$$
$$r = R\Big[1 - \frac{(x-y')^2(y-x')^2}{(x-x')^2(y-y')^2} + \frac{1}{R}\Big]^2$$

In the Regge limit R scales as $\rho^2\rho'^2$ while r does not depend on ρ or ρ'.

NLO Amplitude in $\mathcal{N}$=4 SYM theory

The pomeron contribution in a conformal theory can be represented as an integral over one real variable ν Cornalba (2007)

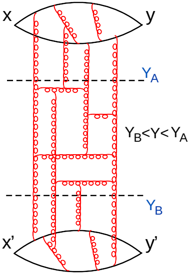
$$
\begin{aligned}
&(x-y)^4(x'-y')^4\langle\mathcal{O}(x)\mathcal{O}^\dagger(y)\mathcal{O}(x')\mathcal{O}^\dagger(y')\rangle \\
&= \frac{i}{2}\int d\nu\,\tilde{f}_+(\nu)\frac{\tanh\pi\nu}{\nu}F(\nu)\Omega(r,\nu)R^{\frac{1}{2}\omega(\nu)}
\end{aligned}
$$

$\omega(\nu)\equiv\omega(0,\nu)$ is the pomeron intercept,
$\tilde{f}_+(\omega)=(e^{i\pi\omega}-1)/\sin\pi\omega$ is the signature factor in the coordinate space.
$F(\nu)$ is the "pomeron residue".
The conformal function $\Omega(r,\nu)$ is given

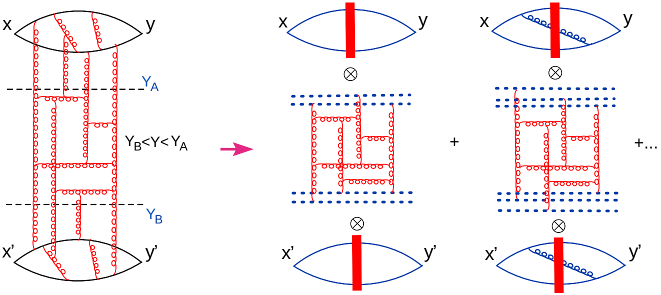
$$
\Omega(r,\nu) = \frac{\nu^2}{\pi^3}\int d^2z\left(\frac{\kappa^2}{(2\kappa\cdot\zeta)^2}\right)^{\frac{1}{2}+i\nu}\left(\frac{\kappa'^2}{(2\kappa'\cdot\zeta)^2}\right)^{\frac{1}{2}-i\nu}
$$

$\zeta\equiv p_1+\frac{z_\perp^2}{s}p_2+z_\perp$

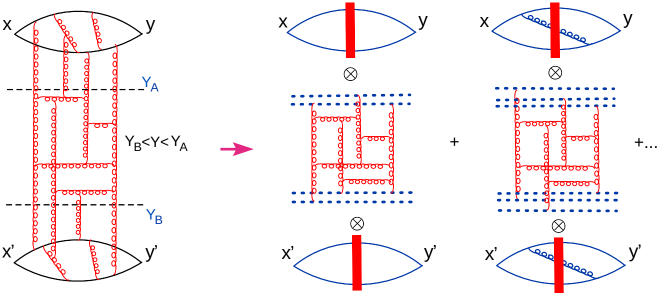
NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



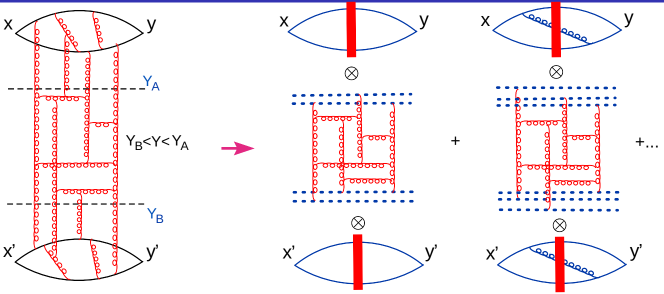
NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



$$(x-y)^4(x'-y')^4\langle T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}^\dagger(y)\hat{\mathcal{O}}(x')\hat{\mathcal{O}}^\dagger(y')\}\rangle$$
$$= \int d^2z_{1\perp}d^2z_{2\perp}d^2z'_{1\perp}d^2z'_{2\perp}\,\mathrm{IF}^{a_0}(x,y;z_1,z_2)[\mathrm{DD}]^{a_0,b_0}(z_1,z_2;z'_1,z'_2)\mathrm{IF}^{b_0}(x',y';z'_1,z'_2)$$

