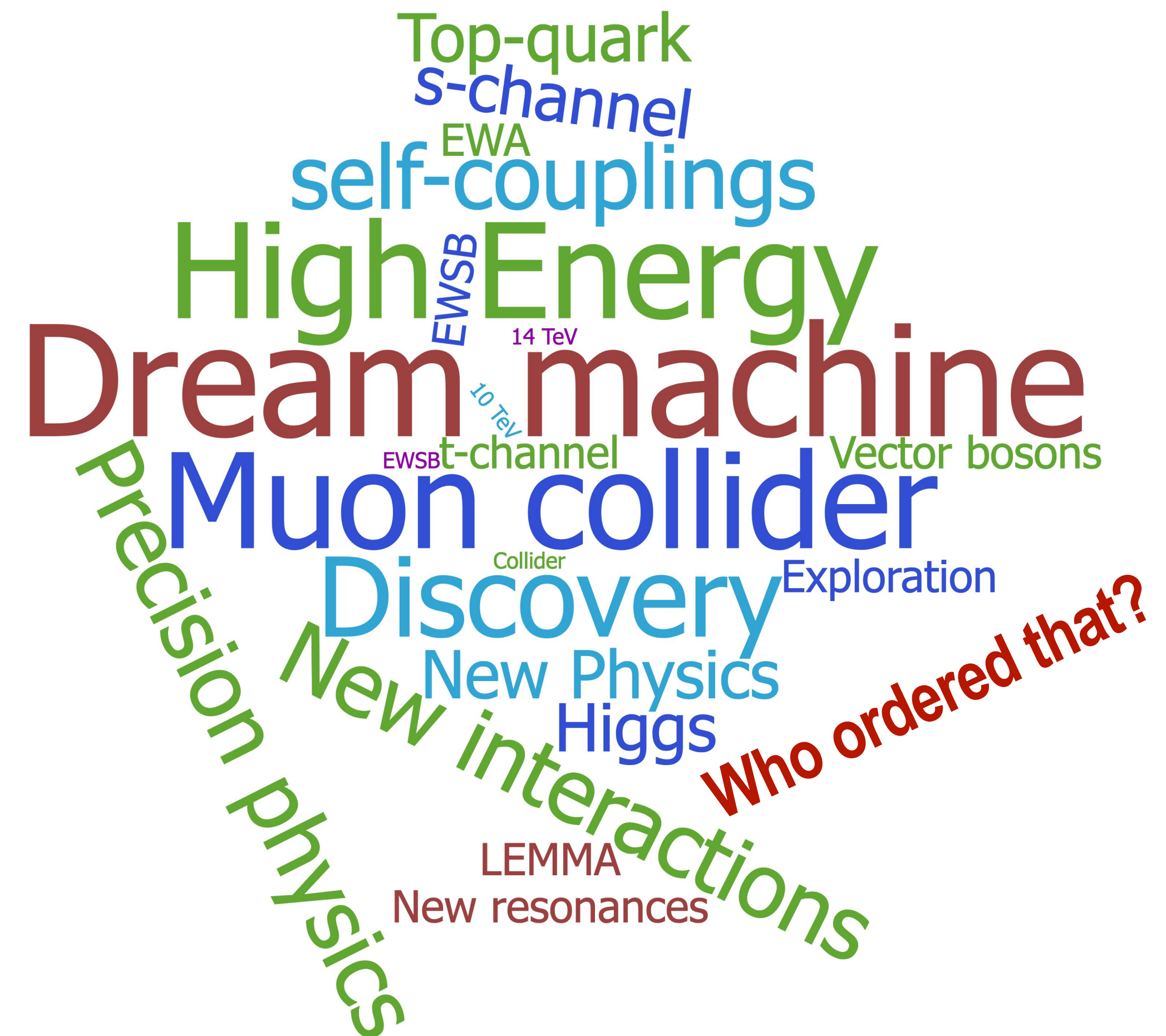


Theoretical predictions for a muon collider

Challenges and opportunities

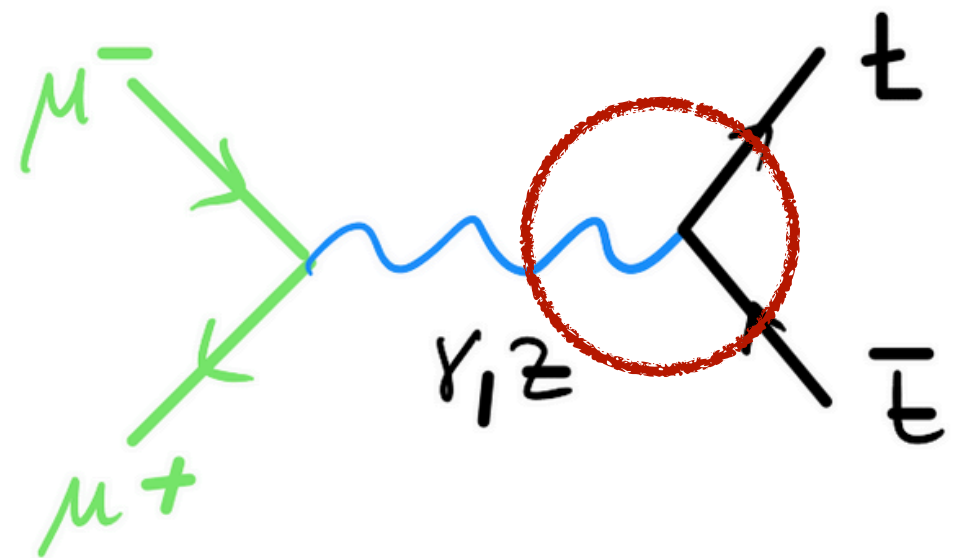
Fabio Maltoni
Università di Bologna
Université catholique de Louvain



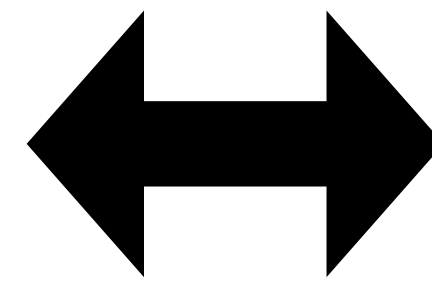
Muon collider production mechanisms

Annihilation vs scattering

$$\sqrt{s} \lesssim 1-5 \text{ TeV}$$

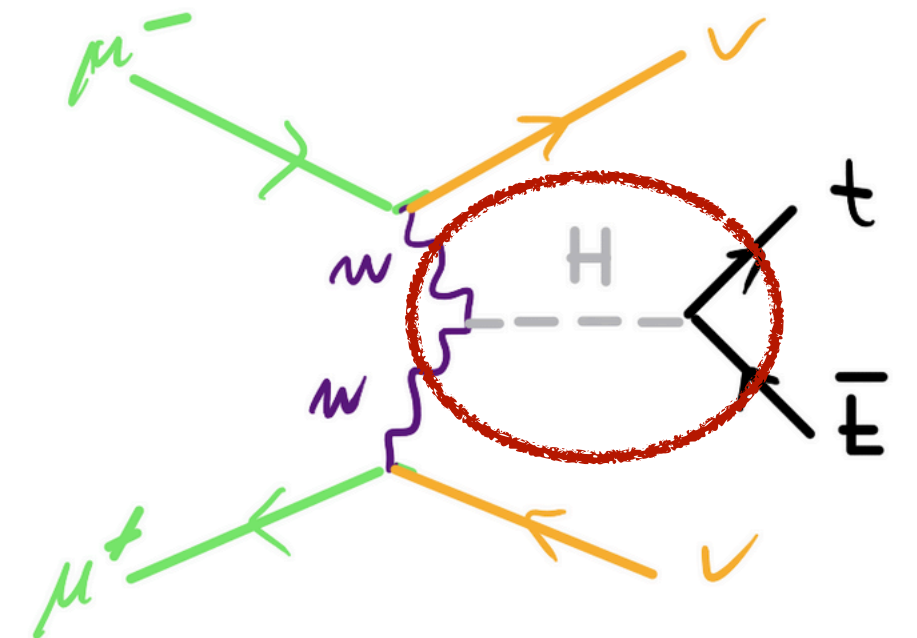


$$\sigma_s \sim \frac{1}{s}$$



$$\sigma_s \sim \frac{1}{M^2} \log^n \frac{s}{M}$$

$$\sqrt{s} \gtrsim 1-5 \text{ TeV}$$



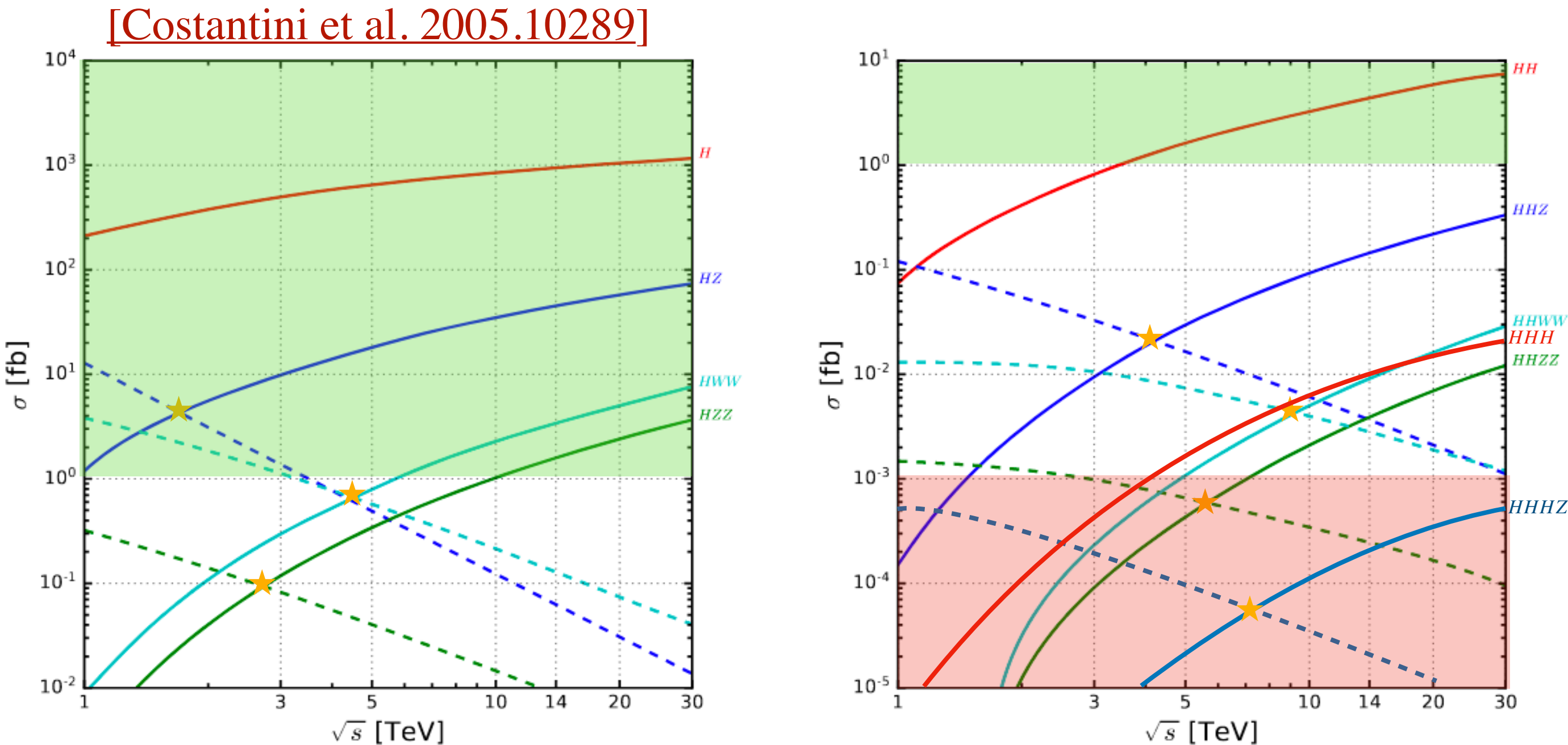
A completely new regime opening for a multi-TeV muon collider

Different physics being probed in the two channels

Muon collider production mechanisms

Annihilation vs scattering

— WW fusion
 - - - - - s-channel



- Higgs production dominated by VBF.
- H production above 100 fb in all ranges $\sqrt{s} \geq 1$ TeV.
- HH production above 1 fb for $\sqrt{s} \gtrsim 3$ TeV.
- HHH production above 10 ab for $\sqrt{s} \gtrsim 14$ TeV
- HZ/HWW/HZZ above 1 fb
- Charged final states have comparable rates ($p_T^\ell > 30$ GeV):

σ [fb]	$\sqrt{s} = 1$ TeV	$\sqrt{s} = 3$ TeV	$\sqrt{s} = 14$ TeV	$\sqrt{s} = 30$ TeV
WH	$8.4 \cdot 10^{-1}$	$7.2 \cdot 10^0$	$3.3 \cdot 10^1$	$5.5 \cdot 10^1$
WZH	$1.7 \cdot 10^{-3}$	$8.0 \cdot 10^{-2}$	$1.1 \cdot 10^0$	$2.5 \cdot 10^0$
WHH	$9.5 \cdot 10^{-5}$	$6.2 \cdot 10^{-3}$	$9.7 \cdot 10^{-2}$	$2.3 \cdot 10^{-1}$

