Soft photon production from gauge/string duality

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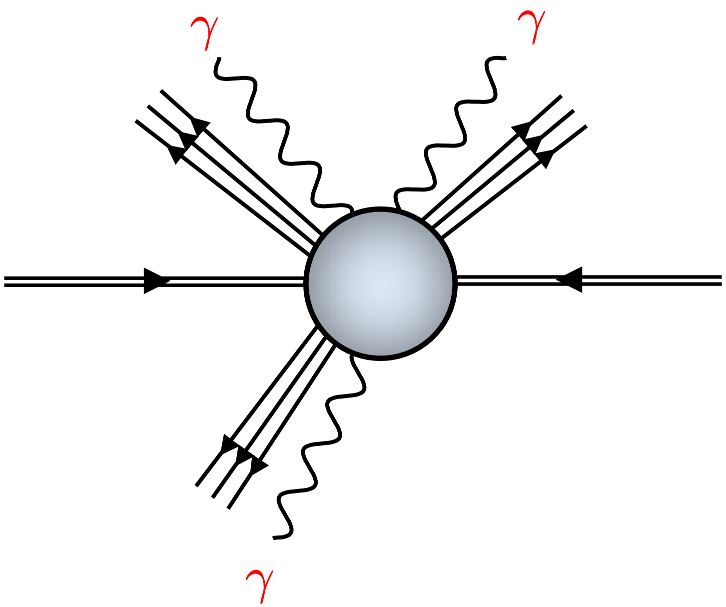
- AdS/CFT
- Soft photon production in $\mathcal{N} = 4$ SYM

Introduction

- Direct soft photon production associated with hadron production



- Soft photons cannot resolve short distance processes occurring at the QCD scale
- Entirely understood by the QED Bremsstrahlung from initial/final state charged particles

Landau & Pomeranchuk '53
Low '58

Anomalous Soft Photons

- Excess of direct soft photons as compared to the prediction from the Bremsstrahlung
  - In forward kinematic region
  - Very low-p_T ($\lesssim$ 100MeV)
  - In $K^+p, \pi^\pm p$, pp, e^+e^-,... collisions – known for more than 20 years
- Recent report of DELPHI experiment @ LEP1
  - Analysis of hadronic Z^0-decay: Abdallah *et al.* EPJC47 (2006) 273, EPJC67 (2010) 343

