High Energy Bounds on Soft $\mathcal{N}=4$ SYM Amplitudes from AdS/CFT

Matteo Giordano

Institut de Physique Théorique - CEA/Saclay

Low x Meeting
Kavala, 23rd June 2010

Outline

Outline

Soft High Energy Scattering

- Soft high energy scattering is one of the oldest unsolved problems of strong interactions (energy regime: $s \to \infty$, $\sqrt{-t} \leq 1\mathrm{GeV}$)
- Optical theorem: imaginary part of the amplitude at $t = 0 \propto$ total cross section $\to$ rising total cross section problem
- Pomeron intercept must respect the Froissart bound (unitarity + mass gap) [Froissart (1961)]

$$\sigma_{tot}(s) \leq \frac{\pi}{m_\pi^2} \log^2 \left(\frac{s}{s_0}\right)$$

- QCD is believed to be the fundamental theory of strong interactions
  - nonperturbative regime of the theory involved [Nachtmann (1991)]
  - satisfactory explanation from first principles still lacking

Gauge/Gravity Duality

- $\mathcal{N} = 4$ SYM: "laboratory" for further developments in QCD, provided with a powerful nonperturbative technique
- AdS/CFT correspondence: duality between $\mathcal{N} = 4$ SYM at large N_c and strong 't Hooft coupling, and type IIB superstring theory on $AdS_5 \times S^5$ at weak coupling ($\sim$ supergravity) [Maldacena (1998a)]
- Conformal theory, no mass gap, no Froissart bound: what about high-energy behaviour of total cross sections?

Outline

Onium–Onium Scattering from Dipole–Dipole Scattering

- Elastic onium-onium scattering from dipole–dipole scattering

$$\mathcal{M}_{(12)}(s,t) = \int d^2\vec{R}_1 d^2\vec{R}_2 |\psi_1(\vec{R}_1)|^2 |\psi_2(\vec{R}_2)|^2 \mathcal{M}_{(dd)}(s,t;\vec{R}_1,\vec{R}_2)$$

- Dipole-dipole scattering amplitude [Dosch *et al.* (1996)]

$$\mathcal{M}_{(dd)}(s,t;\vec{R}_1,\vec{R}_2) \underset{s\to\infty}{=} -i\,2s \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} \mathcal{C}_M(\chi;\vec{b},\vec{R}_1,\vec{R}_2) \quad [t=-|\vec{q}|^2]$$

- Wilson–loop correlation function in Minkowski spacetime

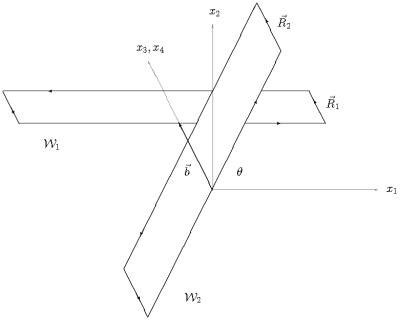
$$\mathcal{G}_M(\chi;T;\vec{b}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp}) = \frac{\langle \mathcal{W}_{\mathcal{C}_1}\mathcal{W}_{\mathcal{C}_2}\rangle}{\langle \mathcal{W}_{\mathcal{C}_1}\rangle\langle \mathcal{W}_{\mathcal{C}_2}\rangle} - 1, \quad \mathcal{C}_M \equiv \lim_{T\to\infty} \mathcal{G}_M$$

- $\mathcal{C}_{1,2}$: dipole classical trajectories, at a hyperbolic angle $\chi \simeq \log \frac{s}{m^2}$ in the longitudinal plane, closed by straight "links" in the transverse plane at $\tau = \pm T$

Euclidean Correlation Functions

- Nonperturbative techniques available in Euclidean space $\Rightarrow$ Euclidean correlation functions

$$\mathcal{G}_E(\theta; T; \vec{b}, \vec{R}_1, \vec{R}_2) = \frac{\langle \mathcal{W}_{\mathcal{C}_1} \mathcal{W}_{\mathcal{C}_2} \rangle}{\langle \mathcal{W}_{\mathcal{C}_1} \rangle \langle \mathcal{W}_{\mathcal{C}_2} \rangle} - 1, \quad \mathcal{C}_E \equiv \lim_{T \to \infty} \mathcal{G}_E$$



$$\mathcal{C}_1 : \; X^{(1)}(\tau, \sigma) = b + u_1 \tau + R_1 \sigma$$
$$\mathcal{C}_2 : \; X^{(2)}(\tau, \sigma) = u_2 \tau + R_2 \sigma$$
$$u_1 = (\cos\theta, \sin\theta, \vec{0}\,), \; u_2 = (1, 0, \vec{0})$$
$$R_i = (0, 0, \vec{R}_i), \quad b = (0, 0, \vec{b}\,)$$
$$\tau \in [-T, T], \quad \sigma \in [0, 1]$$

Euclidean–Minkowskian Duality

- Correlation functions in Minkowski space can be reconstructed from Euclidean correlation functions [Meggiolaro (2005), MG, Meggiolaro (2009)]

$$\begin{aligned} \mathcal{G}_M(\chi; T) &= \mathcal{G}_E(\theta \to -i\chi; T \to iT) \\ \mathcal{C}_M(\chi) &= \mathcal{C}_E(\theta \to -i\chi) \end{aligned}$$

- Combined with the symmetries of the Euclidean theory $\Rightarrow$ crossing symmetry relations [MG, Meggiolaro (2006)]

$$\mathcal{C}_M(i\pi - \chi; \vec{R}_1, \vec{R}_2) = \mathcal{C}_M(\chi; \vec{R}_1, -\vec{R}_2) = \mathcal{C}_M(\chi; -\vec{R}_1, \vec{R}_2)$$

