# Overview of ionization cooling for a Muon Collider

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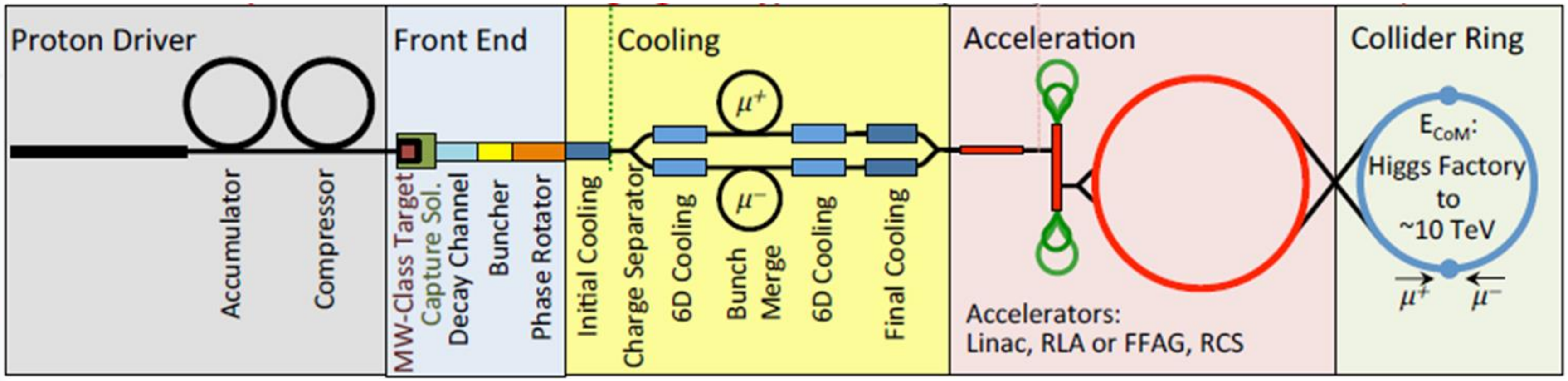
PITT PACC Workshop: Muon Collider Physics (online only)
December 01, 2020

# Outline

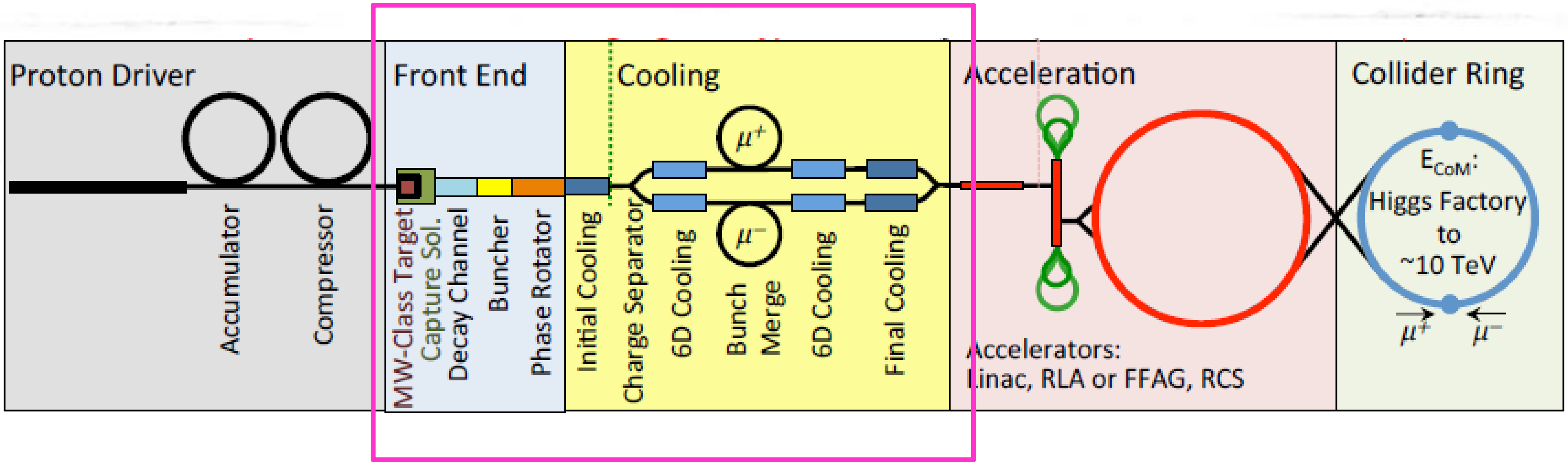
- Muon Collider overview
- Ionization cooling
  - Basic theory
  - Application to a Muon Collider
  - Simulation results
- Future work and possible improvements
- Summary

# Muon Collider components




- The desired 6D emittance for a Muon Collider (MC) is 5-6 orders of magnitude less from the emittance of the muon beam at the production target
- As a result, significant “muon cooling” is required.

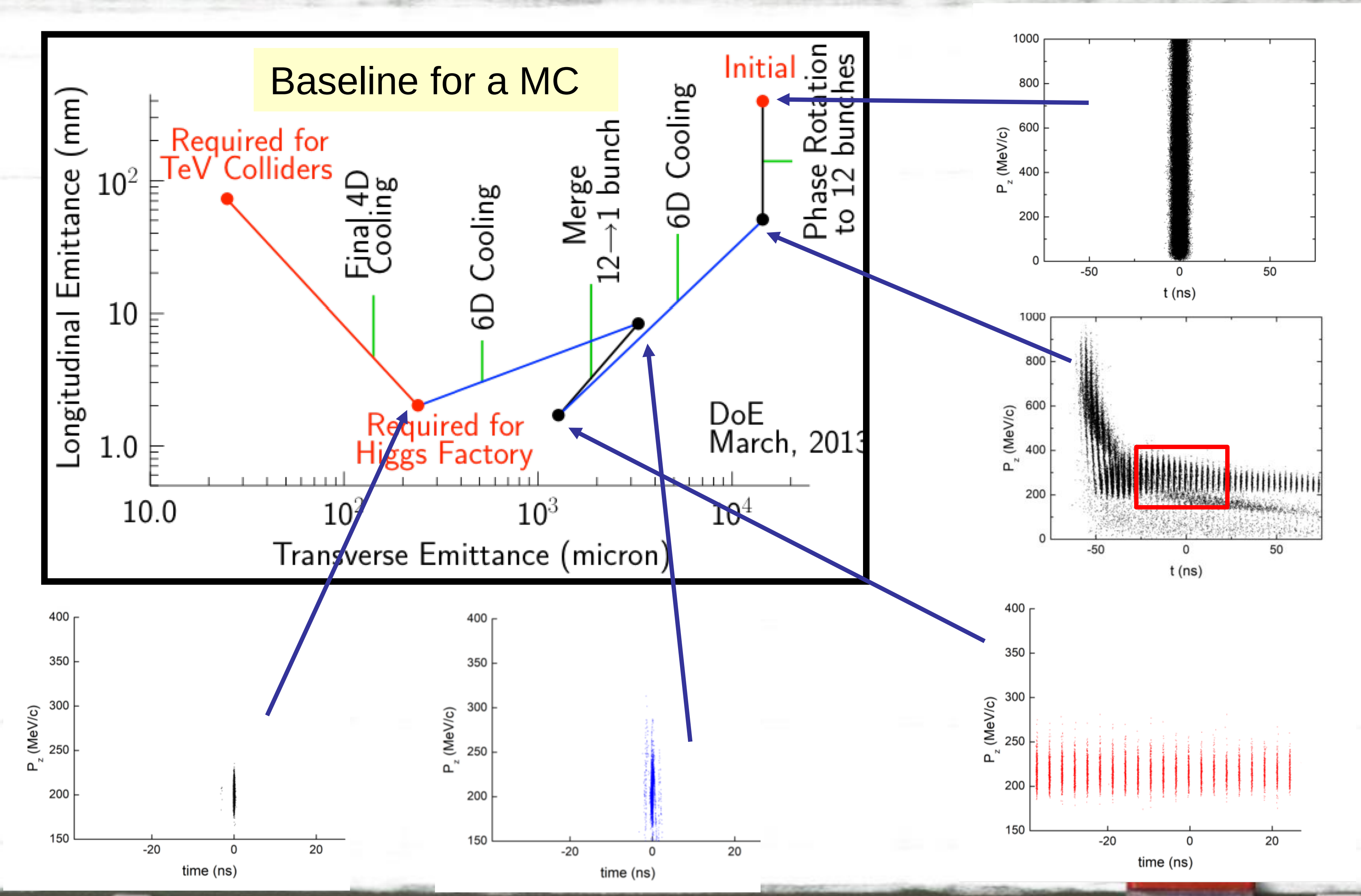
# Cooling scheme for a Muon Collider



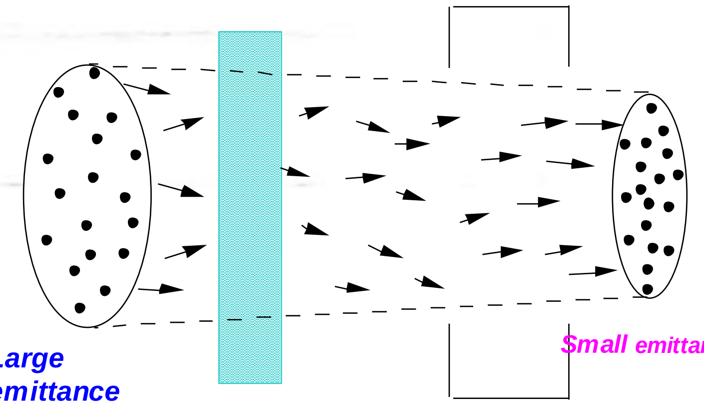

- Front-end produces 21 well aligned muon bunches
- Two sets of 6D cooling schemes
  - One before recombination (trans ε≈1.5 mm)
  - One after recombination (trans ε≈ 0.30 mm or less)
- Final cooling (if necessary)

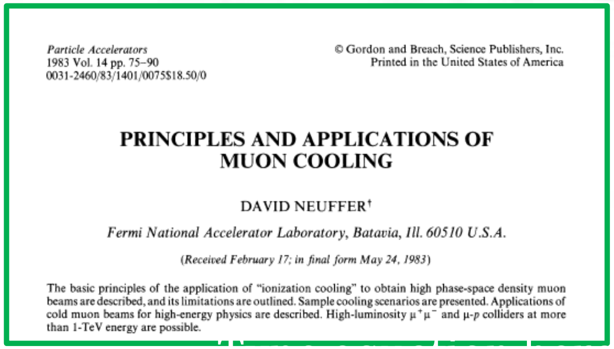
Focus of this talk

# Muon Collider cooling baseline



# Ionization cooling formalism (1)



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PRINCIPLES AND APPLICATIONS OF MUON COOLING

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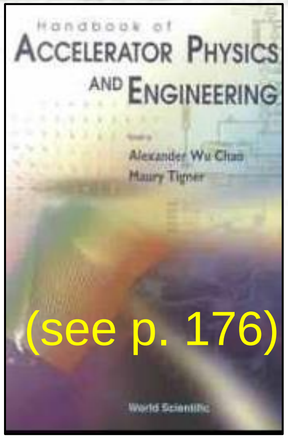
(Received February 17; in final form May 24, 1983)

The basic principles of the application of "ionization cooling" to obtain high phase-space density muon beams are described, and its limitations are outlined. Sample cooling scenarios are presented. Applications of cold muon beams for high-energy physics are described. High-luminosity $\mu^+\mu^-$ and $\mu$-$p$ colliders at more than 1-TeV energy are possible.