    $e^+e^- \longrightarrow Z^0 \longrightarrow$ hadrons + direct soft γs

    # of soft photons was about 4 times larger than theory prediction
  - Analysis of dimuon events: EPJC57 (2008) 499

    $e^+e^- \longrightarrow Z^0 \longrightarrow \mu^+\mu^-$ + direct soft γs

    No deviation from the theoretical expectation

⇒ Soft photon anomaly comes from the strong interaction

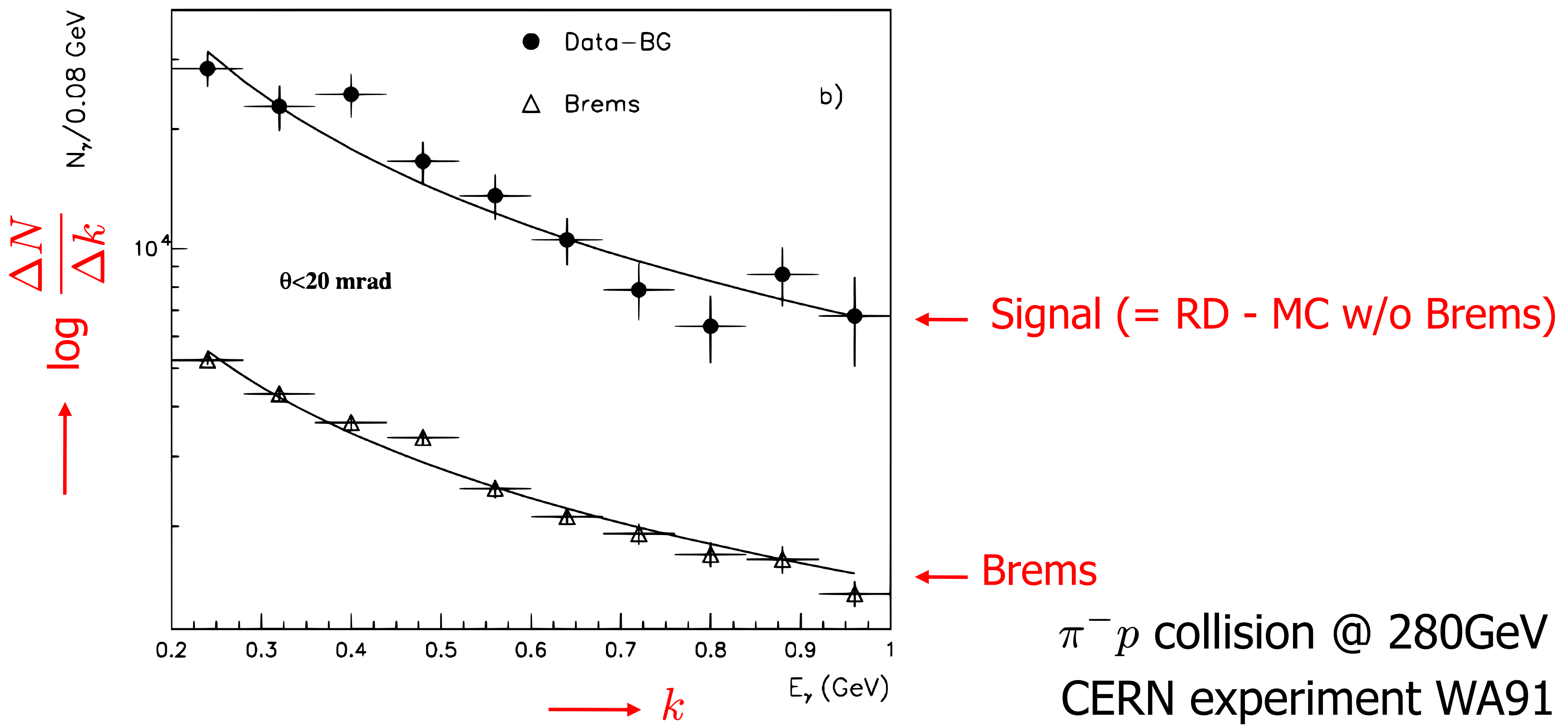
Theoretical Challenges

- Many models
  - String fragmentation — Andersson & Dahlqvist & Gustafson '89
  - Backward reflection at the boundary — Shuryak '89
  - Cold quark gluon plasma — Lichard & Van Hove '90
  - Synchrotron radiation in the stochastic nonperturbative QCD vaccum — Botz & Haberl & Nachtmann '95
  - Hanbury Brown and Twiss effect — Pišút & Pišútová & Tomášik '96
  - Closed quark-antiquark loop — Simonov & Veselov '08, '09
  - QED2 photons associated with QCD string fragmentation — Wong '10
  - ...
- No model can reproduce the anomalous events as a whole
- The problem is nonperturbative, first principle analytic calculation not available

Energy Distribution

- The shape of photon energy distribution is consistent with the Bremsstrahlung:

$$\frac{dN}{dk} \propto \frac{1}{k^\alpha} \qquad \alpha \approx 1 \qquad k = |\vec{k}| \ : \ \text{photon energy}$$



$\pi^- p$ collision @ 280GeV

CERN experiment WA91

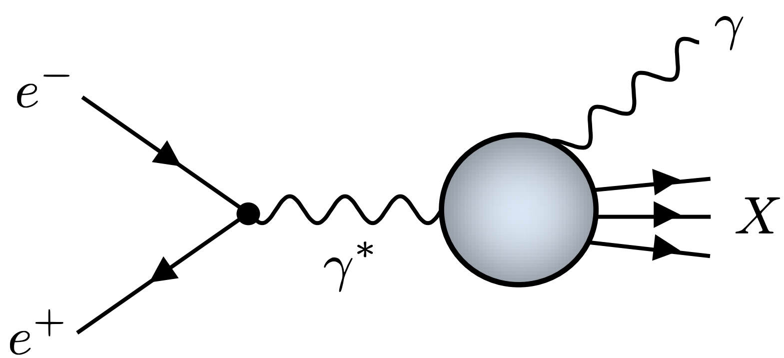
Fig. 2(b) in Belogianni *et al*. PLB548 (2002) 122

Source of Anomalous Photons ?