- Opens the way to investigations with nonperturbative techniques:
  - Instanton Liquid Model [Shuryak, Zahed (2000), MG, Meggiolaro (2010)]
  - AdS/CFT [Janik, Peschanski (2000a,b), MG, Peschanski (2010)]
  - Stochastic Vacuum Model [Shoshi *et al.* (2003)]
  - Lattice Gauge Theory [MG, Meggiolaro (2008), MG, Meggiolaro (2010)]

Outline

AdS/CFT Correspondence

- Wilson loop expectation value: $\langle \mathcal{W} \rangle = e^{-\frac{1}{2\pi\alpha'}A_{\min}}$ [Maldacena (1998b)]
- Wilson loop correlation function at large distance: disconnected minimal surface, connected by supergravity interaction [Berenstein *et al.* (1999)]
- Large distance $L \gg R_1, R_2$, weak gravitational field domain $[L = |\vec{b}|,\ R_i = |\vec{R}_i|]$ [Janik, Peschanski (2000a), MG, Peschanski (2010)]

Euclidean correlation function

$$\mathcal{C}_E(\theta) = \exp\left[\sum_\psi \tilde{\delta}_\psi(\theta)\right] - 1 \qquad \tilde{\delta}_\psi : \text{ contribution of field } \psi$$

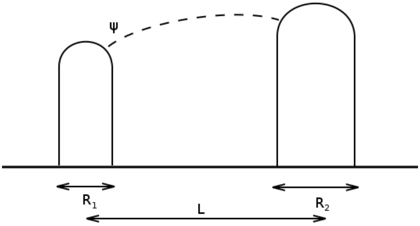
Minkowskian correlation function

$$\mathcal{C}_M(\chi) = \exp\left[i\sum_\psi \delta_\psi(\chi)\right] - 1, \qquad i\delta_\psi(\chi) \equiv \tilde{\delta}_\psi(\theta \to -i\chi)$$

Wilson Loop Correlator and AdS/CFT

- AdS_5 with Euclidean signature, large distance $L \gg R_1, R_2$, weak gravitational field domain [Janik, Peschanski (2000a), MG, Peschanski (2010)]

$$\tilde{\delta}_\psi \equiv \frac{1}{\pi^2 \alpha'^2} \int d\mathcal{A}_1 d\mathcal{A}_2 \, \frac{\delta S_{NG}}{\delta \psi}(X) G_\psi(X, X') \frac{\delta S_{NG}}{\delta \psi}(X')$$



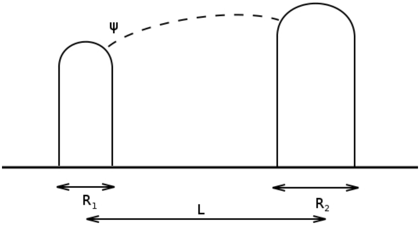
$\tilde{\delta}_\psi$: contribution from the exchange of field ψ between the two world-sheets

$$X = \left(X^{(1)}(\tau,\sigma), z_1(\sigma)\right)$$
$$X' = \left(X^{(2)}(\tau,\sigma), z_2(\sigma)\right)$$

Wilson Loop Correlator and AdS/CFT

- AdS_5 with Euclidean signature, large distance $L \gg R_1, R_2$, weak gravitational field domain [Janik, Peschanski (2000a), MG, Peschanski (2010)]

$$\tilde{\delta}_\psi \equiv \frac{1}{\pi^2 \alpha'^2} \int d\mathcal{A}_1 d\mathcal{A}_2 \, \frac{\delta S_{NG}}{\delta\psi}(X) G_\psi(X, X') \frac{\delta S_{NG}}{\delta\psi}(X')$$



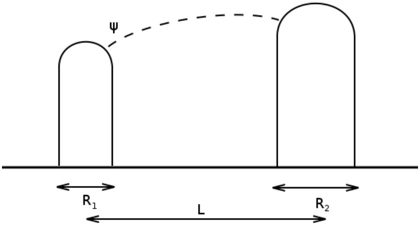
$$\alpha' = \frac{1}{\sqrt{4\pi g_s N_c}} = \frac{1}{\sqrt{2\lambda}}$$

$$g_{\rm YM}^2 = 2\pi g_s \quad \lambda = \frac{g_{\rm YM}^2 N_c}{4\pi}$$

Wilson Loop Correlator and AdS/CFT

- AdS_5 with Euclidean signature, large distance $L \gg R_1, R_2$, weak gravitational field domain [Janik, Peschanski (2000a), MG, Peschanski (2010)]

$$\tilde{\delta}_\psi \equiv \frac{1}{\pi^2 \alpha'^2} \int d\mathcal{A}_1 d\mathcal{A}_2 \, \frac{\delta S_{NG}}{\delta \psi}(X) G_\psi(X, X') \frac{\delta S_{NG}}{\delta \psi}(X')$$



$d\mathcal{A}_i$: area element on world-sheet i

$$d\mathcal{A}_i = d\tau \frac{dz}{z_{ix}}$$

$$z_{ix} = \left(\frac{z_{i\,\max}}{z_i}\right)^2 \sqrt{1 - \left(\frac{z_i}{z_{i\,\max}}\right)^4}$$

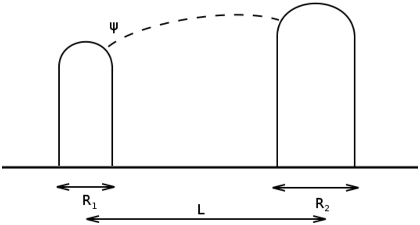
$$z_{i\,\max} = R_i \, \frac{[\Gamma(1/4)]^2}{(2\pi)^{3/2}}$$

[Maldacena (1998b)]