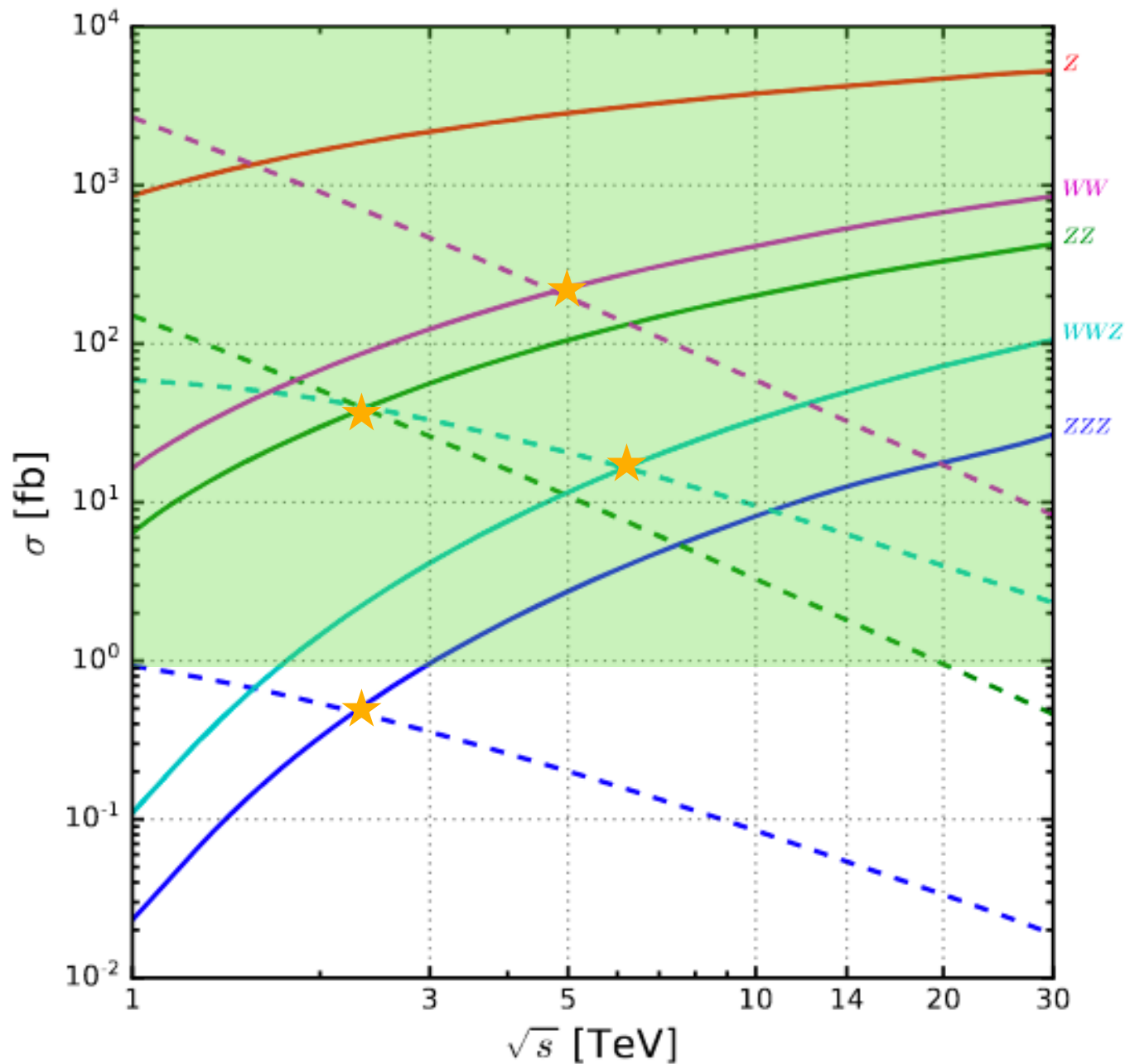
Higgs boson cross sections

Muon collider production mechanisms

Annihilation vs scattering

— WW fusion
 - - - - - s-channel

[Costantini et al. 2005.10289]



- Multi-boson production displays large cross sections. EW radiation...!
- For neutral final states, cross sections overcome annihilation at values of $\sqrt{s} \lesssim 5 \text{ TeV}$.
- Charged final state cross sections in VBF are of similar size ($p_T^\ell > 30 \text{ GeV}$):

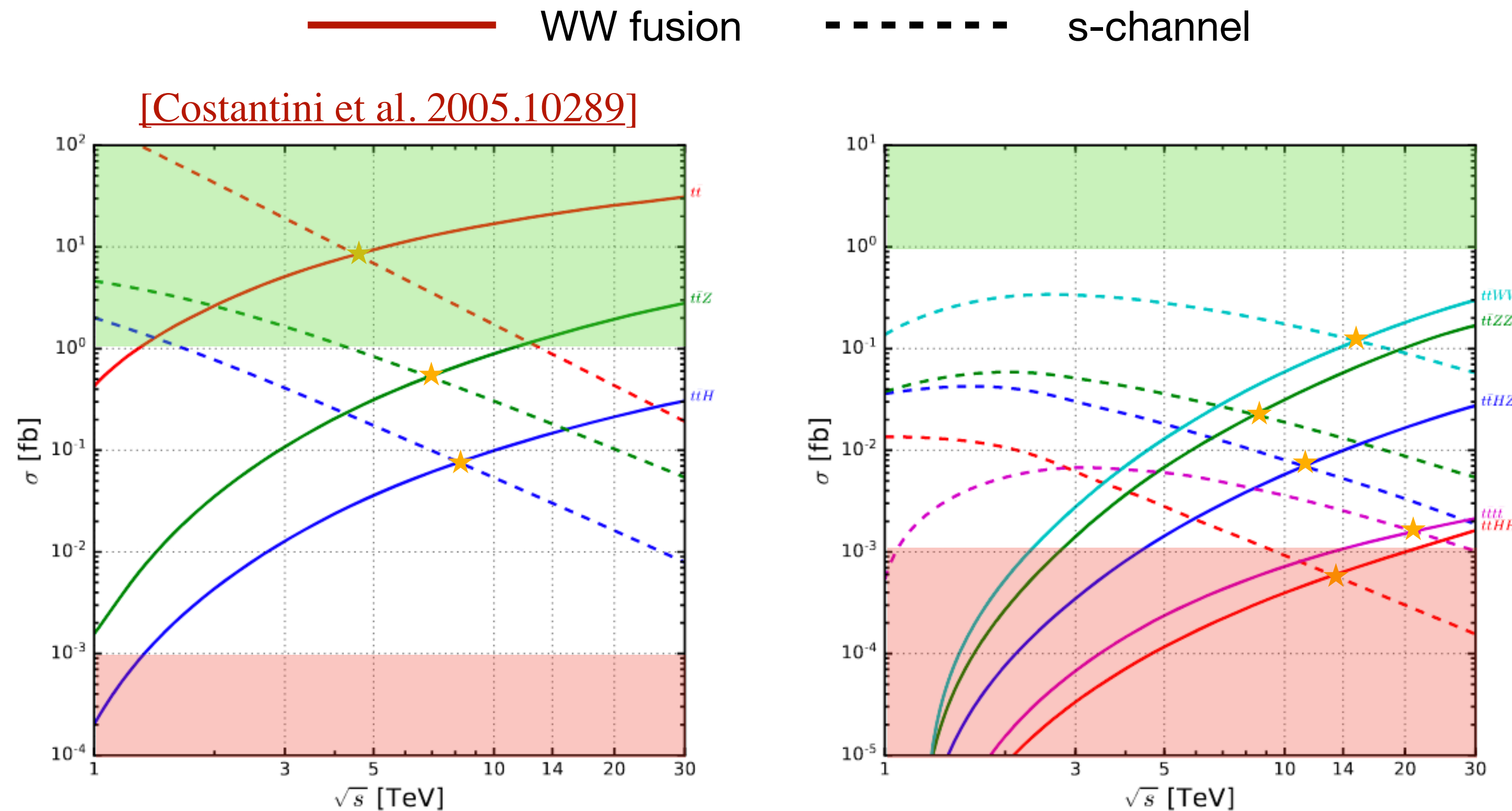
$\sigma \text{ [fb]}$	$\sqrt{s} = 1 \text{ TeV}$	$\sqrt{s} = 3 \text{ TeV}$	$\sqrt{s} = 14 \text{ TeV}$	$\sqrt{s} = 30 \text{ TeV}$
W	$9.9 \cdot 10^2$	$2.4 \cdot 10^3$	$4.6 \cdot 10^3$	$5.7 \cdot 10^3$
WZ	$5.8 \cdot 10^0$	$5.0 \cdot 10^1$	$2.3 \cdot 10^2$	$3.7 \cdot 10^2$
WWW	$1.4 \cdot 10^{-1}$	$4.2 \cdot 10^0$	$4.4 \cdot 10^1$	$1.0 \cdot 10^2$
WZZ	$1.8 \cdot 10^{-2}$	$8.0 \cdot 10^{-1}$	$1.0 \cdot 10^1$	$2.3 \cdot 10^1$

Vector bosons (neutral final states)

- Photon-induced production (not shown here) provides a large contribution to W final states.

Muon collider production mechanisms

Annihilation vs scattering

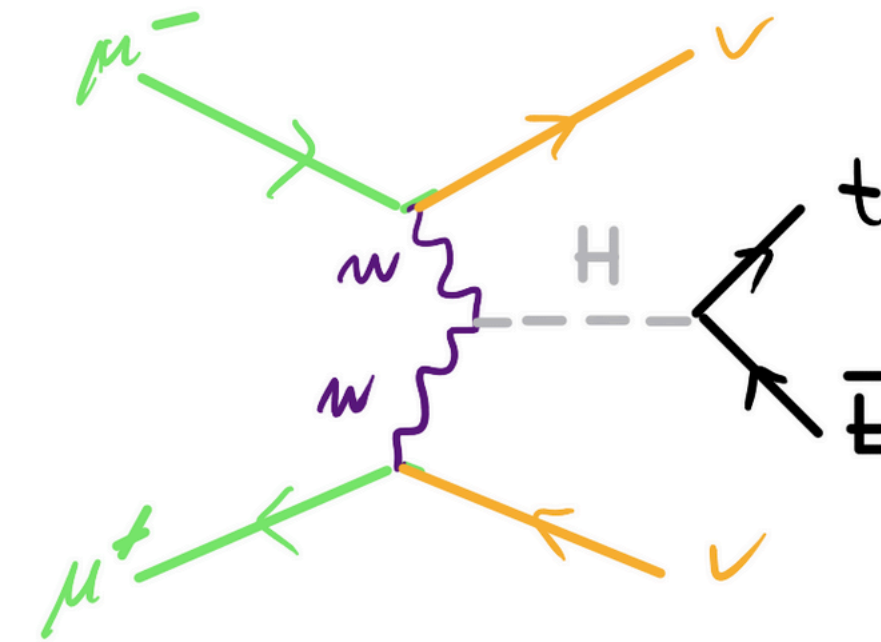
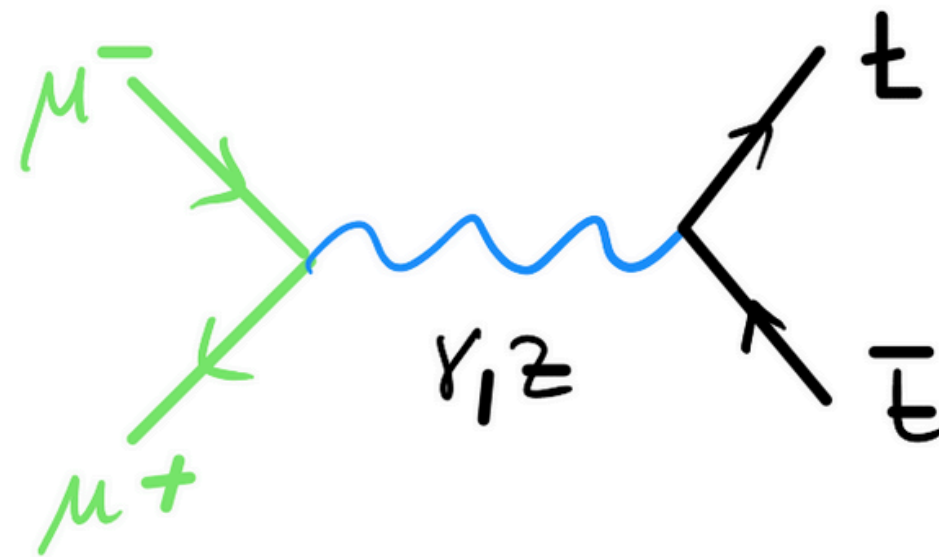


- Top-quark production can be taken as a template for heavy fermion production.
- Around $\sqrt{s} \simeq 5$ TeV, $t\bar{t}$ in VBF overcomes annihilation, with a cross section of about 10 fb.
- $t\bar{t}H, t\bar{t}Z$ production cross-sections are dominated by s-channel.
- Multi-boson associated production and four tops peak at several TeV in the annihilation channel and rise very quickly for VBF. They always remain with cross sections less than 1fb.