Energy loss term

Multiple scattering term

- Transverse cooling:

$$\frac{d\varepsilon_T}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_T + \frac{\beta\gamma\beta_T}{2}\frac{d\theta_0^2}{ds}$$

- Minimum emittance:

$$\varepsilon_T^{\rm eq} = \left(\frac{dE}{ds}\right)^{-1}\frac{\beta_T(13.6\ \mathrm{MeV})^2}{2\beta m_\mu c^2 L_R}$$

$L_R$: Radiation length

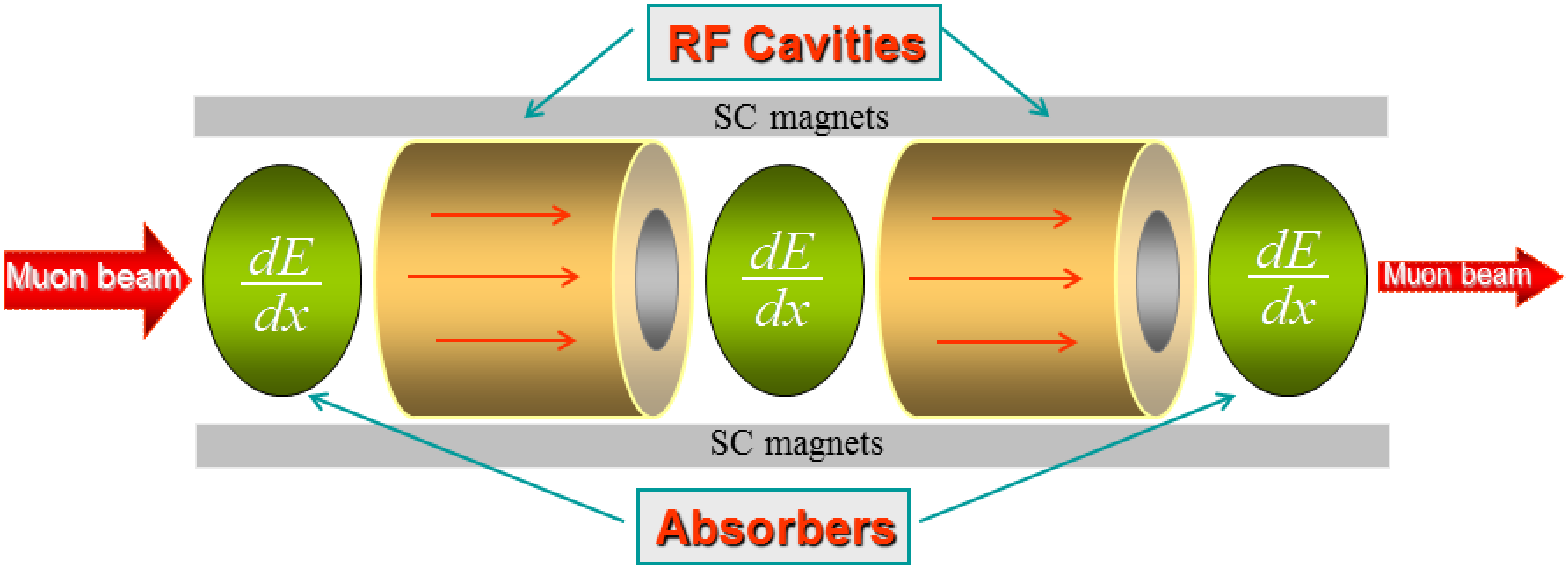
E: Muon energy

$\beta_T$: Transverse beta function

$\frac{dE}{ds}$: Energy loss

- Cooling can be controlled by material and magnetic focusing properties

# Sample of a cooling channel



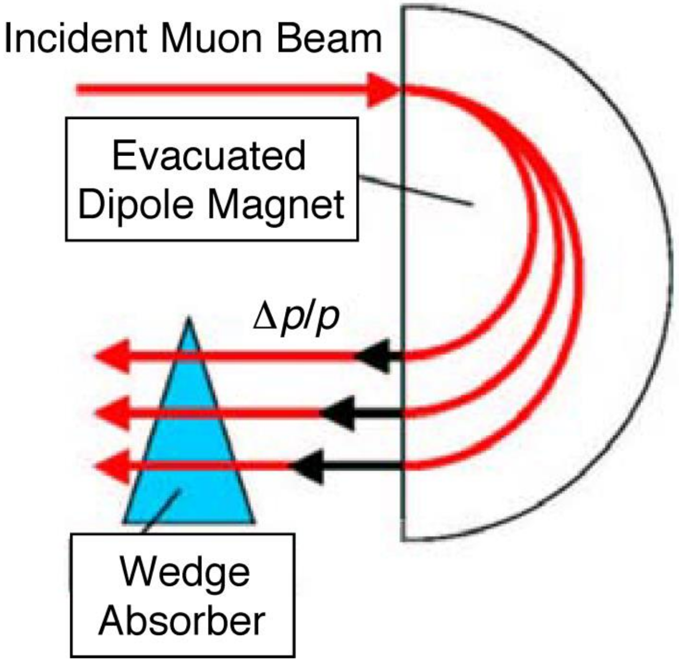

- Energy loss through ionization in the absorbers
- Restore the lost momentum in z with a longitudinal E-field
- A pillbox cavity is placed adjacent to the absorber
- This scheme cools the beam transversely only!

# Ionization cooling formalism (2)

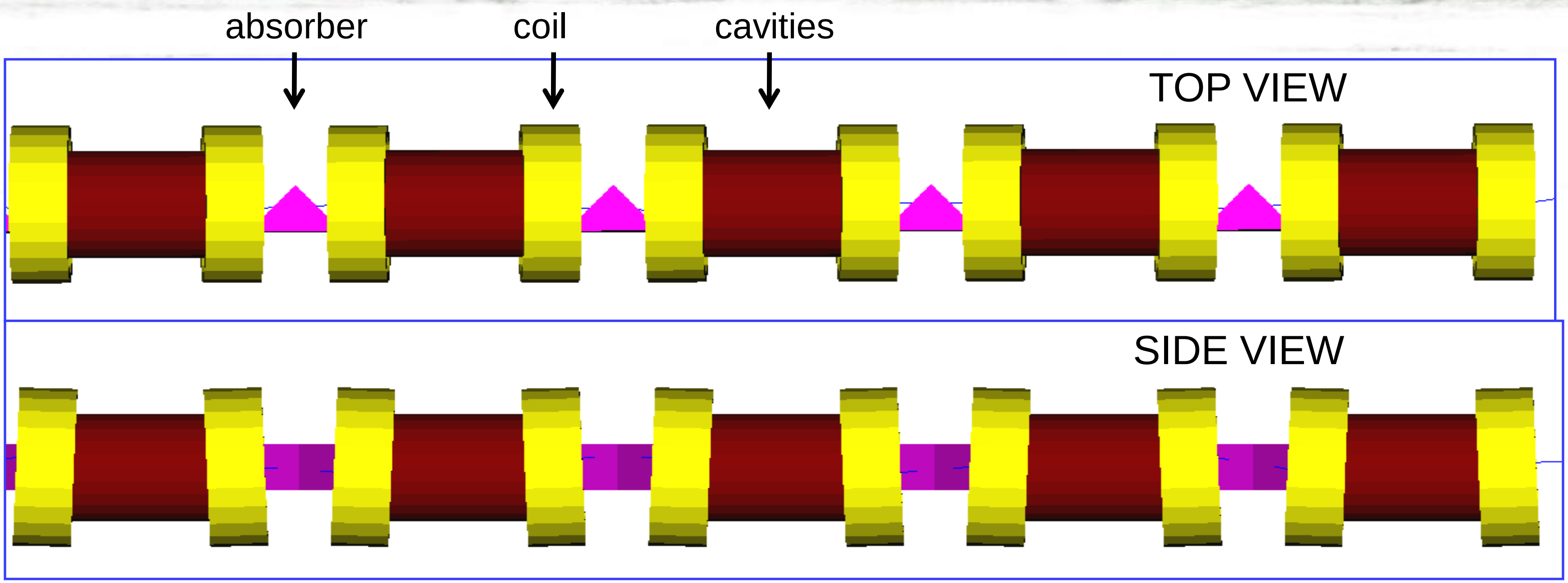
Cooling term — Straggling term

- Longitudinal cooling: $$\frac{d\sigma_E^2}{ds} = -2\frac{\partial\left(\frac{dE}{ds}\right)}{\partial E}\sigma_E^2 + \frac{d < \Delta E_{rms}^2 >}{ds}$$

- Cooling occurs only if derivative: $$\frac{\partial\left(\frac{dE}{ds}\right)}{\partial E} > 0$$
- Ionization loss does not naturally provide adequate longitudinal cooling
- Can be enhanced, if it is arranged that high energy muons lose more energy than low energy ones.



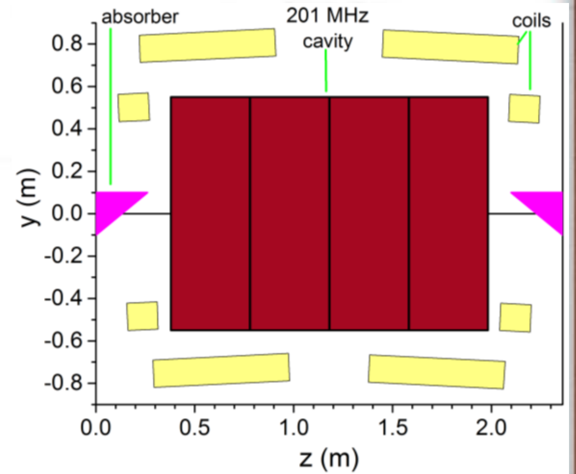
# Cooling channel for a Muon Collider

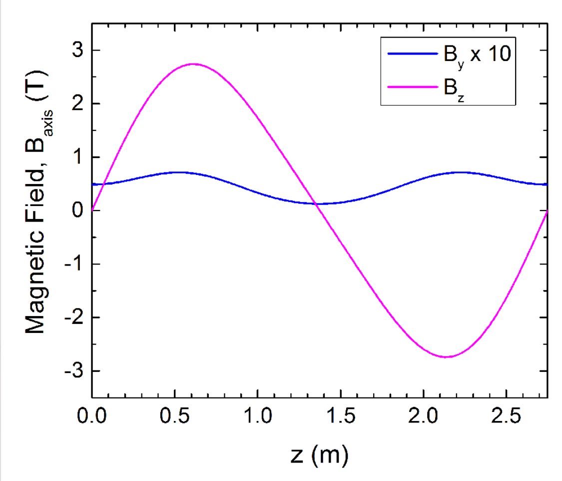


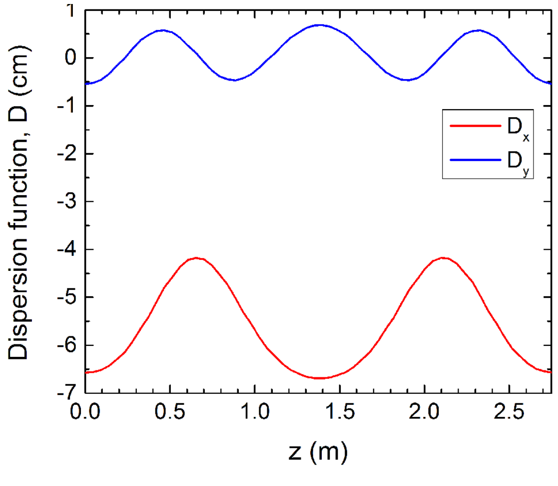

- Straight geometry simplifies construction and relaxes several technological challenges
- Multiple stages with different cell lengths, focusing fields, rf frequencies to ensure fast cooling
- .