Wilson Loop Correlator and AdS/CFT

- AdS_5 with Euclidean signature, large distance $L \gg R_1, R_2$, weak gravitational field domain [Janik, Peschanski (2000a), MG, Peschanski (2010)]

$$\tilde{\delta}_\psi \equiv \frac{1}{\pi^2 \alpha'^2} \int d\mathcal{A}_1 d\mathcal{A}_2 \frac{\delta S_{NG}}{\delta \psi}(X) G_\psi(X, X') \frac{\delta S_{NG}}{\delta \psi}(X')$$

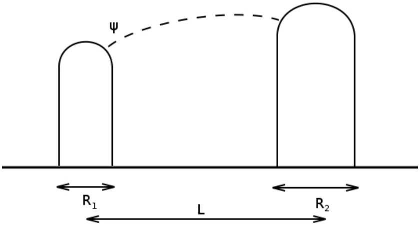


$\frac{\delta S_{NG}}{\delta \psi}$: coupling of world-sheets to supergravity field ψ

Wilson Loop Correlator and AdS/CFT

- AdS_5 with Euclidean signature, large distance $L \gg R_1, R_2$, weak gravitational field domain [Janik, Peschanski (2000a), MG, Peschanski (2010)]

$$\tilde{\delta}_\psi \equiv \frac{1}{\pi^2 \alpha'^2} \int d\mathcal{A}_1 d\mathcal{A}_2 \, \frac{\delta S_{NG}}{\delta \psi}(X) \, G_\psi(X, X') \, \frac{\delta S_{NG}}{\delta \psi}(X')$$



$G_\psi(X, X')$: Green function for field ψ

Supergravity Contributions

- Contributions from lowest-lying states in the spectrum, case $\vec{R}_1 \parallel \vec{R}_2 \parallel \vec{b}$
- Leading dependence on χ, L and R_i after analytic continuation

KK scalar: $$\delta_S = \kappa_S \frac{1}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right)$$

dilaton: $$\delta_D = \kappa_D \frac{1}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right)^3$$

a.s. tensor: $$\delta_B = \kappa_B \frac{\cosh\chi}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right)^2$$

graviton: $$\delta_G = \kappa_G \frac{(\cosh\chi)^2}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right)^3$$

- $\kappa_\psi = \mathcal{O}(\frac{\lambda}{N_c^2})$, dependence on N_c as expected from topology

Supergravity Contributions

- Contributions from lowest-lying states in the spectrum, case $\vec{R}_1 \parallel \vec{R}_2 \parallel \vec{b}$
- Leading dependence on χ, L and R_i after analytic continuation at high energy ($\varsigma = s/2m^2$)

KK scalar: $$\delta_S = \kappa_S \frac{1}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right) \qquad \sim \varsigma^{-1}$$

dilaton: $$\delta_D = \kappa_D \frac{1}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right)^3 \qquad \sim \varsigma^{-1}$$

a.s. tensor: $$\delta_B = \kappa_B \frac{\cosh\chi}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right)^2 \qquad \sim \varsigma^{0}$$

graviton: $$\delta_G = \kappa_G \frac{(\cosh\chi)^2}{\sinh\chi}\left(\frac{R_1 R_2}{L^2}\right)^3 \qquad \sim \varsigma$$

- $\kappa_\psi = \mathcal{O}(\frac{\lambda}{N_c^2})$, dependence on N_c as expected from topology

Outline

AdS/CFT Domain of Validity

- Impact-parameter partial amplitude $a(\chi, \vec{b}) = -i\mathcal{C}_M(\chi, \vec{b})$
- At large L the AdS/CFT contribution reads

$$a_{tail}(\chi, \vec{b}) = i \left[1 - \exp \left(i \sum_{\psi} \delta_{\psi} \right) \right]$$

- Purely elastic amplitude, in agreement with the planar limit $N_c \to \infty$
- Reliable as long as the minimal surface is disconnected and the gravitational field is weak

Weak Field Constraint

- Gravitational perturbation δG_{tt} generated by each of the string world-sheets on the other one smaller than the background metric G_{tt}

$$\delta\psi(X) = \frac{1}{\pi\alpha'} \int d\mathcal{A}\ G_{\psi}(X, X')\ \frac{\delta S_{NG}}{\delta\psi}(X')$$

- For the graviton

$$\frac{\delta G_{tt}}{G_{tt}} \ll 1, \quad G_{tt} \equiv \frac{1}{z^2}$$

- Lower limit in impact parameter space

$$L^2 \gg L_{max}^2 \equiv \frac{R_1 R_2 \varsigma^{\frac{4}{7}}}{\left[\min\left(\sqrt{\frac{R_1}{R_2}}, \sqrt{\frac{R_2}{R_1}}\right)\right]^{\frac{2}{7}}}$$

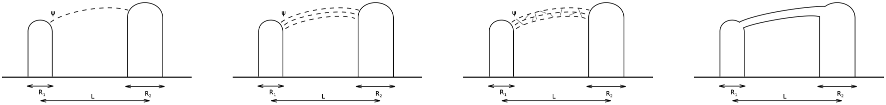
S-Matrix Constraint

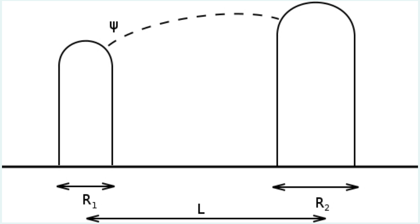
- Form of $a_{tail}(\chi, \vec{b})$ typical of a resummation of independent colorless exchanges (on the gauge theory side), suggesting the possibility to enlarge the domain of validity up to some L_{tail}
- Minimal request: small graviton-induced phase shift δ_G (and $\mathrm{Im}\, a_{tail}(\chi, \vec{b})$ not oscillating with L)

$$L^2 > L^2_{min} \equiv R_1 R_2 \left(\frac{\kappa_G}{\pi} \varsigma \right)^{\frac{1}{3}}$$