- Find new source of soft photons with $1/k$ spectrum not associated with final state Bremsstrahlung

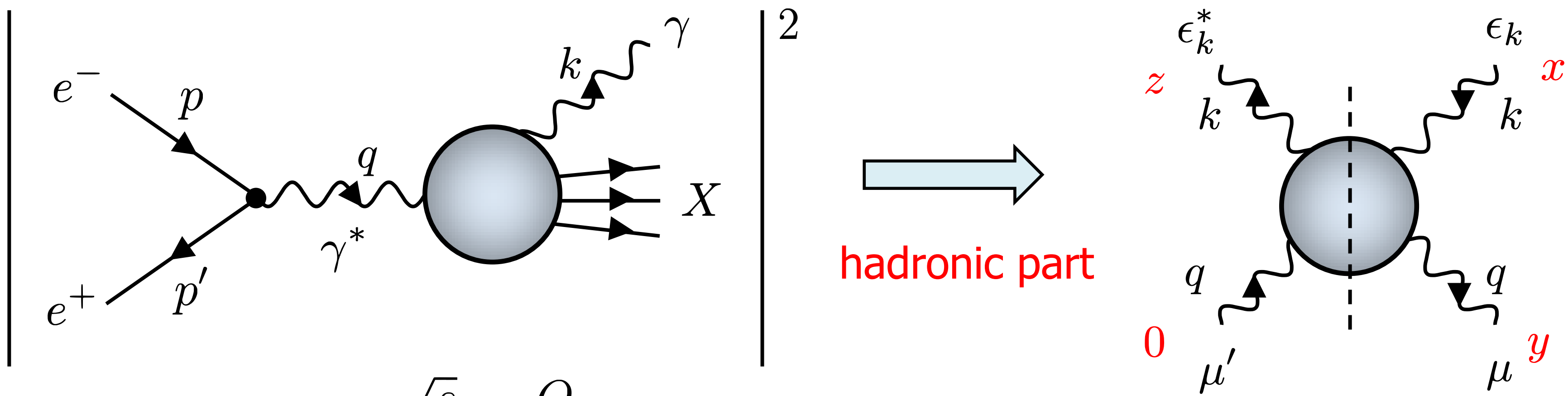
- AdS/CFT allows exact non-perturbative calculations in a class of non-Abelian gauge theories closely related to QCD

- The goal is to calculate the inclusive cross section of photon in e^+e^- annihilation in strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory



Inclusive Photon Production

- Inclusive cross section of photon in e^+e^- annihilation



$$\sqrt{s} = Q$$

Usual formula:

leptonic part

$$d\sigma = \frac{e^6}{2Q^6}\frac{d^3\vec{k}}{2k(2\pi)^3}\frac{1}{4}\sum_{spin}\bar{u}(p)\gamma_\mu v(p')\bar{v}(p')\gamma_{\mu'}u(p)$$

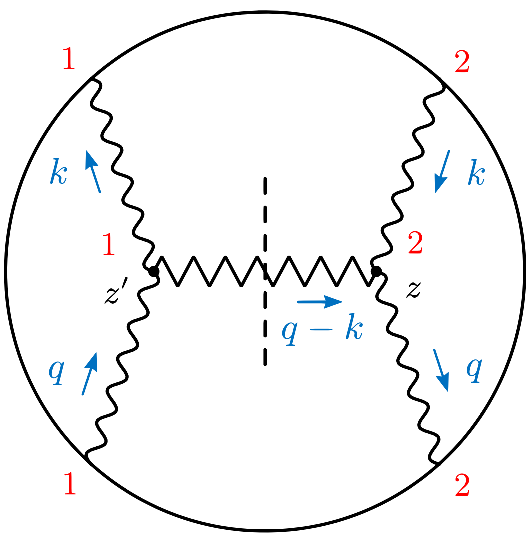
$$\times\sum_{pol}\int d^4x d^4y d^4z\, e^{-iq\cdot x+ik\cdot(y-z)}\langle 0|\tilde{\mathrm{T}}\{J^\mu(x)\,\varepsilon_k\cdot J(y)\}\,\mathrm{T}\{\varepsilon_k^*\cdot J(z)J^{\mu'}(0)\}|0\rangle$$

hadronic part

J^3_μ : Current associated with a $U(1)$ subgroup of $SU(4)\,\mathcal{R}$ -symmetry

Four-point Correlation Function

$$\sum_{pol} \int d^4x d^4y d^4z \, e^{-iq\cdot x + ik\cdot(y-z)} \langle 0|\tilde{\mathrm{T}}\{J^\mu(x)\, \varepsilon_k \cdot J(y)\}\, \mathrm{T}\{\varepsilon_k^* \cdot J(z) J^{\mu'}(0)\}|0\rangle$$



Current operator J_μ^3 on the Minkowski boundary excites a component of the gauge boson A_m^3 in the bulk AdS_5