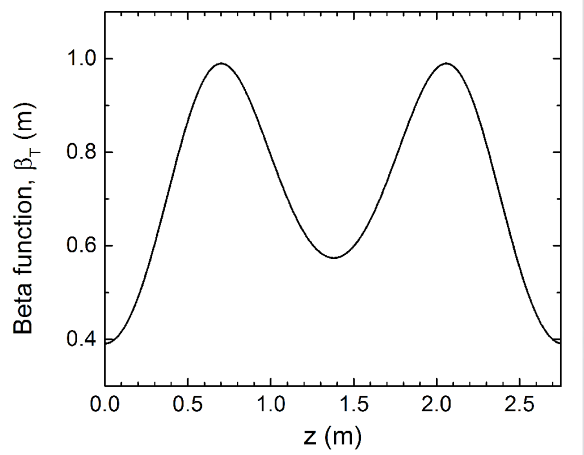
# Cooling channel: How it works

- Coils are slightly tilted to generate a $B_y$ component
- This leads to dispersion, primarily in x.
- 6D cooling on wedge absorber
- Better, if beta is minimum at the absorber

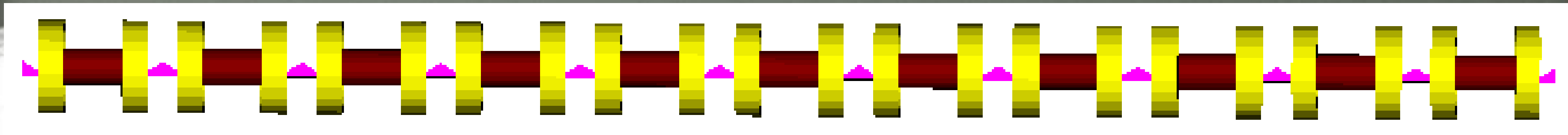


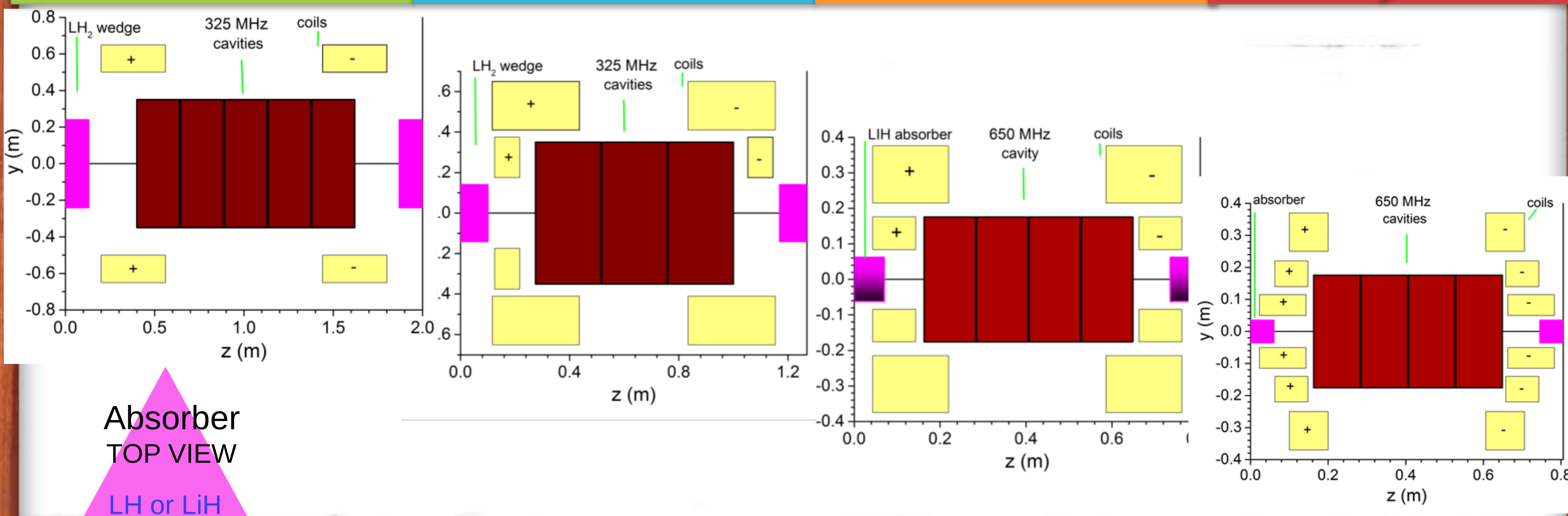



# MC cooling channel (8 stages)



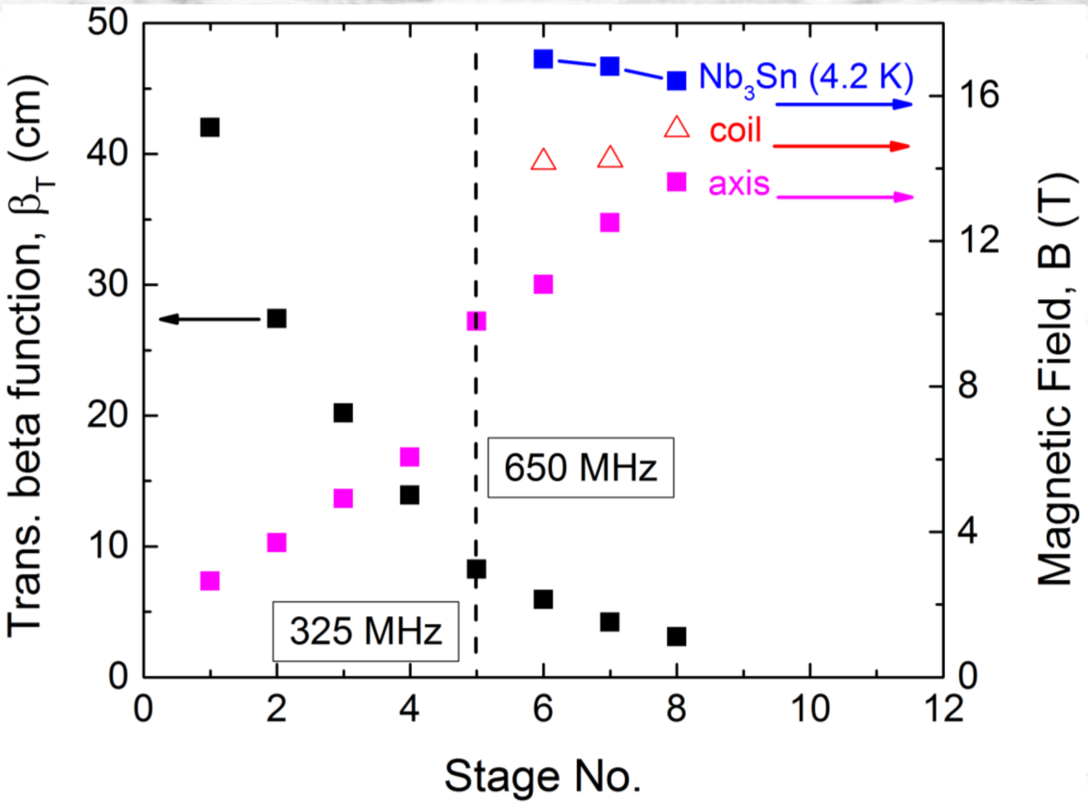
| STAGE 2 | STAGE 4 | STAGE 6 | STAGE 8 |
|---|---|---|---|
| 64 m (32 cells) | 62.5 m (50 cells) | 62 m (77 cells) | 41.1 m (51 cells) |




3.7 T (8.4 T) | 6.0 T (9.2 T) | 10.8 T (14.2 T) | 13.6 T (15.0 T)

Peak B-field on axis (coil)

# Tapered lattice design: 8 stages

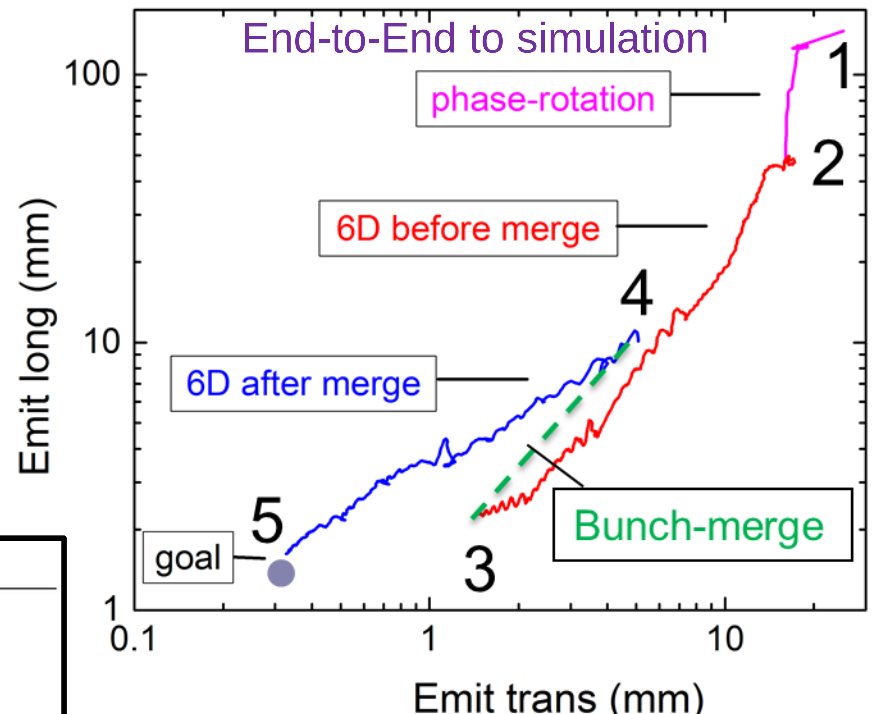


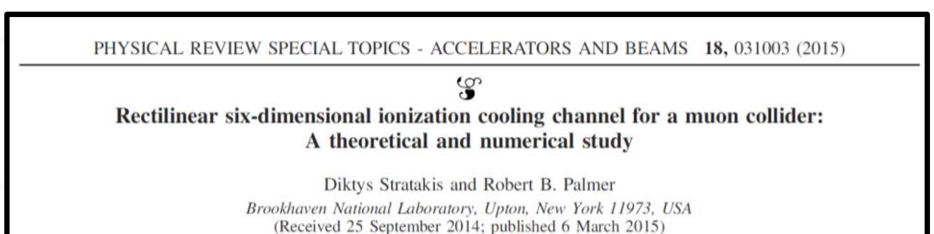

- We set two constrains in our (initial) design:
  - Peak fields on coils don't exceed Niobium Tin limits
  - Cavities within> 1 T operate at 50% of the achievable gradient at 0 T