Associated top-quark production

Muon collider production mechanisms

Annihilation vs scattering

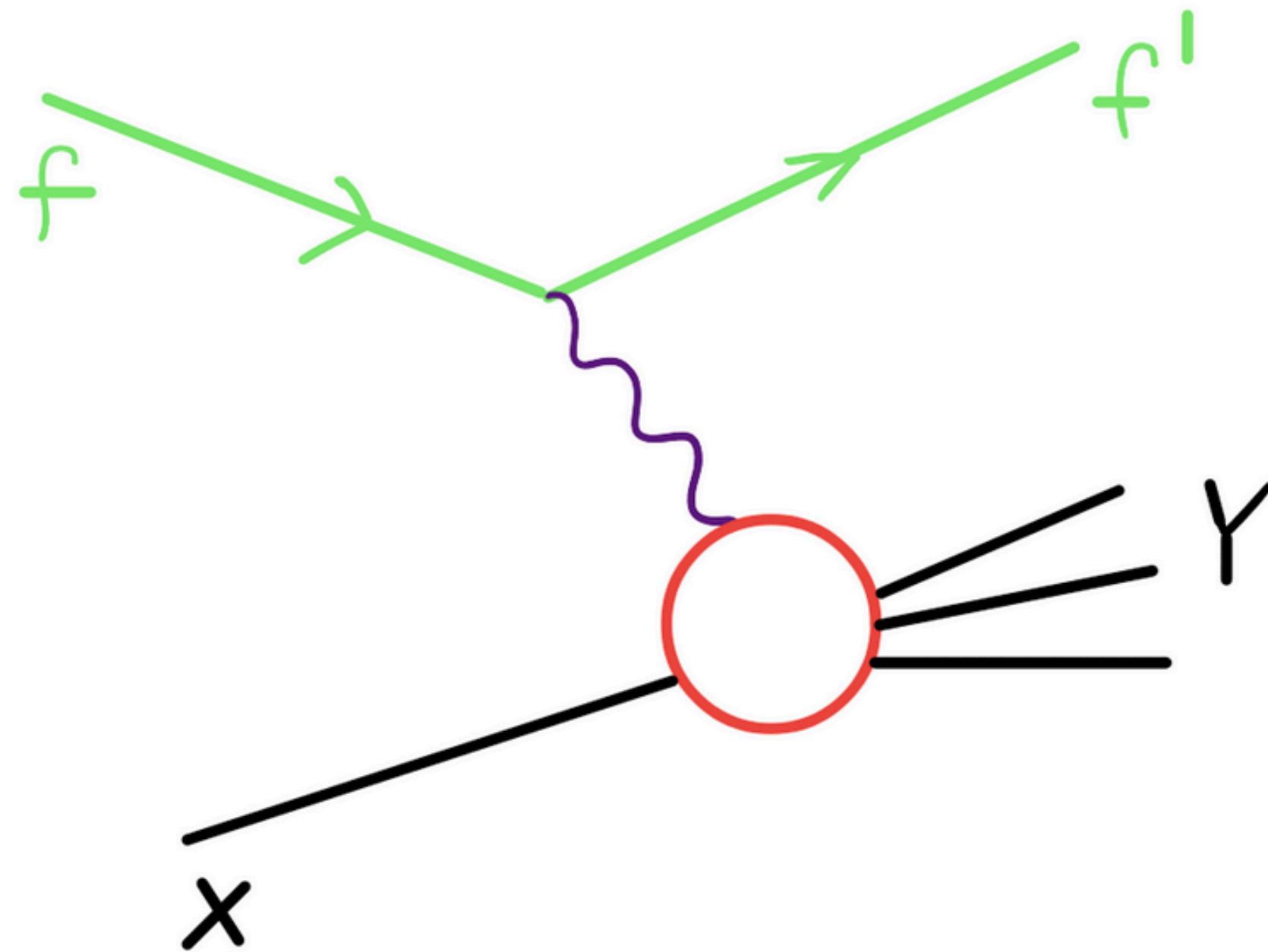


- Annihilation processes (s or t-channel or both) are the way we normally think about physics at e^+e^- colliders.
- QFT technology has been developed since the LEP times to perform accurate predictions for the main production processes.
- A lot of progress made over the LHC times (e.g. NLO automation, two loop,...PS matching/merging) is available, yet it has not been yet applied extensively.

- Vector boson fusion starts at $2 \rightarrow 3$. Dominates at high energy. Its new physics potential up to 3 TeV has been studied in the context of CLIC [[De Blas et al. 1812.02093](#)].
- Possibly complicated as it mixes with annihilation channels, even though at high energy it is de facto separable. Simulations of multiparticle final states, including “jets”, can be time consuming already at tree-level. NLO EW computations for simple final states are in principle possible.
- Considerable simplifications and possible improvements via the EWA.

W,Z, γ as partons

EW⁺⁺



[Kane Repko Rolnick, 1984] [Dawson 1985]

$$E \sim xE \sim (1-x)E, \quad \frac{m}{E} \ll 1, \quad \frac{p_{\perp}}{E} \ll 1$$

$$f_+ = \frac{(1-x)^2}{x} \frac{p_{\perp}^3}{(m^2(1-x) + p_{\perp}^2)^2}, \quad \Rightarrow \log \left(\frac{\mu_F^2}{M_V^2} \right)$$

$$f_- = \frac{1}{x} \frac{p_{\perp}^3}{(m^2(1-x) + p_{\perp}^2)^2},$$

$$f_0 = \frac{(1-x)^2}{x} \frac{2m^2 p_{\perp}}{(m^2(1-x) + p_{\perp}^2)^2}, \quad \Rightarrow \text{const.}$$

$$\frac{d\sigma_{EWA}}{dx dp_{\perp}} (fX \rightarrow f'Y) = \frac{C^2}{2\pi^2} \sum_{i=+,-,0} f_i \times d\sigma(W_i X \rightarrow Y)$$

W,Z, γ as partons

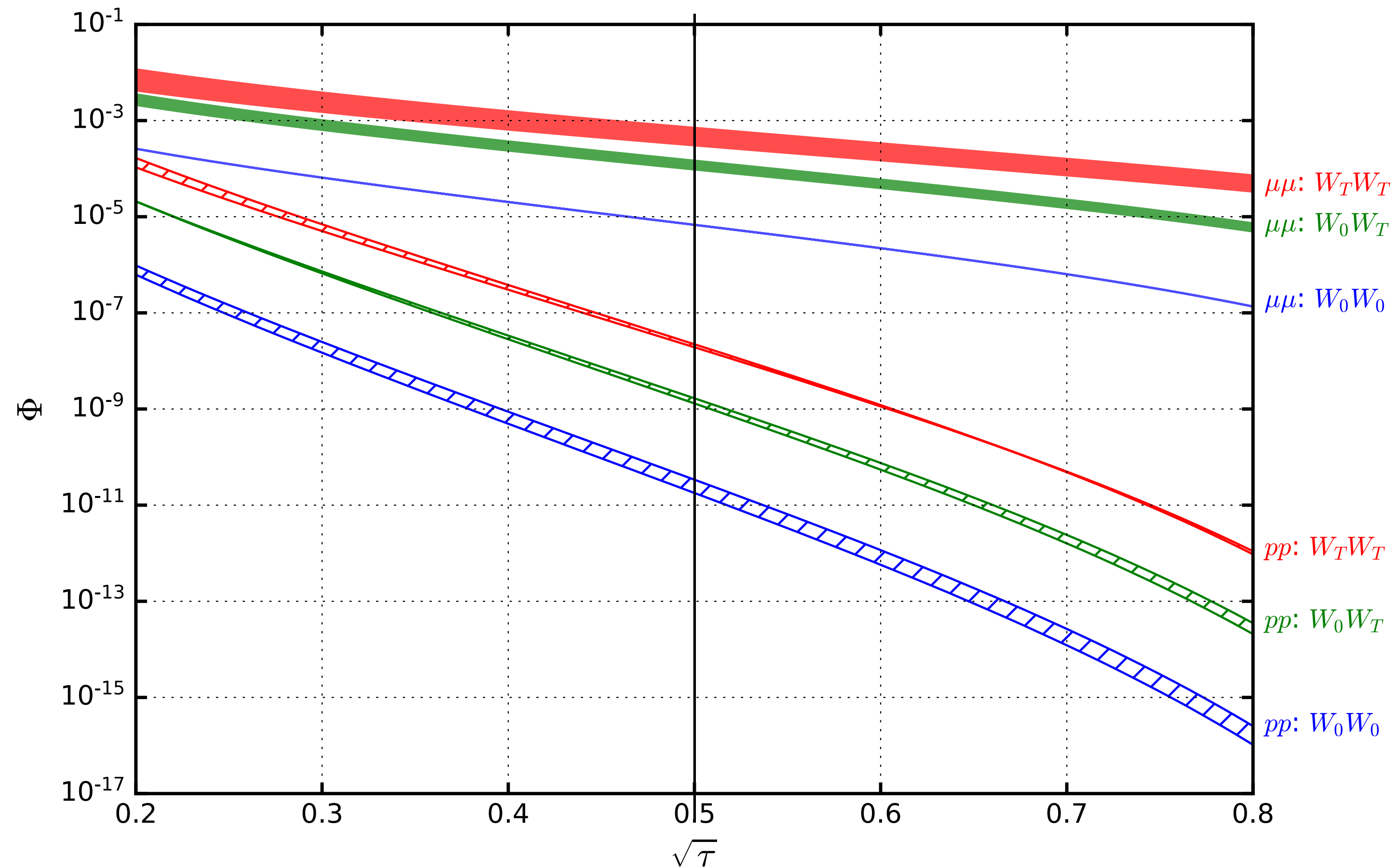
pp vs $\mu\mu$

$$\Phi_{W_{\lambda_1}^+ W_{\lambda_2}^-}(\tau, \mu_f) = \int_{\tau}^1 \frac{d\xi}{\xi} f_{W_{\lambda_1}/\mu}(\xi, \mu_f) f_{W_{\lambda_2}/\mu}\left(\frac{\tau}{\xi}, \mu_f\right)$$

This plot can be used in any case, but it is particularly simple when considering a muon-collider in the same ring of a proton collider, $\sqrt{s}_{\mu\mu} = \sqrt{s}_{pp}$.

For 2->1, let's take for example $\sqrt{\tau} = \frac{M}{\sqrt{s}} = \frac{1}{2}$

the luminosity ratio $\mu\mu/pp$ is larger than 10^4 !



W,Z, γ as partons

$\gamma\gamma$ initial state

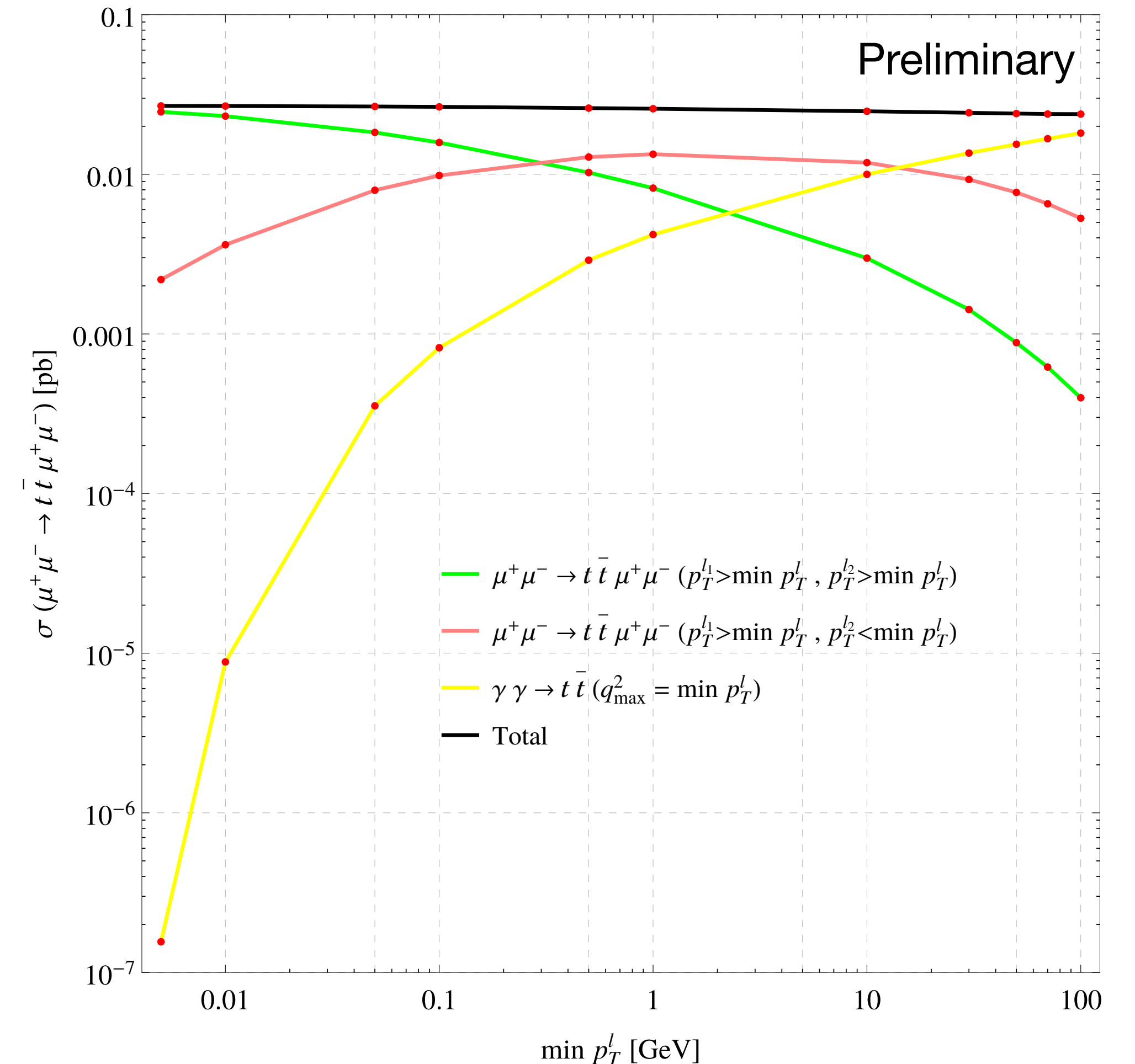
Neutral final states can be obtained from quasi-real IS photons. There are now two cases:

- 1) The final state leptons are detected (min p_T , max η) then q^2 of the photon is sizable $\Rightarrow \mu^+\mu^- \rightarrow X\mu^+\mu^-$ calculation.
- 2) The final state leptons are “lost” \Rightarrow equivalent photon approximation via Weizsacker-Williams PDF:

$$f_{\gamma}^{(\ell)}(y) = \frac{\alpha_{\text{em}}}{2\pi} \left[2m_{\ell}^2 y \left(\frac{1}{q_{\text{max}}^2} - \frac{1}{q_{\text{min}}^2} \right) + \frac{1 + (1 - y)^2}{y} \log \frac{q_{\text{min}}^2}{q_{\text{max}}^2} \right]$$

Note that the two approaches can be used together if q_{max}^2 is matched.

