Non-linear evolution in CCFM and the saturation of the saturation scale

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Motivation and Background

Aim: To implement a generic and simple method of enforcing non-linear effects in linear evolution equations at small-x, in particular CCFM.

Transition between linear and non-linear region around $k \sim Q_s$, but non-linear physics also affects region $k > Q_s$, shape of gluon distribution changed.

Exact non-linear dynamics is not necessary for finding correct Q_s. One can emulate non-linear dynamics via some boundary condition in k on the evolution (Mueller, Triantafyllopoulos hep-ph/0205167)

Implementation of saturation boundary of possible practical interest for any small-x observable sensitive to unintegrated pdfs, but also interesting theoretically.

Saturation in BFKL

Gluon distribution in Mellin representation ($Y \equiv \ln 1/x$):

$$\mathcal{A}(Y,k) = \int \frac{d\gamma}{2\pi\mathrm{i}} e^{\omega Y - (1-\gamma)\rho} \tilde{\mathcal{A}}(\gamma)\,, \quad \rho \equiv \ln(k^2/Q_0^2)$$

Saddle point equations relevant for saturation problem:

$$\omega_s Y - (1-\gamma_s)\rho_s = 0$$
$$\omega_s' Y + \rho_s = 0$$

From these one immediately finds $Q_s^2 = Q_0^2 e^{\frac{\omega_s}{1-\gamma_s} Y}$ where $\omega_s/(1-\gamma_s)$ pure number, $\gamma_s \approx 0.37$. Moreover, $\mathcal{A}$ for $\rho \gtrsim \rho_s$ given by

$$\mathcal{A} \sim \frac{1}{\sqrt{2\pi\omega_s'' Y}} e^{-(1-\gamma_s)(\rho-\rho_s)} e^{-\frac{(\rho-\rho_s)^2}{2\omega_s'' Y}}$$

Characteristics: Geometric scaling and diffusion.

Overview of Method

Q_s essentially line of constant density. Basic idea then simple: Follow a particular line of constant density and do not let $\mathcal{A}$ grow "too large".

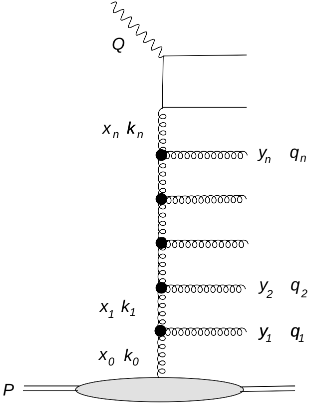
In practice, define "critical line" by $\mathcal{A}(Y, \rho_c) = c$. First part. Second part: Apply boundary at $\rho \leq \rho_c - \Delta$. c and Δ numbers.

One can let $\mathcal{A}(Y, \rho) = C$ for $C = 0$ (absorptive boundary) or $C \neq 0$ for $\rho \leq \rho_c - \Delta$. Or one can freeze $\mathcal{A}(Y, \rho) = \mathcal{A}(Y, \rho_c - \Delta)$ for $\rho \leq \rho_c - \Delta$.

Different choices will change normalization but shape and Y dependence of $\mathcal{A}$ and Q_s should not change.

In arXiv:0901.2873 it was demonstrated that this method is for BFKL equivalent to solving non-linear BK (for $k \geq Q_s$) for all Y and for both fixed and running $\bar{\alpha}_s$.

CCFM



$x_i = z_i x_{i-1}$, $y_i = (1 - z_i)x_{i-1}$. Emission of real gluons ordered in angle: $\theta_1 < \theta_2 < \cdots < \theta_n < \bar{\theta}$ (from coherence). Maximum angle set by kinematics of hard scattering.