# Simulation results

- Complete end-to-end simulation from the target (point 1)
- 6D emittance reduction by five orders of magnitude (point 5)
- Achieved emittances and transmissions specified by MAP
- Overall distance ~ 900 m

| Parameters end of cooling channel | MAP Goal | Channel |
|---|---|---|
| Emit., Trans. (mm) | 0.30 | 0.28 |
| Emit., Long. (mm) | 1.50 | 1.57 |
| Particles # | $4.7x10^{12}$ | $5.9x10^{12}$ |



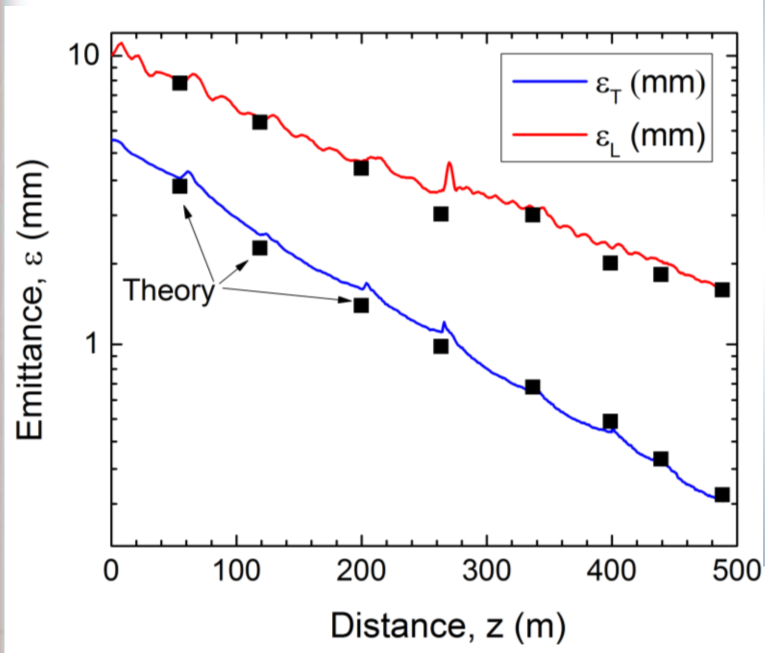


PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **18,** 031003 (2015)

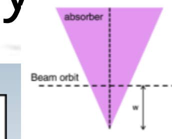
**Rectilinear six-dimensional ionization cooling channel for a muon collider: A theoretical and numerical study**

Diktys Stratakis and Robert B. Palmer
*Brookhaven National Laboratory, Upton, New York 11973, USA*
(Received 25 September 2014; published 6 March 2015)

# Theory and simulation

- Cooling channel performance can be predicted theoretically
- We found good agreement between simulation & theory




Transverse Cooling: $\frac{d\varepsilon_T}{ds} = -\frac{g_T}{\beta^2 E}\frac{dE}{ds}\varepsilon_T + \frac{\beta_T E_s^2}{2\beta^3 m_\mu c^2 L_R E}$

$g_T = 1 - D/w$

$\varepsilon_T^{eq} = \left(\frac{dE}{ds}\right)^{-1} \frac{\beta_T E_s^2}{2\beta g_T m_\mu c^2 L_R}.$

Longitudinal Cooling: $\frac{d\varepsilon_L}{ds} = -\frac{g_L}{\beta^2 E}\frac{dE}{ds}\varepsilon_L + \frac{\beta_L}{2}\frac{d<\Delta E^2>}{ds}$

$g_L = \frac{2\gamma^2 - 2\ln[K(\gamma^2-1)]}{\gamma^2 \ln[K(\gamma^2-1)] - (\gamma^2-1)} + \frac{D}{w}.$

$\varepsilon_L^{eq} = \left(\frac{dE}{ds}\right)^{-1} \frac{\beta^2 E \beta_L}{2 g_L}\frac{d\langle\Delta E^2\rangle}{ds}$

Emittance evolution: $\varepsilon^{calc}(s) = \varepsilon^{eq} + (\varepsilon^0 - \varepsilon^{eq})\exp\left(-\frac{s}{s^{calc}}\right)$

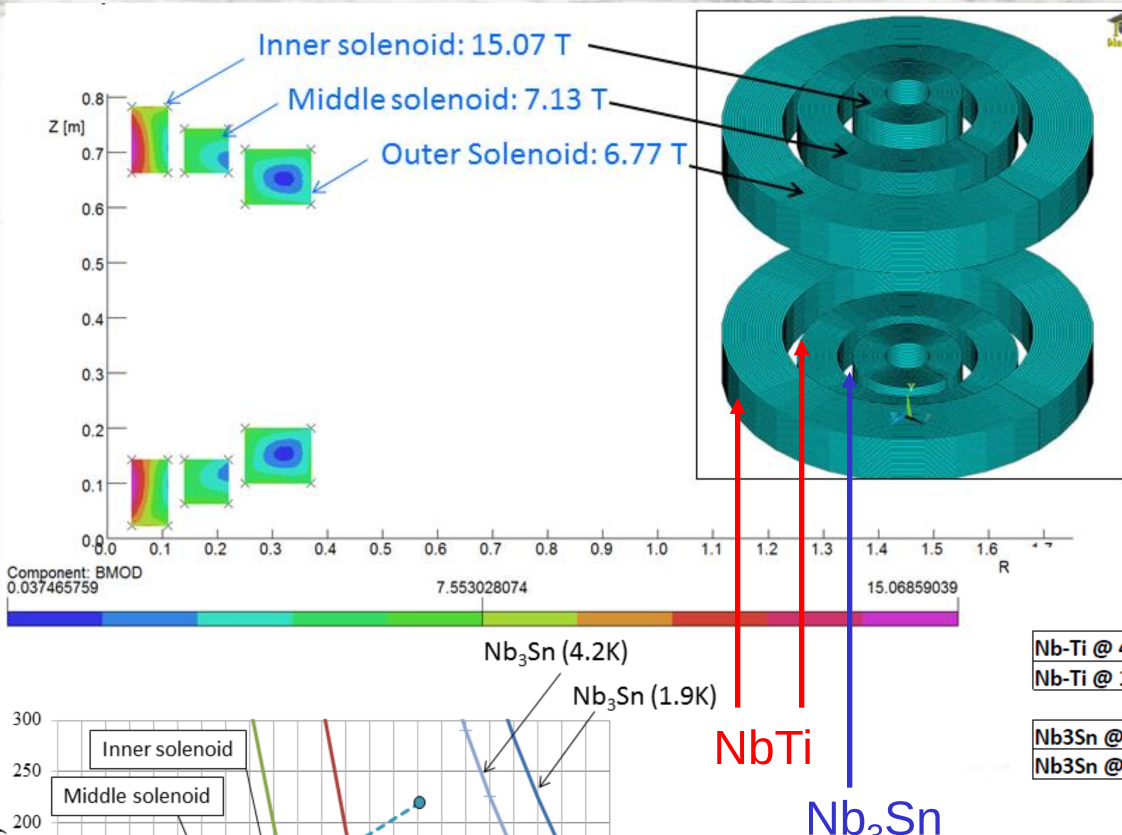
$s_T^{calc} = \frac{\beta^2 E}{g_T}\langle\frac{dE}{ds}\rangle^{-1}$ $s_L^{calc} = \frac{\beta^2 E}{g_L}\langle\frac{dE}{ds}\rangle^{-1}$

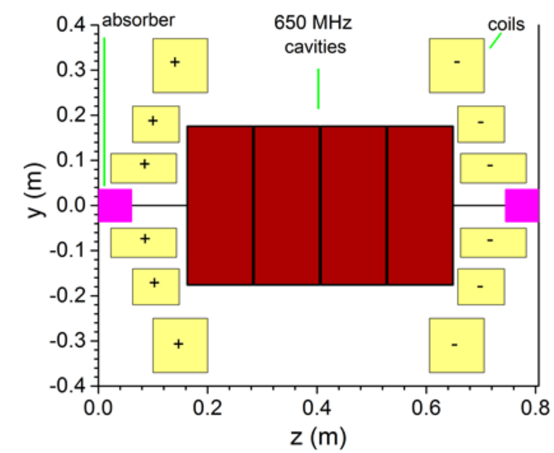
TUPME021 Proceedings of IPAC2014, Dresden, Germany

THEORETICAL FRAMEWORK TO PREDICT EFFICIENCY OF IONIZATION COOLING LATTICES*

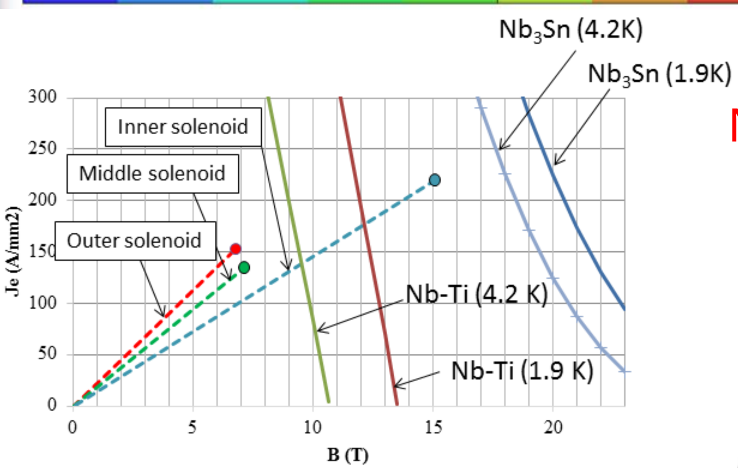
Diktys Stratakis#, Brookhaven National Laboratory, Upton, NY, USA
David Neuffer, Fermi National Accelerator Laboratory, Batavia, IL, USA

# Magnet technology




| | % of the load line at operational current | | |
|---|---|---|---|
| | Inner solenoid | Middle solenoid | Outer solenoid |
| **Nb-Ti @ 4.2 K** | - | 76% | 74% |
| **Nb-Ti @ 1.9 K** | - | 59% | 58% |
| **Nb3Sn @ 4.2 K** | 88% | - | - |
| **Nb3Sn @ 1.9 K** | 81% | - | - |