Calculate Witten diagrams in real-time (Keldysh) formalism

Exchange gravition, $SO(6)$ gauge boson, dilaton

g_{mn} $\qquad A_m^a$ $\qquad \phi$

$$S = \frac{1}{2\kappa^2}\int d^5x \sqrt{-g}\left(\mathfrak{R} - \frac{4}{3}\partial_m\phi\partial^m\phi\right) - \frac{1}{4g_{YM}^2}\int d^5x\sqrt{-g}\; F_{mn}^a F_a^{mn} e^{-\frac{4}{3}\phi}$$
$$+ \frac{N_c^2}{96\pi^2}\int d^5x\, \epsilon^{mnpqr} d^{abc}\partial_m A_n^a \partial_p A_q^b A_r^c$$

Chern-Simons term

Graviton Exchange

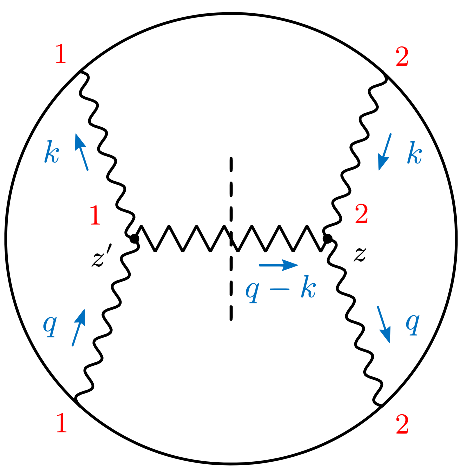
$$\frac{2\kappa^2}{g_{YM}^4}\int\frac{dz}{z^5}\int\frac{dz'}{z'^5}T_{(2)}^{mn}(z,q,-k)G^{(21)}_{mn;m'n'}(z,z',q-k)T_{(1)}^{m'n'}(z',-q,k)$$

T^{mn} : Energy-momentum tensor

$G_{mn,m'n'}$: Graviton propagator

Decomposed as

$$\begin{aligned}\int_{z,z'}T^{mn}G_{mn,m'n'}T^{m'n'} &= \int_{z,z'}T^{\mu\nu}G_{\mu\nu,\mu'\nu'}T^{\mu'\nu'}\\ &+\int_{z,z'}\left(T^{\mu\nu}G_{\mu\nu,z'z'}T^{z'z'}+T^{zz}G_{zz,\mu'\nu'}T^{\mu'\nu'}+T^{zz}G_{zz,z'z'}T^{z'z'}\right)\\ &+4\int_{z,z'}T^{\mu z}G_{\mu z,\mu' z}T^{\mu' z}\\ &+2\int_{z,z'}\Big(T^{\mu\nu}G_{\mu\nu,\mu' z'}T^{\mu' z'}+T^{\mu z}G_{\mu z,\mu'\nu'}T^{\mu'\nu'}\\ &\qquad +T^{\mu z}G_{\mu z,z'z'}T^{z'z'}+T^{zz}G_{zz,\mu' z'}T^{\mu' z'}\Big)\end{aligned}$$



We need all components of $G_{mn,m'n'}$ (not only $G_{++,--}$)

Graviton Exchange (cont'd)

$$\int_{z,z'} T^{\mu\nu} G_{\mu\nu,\mu'\nu'} T^{\mu'\nu'}$$

$$T^{\mu\nu}(z,-q,k) = iz^7 \frac{\pi Q}{2} H_1^{(1)}(Qz) \left(A^{\mu\nu} - \frac{\eta^{\mu\nu}}{4} A^{\rho}{}_{\rho} \right)$$

$$A^{\mu\nu} = \varepsilon_q \cdot \varepsilon_k^* \, q^\mu k^\nu - q \cdot \varepsilon_k^* \, \varepsilon_q^\mu k^\nu - k \cdot \varepsilon_q \, q^\mu \varepsilon_k^{*\nu} + q \cdot k \, \varepsilon_q^\mu \varepsilon_k^{*\nu}$$

$$G_{\mu\nu;\mu'\nu'} = \frac{\eta_{\mu\mu'}\eta_{\nu\nu'} + \eta_{\mu\nu'}\eta_{\nu\mu'}}{z^2 z'^2} \times \pi z^2 z'^2 \int_0^\infty d\omega^2 \delta(\omega^2 + (q-k)^2) J_2(\omega z) J_2(\omega z') + \cdots$$

cut propagator

- Integration over z, z' and ω

$$\frac{\pi Q}{2} \int_0^\infty dz z^2 J_2(\omega z) H_1^{(1)}(Qz) = \frac{-2i\omega^2}{(Q^2 - \omega^2)^2}$$

$$\pi \int_0^\infty d\omega^2 \delta(\omega^2 + (q-k)^2) \frac{4\omega^4}{(Q^2-\omega^2)^4} = \frac{\pi}{4k^4} \left(1 - \frac{2k}{Q}\right)^2$$

