Shockwaves and DIS within the Gauge/Gravity Duality

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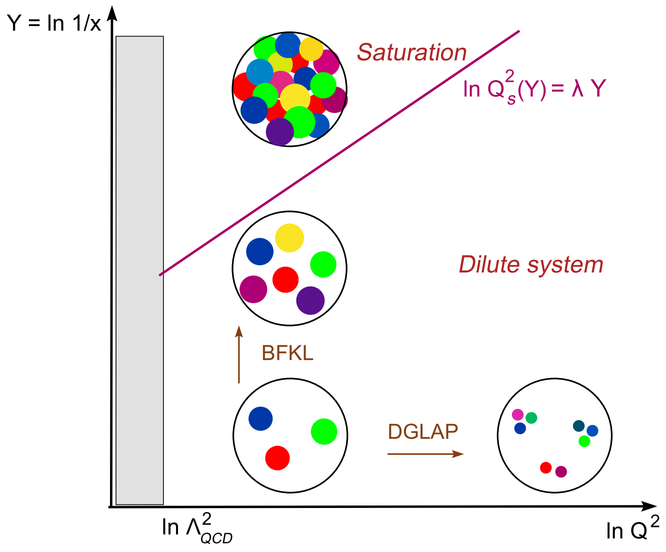
Outline

- Parton saturation in QCD at weak coupling
- Gravitational shockwaves and “nuclei” in “modified” $\mathcal{N} = 4$ SYM
- DIS off “nuclei” at strong coupling
- Parton saturation at strong coupling

Logarithmic Plane in QCD



Nucleus Properties

- Energy density of a fast right mover along x^+?
- Characteristic scale Λ
- Energy conservation $\rightsquigarrow$ x^+-independence

$$T_{--} = \gamma^2 \, N_c^2 \Lambda^4 \, f(x_\perp \Lambda) \, \frac{L}{\gamma} \delta(x^-) \Rightarrow E = \gamma N_c^2 \Lambda^2 L$$

- N_c^2 constituents per unit volume to use AdS/CFT

Gravity Dual

- Deform AdS_5 metric $\leftrightarrow$ vacuum

$$\mathrm{d}s^2 = \frac{R^2}{z^2}\left[\mathrm{d}z^2 - 2\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}x_\perp^2 + h(z, x^-, x_\perp)(\mathrm{d}x^-)^2\right]$$

  Boundary gauge theory at $z = 0$

- Correspondence relates metric to energy momentum tensor (nucleus)

$$T_{--} = \frac{R^3}{4\pi G_5}\, h^{(4)} = \frac{N_c^2}{2\pi^2}\, h^{(4)}$$

  Need T_{--} to scale like N_c^2, otherwise perturbation parametrically small

Einstein Equations

- Solve Einstein eqns in transverse momentum space

$$h = \frac{1}{2}(k_\perp z)^2 \left[c_1(x^-, k_\perp)\mathrm{K}_2(k_\perp z) + c_2(x^-, k_\perp)\mathrm{I}_2(k_\perp z)\right]$$

- Which solution to accept?

  Choices in similar situations

  - Janik, Peschanski: $c_1 = 0$, $k_\perp = 0 \leftrightarrow$ homogeneous
  - Gubser et al. Sourced Einstein eqns $\rightarrow z^4$ close to the boundary, $1/z^2$ deep in the bulk
  - Beuf $c_2 = 0$ to kill divergence at large z

Putting the Cutoff

- Choose I_2 solution with cutoff such that $zk_\perp \lesssim 1$
- h encodes momentum spectrum of quantum fluctuations
  $h \to \mathrm{const}$ as $z \to 0$ means flat spectrum at high $k_\perp$
  $1/k_\perp^4$ is not ideal, but seems better
- Back to transverse coordinate space with only I_2 part

$$h = 2\pi^2 \gamma \Lambda^4 L f(x_\perp \Lambda) \delta(x^-) z^4$$

  approximate solution for $z \lesssim 1/\Lambda$
  $f(x_\perp \Lambda)$ dim-less, vanishes for $x_\perp \gtrsim 1/\Lambda$
- Cutoff $\leftrightarrow$ IR scale of gauge theory

Deep Inelastic Scattering

- Gauge theory: Left moving $\mathcal{R}$-current or $J = \frac{1}{4}(F^a_{\mu\nu})^2$ off nucleus or plasma
- Lorentz invariants of kinematics x, Q^2
  $Q^2 > 0$ virtuality of spacelike current

$$x = \frac{Q^2}{s} = \frac{-2q^+q^-}{2q^-\gamma\Lambda}$$

- Structure function

$$F(x, Q^2) = \mathrm{Im}\Pi(x, Q^2) = \mathrm{FT}\langle J(x)J(y)\rangle$$

- We shall be interested in the regime $Q \gg \Lambda$, $x \ll 1$.