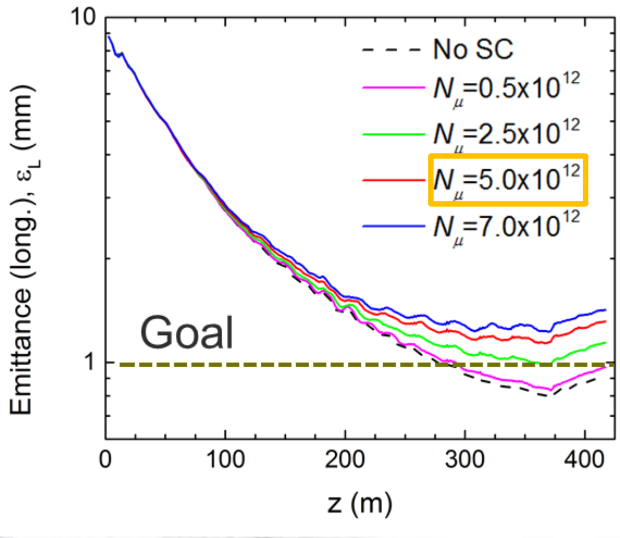

WEPRI103 Proceedings of IPAC2014, Dresden, Germany

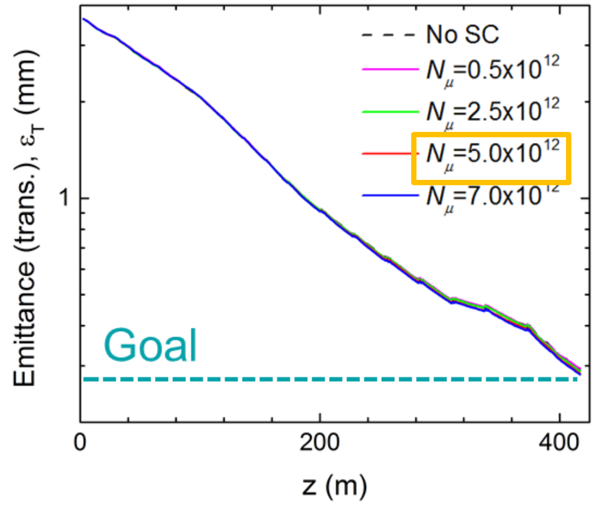
**MAGNET DESIGN FOR A SIX-DIMENSIONAL RECTILINEAR COOLING CHANNEL - FEASIBILITY STUDY***

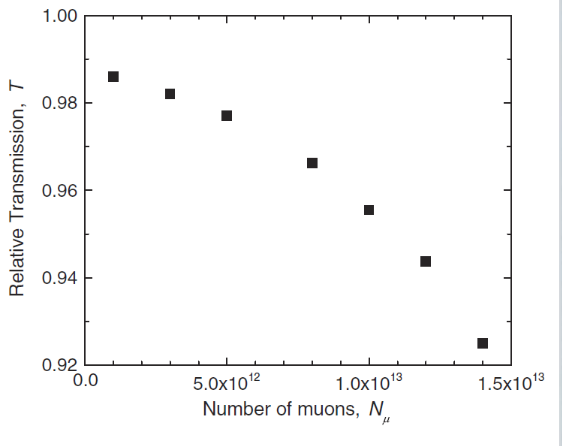
H. Witte†, D. Stratakis, J. S. Berg, R. B. Palmer, Brookhaven National Laboratory, Upton, NY, USA
F. Borgnolutti, Lawrence Berkeley National Laboratory, Berkeley, CA, USA

# Influence of space-charge

- At the end of cooling, $5 \times 10^{12}$ muons are squeezed within a 2 cm rms bunch. There is a concern for space-charge (SC)
- Simulation revealed that SC causes particle loss & longitudinal emittance growth

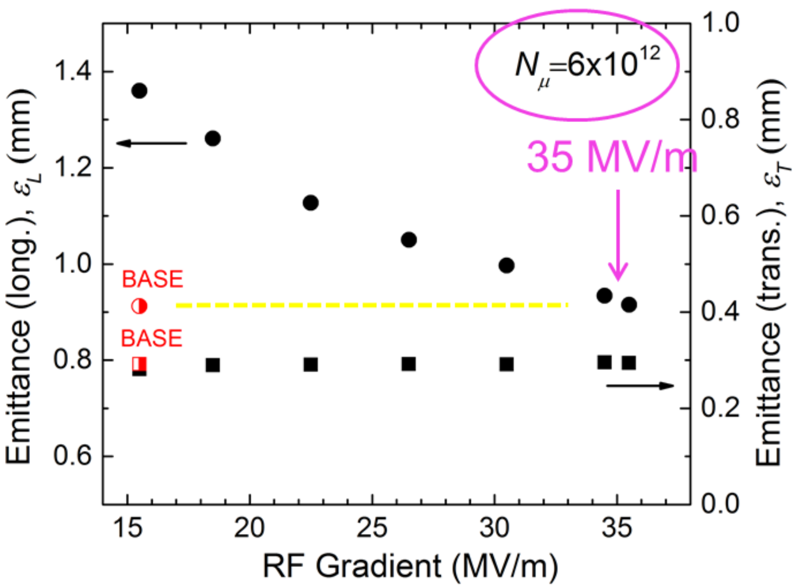


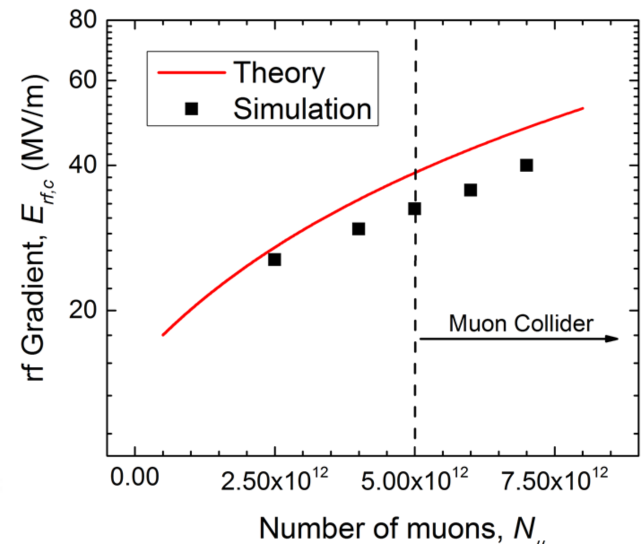



# Space-charge compensation

- Increasing the rf gradient can compensate SC. Compensation gradient is coupled to the beam intensity
- For a MC to obtain a $\varepsilon_L$ ~ 1.0 mm the rf gradient of a 805 MHz cavity needs to surpass 32 MV/m




PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **18,** 044201 (2015)

**Influence of space-charge fields on the cooling process of muon beams**

Diktys Stratakis and Robert B. Palmer
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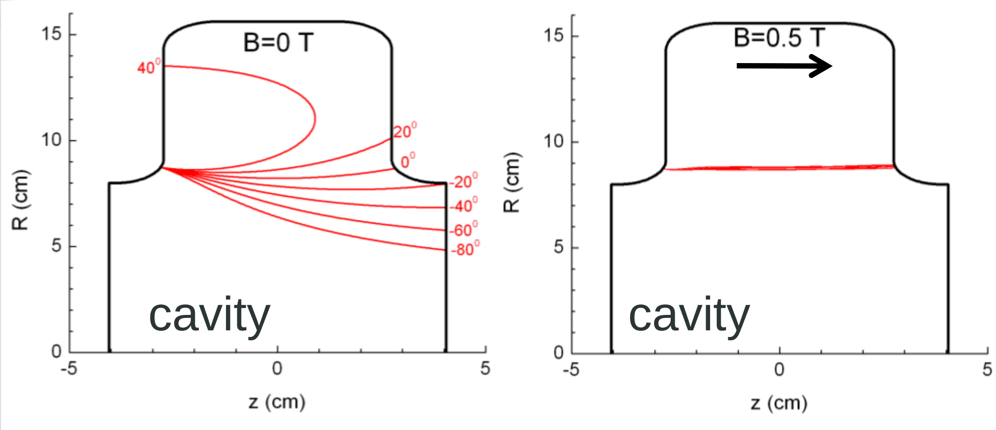
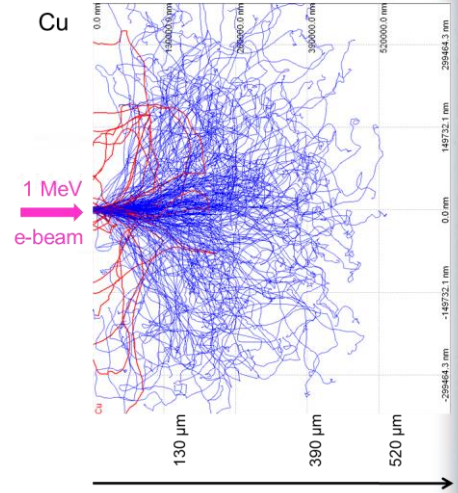
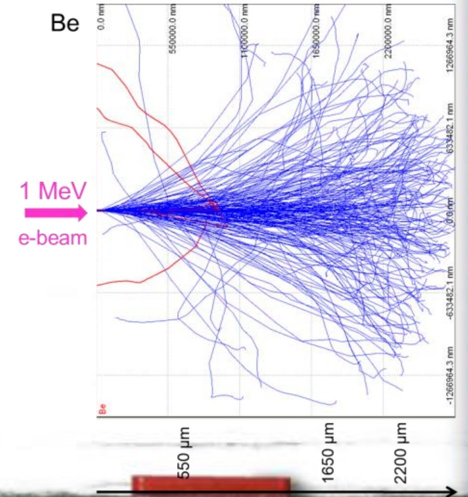
David P. Grote
*Lawrence Livermore National Laboratory, Livermore, California 94550, USA*
(Received 15 November 2014; published 7 April 2015)

# Operation of rf cavities in B-fields



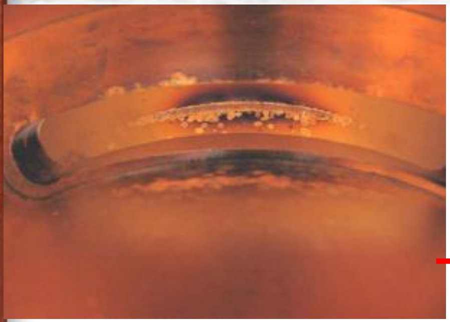

Damage on a 805 MHz rf cavity immersed in a multi-T magnetic field.