- Weaker constraint, strong gravitational field begins to appear in the bulk near the relevant minimal surfaces, signaling the opening of inelastic channels

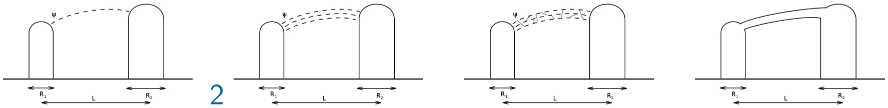
Characteristic Impact-Parameter Scales

1 

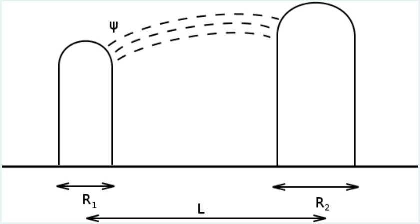


Large distances ($L > L_{max}$): weak gravitational field in the bulk, holographic determination of the impact-parameter tail of the scattering amplitude from the contribution of the disconnected minimal surface.

Characteristic Impact-Parameter Scales

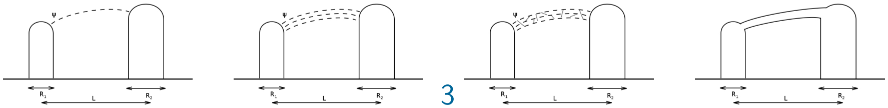


2

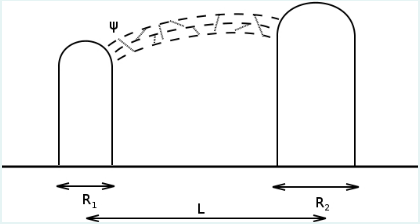


Moderately large distances ($L_{min} < L < L_{max}$): minimal surface still disconnected, gravitational field begins to become strong in some region in the bulk. Elastic eikonal expression approximately valid up to L_{tail}, with $L_{min} \leq L_{tail} \leq L_{max}$

Characteristic Impact-Parameter Scales

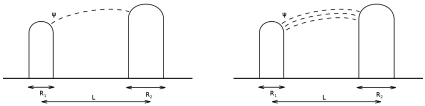


3

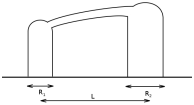


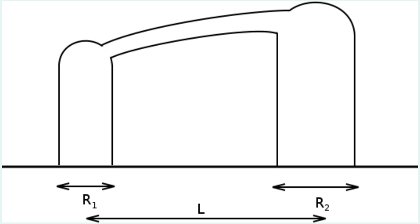
Moderately small distances ($L_{connect} < L < L_{min}$): minimal surface still made of disconnected surfaces joined by supergravity fields, but elastic eikonal expression no more reliable (even from the S-matrix point-of-view)

Characteristic Impact-Parameter Scales



4

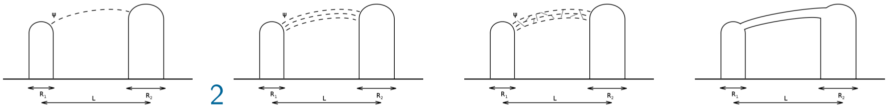


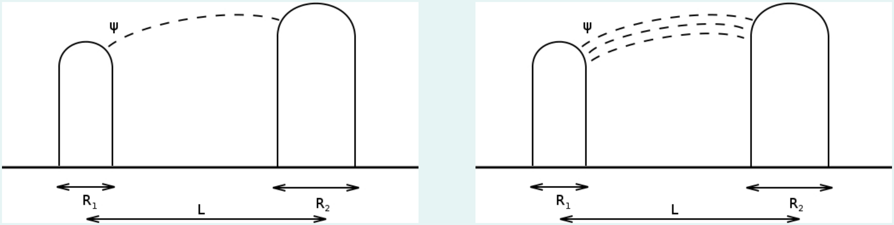


Small distances ($L \leq L_{connect}$): connected minimal surface (Gross-Ooguri transition), AdS/CFT description goes beyond the interaction through exchange supergravity fields

Characteristic Impact-Parameter Scales

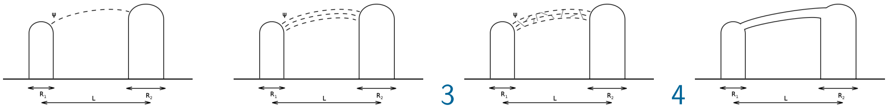
1 2

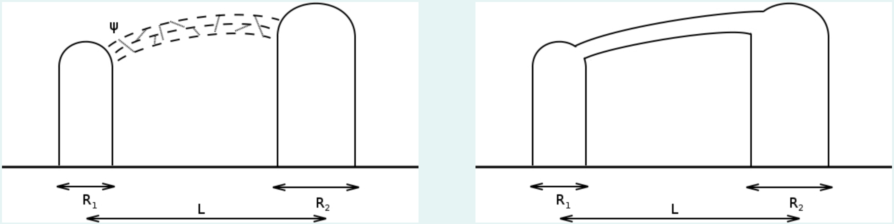




- Region 1 and (possibly) part of Region 2: *tail* region ($L \geq L_{tail}$)
- Regions 3 and 4: *core* region ($L < L_{tail}$)

Characteristic Impact-Parameter Scales





- Region 1 and (possibly) part of Region 2: *tail* region ($L \geq L_{tail}$)
- Regions 3 and 4: *core* region ($L < L_{tail}$)