$a_0 = \frac{x_+y_+}{(x-y)^2}$, $b_0 = \frac{x'_-y'_-}{(x'-y')^2}$ $\Leftrightarrow$ impact factors do not scale with energy
$\Rightarrow$ all energy dependence is contained in $[\mathrm{DD}]^{a_0,b_0}$

NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



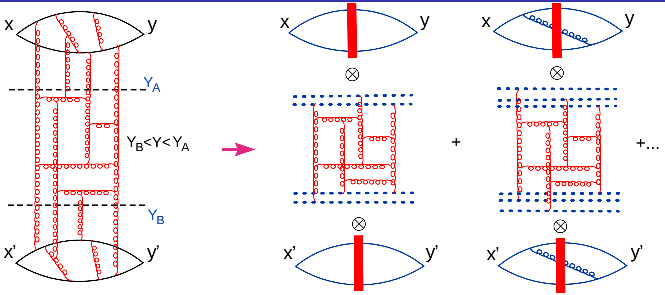
Projection onto conformal transverse eigenfunctions $\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^\gamma$ $(\gamma = \frac{1}{2} + i\nu)$:

$$\int dz_1 dz_2 (x-y)^4 T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(y)\}\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^\gamma = \left(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\right)^\gamma [I^A_{\rm LO}(\gamma) + I^A_{\rm NLO}(\gamma)]\hat{\mathcal{U}}(z_0,\gamma),$$

$$\hat{\mathcal{U}}(z_0,\gamma) = \int d^2z_1 d^2z_2 (z_{12}^2 z_{10}^{-2} z_{20}^{-2})^\gamma \hat{\mathcal{U}}(z_1,z_2)$$

$$I^A_{\rm LO}(\gamma) = B(1-\gamma,1-\gamma)\Gamma(1+\gamma)\Gamma(2-\gamma), \qquad I^A_{\rm NLO}(\gamma) = \frac{\lambda}{8\pi^2} I^A_{\rm LO}\Phi_1(\gamma)$$

$$\Phi_1(\gamma) = -2\psi'(\gamma) - 2\psi'(1-\gamma) + \frac{2}{3}\pi^2 + \frac{\chi(\gamma)-2}{\gamma(1-\gamma)} + 2C\chi(\gamma)$$



$$(x-y)^4(x'-y')^4\langle T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}^\dagger(y)\hat{\mathcal{O}}(x')\hat{\mathcal{O}}^\dagger(y')\}\rangle$$

$$= \int d\nu d\nu' \int d^2z_0 d^2z_0' \, \frac{\nu^2(1+4\nu^2)}{4\pi\cosh\pi\nu}\frac{\Gamma^2\left(\frac{1}{2}-i\nu\right)}{\Gamma(1-2i\nu)}\left(\frac{-\kappa^2}{(-2\kappa\cdot\zeta_0)^2}\right)^{\frac{1}{2}+i\nu}\left[1+\frac{\alpha_s N_c}{2\pi}\Phi_1(\nu)\right]$$

$$\times \, \frac{\nu'^2(1+4\nu'^2)}{4\pi\cosh\pi\nu'}\frac{\Gamma^2\left(\frac{1}{2}-i\nu'\right)}{\Gamma(1-2i\nu')}\left(\frac{-\kappa'^2}{(-2\kappa'\cdot\zeta_0')^2}\right)^{\frac{1}{2}+i\nu'}\left[1+\frac{\alpha_s N_c}{2\pi}\Phi_1(\nu')\right]$$

$$\times \, \langle\hat{\mathcal{U}}^{a_0}_{\rm conf}(\nu,z_0)\hat{\mathcal{V}}^{b_0}_{\rm conf}(\nu',z_0')\rangle$$