CCFM equation

Integral equation:

$$\mathcal{A}(Y,k,p) = \int_0^Y dy \int \frac{d^2q}{\pi q^2}\, \bar{\alpha}_s\, \Delta_{ns}(Y-y,k,q)\, \theta(\log(p/q)+Y-y)\, \mathcal{A}(y,|\boldsymbol{k}+\boldsymbol{q}|,q)$$

Here $Y-y=\ln 1/z$. Δ_{ns} is "non-Sudakov" form factor, θ encodes the angular ordering, and p is related to maximal angle $\bar{\theta}$ as $\bar{\theta}=p/(x_nE)$.

In addition there are also soft emissions: $1/(1-z)$ with associated "Sudakov" Δ_s. These are "probability conserving", but in practice important to include them. For future work. Included in CASCADE MC.

A note on the non-Sudakov

"non-Sudakov" form factor in CCFM given by ($Y - y = \ln 1/z$)

$$\Delta_{ns} = \exp\left(- \int_z^1 \frac{dz'}{z'} \int_{z'^2 q^2}^{k^2} \frac{dq'^2}{q'^2} \, \bar{\alpha}_s \right)$$

If $k^2 < zq^2$ then $\Delta_{ns} > 1$. $k^2 \geq zq^2$ is kinematical constraint, necessary for internal consistency. Without it, linear evolution is unstable.

Another version of Δ_{ns} given by (Kwiecinski et al. hep-ph/9503266)

$$\Delta_{ns} = \exp\left(- \int_z^1 \frac{dz'}{z'} \int \frac{dq'^2}{q'^2} \, \bar{\alpha}_s \, \theta(k - q')\theta(q' - z'q) \right) \leq 1$$

With saturation however we will see that there is no difference in using either of them.

Saturation problem in CCFM

Mellin representation

$$\mathcal{A}(Y,k,p) = \int \frac{d\gamma}{2\pi\mathrm{i}}\, e^{\omega(\gamma)Y-(1-\gamma)\rho}\, H(p/k)$$

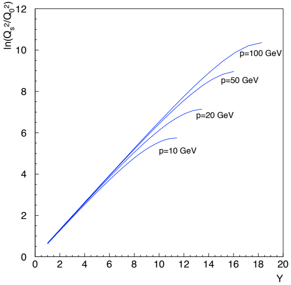
H encodes effect of angular ordering, can be found by differentiating integral equation wrt p.

If $p \to \infty$, $H(p/k) \to 1$, then same type solution as in BFKL, *but*, ω still different. BFKL recovered only when $\bar{\alpha}_s \to 0$. For $p > Q_s$

$$\rho_s = \frac{\omega_s^{(0)}}{1-\gamma_s^{(0)}}\, Y - \frac{\bar{\alpha}_s H(1)}{(1-\gamma_s^{(0)}) f_s^{(0)}}\, e^{-(1-\gamma_s^{(0)})\rho_p + \omega_s^{(0)} Y} + \cdots$$

As $p/Q_s \to 1$ more complicated behavior with p and Y, formula then not valid anymore.

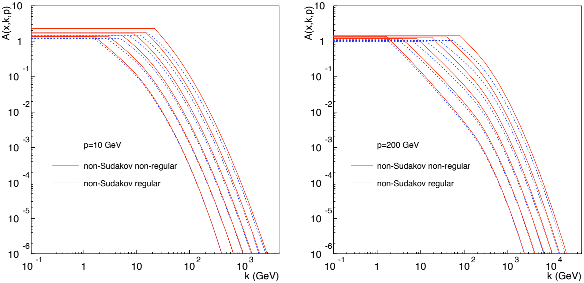
Saturation scale: Plot



The case $p/Q_s < 1$ is more difficult to treat analytically. In that case we only do numerical analysis.

We will apply same method in the numerical solution of CCFM. In arXiv:0906.2683 only the case $k \ll p$ was studied. Then p dependence could be dropped.