Unitarity Bound

- To incorporate Regions 3 and 4: unitarity constraint

$$\mathrm{Im}\; a(\chi, \vec{b}) \leq 2$$

- Full amplitude $\mathcal{A} \equiv \mathcal{A}_{core} + \mathcal{A}_{tail}$
  - Tail region: large impact-parameter contribution $\mathcal{A}_{tail}$,

$$\mathcal{A}_{tail}(s, t; \vec{R}_1, \vec{R}_2) = 2is \int_{L \geq L_{tail}} d^2\vec{b}\; e^{i\vec{q}\cdot\vec{b}} \left[1 - e^{\left(i \sum_\psi \delta_\psi\right)}\right]$$

  - Core region: $\mathcal{A}_{core}$ constrained by the unitarity bound

$$\mathrm{Im}\, \mathcal{A}_{core} \leq 4\pi s L_{tail}^2$$

- We will be able to set a lower and an upper bound on the large-s behavior of the full amplitude depending on the s-dependence of L_{tail}

Outline

Total Cross Section I

- Contribution σ_{tail} to the total cross section as obtained from AdS/CFT (optical theorem)

$$\sigma_{tail} = 4\pi(\sinh\chi)^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\,\lambda\,\times$$
$$\times\left[1 - \cos\left(\frac{\kappa_S}{\lambda^2}\frac{1}{(\sinh\chi)^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{(\sinh\chi)^2} + \frac{\kappa_B}{\lambda^4}\frac{\coth\chi}{(\sinh\chi)^{\frac{2}{3}}} + \frac{\kappa_G}{\lambda^6}(\coth\chi)^2\right)\right]$$

- Hierarchy between the different contributions

$$L \to \lambda \equiv (\sinh\chi)^{-\frac{1}{6}}\frac{L}{\sqrt{R_1 R_2}}$$

- Rescaling with $\sinh\chi$ allows to keep manifest the symmetry under crossing ($\chi \to i\pi - \chi$) of the various phase shifts

Total Cross Section I

- Contribution σ_{tail} to the total cross section as obtained from AdS/CFT (optical theorem)

$$\sigma_{tail} = 4\pi(\sinh\chi)^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\,\lambda\,\times$$

$$\times\left[1-\cos\left(\frac{\kappa_S}{\lambda^2}\frac{1}{(\sinh\chi)^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{(\sinh\chi)^2} + \frac{\kappa_B}{\lambda^4}\frac{\coth\chi}{(\sinh\chi)^{\frac{2}{3}}} + \frac{\kappa_G}{\lambda^6}(\coth\chi)^2\right)\right]$$

$$\oplus \qquad \oplus \qquad \ominus \qquad \oplus$$

- Hierarchy between the different contributions

$$L \to \lambda \equiv (\sinh\chi)^{-\frac{1}{6}} \frac{L}{\sqrt{R_1 R_2}}$$

- Rescaling with $\sinh\chi$ allows to keep manifest the symmetry under crossing $(\chi \to i\pi - \chi)$ of the various phase shifts

Total Cross Section I

- Contribution σ_{tail} to the total cross section as obtained from AdS/CFT (optical theorem)

$$\sigma_{tail} = 4\pi(\sinh\chi)^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\,\lambda \times$$

$$\times \left[1 - \cos\left(\frac{\kappa_S}{\lambda^2}\frac{1}{(\sinh\chi)^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{(\sinh\chi)^2} + \frac{\kappa_B}{\lambda^4}\frac{\coth\chi}{(\sinh\chi)^{\frac{2}{3}}} + \frac{\kappa_G}{\lambda^6}(\coth\chi)^2\right)\right]$$

- Leading term from graviton exchange, crossing-symmetric (Pomeron)
- Subleading term from antisymmetric-tensor exchange, crossing-antisymmetric (Odderon)

Total Cross Section I

- Contribution σ_{tail} to the total cross section as obtained from AdS/CFT (optical theorem)

$$\sigma_{tail} = 4\pi(\sinh\chi)^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\,\lambda \times$$

$$\times\left[1 - \cos\left(\frac{\kappa_S}{\lambda^2}\frac{1}{(\sinh\chi)^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{(\sinh\chi)^2} + \frac{\kappa_B}{\lambda^4}\frac{\coth\chi}{(\sinh\chi)^{\frac{2}{3}}} + \frac{\kappa_G}{\lambda^6}(\coth\chi)^2\right)\right]$$

- Leading term from graviton exchange, crossing-symmetric (Pomeron)
- Subleading term from antisymmetric-tensor exchange, crossing-antisymmetric (Odderon)

Total Cross Section I

- Contribution σ_{tail} to the total cross section as obtained from AdS/CFT (optical theorem)

$$\sigma_{tail} = 4\pi(\sinh\chi)^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\,\lambda \times$$

$$\times\left[1 - \cos\left(\frac{\kappa_S}{\lambda^2}\frac{1}{(\sinh\chi)^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{(\sinh\chi)^2} + \frac{\kappa_B}{\lambda^4}\frac{\coth\chi}{(\sinh\chi)^{\frac{2}{3}}} + \frac{\kappa_G}{\lambda^6}(\coth\chi)^2\right)\right]$$

- At large energy the graviton dominates

$$\sigma_{tail} \underset{s\to\infty}{\simeq} 4\pi R_1 R_2 s^{\frac{1}{3}} \int_{\lambda_{tail}}^{\infty} d\lambda\,\lambda\left[1 - \cos\left(\frac{\kappa_G}{\lambda^6}\right)\right]$$

Total Cross Section II

- We parameterise the dependence of L_{tail} on s

$$L_{tail} = \lambda_0 \sqrt{R_1 R_2}\, \varsigma^{\beta} \Rightarrow \lambda_{tail} = \lambda_0\, \varsigma^{\beta - \frac{1}{6}}$$