Assembling NLO $F(\nu)$

The last ingredient is the amplitude of scattering of two conformal dipoles ($\gamma \equiv \frac{1}{2} + i\nu$)

$$\langle \hat{\mathcal{U}}^a(z_0,\gamma)\hat{\mathcal{V}}^b(z_0',\gamma)\rangle = \delta(\nu-\nu')\delta(z_0-z_0')\ (ab)^{\frac{1}{2}\omega(\nu)}[A_{\rm LO}(\gamma)+A_{\rm NLO}(\gamma)]$$

$$A_{\rm LO}(\gamma) = \frac{\Gamma(-\gamma)\Gamma(\gamma-1)}{\Gamma(1+\gamma)\Gamma(2-\gamma)}, \quad A_{\rm NLO}(\gamma) = -\frac{\lambda}{4\pi^2}A_{\rm LO}\Big[\frac{\chi(\gamma)}{\gamma(1-\gamma)} - \frac{\pi^2}{3}\Big]$$

With our choice $a = \frac{x_+y_+}{(x-y)^2}, \quad b = \frac{x'_-y'_-}{(x'-y')^2}, \quad ab = R \Rightarrow$

$$\langle \hat{\mathcal{U}}(z_0,\gamma)\hat{\mathcal{V}}(z_0',\gamma)\rangle = \delta(\nu-\nu')\delta(z_0-z_0')\ R^{\frac{1}{2}\omega(\nu)}[A_{\rm LO}(\gamma)+A_{\rm NLO}(\gamma)]$$

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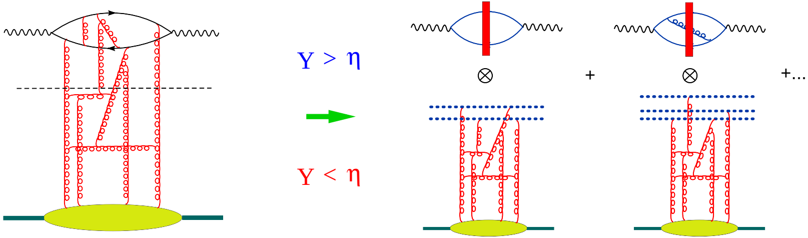
Now one can assemble $F(\nu)$ in the next-to-leading order

$$F(\nu) = F_{\rm LO}(\nu) + \lambda F_{\rm NLO}(\nu) + O(\lambda^2)$$

$$F_{\rm LO}(\nu) = I^A_{\rm LO}(\nu)A_{\rm LO}(\nu)I^B_{\rm LO}(\nu),$$

$$F_{\rm NLO}(\nu) = I^A_{\rm NLO}(\nu)A_{\rm LO}(\nu)I^B_{\rm LO} + I^A_{\rm LO}(\nu)A_{\rm NLO}(\nu)I^B_{\rm LO} + I^A_{\rm LO}(\nu)A_{\rm LO}(\nu)I^B_{\rm NLO}(\nu)$$



DIS structure function $F_2(x)$: photon impact factor + evolution of color dipoles+ initial conditions for the small-x evolution

Composite "conformal" dipole

$$[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]^{\mathrm{conf}}_a$$
$$= \mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\} - \frac{\alpha_s}{4\pi^2}\int d^2z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \ln \frac{a e^{2\eta} z_{12}^2}{z_{13}^2 z_{23}^2} [\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c \mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

Photon impact factor in the LO

$$(x-y)^6 T\{j_\mu(x) j_\nu(y)\} = \frac{1}{\pi^2}\int \frac{d^2z_1 d^2z_2}{z_{12}^4} \mathcal{R}^3 \hat{\mathcal{U}}^{\mathrm{conf}}(z_1,z_2) \frac{\partial^2}{\partial x^\mu \partial y^\nu}\left[-2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) + \kappa^2(\zeta_1\cdot\zeta_2)\right]$$

$$\Delta \equiv (x-y), \qquad x_* = x^+\sqrt{s/2}, \qquad y_* = x^+\sqrt{s/2}, \qquad R \equiv -\frac{\Delta^2 z_{12\perp}^2}{x_* y_* \mathcal{Z}_1 \mathcal{Z}_2}$$