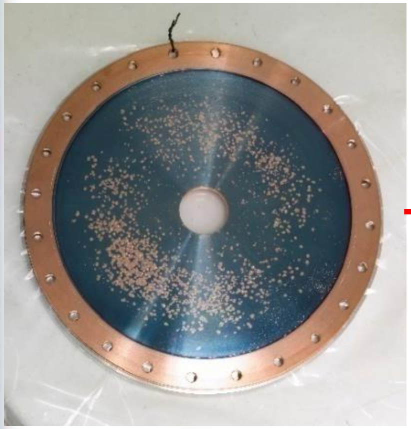


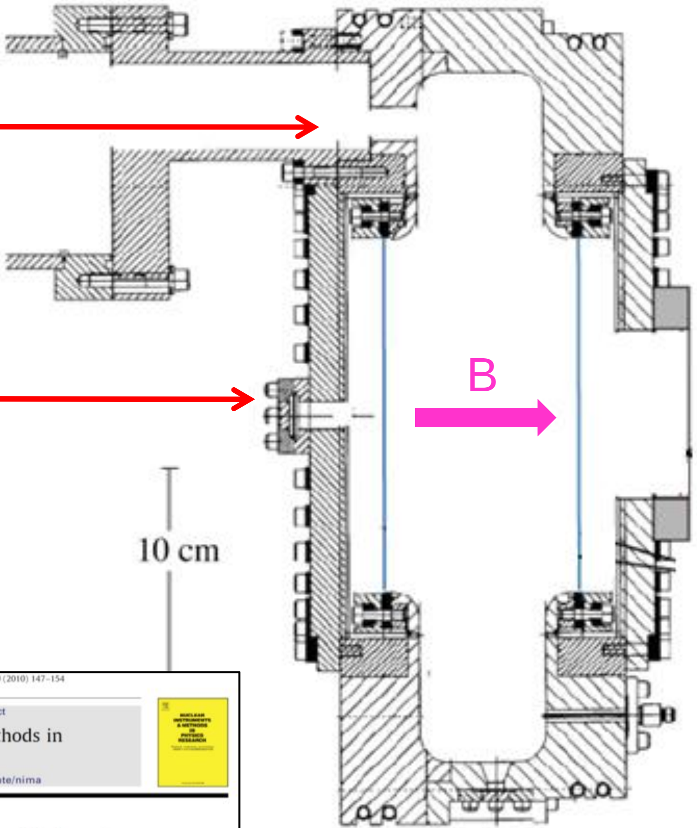

- Electrons impact rf surface and deposit heat in a small volume
- Surface damage via pulsed heating
- Effect is amplified with a B-field: A Cu made cavity may damaged when B> 1T
- Solution: Use dense materials like Be

# Data vs. model predictions





805 MHz
Cu made cavity

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ELSEVIER

Effects of external magnetic fields on the operation of high-gradient accelerating structures

Diktys Stratakis*, Juan C. Gallardo, Robert B. Palmer

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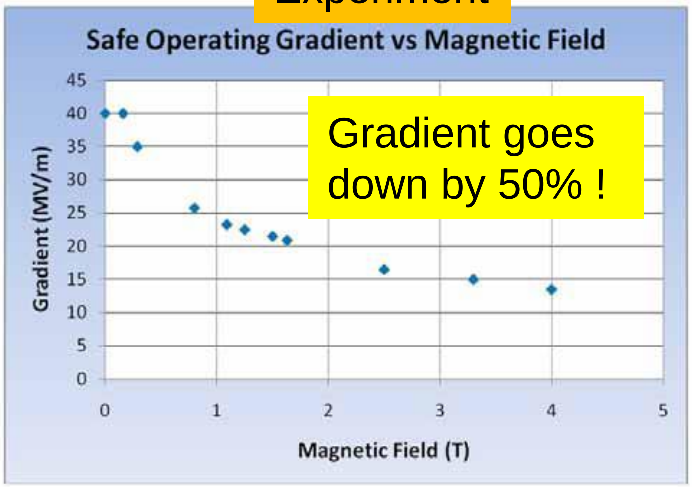
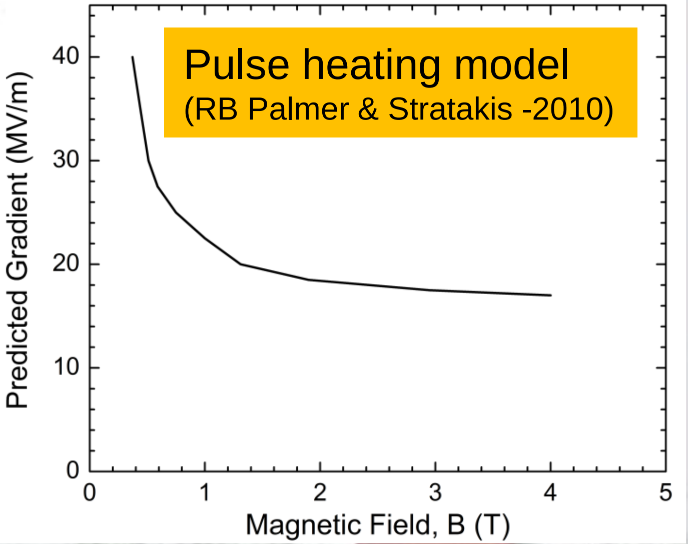
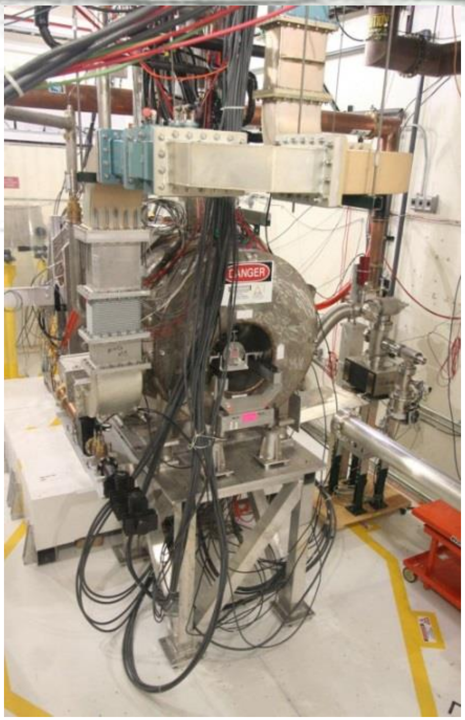
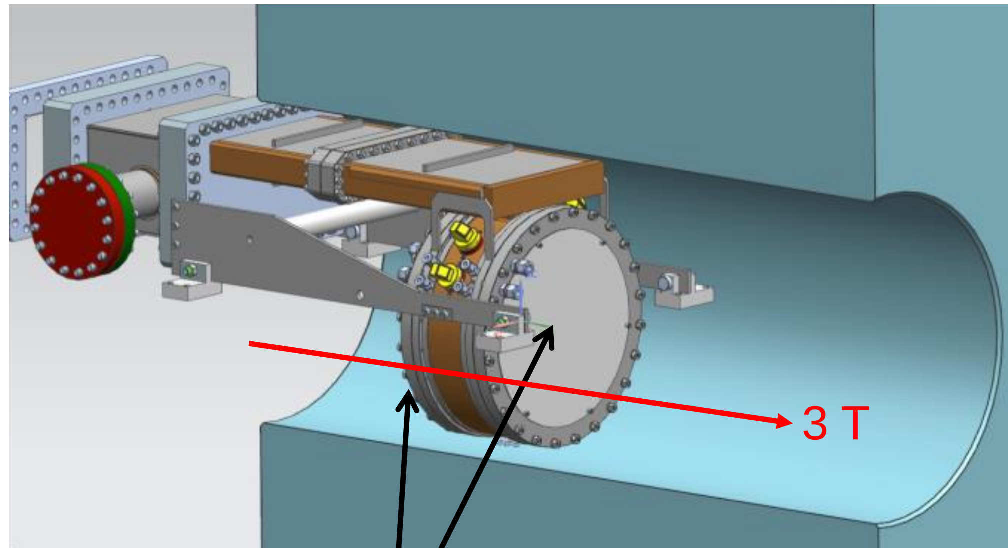
Experiment



# Modular cavity test: A game changer






removable plates (Cu, Al, Be)

| Material | $B$-field (T) | SOG (MV/m) | BDP ($\times 10^{-5}$) |
|---|---|---|---|
| Cu | 0 | $24.4 \pm 0.7$ | $1.8 \pm 0.4$ |
| Cu | 3 | $12.9 \pm 0.4$ | $0.8 \pm 0.2$ |
| Be | 0 | $41.1 \pm 2.1$ | $1.1 \pm 0.3$ |
| Be | 3 | $> 49.8 \pm 2.5$ | $0.2 \pm 0.07$ |
| Be/Cu | 0 | $43.9 \pm 0.5$ | $1.18 \pm 1.18$ |
| Be/Cu | 3 | $10.1 \pm 0.1$ | $0.48 \pm 0.14$ |

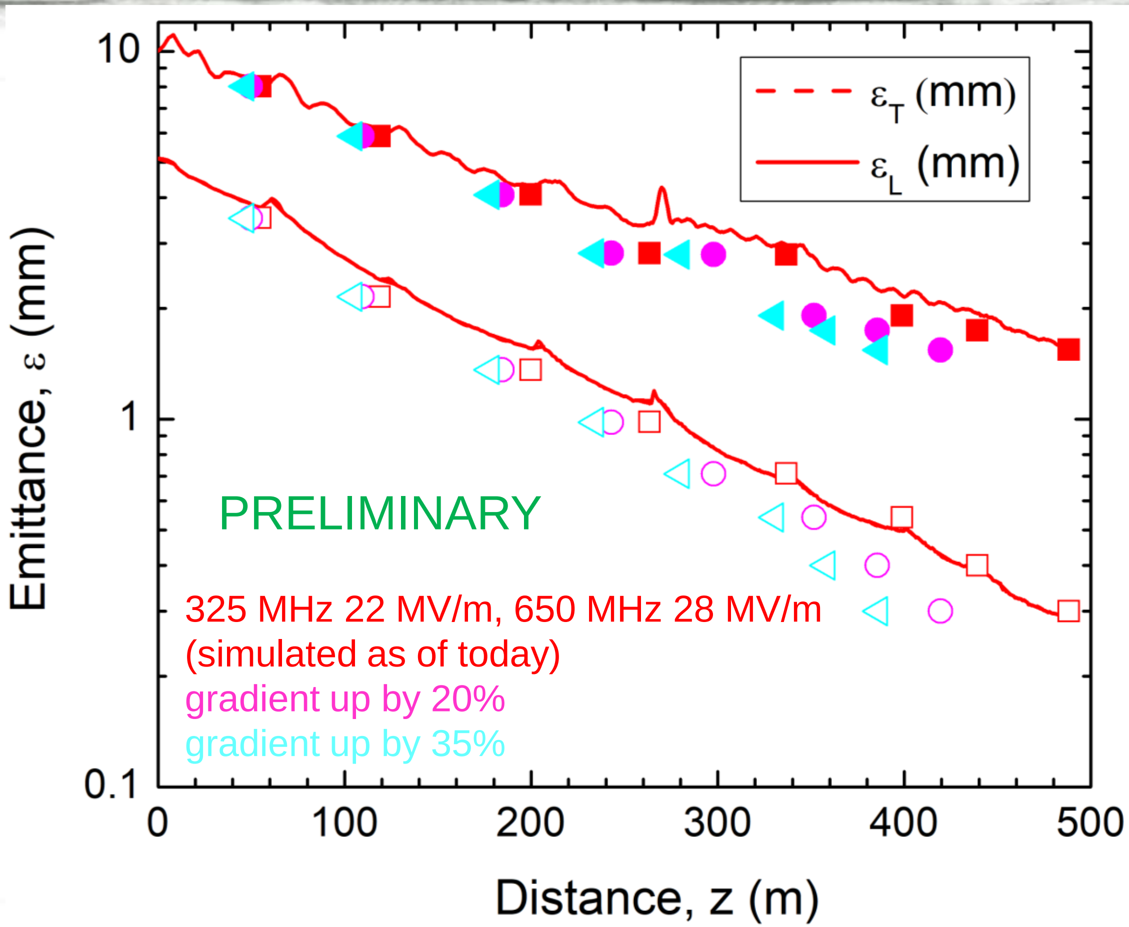
PHYSICAL REVIEW ACCELERATORS AND BEAMS **23,** 072001 (2020)

**Operation of normal-conducting rf cavities in multi-Tesla magnetic fields for muon ionization cooling: A feasibility demonstration**

D. Bowring,* A. Bross, P. Lane, M. Leonova, A. Moretti, D. Neuffer, R. Pasquinelli, D. Peterson, M. Popovic, D. Stratakis, and K. Yonehara
*Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*

- A Beryllium based cavity sustained a high gradient in the presence of multi-tesla B-fields!