- Total cross section

$$\sigma_{tail} \simeq \frac{2\pi}{3}\lambda_0^2 R_1 R_2 \varsigma^{2\beta} \int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} \varsigma^{1-6\beta} x\right)\right]$$

$$\underset{s\to\infty}{\sim} \frac{2\pi}{3} R_1 R_2 \begin{cases} \varsigma^{\frac{1}{3}} \dfrac{3\pi \kappa_G^{\frac{1}{3}}}{\Gamma(1/3)} & \beta < \dfrac{1}{6} \\ \varsigma^{\frac{1}{3}} \lambda_0^2 \displaystyle\int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} x\right)\right] & \beta = \dfrac{1}{6} \\ \varsigma^{2-10\beta} \dfrac{1}{2} \kappa_G^2 \lambda_0^{-10} & \beta > \dfrac{1}{6} \end{cases}$$

Total Cross Section II

- We parameterise the dependence of L_{tail} on s

$$L_{tail} = \lambda_0 \sqrt{R_1 R_2}\, \varsigma^{\beta} \Rightarrow \lambda_{tail} = \lambda_0\, \varsigma^{\beta - \frac{1}{6}}$$

- Total cross section

$$\sigma_{tail} \simeq \frac{2\pi}{3}\lambda_0^2 R_1 R_2 \varsigma^{2\beta} \int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} \varsigma^{1-6\beta} x\right)\right]$$

$$\underset{s\to\infty}{\sim} \frac{2\pi}{3} R_1 R_2 \begin{cases} \varsigma^{\frac{1}{3}} \dfrac{3\pi \kappa_G^{\frac{1}{3}}}{\Gamma(1/3)} & \beta < \dfrac{1}{6} \\ \varsigma^{\frac{1}{3}} \lambda_0^2 \displaystyle\int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} x\right)\right] & \beta = \dfrac{1}{6} \\ \varsigma^{2-10\beta} \dfrac{1}{2} \kappa_G^2 \lambda_0^{-10} & \beta > \dfrac{1}{6} \end{cases}$$

Total Cross Section II

- We parameterise the dependence of L_{tail} on s

$$L_{tail} = \lambda_0 \sqrt{R_1 R_2}\, \varsigma^{\beta} \Rightarrow \lambda_{tail} = \lambda_0\, \varsigma^{\beta - \frac{1}{6}}$$

- Total cross section

$$\sigma_{tail} \simeq \frac{2\pi}{3}\lambda_0^2 R_1 R_2 \varsigma^{2\beta} \int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} \varsigma^{1-6\beta} x\right)\right]$$

$$\underset{s\to\infty}{\sim} \frac{2\pi}{3} R_1 R_2 \begin{cases} \varsigma^{\frac{1}{3}} \dfrac{3\pi \kappa_G^{\frac{1}{3}}}{\Gamma(1/3)} & \beta < \dfrac{1}{6} \\ \varsigma^{\frac{1}{3}} \lambda_0^2 \displaystyle\int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} x\right)\right] & \beta = \dfrac{1}{6} \\ \varsigma^{2-10\beta} \dfrac{1}{2} \kappa_G^2 \lambda_0^{-10} & \beta > \dfrac{1}{6} \end{cases}$$

Total Cross Section II

- We parameterise the dependence of L_{tail} on s

$$L_{tail} = \lambda_0 \sqrt{R_1 R_2}\, \varsigma^{\beta} \Rightarrow \lambda_{tail} = \lambda_0\, \varsigma^{\beta - \frac{1}{6}}$$

- Total cross section

$$\sigma_{tail} \simeq \frac{2\pi}{3}\lambda_0^2 R_1 R_2 \varsigma^{2\beta} \int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} \varsigma^{1-6\beta} x\right)\right]$$

$$\underset{s\to\infty}{\sim} \frac{2\pi}{3} R_1 R_2 \begin{cases} \varsigma^{\frac{1}{3}} \dfrac{3\pi \kappa_G^{\frac{1}{3}}}{\Gamma(1/3)} & \beta < \dfrac{1}{6} \\ \varsigma^{\frac{1}{3}} \lambda_0^2 \displaystyle\int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} x\right)\right] & \beta = \dfrac{1}{6} \\ \varsigma^{2-10\beta} \dfrac{1}{2} \kappa_G^2 \lambda_0^{-10} & \beta > \dfrac{1}{6} \end{cases}$$

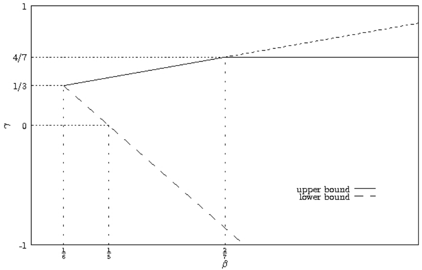
- Since $\sigma_{tot} > \sigma_{tail}$ the previous equation provides a *lower* bound
- The unitarity constraint gives an *upper* bound on the contribution from the "core" region and thus on the whole total cross section

$$\begin{aligned}\sigma_{tot} &= \sigma_{core} + \sigma_{tail} \\ &< 4\pi\lambda_0^2 R_1 R_2 \varsigma^{2\beta} \left\{ 1 + \frac{1}{6}\int_0^1 dx\, x^{-\frac{4}{3}} \left[1 - \cos\left(\kappa_G \lambda_0^{-6} \varsigma^{1-6\beta} x \right) \right] \right\}\end{aligned}$$

- More rigorously

$$\min\left(\frac{1}{3}, 2-10\beta\right) \leq \lim_{\varsigma\to\infty} \frac{\log \sigma_{tot}}{\log \varsigma} \leq \max\left(\frac{1}{3}, 2\beta\right)$$