$\sqrt{s} = Q$: CM energy

k : photon energy

θ : angle of e^- & γ

$k \ll Q$

- Contraction with leptonic part

$$k \frac{dN_{G1}}{d^3\vec{k}} = \frac{\alpha_{em}}{32\pi^2 k^2} \left(1 - \frac{4k}{Q}\right) (1 + \cos^2\theta)$$

- Checked by FORM

Vermaseren '84 –

The Results

graviton exchange:

$$k\frac{dN_G}{d^3\vec{k}} = \frac{\alpha_{em}}{16\pi^2k^2}\left\{\left(\frac{7}{12}-\frac{2k}{Q}\right)(1+\cos^2\theta)+\left(\frac{1}{2}-\frac{k}{Q}\right)(1-\cos^2\theta)\right\}$$

gauge boson exchange:

$$k\frac{dN_A}{d^3\vec{k}} = \frac{\alpha_{em}}{16\pi^2k^2}\left\{\frac{1}{4}(1+\cos^2\theta)+\left(\frac{1}{2}-\frac{k}{Q}\right)(1-\cos^2\theta)\right\}$$

dilaton exchange:

$$k\frac{dN_\phi}{d^3\vec{k}} = \frac{\alpha_{em}}{16\pi^2k^2}\left(\frac{1}{6}-\frac{2k}{3Q}\right)(1+\cos^2\theta)$$

$\sqrt{s} = Q$: CM energy
k : photon energy
θ : angle of e^- & γ

sum:

$$k\frac{dN}{d^3\vec{k}} = \frac{\alpha_{em}}{8\pi^2k^2}\left(1-\frac{k}{3Q}(7+\cos^2\theta)\right) \approx \frac{\alpha_{em}}{8\pi^2k^2}$$

$k \ll Q$

Leading term:

$1/k$ spectrum !!

Spherical !! Angular dependence miraculously cancels after summing over all components of graviton, gauge boson, dilaton exchanges

Remarks

- Leading term ($1/k$) is spherical

  In $\mathcal{N}=4$ SYM the integrated energy distribution in the final state of e^+e^- annihilation is known to be exactly spherical

  Hofman & Maldacena '08

- However, subleading term (k^0) is not spherical

- At weak coupling ($g=0$)

$$k\frac{dN}{d^3\vec{k}} \approx \frac{\alpha_{em}}{2\pi^2 k^2}\ln\frac{Q^2}{m^2}$$

  m: parton mass

  Also at weak coupling, the leading term is spherical but subleading term not

- Collinear singularity at weak coupling disappears at strong coupling

Summary

- Motivated by the soft photon anomaly, we exactly calculated the inclusive cross section of soft photon production in strongly coupled $\mathcal{N} = 4$ SYM

- We got the Bremsstrahlung ($1/k$) spectrum
  Could be the origin of the `anomalous soft photons'

- Much work is needed in order to compare with experiments
  Spherical distribution is most likely an artifact of $\mathcal{N} = 4$ SYM
  Use more realistic AdS/QCD models