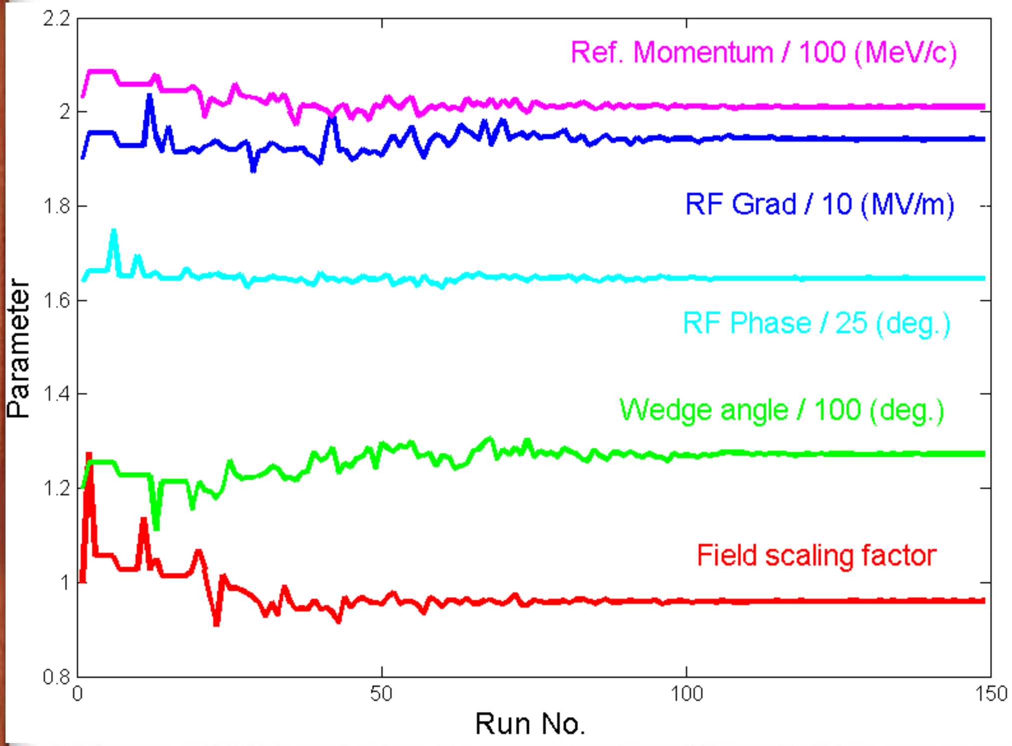
# Future: Simulate with higher gradients

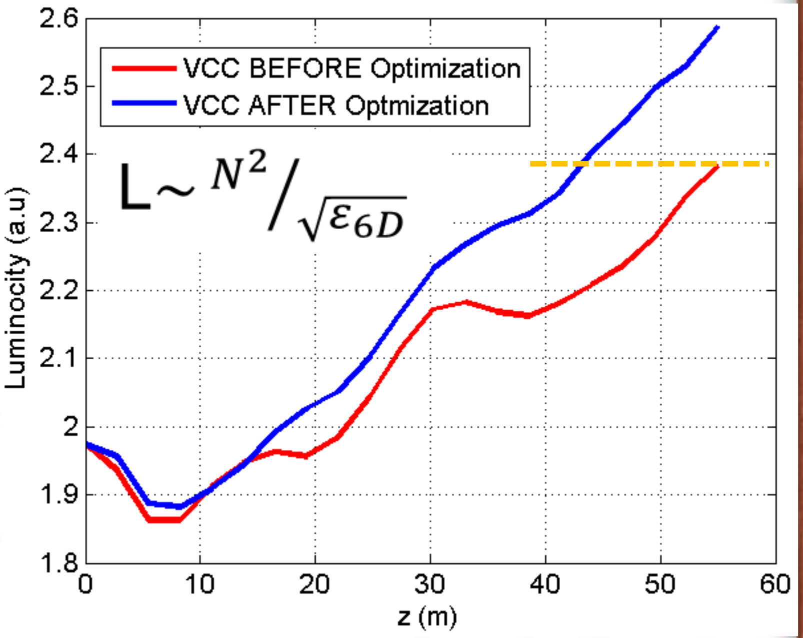



- Increasing the rf gradient can reduce the length of the cooling channel

# Future: Multivariable optimization

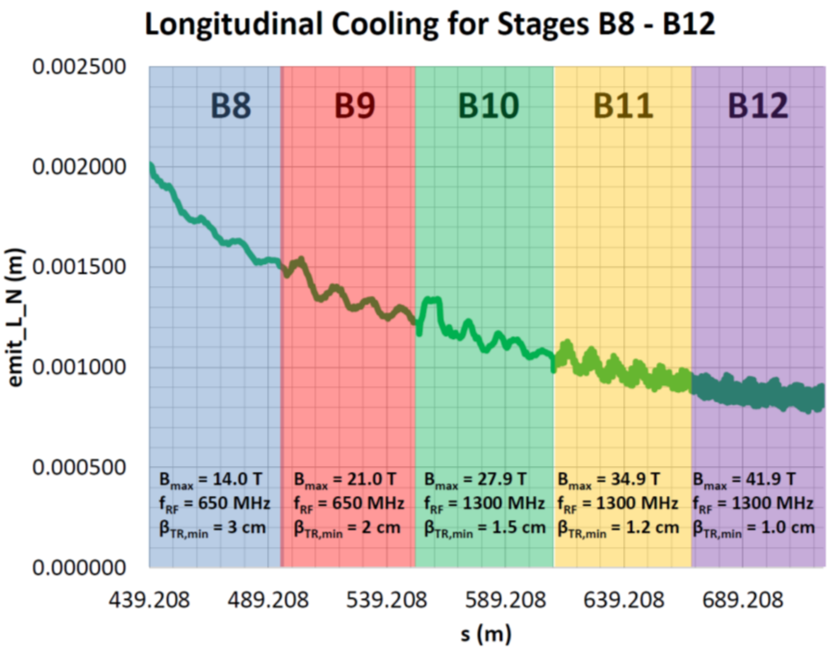
- Nelder-Mead algorithm: Objective is to maximize luminosity.
- Applied for VCC optimization: 8 parameters each time
- Promising results for first stage: 25% shorter channel!

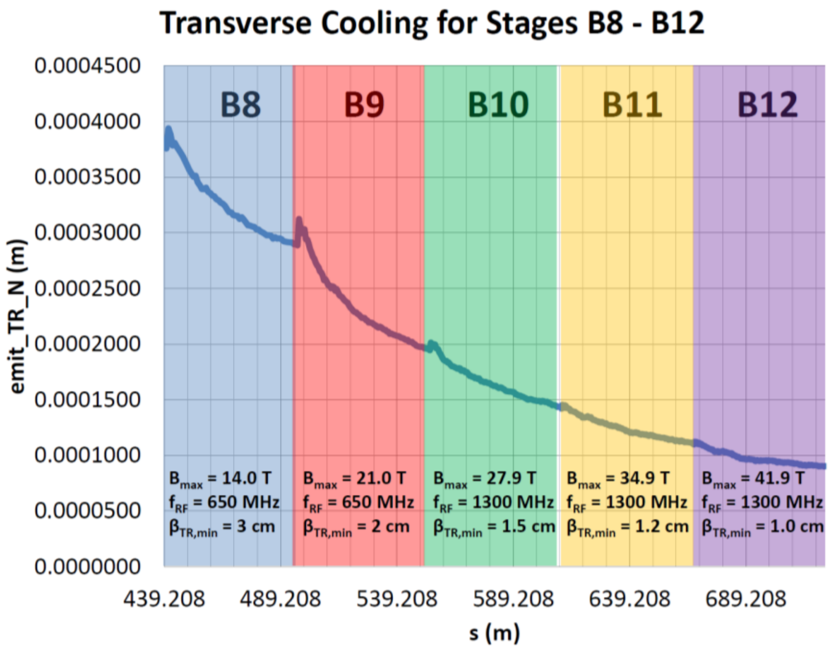


# Future: Move towards higher fields

- The proposed channel can achieve substantial cooling if HTS magnet technology is considered