High Energy Bounds II

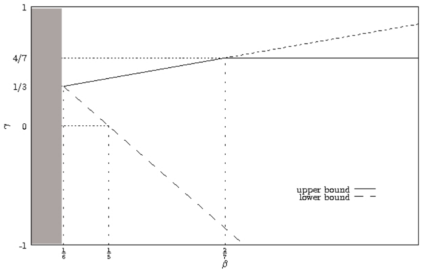


$L_{tail} \propto s^{\beta}$

$\sigma_{tot} \propto s^{\gamma}$

1. For $\beta < \frac{1}{6}$ ($L_{tail} < L_{min}$) the total cross section would become purely elastic at high energy
2. At $\beta = \frac{1}{6}$ ($L_{tail} = L_{min}$) the *tail* and *core* contributions have the same high-energy behavior
3. For $\frac{1}{6} < \beta \leq \frac{2}{7}$ ($L_{min} < L_{tail} < L_{max}$) the *core* region gives the dominant contribution
4. For $\beta = \frac{2}{7}$ ($L_{tail} = L_{max}$) rigorous AdS/CFT bound

High Energy Bounds II

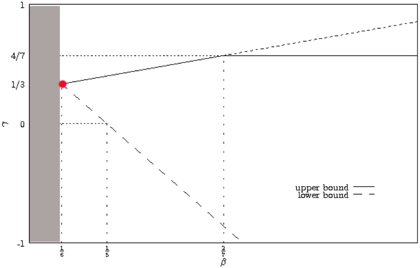


$L_{tail} \propto s^{\beta}$

$\sigma_{tot} \propto s^{\gamma}$

1. For $\beta < \frac{1}{6}$ ($L_{tail} < L_{min}$) the total cross section would become purely elastic at high energy unphysical
2. At $\beta = \frac{1}{6}$ ($L_{tail} = L_{min}$) the *tail* and *core* contributions have the same high-energy behavior
3. For $\frac{1}{6} < \beta \leq \frac{2}{7}$ ($L_{min} < L_{tail} < L_{max}$) the *core* region gives the dominant contribution
4. For $\beta = \frac{2}{7}$ ($L_{tail} = L_{max}$) rigorous AdS/CFT bound

High Energy Bounds II

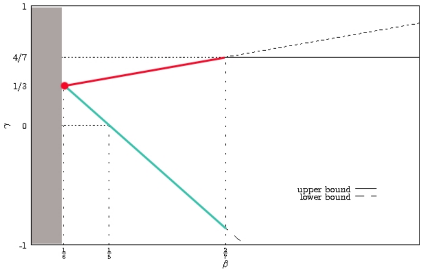


$L_{tail} \propto s^{\beta}$

$\sigma_{tot} \propto s^{\gamma}$

1. For $\beta < \frac{1}{6}$ ($L_{tail} < L_{min}$) the total cross section would become purely elastic at high energy unphysical
2. At $\beta = \frac{1}{6}$ ($L_{tail} = L_{min}$) the *tail* and *core* contributions have the same high-energy behavior $\sigma_{tot} \propto s^{\frac{1}{3}}$
3. For $\frac{1}{6} < \beta \leq \frac{2}{7}$ ($L_{min} < L_{tail} < L_{max}$) the *core* region gives the dominant contribution
4. For $\beta = \frac{2}{7}$ ($L_{tail} = L_{max}$) rigorous AdS/CFT bound

High Energy Bounds II

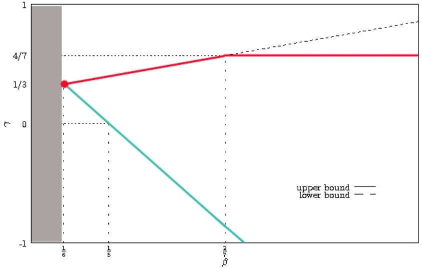


$L_{tail} \propto s^{\beta}$

$\sigma_{tot} \propto s^{\gamma}$

1. For $\beta < \frac{1}{6}$ ($L_{tail} < L_{min}$) the total cross section would become purely elastic at high energy unphysical
2. At $\beta = \frac{1}{6}$ ($L_{tail} = L_{min}$) the *tail* and *core* contributions have the same high-energy behavior $\sigma_{tot} \propto s^{\frac{1}{3}}$
3. For $\frac{1}{6} < \beta \leq \frac{2}{7}$ ($L_{min} < L_{tail} < L_{max}$) the *core* region gives the dominant contribution window of possible power-law behaviors
4. For $\beta = \frac{2}{7}$ ($L_{tail} = L_{max}$) rigorous AdS/CFT bound

High Energy Bounds II



$L_{tail} \propto s^{\beta}$

$\sigma_{tot} \propto s^{\gamma}$

1. For $\beta < \frac{1}{6}$ ($L_{tail} < L_{min}$) the total cross section would become purely elastic at high energy unphysical
2. At $\beta = \frac{1}{6}$ ($L_{tail} = L_{min}$) the *tail* and *core* contributions have the same high-energy behavior $\sigma_{tot} \propto s^{\frac{1}{3}}$
3. For $\frac{1}{6} < \beta \leq \frac{2}{7}$ ($L_{min} < L_{tail} < L_{max}$) the *core* region gives the dominant contribution window of possible power-law behaviors
4. For $\beta = \frac{2}{7}$ ($L_{tail} = L_{max}$) rigorous AdS/CFT bound $\sigma_{tot} \lesssim s^{\frac{4}{7}}$

Subleading Contributions

- Subleading part of the total cross section at large s (keeping only the leading contribution from each field)

$$\sigma_{tail}^{subleading} \simeq 4\pi\varsigma^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\, \lambda \sin\left(\frac{\kappa_G}{\lambda^6}\right) \left(\frac{\kappa_S}{\lambda^2}\frac{1}{\varsigma^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{\varsigma^2} + \frac{\kappa_B}{\lambda^4}\frac{1}{\varsigma^{\frac{2}{3}}}\right)$$