Backup Slides

Anomalous Soft Photons

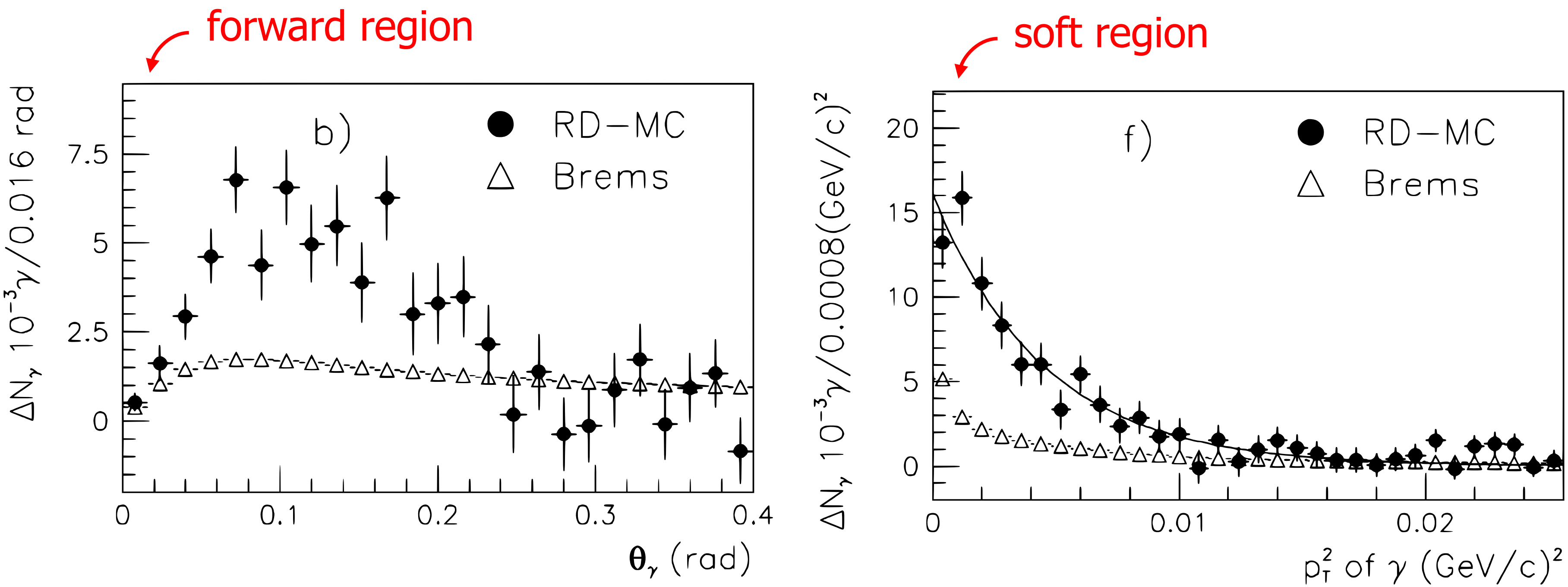


Fig. 4(b) and (f) in Abdallah *et al*. EPJC47 (2006) 273

DELPHI experiment @ LEP1