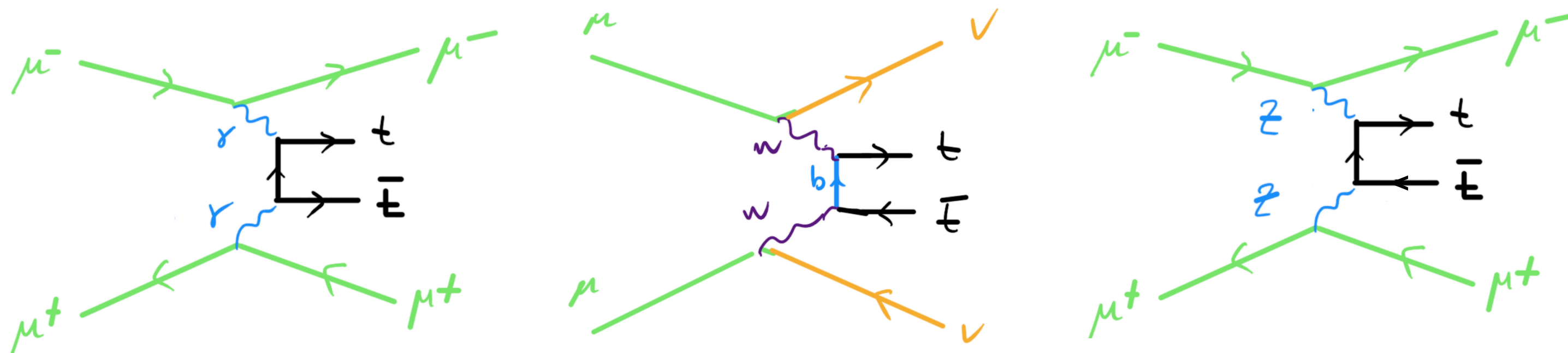
μ Collider @ 14 TeV – γ Boson Fusion



W,Z, γ as partons

$\gamma\gamma$ initial state

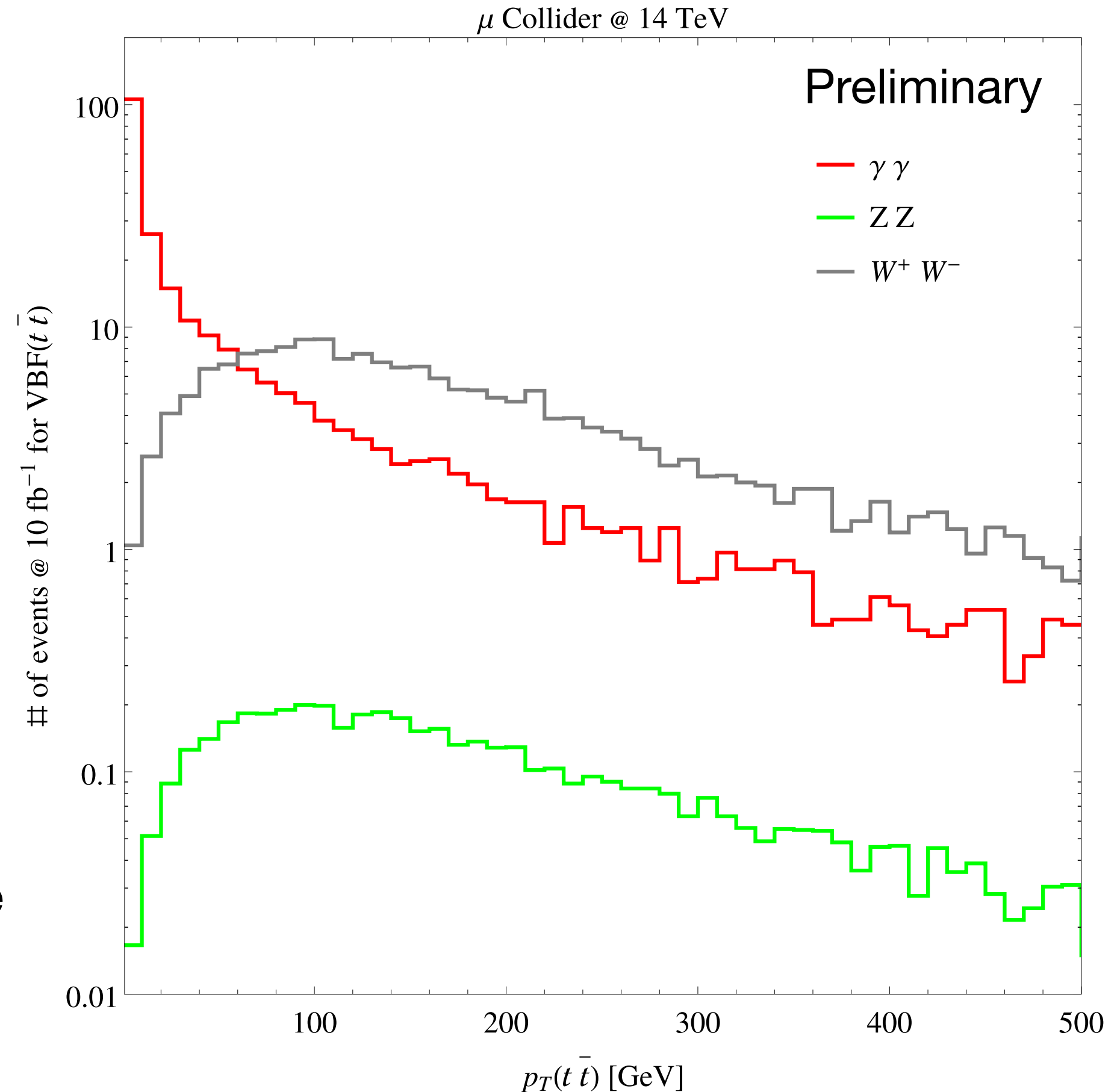
These effects are large for charged final states. A factor 5 for WW, and a factor 2 for tt [[Han, Ma, Xie, 2007.14300](#)].



$$\sigma_{\gamma\gamma}(t\bar{t}) = 23 \text{ fb} \quad \sigma_{WW}(t\bar{t}) = 21 \text{ fb} \quad \sigma_{ZZ}(t\bar{t}) < 1 \text{ fb}$$

However, $\gamma\gamma$ and WW channels can be easily separated by looking at the $p_T(X)$ of the produced system X : low p_T is dominated by $\gamma\gamma$.

Could be interesting for accessing $\gamma\gamma \rightarrow H$ production/couplings? Cross section O(fb)