- Bare secondary contributions are shielded by graviton exchanges
- For $\beta > \frac{1}{6}$, in which case $\lambda_{tail}^{-1} \to 0$

graviton	$\longrightarrow$	$2 - 10\beta$
antisymmetric tensor	$\longrightarrow$	$1 - 8\beta$
KK scalar	$\longrightarrow$	-6β
dilaton	$\longrightarrow$	-10β

Subleading Contributions

- Subleading part of the total cross section at large s (keeping only the leading contribution from each field)

$$\sigma_{tail}^{subleading} \simeq 4\pi\varsigma^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\, \lambda \sin\left(\frac{\kappa_G}{\lambda^6}\right) \left(\frac{\kappa_S}{\lambda^2}\frac{1}{\varsigma^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{\varsigma^2} + \frac{\kappa_B}{\lambda^4}\frac{1}{\varsigma^{\frac{2}{3}}}\right)$$

- Bare secondary contributions are shielded by graviton exchanges
- For $\beta > \frac{1}{6}$, in which case $\lambda_{tail}^{-1} \to 0$

graviton	$\longrightarrow$	$2 - 10\beta$
antisymmetric tensor	$\longrightarrow$	$1 - 8\beta$
KK scalar	$\longrightarrow$	-6β
dilaton	$\longrightarrow$	-10β

Subleading Contributions

- Subleading part of the total cross section at large s (keeping only the leading contribution from each field)

$$\sigma_{tail}^{subleading} \simeq 4\pi\varsigma^{\frac{1}{3}} R_1 R_2 \int_{\lambda_{tail}}^{\infty} d\lambda\,\lambda\,\sin\left(\frac{\kappa_G}{\lambda^6}\right)\left(\frac{\kappa_S}{\lambda^2}\frac{1}{\varsigma^{\frac{4}{3}}} + \frac{\kappa_D}{\lambda^6}\frac{1}{\varsigma^2} + \frac{\kappa_B}{\lambda^4}\frac{1}{\varsigma^{\frac{2}{3}}}\right)$$

- Bare secondary contributions are shielded by graviton exchanges
- For $\beta = \frac{1}{6}$, in which case $\lambda_{tail}^{-1} \to const.$

graviton (*tail* and *core*)	$\longrightarrow$	$2 - 10\beta$	$= \frac{1}{3}$
antisymmetric tensor	$\longrightarrow$	$1 - 8\beta$	$= -\frac{1}{3}$
KK scalar	$\longrightarrow$	-6β	$= -1$
dilaton	$\longrightarrow$	-10β	$= -\frac{5}{3}$

Outline

Conclusions and Outlook

- $\mathcal{N} = 4$ SYM: laboratory for QCD in the gauge/gravity duality approach
- High energy bounds on the total cross section in $\mathcal{N} = 4$ SYM obtained using unitarity and the AdS/CFT result for the tail
  - at $\beta = 1/6$ prediction $\sigma_{tot} \sim s^{\frac{1}{3}}$ (see also [Levin, Potashnikova (2009)])
  - absolute holographic bound $\sigma_{tot} \lesssim s^{\frac{4}{7}}$

- Open issues:
  - grey and black disk model for the core
  - subleading contributions
  - nonconformal, confining case

Conclusions and Outlook

- $\mathcal{N} = 4$ SYM: laboratory for QCD in the gauge/gravity duality approach
- High energy bounds on the total cross section in $\mathcal{N} = 4$ SYM obtained using unitarity and the AdS/CFT result for the tail
  - at $\beta = 1/6$ prediction $\sigma_{tot} \sim s^{\frac{1}{3}}$ (see also [Levin, Potashnikova (2009)])
  - absolute holographic bound $\sigma_{tot} \lesssim s^{\frac{4}{7}}$

- Open issues:
  - grey and black disk model for the core
  - subleading contributions
  - nonconformal, confining case

References

- M. Froissart,
  Phys. Rev. **123** (1961) 1053
- O. Nachtmann,
  Ann. Phys. **209** (1991) 436
- J. Maldacena,
  Adv. Theor. Math. Phys. **2** (1998) 231
- H. G. Dosch, E. Ferreira and A. Krämer,
  Phys. Rev. D **50** (1994) 1992
- E. Meggiolaro,
  Nucl. Phys. **B707** (2005) 199
- M. Giordano and E. Meggiolaro,
  Phys. Lett. **B675** (2009) 123
- M. Giordano and E. Meggiolaro,
  Phys. Rev. **74** (2006) 016003
- E. Shuryak and I. Zahed,
  Phys. Rev. D **62** (2000) 085014
- M. Giordano and E. Meggiolaro,
  Phys. Rev. D **81** (2010) 074022
- R. A. Janik and R. Peschanski,
  Nucl. Phys. **B565** (2000) 193
- R. A. Janik and R. Peschanski,
  Nucl. Phys. B **586** (2000) 163
- M. Giordano and R. Peschanski,
  JHEP **05** (2010) 037
- A. I. Shoshi, F. D. Steffen, H. G. Dosch and H. J. Pirner,
  Phys. Rev. D **68** (2003) 074004
- M. Giordano and E. Meggiolaro,
  Phys. Rev. D **78** (2008) 074510
- J. Maldacena,
  Phys. Rev. Lett. **80** (1998) 4859
- D. Berenstein, R. Corrado, W. Fischler and J. Maldacena,
  Phys. Rev. D **59** (1999) 105023
- E. Levin and I. Potashnikova,
  JHEP **031** (2009) 0906