θ_γ and p_T are defined w.r.t. the parent jet

Anomalous Soft Photons

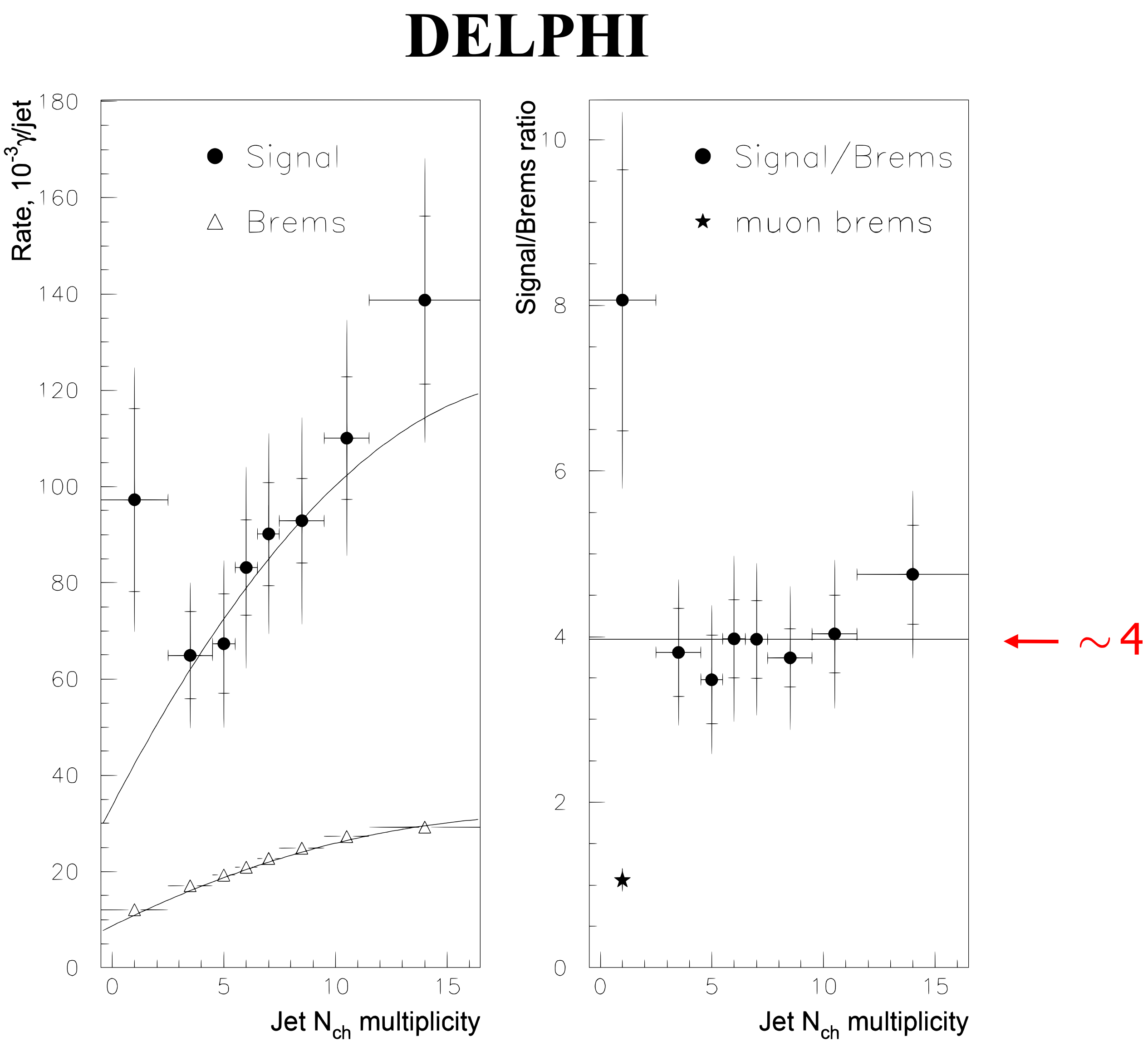
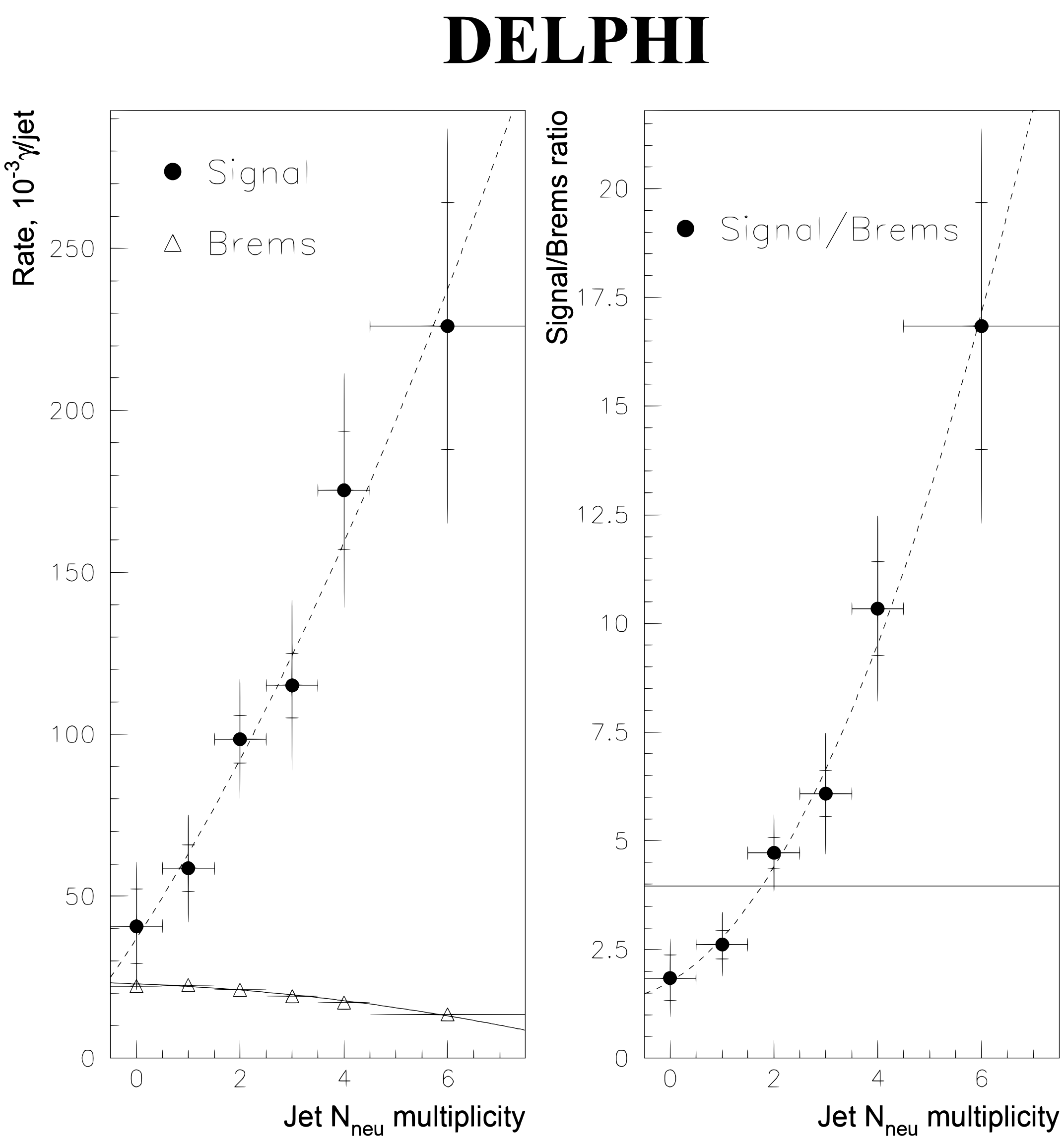