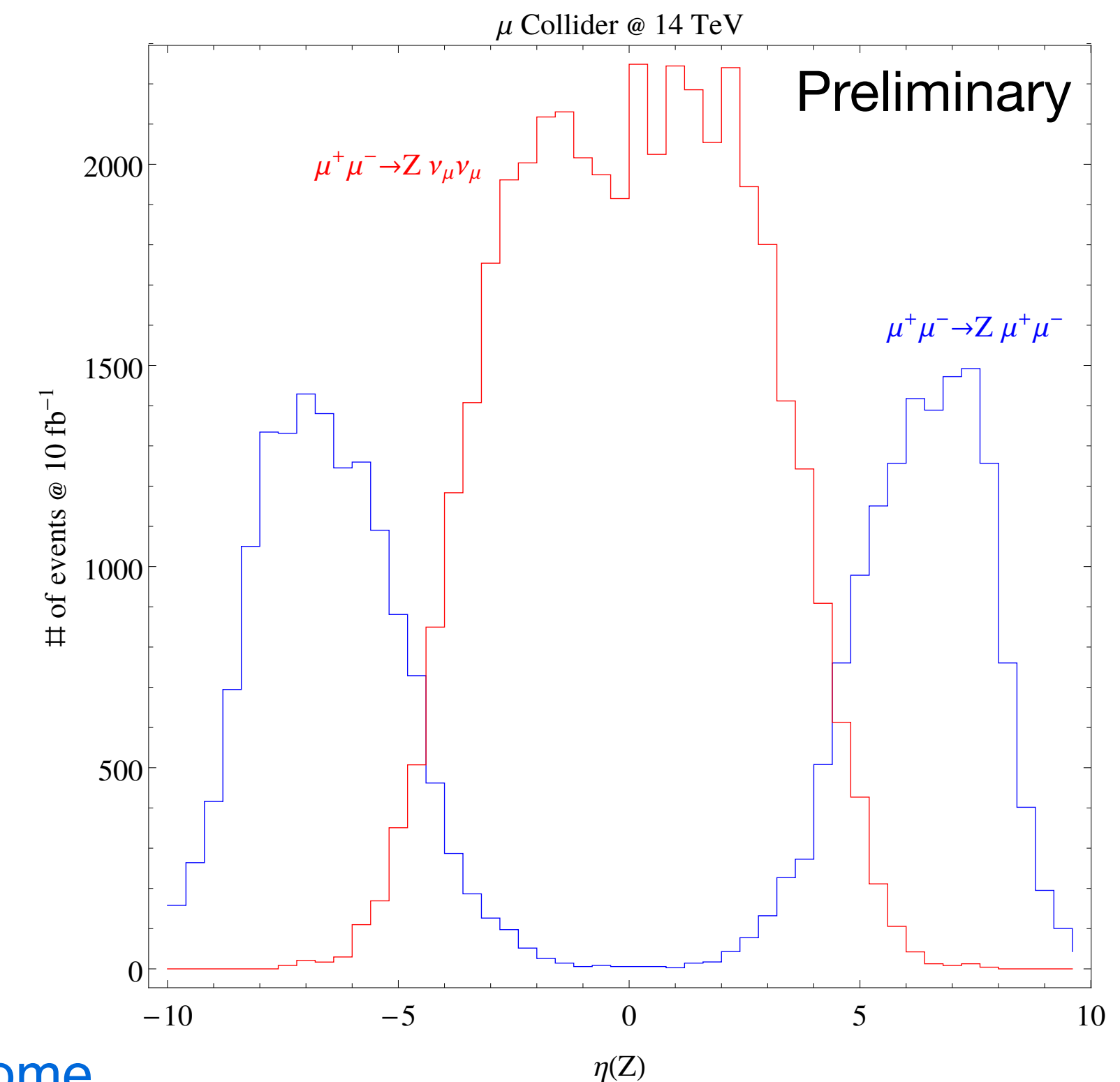
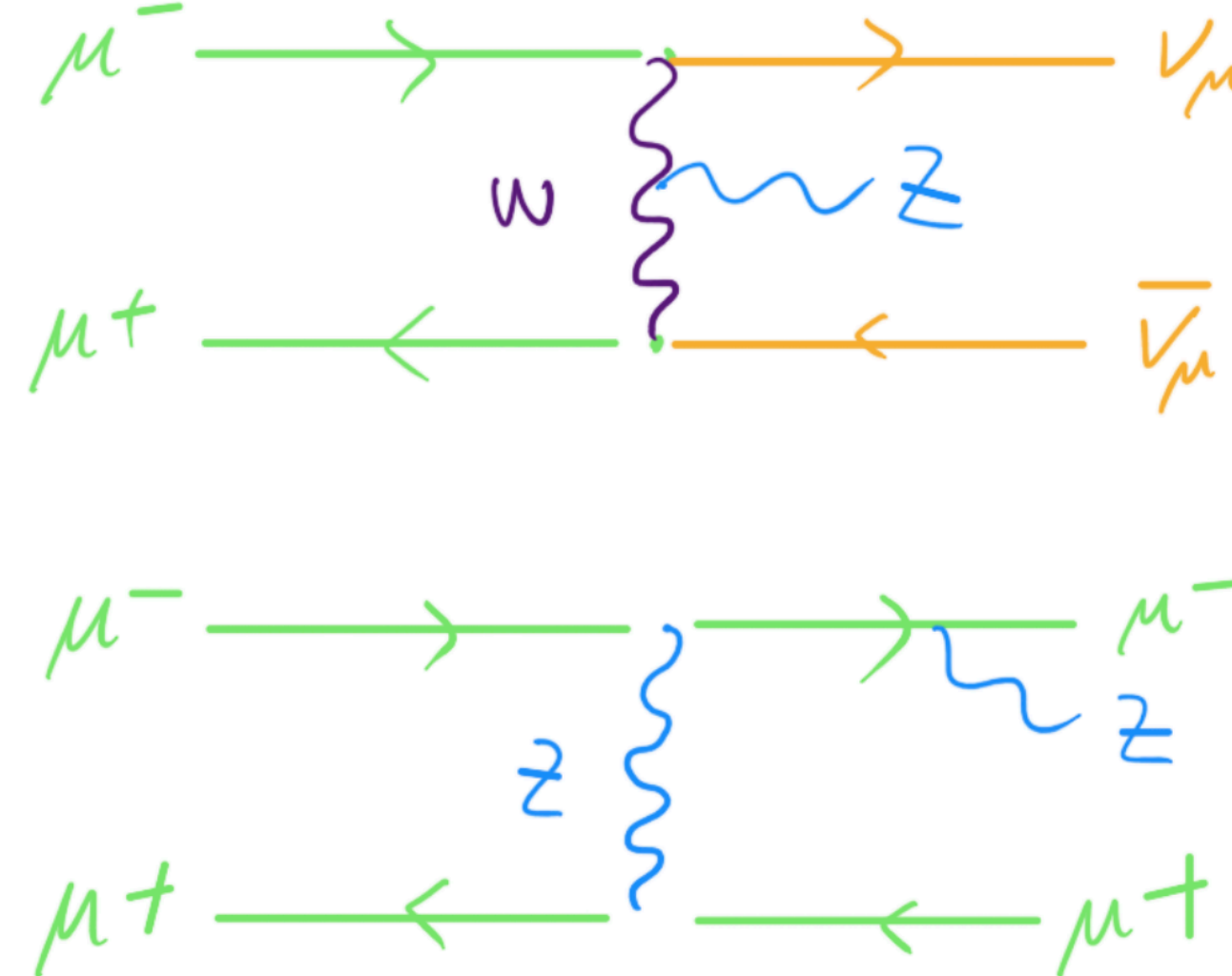
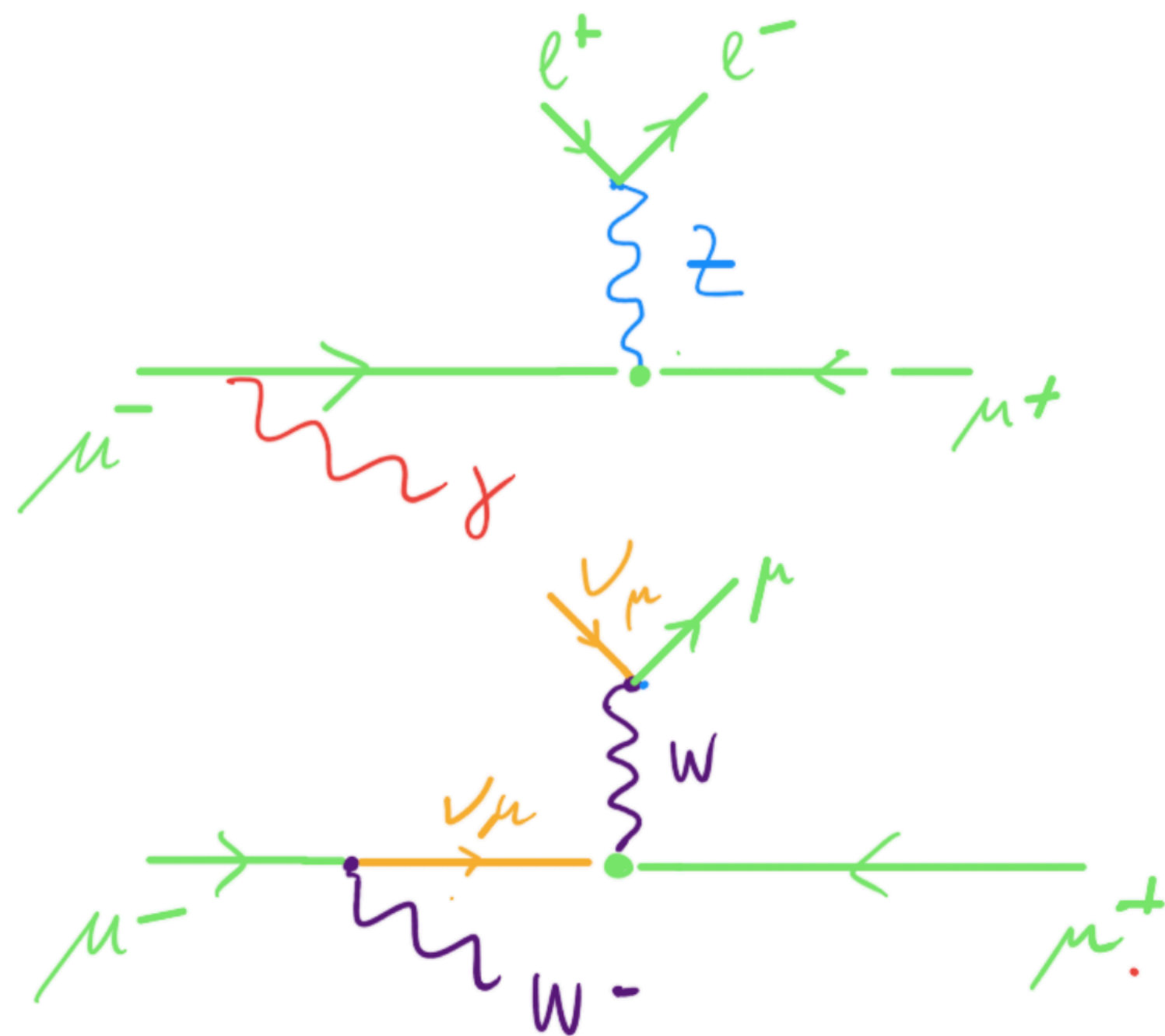


Vector boson radiation

Initial and final radiation

Initial State radiation => radiate return of QED as well as Weak nature: $2 \rightarrow 3$ process factories in to $P_{l \rightarrow V l'} \cdot M(2 \rightarrow 2)$

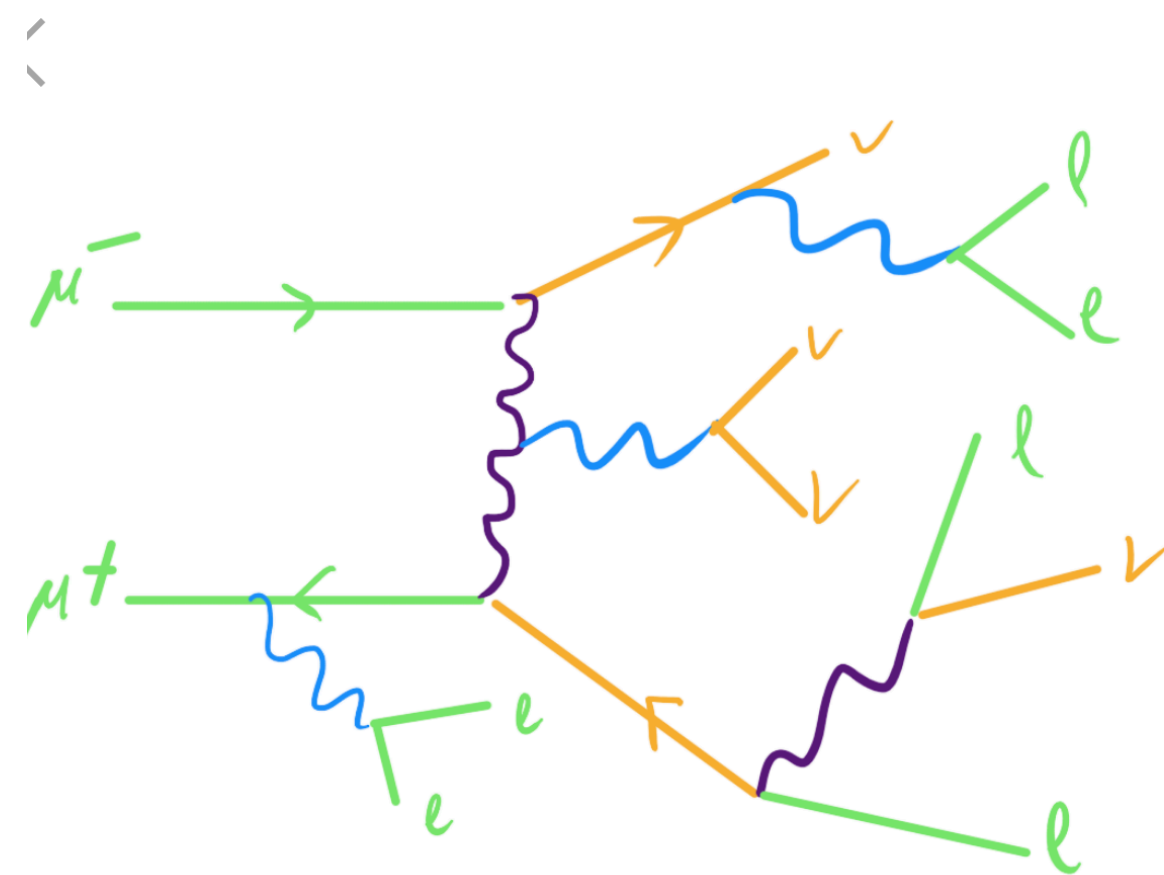
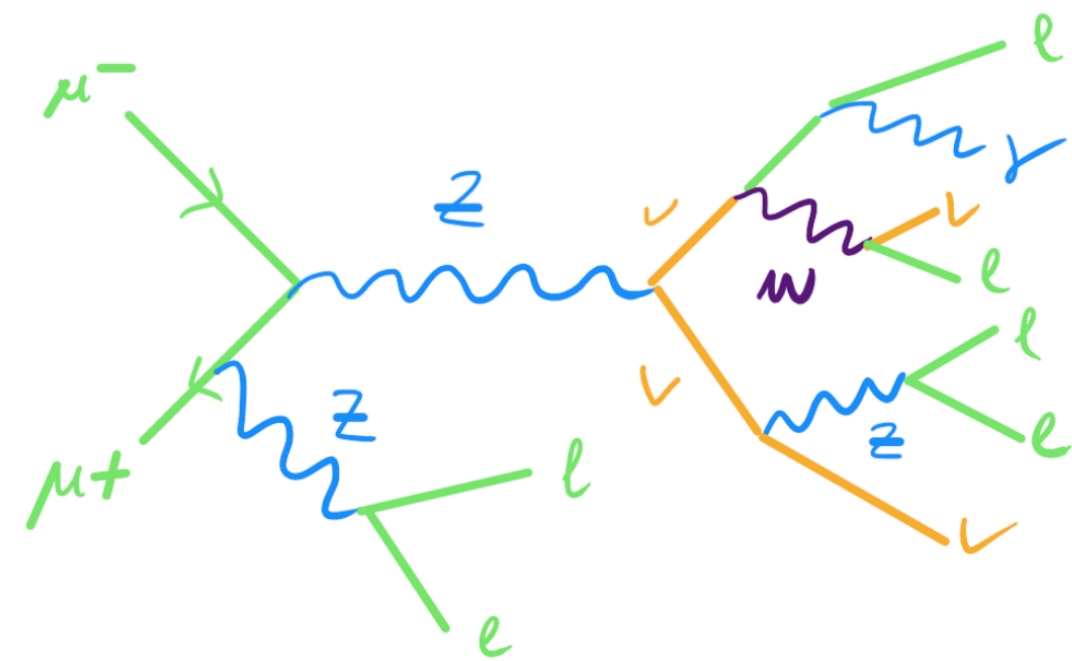
Final State radiation patterns can be very different depending on the actual charge flow. For example in VBF, NC vs CC:



[Systematic study of dynamics of multi-boson production welcome](#)

Vector boson radiation

EW resummation



Various approaches available in the literature:

Using SCET:

[Chiu, Golf, Kelley, Manhoar, 0709.2377, 0712.0396, 0806.1240, 0909.0012, 0909.0947]

Using DGLAP:

[Bauer, Webber et al. 1703.08562, 1712.07147, 1806.10157, 1808.08831, 2007.15001]

Parton showers:

[Christiansen and Sjostrand 1401.5238]

[Kleiss and Verheyen, 2002.09248]