Eikonal gravitational scattering

- For $J \sim (F^a_{\mu\nu})^2$ solve EOM for dilaton field
- Shockwave structure allows iterative solution

  What is meaning of δ-function in scattering?

$$\text{width} \sim \frac{L}{\gamma} \ll t_{\text{coh}} \sim \frac{q^-}{Q^2} = \frac{1}{x\gamma\Lambda} \Rightarrow x\Lambda L \ll 1$$

- Iteration: Resum all multi-graviton exchanges

$$\Pi = \frac{Q^6 \Lambda N_c^2}{32\pi^2 x L} \int \mathrm{d}z \, z K_2^2(Qz) \int \mathrm{d}^2 b_\perp \mathcal{T}(z, b_\perp)$$

  $z \sim$ transverse size of fluctuation

  Typically fluctuations of size $1/Q$ scatter off nucleus

From Eikonal back to Single

- Scattering described by

$$\mathcal{T} = \mathrm{i}\left[1 - \exp\left(\mathrm{i}\frac{q^- \tilde{h}}{2}\right)\right]$$

exponent $\sim$ nucleus energy

- In OPE language: Multi graviton exchange $\leftrightarrow$ higher twist from energy momentum tensor
- In single scattering approximation

$$\Pi(x, Q^2) = \frac{N_c^2 \Lambda^2 Q^2}{10 x^2}$$

Real, structure function vanishes, no partons at high Q^2.

Unitarity (Saturation)

- First contribution to Im part from 2-graviton exchange

  Should keep all orders result when $\mathcal{T}^{(1)} \sim \mathcal{O}(1)$

$$\mathcal{T}^{(1)} = \frac{\pi^2 \Lambda^3 L f(b_\perp \Lambda)}{Q^2 x}$$

- Unitarity (saturation) line

$$Q_s^2(x, b_\perp) = \frac{\pi^2 \Lambda^3 L}{2x} f(b_\perp \Lambda)$$

  Energy dependence of single scattering $\rightarrow$

  x-dependence of saturation line

  Nuclear energy profile $\rightarrow$ saturation profile

Structure function at saturation

- Similar for $\mathcal{R}$-current DIS (closer to E/M current)
  Consider homogeneous case
  $\mathcal{T}$ exactly the same $\to Q_s^2$ the same
  (should not depend on the "probe")

$$F_2 = \frac{N_c^2 Q^4}{16\pi^3 \Lambda L} \int \mathrm{d}z\, z \left[\mathrm{K}_0^2(Qz) + \mathrm{K}_1^2(Qz)\right] \mathrm{Im}\mathcal{T}$$

- Unitarity $\rightsquigarrow F_2$ saturates for $Q \ll Q_s$

$$F_2(Q \lesssim Q_s) \simeq \frac{N_c^2}{64\pi^3} \frac{Q^2}{\Lambda L} \ln \frac{Q_s^2}{Q^2}$$

  Similar to QCD result

Parton Saturation

- Gluon occupation number in QCD at saturation

$$n(x,k_\perp) \equiv \frac{1}{N_c^2 \pi R^2} \frac{\mathrm{d}xG(x,Q^2)}{\mathrm{d}Q^2}\Big|_{Q=k_\perp} \sim \frac{1}{\lambda} \ln \frac{Q_s^2}{k_\perp^2}$$

  Saturates at $\mathcal{O}(1/\lambda)$ modulo mild logarithmic increase

- Occupation number at strong coupling?