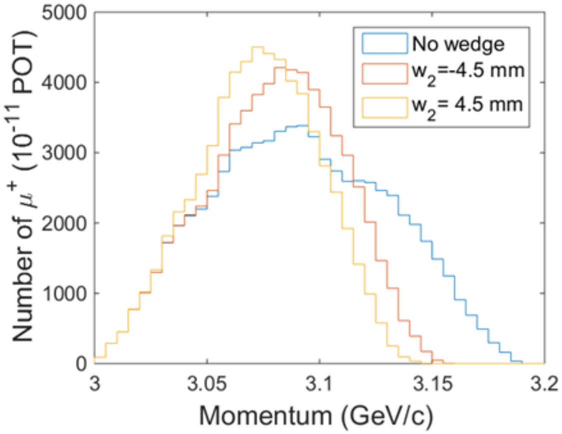


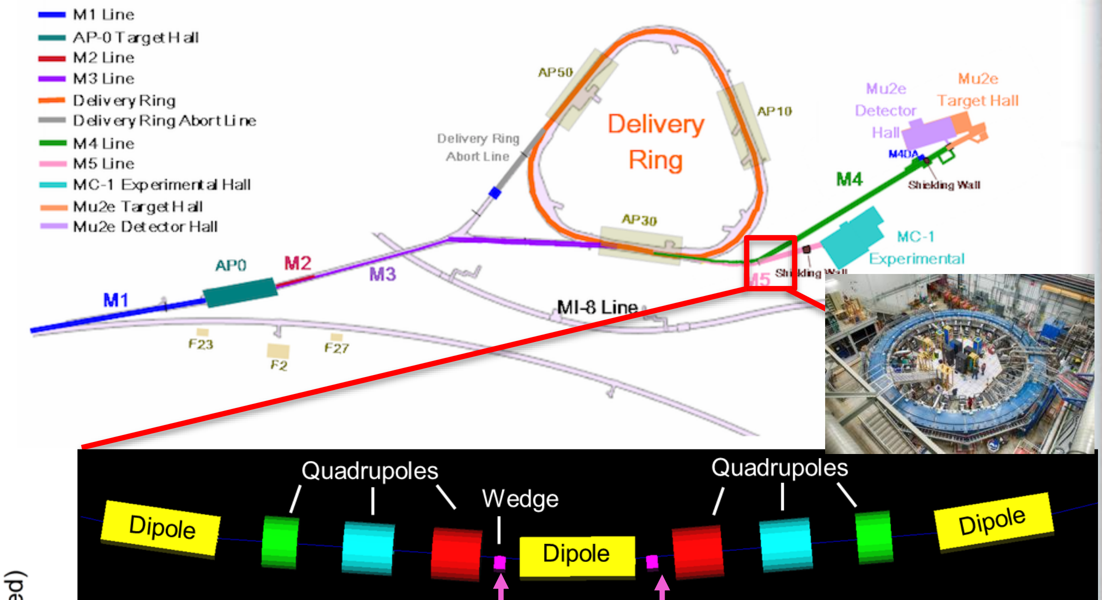



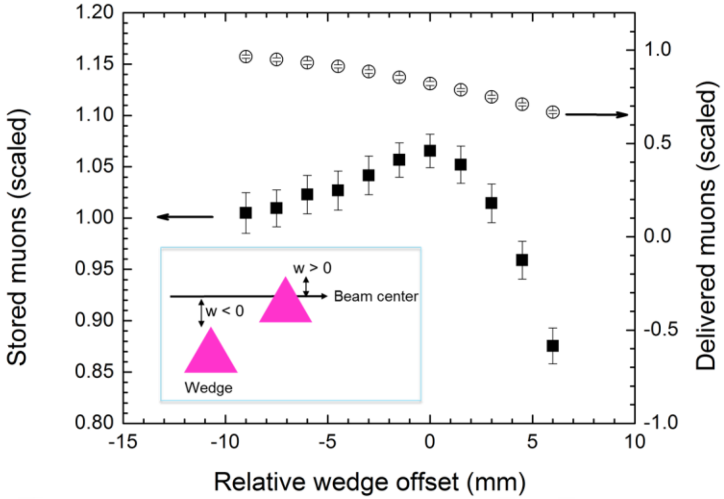

Don Summers, University of Mississippi

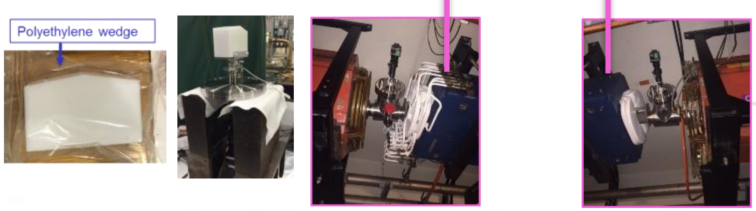
# Longitudinal cooling demonstration for the Muon g-2 Experiment

- Proof-of-principle experiment: Demonstrated 8% gain




PHYSICAL REVIEW ACCELERATORS AND BEAMS **22**, 053501 (2019)

**Application of passive wedge absorbers for improving the performance of precision-science experiments**

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# Summary

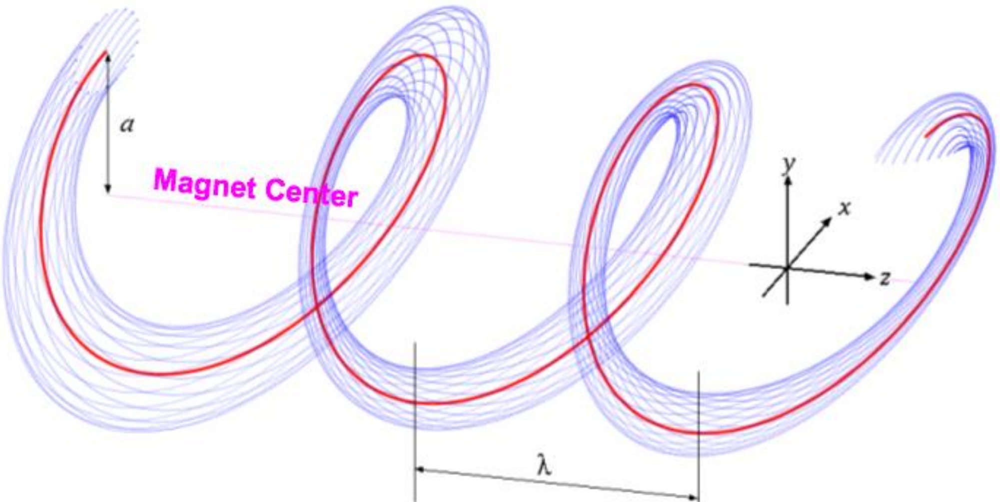
- We have a complete simulation model for a 6D cooling channel for a Muon Collider
- Beam at the end of the channel matches parameters specified by the Muon Accelerator Program within the limits of Nb-Sn magnet technology
- Further cooling is possible with HTS technology
- Operation of rf cavities in B-fields has been demonstrated:
  - Cooling channel can become shorter
  - Space-charge can be compensated -> higher transmission
- It is possible to reduce the channel length by >40% with a combination of higher rf gradients and multi-variable optimization

# Further related work

- Neutrino factory cooling
  - D. Stratakis and D. Neuffer, Journal of Physics G: Nuclear and Particle Physics 41,
- Helical cooling channel
  - K. Yonehara, JINST 13, P09003 (2018)
- Final cooling
  - H. Sayed, Phys.Rev.ST Accel.Beams 18, 091001 (2015)
- Bunch merger
  - Y. Bao, Phys. Rev. Accel. Beams 19, 031001 (2016)
- Helical FOFO Snake
  - Y. Alexahin, JINST 13, P08013 (2018)

# Beam dynamics in Helical Cooling Channel

Ya.S. Derbenev and R.P. Johnson, PRSTAB **8** 041002 (2005)




Helical beam trajectories

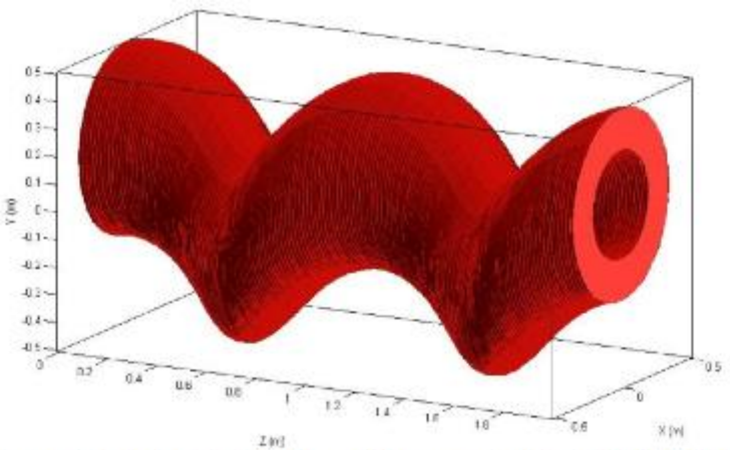


Figure 1: A helical solenoid generated with SolCalc.

Time- and z- independent Hamiltonian is a result from a helical coordinate system

- Helical coordinate system

$$\kappa = \frac{p_\varphi}{p_z} = \frac{2\pi a}{\lambda} = k \cdot a$$

- Two Lorentz forces are applied on a muon and make a balance

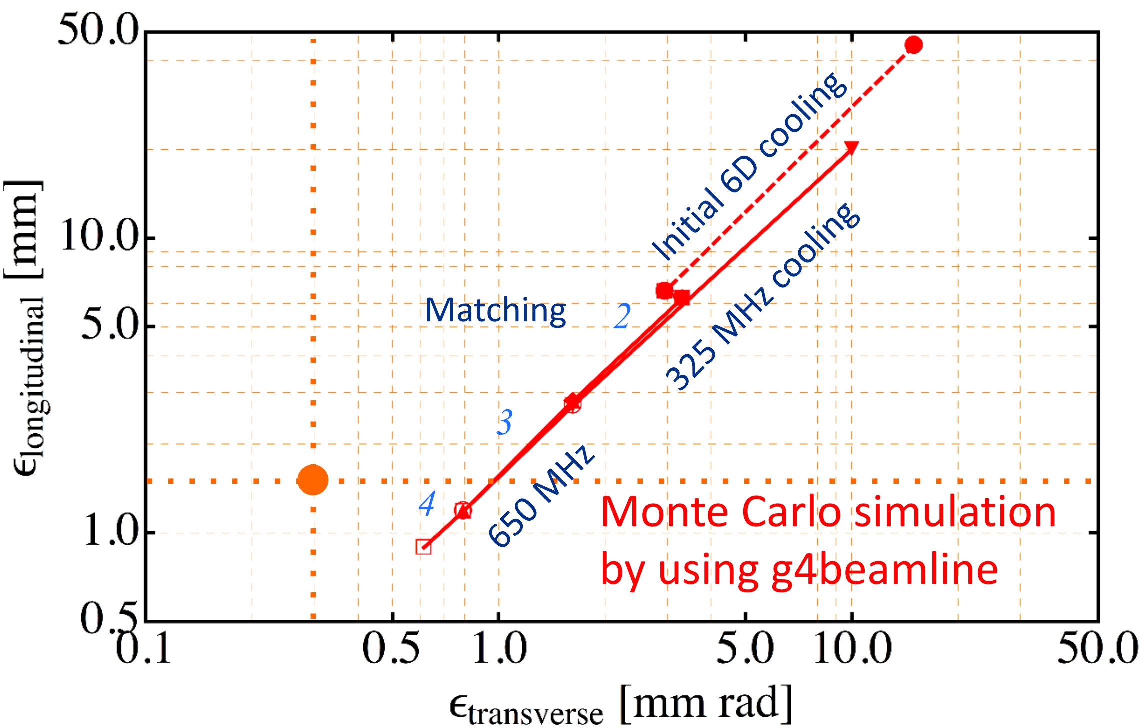
$$f_{balance} = \frac{e}{m_\mu}\left(p_z \cdot b_\varphi - p_\varphi \cdot b_z\right)$$

- Equation of motion of a reference (a red line in Top-Left Figure) is given

$$p(a) = \frac{\sqrt{1+\kappa^2}}{k} \cdot \left(b_z - \frac{1+\kappa^2}{\kappa^2} \cdot b_\varphi\right)$$

- Solenoidal ($b_z$) and helical dipole ($b_\varphi$) fields are generated in a helical solenoid magnet
- Field gradient $b'_\varphi = \frac{db_\varphi}{dr}$ is naturally generated in the magnet and it induces focusing (a blue line in Top-Left Figure)

# Simulated Cooling Performance




- RF parameter
  - E = 20 MV/m
  - ν = 325 & 650 MHz
- Gas pressure
  - 160 atm at 300 K
  - 43 atm at 80 K
- Magnetic fields
  - Bz = 4 – 12 Tesla
- Equilibrium emittance
  - $\varepsilon_T$ = 0.6 mm (goal: 0.3 mm)
  - $\varepsilon_L$ = 0.9 mm (goal: 1.5 mm)
- Transmission (one cooling cycle)
  - 60 %
- Channel length (one cooling cycle)
  - 280 meter

Helical cooling decrement can be explained by an old cooling theory

$$\varepsilon_n(s) = \left(\varepsilon_{n,0} - \varepsilon_{n,equ}\right) e^{-\Lambda_x s} + \varepsilon_{n,equ}$$

$$\Lambda_x = \left(\frac{g_x}{\beta^2 E}\frac{dE}{ds}\right)$$ ; cooling decrement

$$\varepsilon_{n,equ} = \frac{\beta\gamma}{2}\left(\hat{\beta}_x \sigma_x^2\right) / \left(\frac{1}{\beta^2 E}\frac{dE}{ds}\right)$$ ; equilibrium emittance

$$\varphi = \int \frac{ds}{\hat{\beta}_x} \rightarrow \hat{\beta}_x = \frac{1}{\omega \hat{Q}_x}$$ ; beta function in HCC

$g_x$ ; partition of cooling

$\sigma_x^2$ ; rms fraction due to stochastic process

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