EW splitting functions

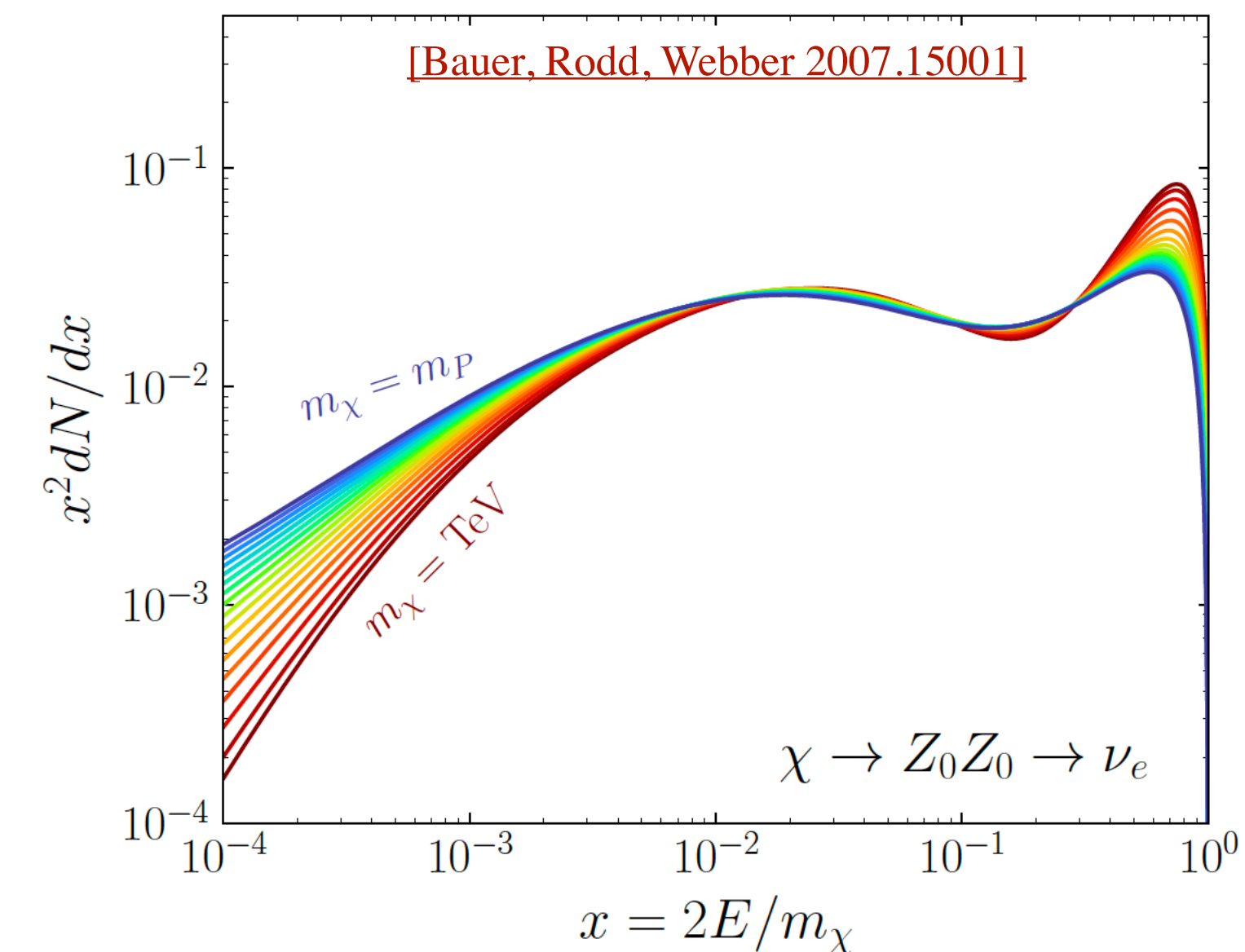
[Chen, Han, Tweedie, 1611.00788]

New gauge formulation

[Cuomo, Vecchi, Wulzer, 1911.12366]

EW PDF at muon collider

[Han, Ma, Xie, 2007.14300]



See a first discussion in Snowmass TF07: <https://indico.fnal.gov/event/45400/> : more to come!!

Available tools

MonteCarlos for muon colliders

- Event generation at LO based on matrix elements available (e.g. MadGraph and Whizard) for annihilation

$$- \mu^+ \mu^- \rightarrow X$$

and VBF

- $\mu^+ \mu^- \rightarrow X + \nu_\mu \bar{\nu}_\mu$ W·W fusion
- $\mu^+ \mu^- \rightarrow X + \mu \bar{\mu}$ Z/ γ^* ·Z/ γ^* fusion
- $\mu^+ \mu^- \rightarrow X + \nu_\mu \mu$ W·Z/ γ^* fusion (charged)

- BSM scenarios including EFT available through FeynRules.

Recent Examples:

- [2008.12204](#) * Han et al. (MadGraph)
- [2006.16277](#) * Capdevilla et al. (MadGraph)
- [2005.10289](#) * Costantini et al. (MadGraph)
- [2003.13628](#) * Chiesa et al (MadGraph and Whizard)
- [2002.12218](#) * Kumar et al. (MadGraph)
- [2001.04431](#) * Bartosik et al. (Pythia8)
- [1910.04170](#) * Ruhdorfer et al (MadGraph)
- [1810.10993](#) * Di Luzio et al. (by hand)
- [1807.04743](#) * Buttazzo et al. (MadGraph)
- more...

Precision SM

NLO EW and QCD

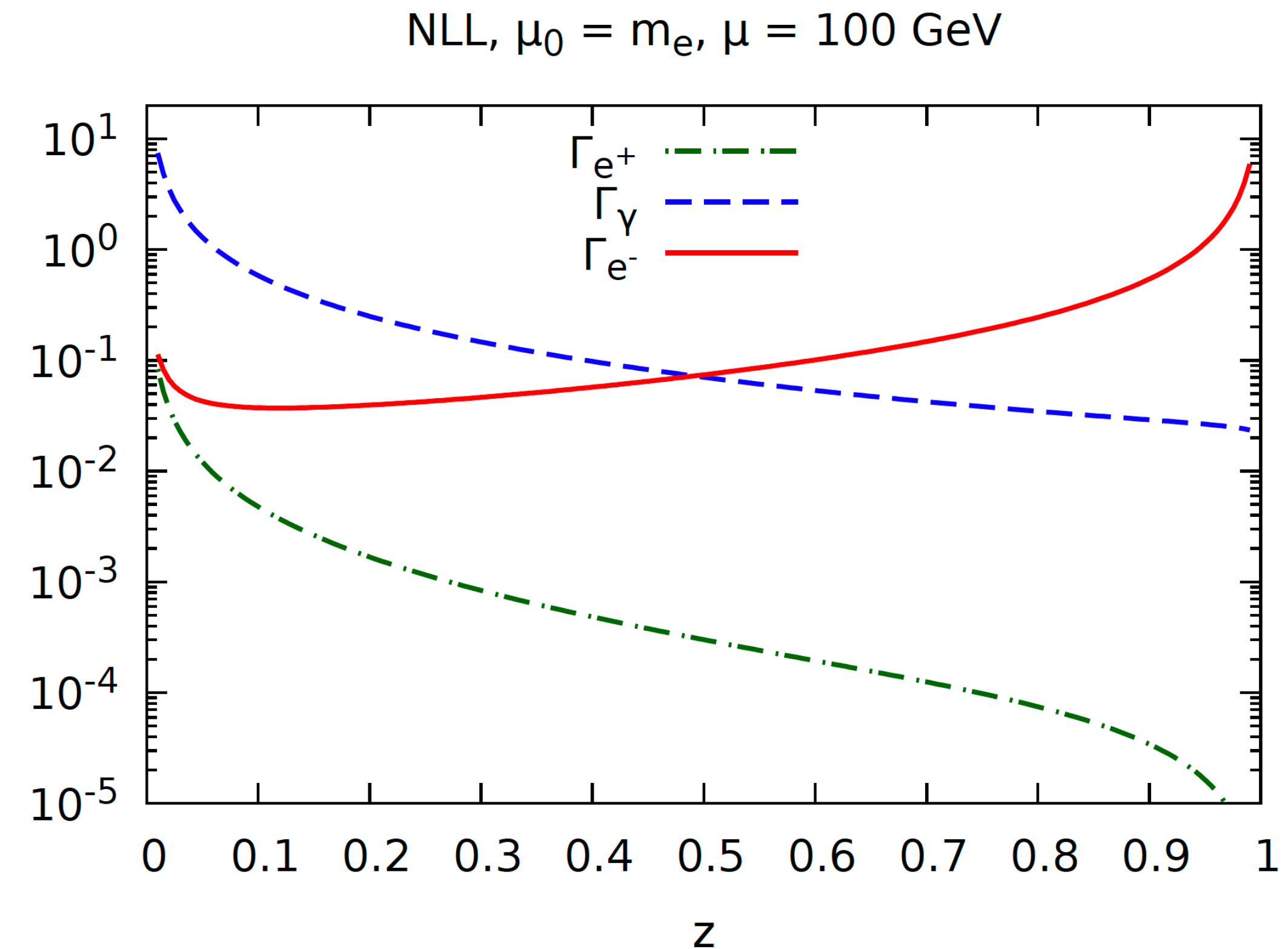
One-loop computations in the SM, QCD and EW are now available in an automatic form (see, e.g., MG5aMC@NLO [\[Frixione et al., 1804.10017\]](#) or OpenLoops [\[Pozzorini et al., 1907.13071\]](#)) and have been extensively used for predictions at the LHC.

Matching with EW+QED PS only available for specific processes.

Recent work has made NLL PDF for the e^- , e^+ , γ available. This opens the way to use the current packages available for automated calculations of EW corrections (for SM processes) at pp, also for e^+e^- colliders. Extension of the same approach to the computation of EW 1-loop effects in $\mu^+\mu^-$ possible.

Automatic Merging of NLO with QED+QCDPS needed.

At very high energy possibly merging with complete EW shower?



[\[Frixione, 1909.03886\]](#) [\[Bertone et al, 1911.12040\]](#)

Precision SM

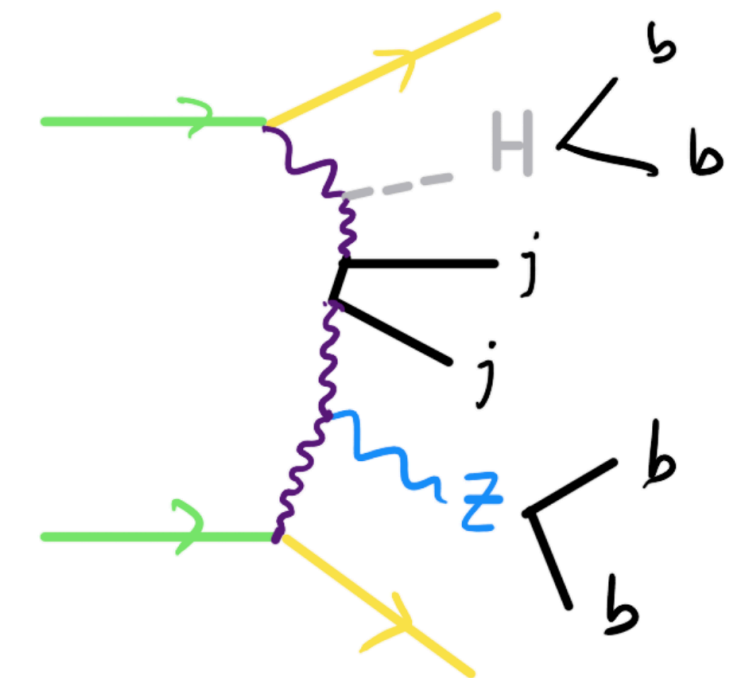
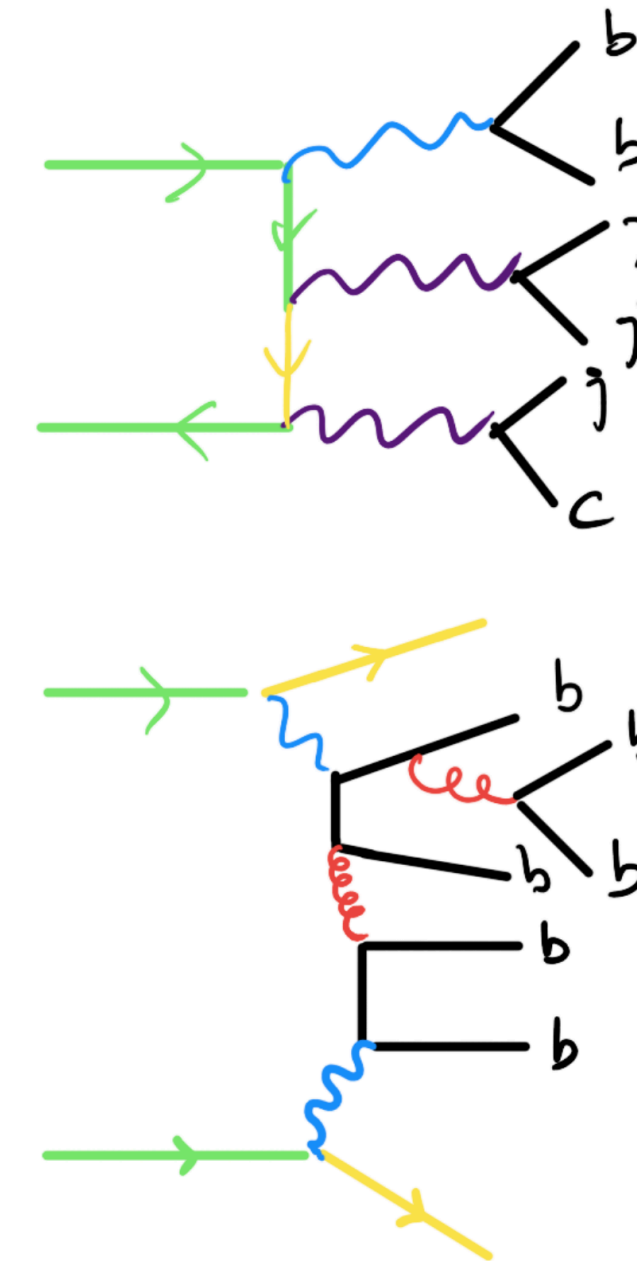
Multi-parton final states : signal and backgrounds

Given the large energy available in the cms, multi-particle final states will be possible and backgrounds from multipartons will arise.