  Energy density in terms of structure function (sum rule)

$$n(x,k_\perp) \sim \frac{1}{x\gamma\Lambda} \frac{1}{N_c^2} \frac{\mathrm{d}E}{\mathrm{d}^2 b_\perp \mathrm{d}^2 k_\perp} \sim \frac{1}{N_c^2} \Lambda L \frac{\mathrm{d}F_2}{\mathrm{d}Q^2}\Big|_{Q=k_\perp} \sim \ln \frac{Q_s^2}{k_\perp^2}$$

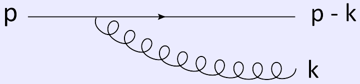
  Saturation with maximal value of $\mathcal{O}(1)$

Conclusions - Outlook

- Introduce cutoff to shockwaves solutions
- How does cutoff affect shockwave scattering?
- Seems we can introduce the concept of parton
- High energy $\rightsquigarrow$ parton saturation
- Saturation momentum $Q_s^2 \sim \Lambda^3 L/x$

Small-x and BFKL Equation

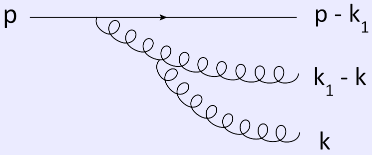
- Gluon emission from gluon or quark at high-energy



$k_z = x\, p_z << p_z$

Emission probability $\mathrm{d}P \simeq \frac{\alpha C_R}{\pi} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{\mathrm{d}x}{x}$

- One intermediate gluon



$k_z << k_{1z} << p_z$

Relative factor from propagator integration $\alpha_s \ln \frac{1}{x}$

Small-x and BFKL Equation

- n-intermediate gluon emissions $\rightsquigarrow$ factor $\left(\alpha_s \ln \frac{1}{x}\right)^n$
- Resum of $(\alpha_s \ln 1/x)^n$ terms of perturbation series

  BFKL evolution equation (Balitsky, Fadin, Kuraev, Lipatov)

  For unintegrated gluon distribution function

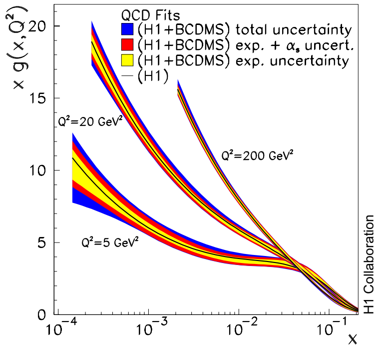
$$\frac{\partial f_g(x, k_\perp^2)}{\partial \ln 1/x} = \bar{\alpha}_s \int \mathrm{d}^2\ell_\perp \, \mathcal{K}(k_\perp^2, \ell_\perp^2) f_g(x, \ell_\perp^2)$$

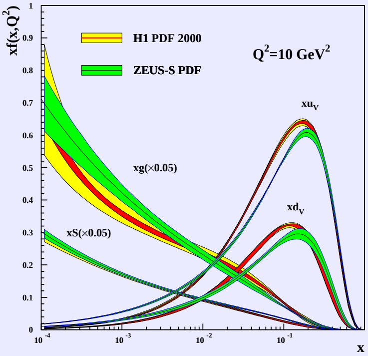
- Emitted partons can have same size

  $\rightsquigarrow$ High partonic density $f_g \sim x^{-\omega_{\mathbb{P}}}$

  $\rightsquigarrow$ Large cross section $\sigma \sim s^{\omega_{\mathbb{P}}}$

Data of Gluon Distribution Function





Saturation and Color Glass Condensate

- Estimation of maximal allowed density in QCD
  $a^\dagger a \sim \mathcal{A}\mathcal{A} \sim 1/g^2 \sim 1/\alpha_s$
- Gluon emitted in presence of strong background field
  Color Glass Condensate
- Coherent effects - saturation of emission rate

$$P(f_g) \sim 1 - \mathrm{e}^{-\alpha_s f_g}$$

  Mild logarithmic, in $1/x$, increase of gluon distribution
- Evolution equations acquire nonlinear terms
  $\sim -f_g^2(x, k_\perp^2), \ldots$ (...)
- Take elegant form in terms of specific observable

Sum Rule and Saturation Momentum

- From single scattering approximation (high Q^2)

  Analyticity and contour integration

  $$\int_0^1 \mathrm{d}x\, F_2(x, Q^2) = \frac{11}{120}\, N_c^2 \Lambda^2$$

  Integral dominated by $x \lesssim x_s(Q^2)$

  $$x_s \underbrace{F_2(x_s, Q^2)}_{\sim Q^2 N_c^2/\Lambda L} \sim N_c^2 \Lambda^2 \Rightarrow Q_s^2 \sim \frac{\Lambda^3 L}{x}$$