Fig. 5 in Abdallah *et al*. EPJC67 (2010) 343

Anomalous Soft Photons



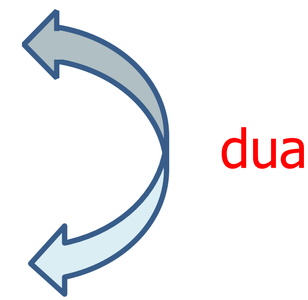
← not constant

Fig. 6 in Abdallah *et al*. EPJC67 (2010) 343

The AdS/CFT Correspondence

- Duality (equivalence) between the string theory in the curved space-time bulk and the gauge filed theory on the conformal boundary
  Maldacena '97

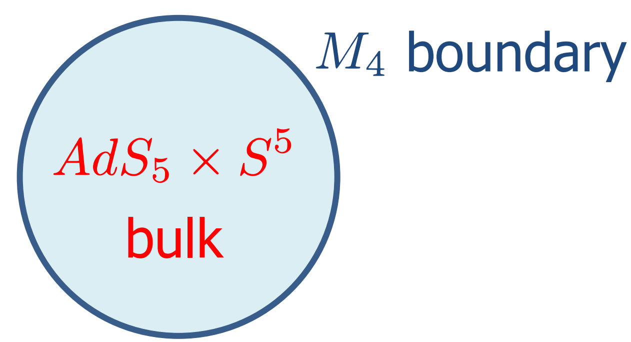
- Best-known example:
  - $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory in 4 dim.
    - Large N_c limit $N_c \to \infty$
    - Strong coupling $\lambda \to \infty$
  - Type IIB superstring theory in $AdS_5 \times S^5$
    - Weak coupling $g_s \sim 1/N_c$



⇩ low energy effective theory

Type IIB supergravity in $AdS_5 \times S^5$

↓ compactification



$\mathcal{N}=4$ Supersymmetric Yang-Mills

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{i=1}^{4}\bar{\psi}^a_i\left(\bar{\sigma}\cdot D\psi_i\right)^a + \frac{1}{2}\sum_{1\le i<j\le 4}(D_\mu\phi_{ij})^{\dagger a}(D^\mu\phi_{ij})^a$$

$$+ 2\sqrt{2}gf^{abc}\sum_{1\le i<j\le 4}\mathrm{Re}\left(\phi^a_{ij}\psi^b_i\psi^c_j\right) - \frac{g^2}{4}\sum_{\substack{1\le i<j\le 4\\ 1\le k<l\le 4}}\left|f^{abc}\phi^b_{ij}\phi^c_{kl}\right|^2$$

	$SU(N_c)$	Global $SU(4)$ $\mathcal{R}$-symmetry
gauge boson (gluon) A^a_μ	adjoint rep.	1
Weyl fermion (quark) ψ^a_i	adjoint rep.	4
scalar ϕ^a_{ij}	adjoint rep.	6
	(color)	(flavor)

β-function is zero (conformal)

't Hooft copluing $\lambda = g^2 N_c$ is a free parameter

Type IIB Supergravity on AdS_5

- The metric of AdS_5 in the Poincaré coordinates

our universe (4-dim. Minkowski)

boundary at $z \to 0$

$$ds^2 = g_{mn} dx^m dx^n = \frac{\overbrace{\eta_{\mu\nu} dx^\mu dx^\nu}^{} + dz^2}{z^2}$$

$$x^m = (x^\mu, z) \qquad \eta^{\mu\nu} = \mathrm{diag}(-1,1,1,1)$$

- The relevant part of gravity action on AdS_5

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left(\mathfrak{R} - \frac{4}{3} \partial_m \phi \partial^m \phi \right) - \frac{1}{4 g_{YM}^2} \int d^5x \sqrt{-g}\, F^a_{mn} F_a^{mn} e^{-\frac{4}{3}\phi} + \frac{N_c^2}{96\pi^2} \int d^5x\, \epsilon^{mnpqr} d^{abc} \partial_m A^a_n \partial_p A^b_q A^c_r$$

involves

	graviton	g_{mn}
	dilaton	ϕ
$SO(6) \cong SU(4)$	gauge boson	$A^a_m \ (1 \leq a \leq 15)$