For example, $t\bar{t}$ production can lead to 6-jet final states, with several flavor tags. Another example are searches for HH and HHH final states that give rise to 4 and 6 b-jet final states, to which many final state can contribute (including H+4jets+vv)

Optimised Matrix Element generators (such as Alpha or Comix) could be used needed to handle such large multiplicities on top of MadGraph and Whizard.

In addition, merging different multiplicities, as well interfaces to QED+QCD/PS will be needed.



BSM explorations

Example #1: resonant BSM

(a) Singlet production

(b) HZ in 2HDM

(c) $\tilde{t}\tilde{t}^*$

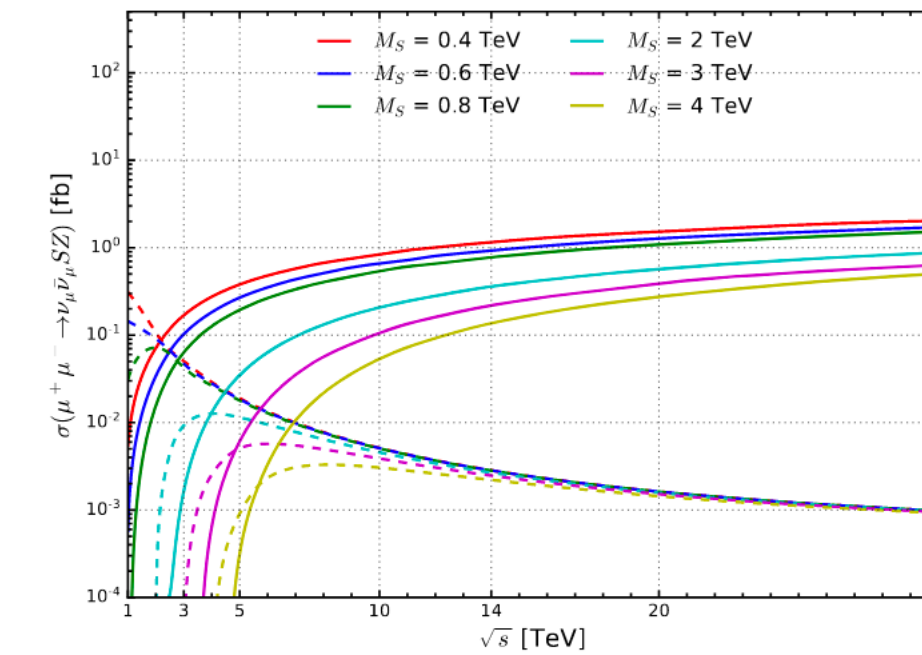
(d) $t'\bar{t}'$

(e) $\tilde{\chi}^0\tilde{\chi}^0$

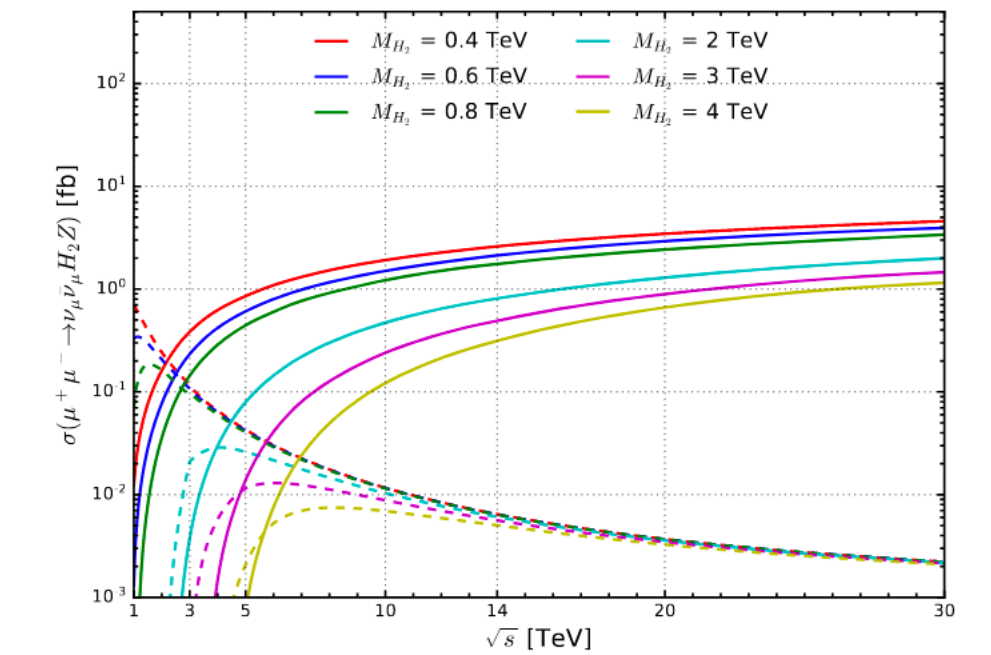
(f) $\chi^+\chi^-$

Sample of BSM processes explored s-channel vs t-channel in specific scenarios. Just scratched the surface. Proof of principle: no technical problems encountered.

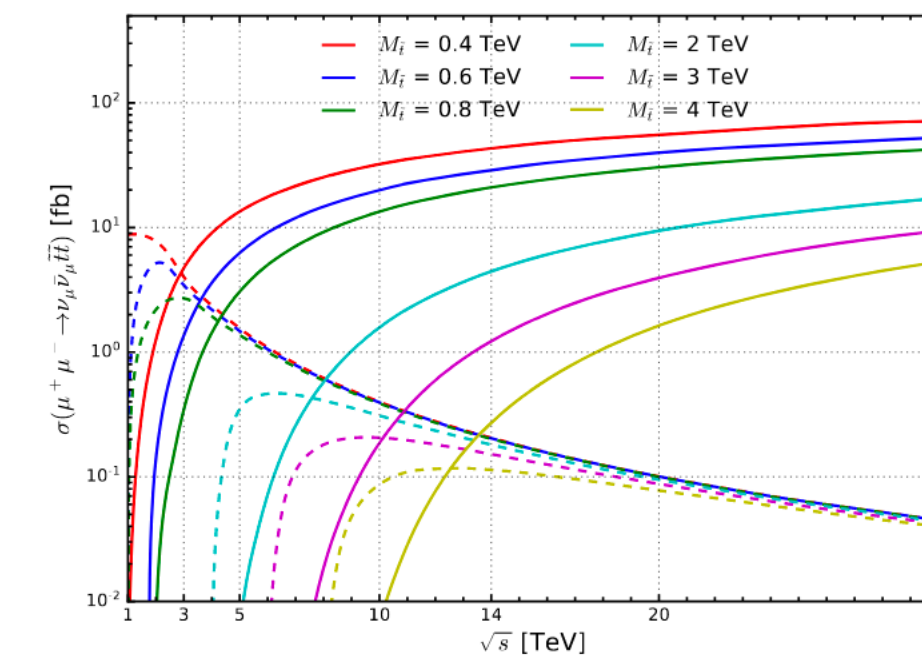
Thorough analysis based on more representative BSM scenarios welcome. Study cases to be identified and used as “official” benchmarks.



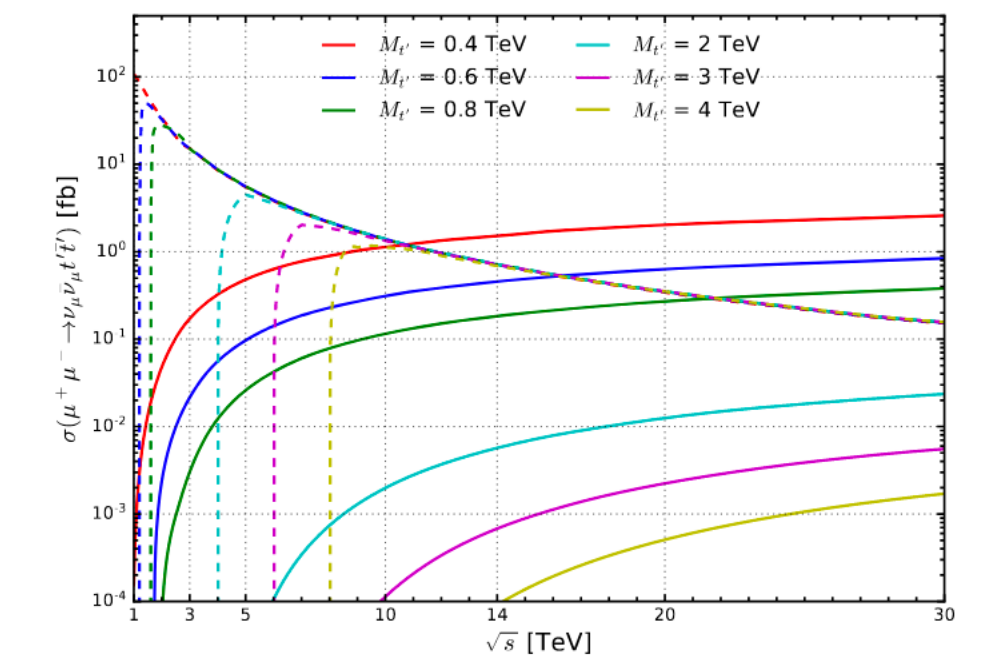
(a)



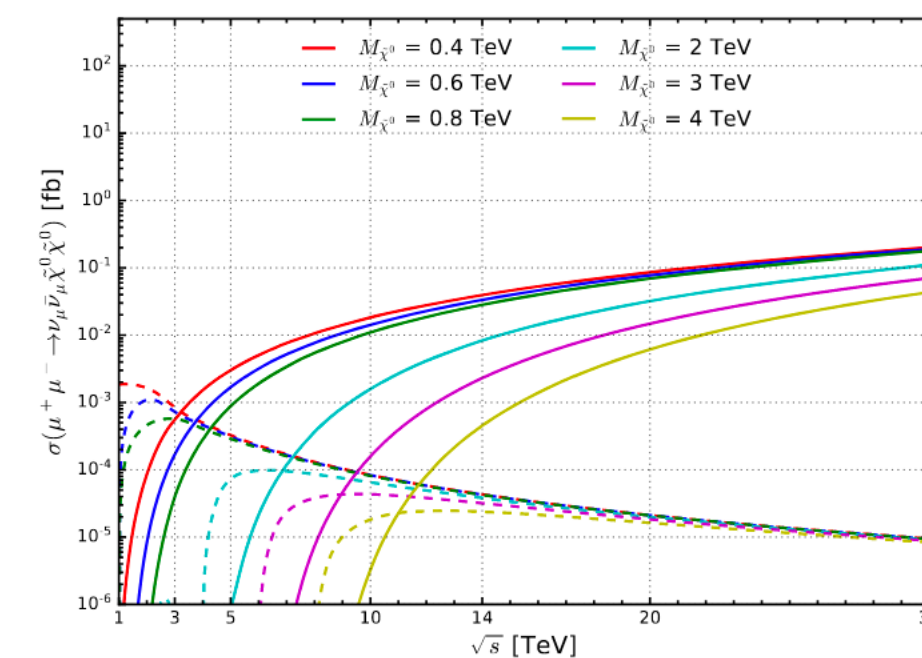
(b)



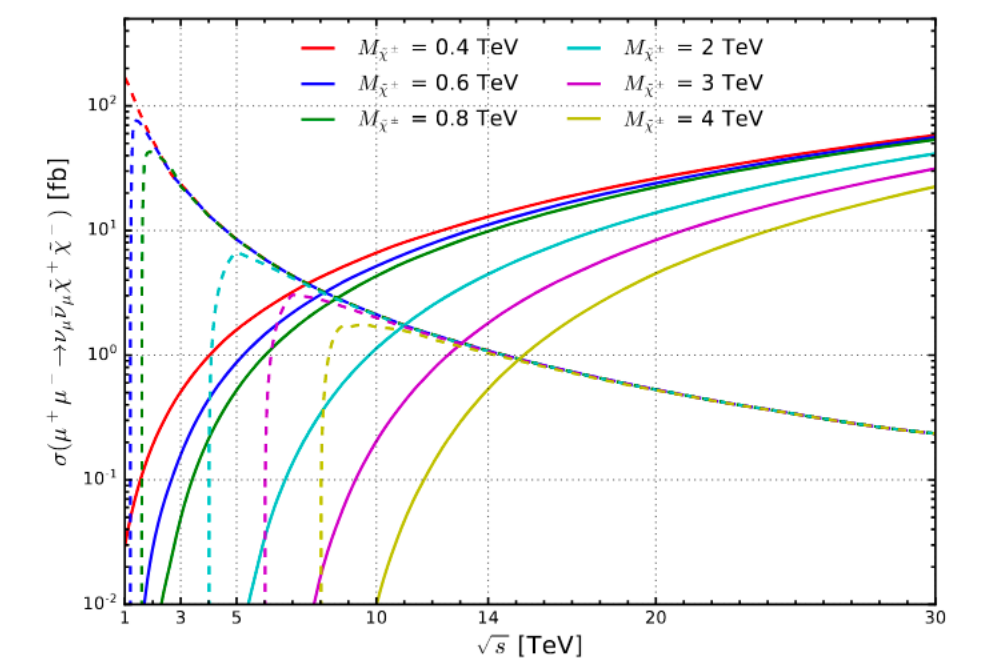
(c)