$$\mathcal{Z}_1 = -\frac{(x-z_1)^2}{x_*} + \frac{(y-z_1)^2}{y_*}, \qquad \mathcal{Z}_2 = -\frac{(x-z_2)^2}{x_*} + \frac{(y-z_2)^2}{y_*}$$

$$\begin{aligned}
I^{NLO}_{\mu\nu}(x,y) = & -\frac{\alpha_s N_c^2}{8\pi^7 x_*^2 y_*^2}\int d^2z_1 d^2z_2\, \mathcal{U}^{\rm conf}(z_1,z_2)\Big\{\Big[\frac{1}{\mathcal{Z}_1^2\mathcal{Z}_2^2}\partial^x_\mu\partial^y_\nu \ln\frac{\Delta^2}{x_*y_*} \\
& + 2\frac{\left(\partial^x_\mu \mathcal{Z}_1\right)\left(\mathcal{Z}_2\partial^y_\nu\right)}{\mathcal{Z}_1^3\mathcal{Z}_1^3}\Big[\ln\frac{1}{R} + \frac{1}{2R} - 2\Big] + \frac{2\left(\partial^x_\mu\mathcal{Z}_1\right)\left(\partial^y_\nu\mathcal{Z}_1\right)}{\mathcal{Z}_1^4\mathcal{Z}_2^2}\Big[\ln\frac{1}{R} - \frac{1}{2R}\Big] \\
& - \frac{1}{2}\Big[\frac{\partial^x_\mu\mathcal{Z}_1}{\mathcal{Z}_1^3\mathcal{Z}_2^2}\partial^y_\nu\ln\frac{\Delta^2}{x_*y_*} + \frac{\partial^y_\nu\mathcal{Z}_1}{\mathcal{Z}_1^3\mathcal{Z}_2^2}\partial^x_\mu\ln\frac{\Delta^2}{x_*y_*}\Big]\Big(1-\frac{1}{R}\Big) - \frac{1}{2\mathcal{Z}_2^2}\Big[\Big(\partial^x_\mu\frac{1}{\mathcal{Z}_1^2}\Big)\partial^y_\nu R + \Big(\partial^y_\nu\frac{1}{\mathcal{Z}_1^2}\Big)\partial^x_\mu R\Big]\frac{\ln R}{1-R} \\
& - \Big(\partial^x_\mu\partial^y_\nu\frac{\Delta^2}{x_*y_*}\Big)\frac{R^3}{z_{12}^4}\Big[\frac{1}{R} + \frac{3}{2R^2} - 2\Big]\Big(\frac{x_*y_*}{\Delta^2}\Big)^3 + \frac{1}{R}\Big[\frac{\partial^x_\mu\mathcal{Z}_1}{\mathcal{Z}_1^3\mathcal{Z}_2^2}\Big(\partial^y_\nu\ln\frac{\Delta^2}{x_*y_*}\Big) + \frac{\partial^y_\nu\mathcal{Z}_1}{\mathcal{Z}_1^3\mathcal{Z}_2^2}\Big(\partial^x_\mu\ln\frac{\Delta^2}{x_*y_*}\Big)\Big] \\
& + 4\frac{\left(\partial^x_\mu\mathcal{Z}_1\right)\left(\partial^y_\nu\mathcal{Z}_2\right)}{\mathcal{Z}_1^3\mathcal{Z}_2^3}\Big[4{\rm Li}_2(1-R) - \frac{2\pi^2}{3} + 2(\ln R - 1)\Big(\ln R - \frac{1}{R}\Big)\Big] \\
& + 2\frac{\left(\partial^x_\mu\mathcal{Z}_1\right)\left(\partial^y_\nu\mathcal{Z}_2\right)}{\mathcal{Z}_1^3\mathcal{Z}_2^3}\Big[\frac{\ln R}{R(1-R)} - \frac{1}{R} + 2\ln R - 4\Big] + 2\frac{\left(\partial^x_\mu\mathcal{Z}_1\right)\left(\partial^y_\nu\mathcal{Z}_1\right)}{\mathcal{Z}_1^4\mathcal{Z}_2^2}\Big[\frac{\ln R}{R(1-R)} - \frac{1}{R}\Big] \\
& - \Big(\frac{\partial^x_\mu\mathcal{Z}_1}{\mathcal{Z}_1^3\mathcal{Z}_2^2}\partial^y_\nu\ln\frac{\Delta^2}{x_*y_*} + \frac{\partial^y_\nu\mathcal{Z}_2}{\mathcal{Z}_2^3\mathcal{Z}_1^2}\partial^x_\mu\ln\frac{\Delta^2}{x_*y_*}\Big)\Big[\frac{\ln R}{R(1-R)} - 2\Big] + (z_1\leftrightarrow z_2)\Big] \\
& - 2\frac{z_{12\perp}^2}{\mathcal{Z}_1^3\mathcal{Z}_2^3}\Big[4{\rm Li}_2(1-R) - \frac{2\pi^2}{3} + 2\Big(\ln\frac{1}{R} + \frac{1}{R} + \frac{1}{2R^2} - 3\Big)\ln\frac{1}{R} - \Big(6 + \frac{1}{R}\Big)\ln R + \frac{3}{R} - 4\Big]\partial^x_\mu\partial^y_\nu\frac{\Delta^2}{x_*y_*}\Big\}
\end{aligned}$$