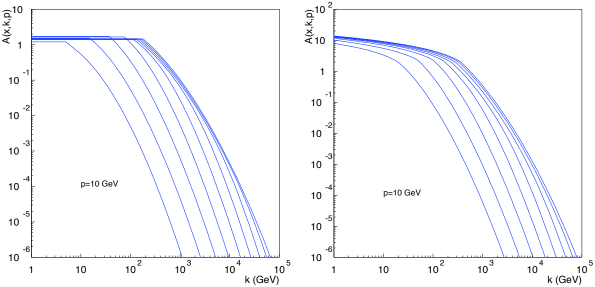
Fixed coupling results



Solutions for $Y = 6 \rightarrow 14$. Coupling $\bar{\alpha}_s = 0.2$. Different solutions rather similar.

Note that growth of Q_s driven by growth at $k > Q_s$. However coherence restricts this growth as $\exists$ very little phase space.

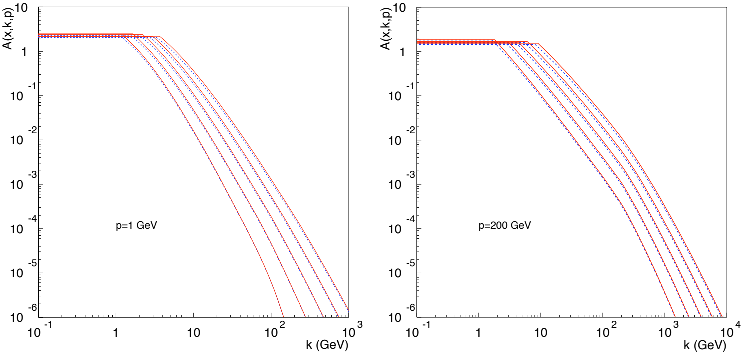
Coherence + Saturation = Saturation of Saturation



In linear evolution strong growth at small k compensates suppression due to phase space. But this growth removed by saturation, thus combined effect implies that $\mathcal{A}$ "stalls" at some point and Q_s itself saturates.

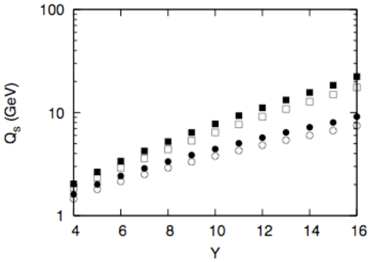
However, this happens at extreme Y, here $Y = 10 \rightarrow 50$, and with running $\bar{\alpha}_s$ even higher Y needed.

Running coupling results



As well know from BFKL studies, growth of Q_s slowed down in this case. When $p \gg k$ result is BFKL-like, but very different when $p \lesssim k$.

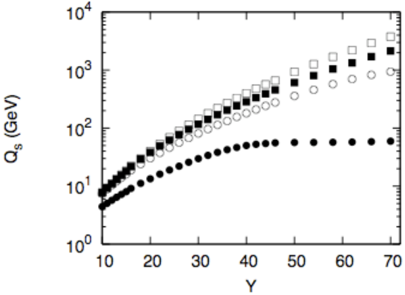
Saturation scale results



Different boundary conditions, and the different Δ_{ns}, and $p = 1$ (circles) and $p = 200$ (squares) GeV. Results very similar.

For $p >> Q_s$, $\rho_s \propto \sqrt{Y}$, but different when $p \sim Q_s$.

In Asymptotia



$p = 1, 10, 40$ and 200 GeV.

Complete saturation of Q_s requires extreme Y. However, even at small Y, when p is small behavior rather different than in BFKL.

Outlook

First study of Q_s in CCFM retaining p dependece. We find that Q_s and $\mathcal{A}$ look rather different than in the corresponding BFKL analysis.

The phase space suppression above p, together with saturation implies that Q_s essentially saturates at large (extreme) Y.

For phenomenology it is important to include the soft emissions. These are known to slow down the evolution.

Results from CASCADE MC show that growth with x is very slow. To understand this better it is valuable to have simple numerical solution of equation.

One should then study the effect of saturation and non-leading effects in the small-x evolution simultaneously, to see the relative importance of the various effects, and whether saturation plays a role for hard processes at the LHC.