(d)



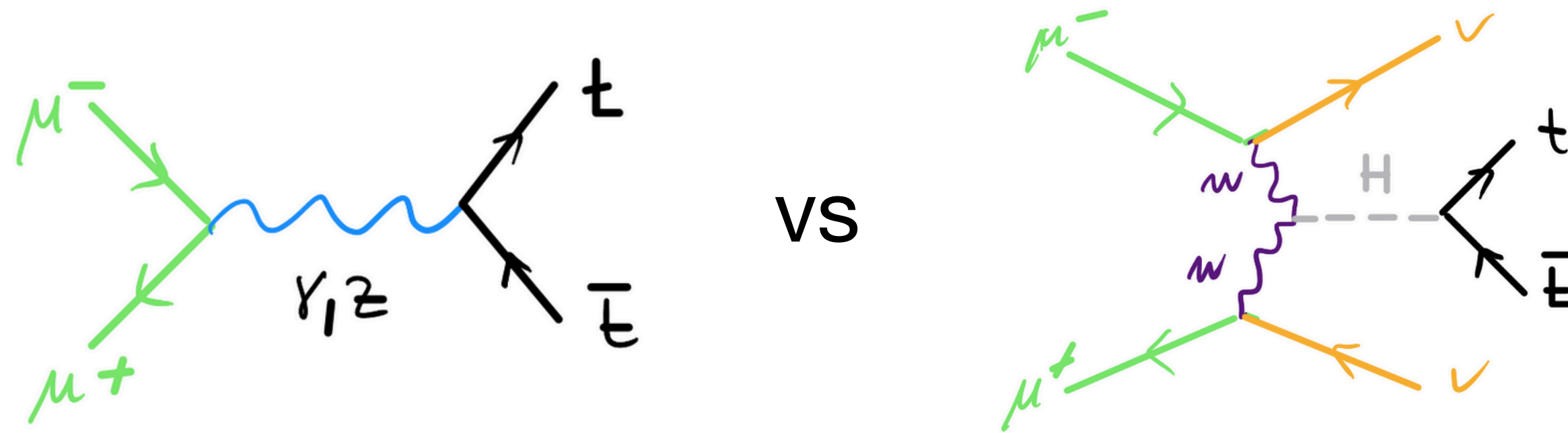
(e)



(f)

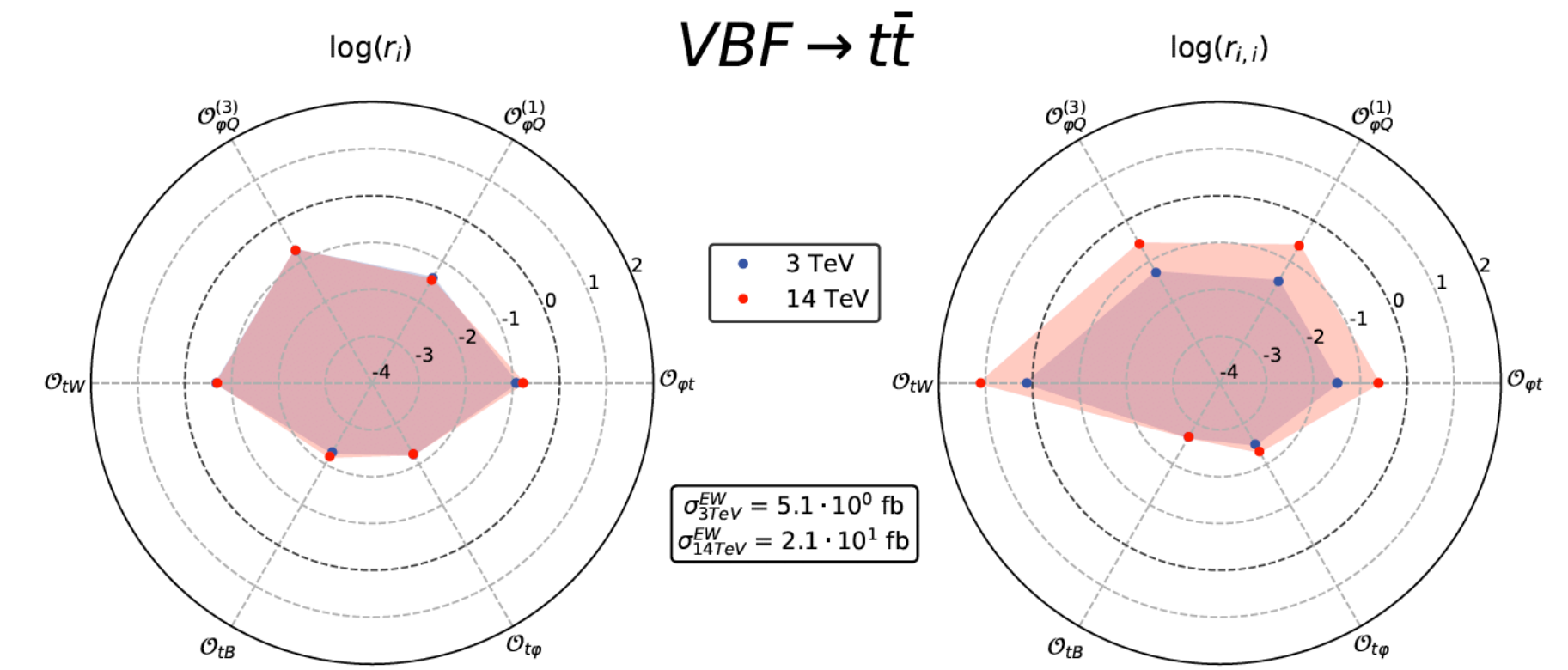
BSM explorations

Example #2: EFT

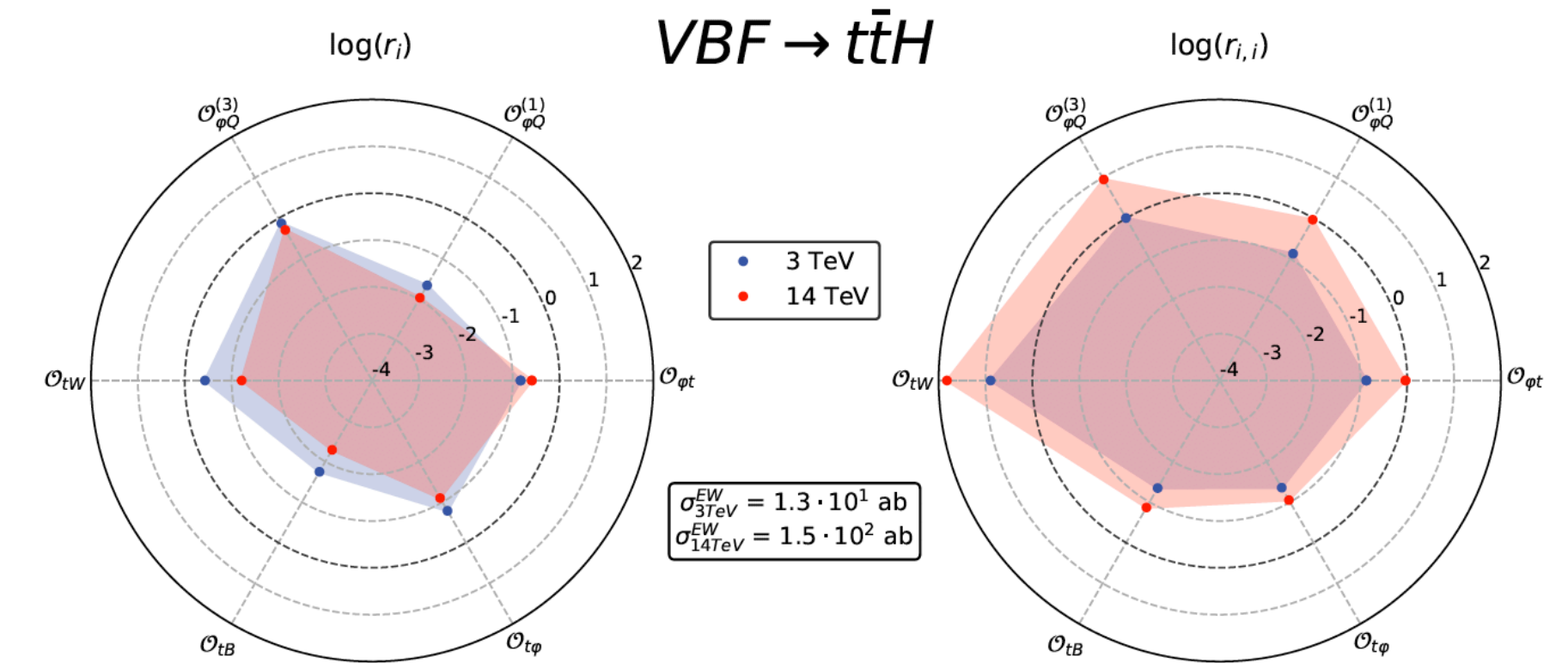


Annihilation can probe $t\bar{t}\gamma, t\bar{t}Z$ couplings at very high energy with a large statistics.

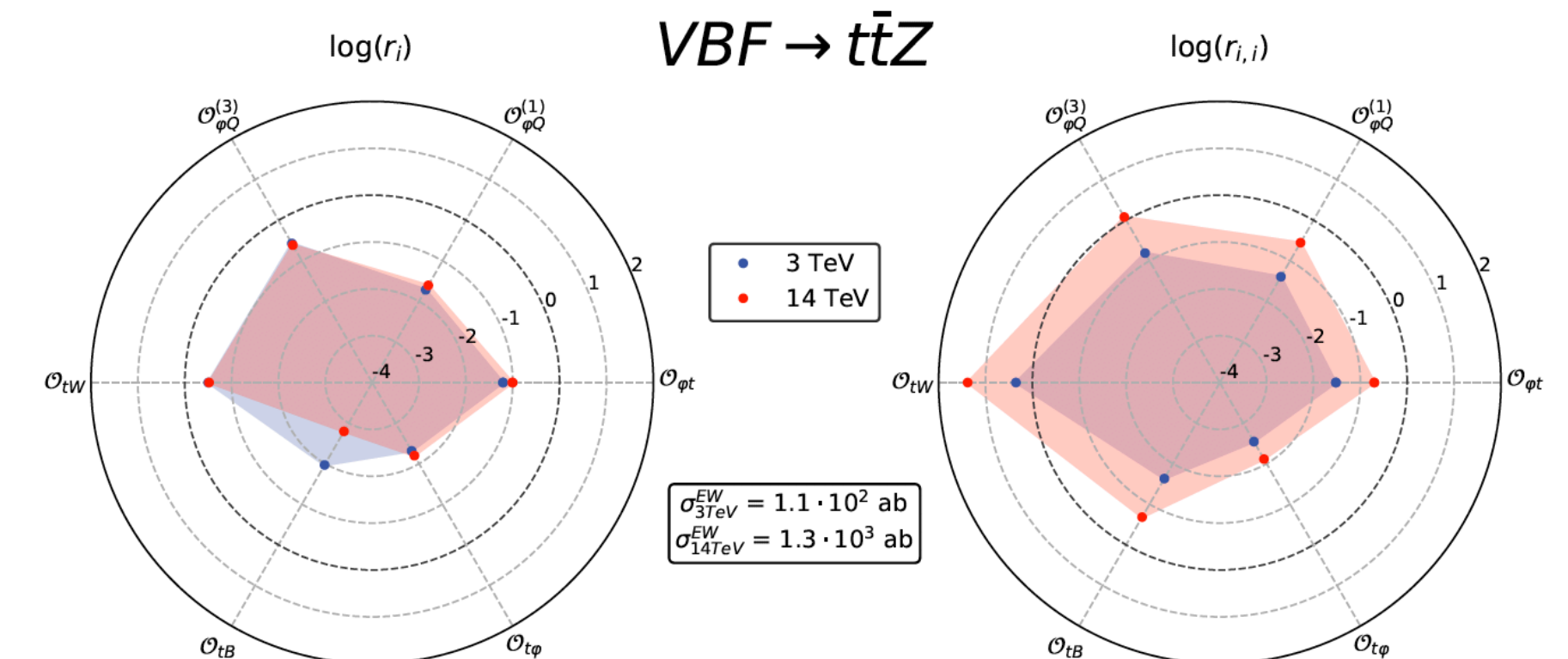
However, VBF can access a much larger set of dim=6 operators with very interesting energy-growth behaviors (net with lower statistics).



(a)



(b)



Outlook

The challenges ahead

- A wide set of predictions for a muon collider can be obtained already with the present tools at the tree-level, both for SM and BSM. While this might be sufficient for this first exploratory phase, better predictions/simulations will be certainly needed.
- Several immediate directions of study/understanding/development:
 1. Exploration of the BSM scenarios that can be accessed at a multi-TeV muon collider (at tree and loop level) and then building of a small library of models and benchmarks.
 2. Inclusion of loops and NLO EW corrections for key SM processes and QED matching for precision measurements and EFT interpretations.
 3. Develop general EW resummation procedures (Soft/IS and FS collinear) and fully exclusive EW/QCD shower initial as well as final state.
 4. Study of backgrounds from multi-particle, multi-jet final states (e.g., resonant vs non-resonant contributions).