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$$a\frac{d}{da}[\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{conf}}_a = \frac{\alpha_s}{2\pi^2}\int d^2z_3\,\Big([\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{conf}}_a$$

$$\times\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\Big(b\ln z_{12}^2\mu^2 + b\frac{z_{13}^2-z_{23}^2}{z_{13}^2z_{23}^2}\ln\frac{z_{13}^2}{z_{23}^2}+\frac{67}{9}-\frac{\pi^2}{3}\Big)\Big]$$

$$+\frac{\alpha_s}{4\pi^2}\int\frac{d^2z_4}{z_{34}^4}\Big\{\Big[-2+\frac{z_{23}{}^2z_{23}^2+z_{24}^2z_{13}^2-4z_{12}^2z_{34}^2}{2(z_{23}{}^2z_{23}^2-z_{24}^2z_{13}^2)}\ln\frac{z_{23}{}^2z_{23}^2}{z_{24}^2z_{13}^2}\Big]$$

$$\times[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_4}\}\{U_{z_4}U^\dagger_{z_2}\} - \mathrm{tr}\{U_{z_1}U^\dagger_{z_3}U_{z_4}U^\dagger_{z_2}U_{z_3}U^\dagger_{z_4}\} - (z_4\to z_3)]$$

$$+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\Big[2\ln\frac{z_{12}^2z_{34}^2}{z_{23}{}^2z_{23}^2}+\Big(1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{23}{}^2z_{23}^2}\Big)\ln\frac{z_{13}^2z_{24}^2}{z_{23}{}^2z_{23}^2}\Big]$$

$$\times[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_4}\}\mathrm{tr}\{U_{z_4}U^\dagger_{z_2}\} - \mathrm{tr}\{U_{z_1}U^\dagger_{z_4}U_{z_3}U^\dagger_{z_2}U_{z_4}U^\dagger_{z_3}\} - (z_4\to z_3)]\Big\}$$

$$b=\frac{11}{3}N_c-\frac{2}{3}n_f$$

$K_{\mathrm{NLO\ BK}}$ = Running coupling part + Conformal "non-analytic" (in j) part + Conformal analytic ($\mathcal{N}=4$) part

Linearized $K_{\mathrm{NLO\ BK}}$ reproduces the known result for the forward NLO BFKL kernel.

Conclusions

- High-energy operator expansion in color dipoles works at the NLO level.

Conclusions

- High-energy operator expansion in color dipoles works at the NLO level.
- The NLO BK kernel in for the evolution of conformal composite dipoles in $\mathcal{N} = 4$ SYM is Möbius invariant in the transverse plane.
- The NLO BK kernel agrees with NLO BFKL eigenvalues.
- The correlation function of four Z^2 operators is calculated at the NLO order.
- The analytic expression for the NLO photon impact factor is calculated (in the coord. space)